UNIVERSITI PUTRA MALAYSIA

SIMULATIONS OF ONSET OF CONVECTION IN A NON-NEWTONIAN LIQUID INDUCED BY UNSTEADY-STATE HEAT CONDUCTION

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FK 2001 23
SIMULATIONS OF ONSET OF CONVECTION IN A NON-NEWTONIAN LIQUID INDUCED BY UNSTEADY-STATE HEAT CONDUCTION

By

TING KEE CHIEN

Thesis Submitted in Fulfilment of the Requirement for the Degree of Master of Science in the Faculty of Engineering Universiti Putra Malaysia

December 2001
SIMULATIONS OF ONSET OF CONVECTION IN A NON-NEWTONIAN LIQUID INDUCED BY UNSTEADY-STATE HEAT CONDUCTION

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Chairman: Associate Professor Tan Ka Kheng, Ph.D.

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The onset of convection in an initially static non-Newtonian liquid under Fixed Surface Temperature (FST) and Constant Heat Flux (CHF) boundary conditions was simulated using a CFD package. Steady-state and unsteady-state simulations were successfully conducted for bottom surface heating of shear thinning non-Newtonian liquids. Simulations on Newtonian liquid water and glycerine were conducted to verify the simulation setup.

Fourier's law of heat conduction was used to validate the steady-state simulation results. Simulations conducted for non-Newtonian liquid with Tien et al.'s (1969) experimental data were found to agree well with Fourier's law at conduction phase. Tien et al.'s definition of non-Newtonian power-law Rayleigh number was found to be inadequate in representing the onset of convection in non-Newtonian liquid. Attempts to determine the Rayleigh number for non-Newtonian liquid using apparent viscosity was successfully carried out. A more realistic critical Rayleigh number for non-Newtonian liquid was successfully determined with local values of Rayleigh number around a convection cell successfully obtained.
For simulations conducted for unsteady-state heat conduction in non-Newtonian liquid, transient heat conduction theory was used to validate the results. Convection was found to occur in a continuous deep fluid bounded by two horizontal rigid surfaces and adiabatic vertical walls. Transient critical Rayleigh number for non-Newtonian liquid under unsteady state heat conduction defined by Tan (1994) was successfully applied. Transient critical Rayleigh number for non-Newtonian liquid was found to vary with flow behavior $n$ of the Power Law model. A more realistic transient critical Rayleigh number for non-Newtonian liquid was successfully determined using apparent viscosity.

Development of thermal plumes in viscous non-Newtonian liquid were found to differ slightly from the development of thermal plumes in non-viscous Newtonian liquid. The $N\eta_{\text{max}}$ for unsteady-state simulations of Newtonian and non-Newtonian liquid were observed to be $3.8 \pm 2.0$ for FST cases and $2.7 \pm 1.8$ for CHF cases.

Effect of boundary condition at interface on onset of transient convection were studied. Velocity boundary condition of a top surface solid were found to be best approximated using top-cooling simulations. Bottom-heating simulations in a deep fluid revealed that the upper interface boundary has the property between a solid and a free surface.
Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Master Sains

SIMULASI PERMULAAN PEROLAKAN DALAM CECAIR BUKAN NEWTONIAN YANG DIARUHI KONDUksi HABA TIDAK MANTAP

Oleh

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Permulaan perolakan dalam cecair bukan Newtonian yang statik pada mulanya dibawah keadaan Permukaan Suhu Tetap (FST) dan Fluks Haba Tetap (CHF) telah disimulasikan dengan pakej Pengiraan Dinamik Bendalir (CFD). Simulasi untuk keadaan mantap dan keadaan tidak mantap telah berjaya dilaksanakan untuk pemanasan cecair bukan Newtonian dari permukaan bawah. Simulasi ke atas cecair Newtonian air dan glycerine telah dilaksanakan untuk mengesahkan penetapan simulasi.

dengan nilai tempatan nombor Rayleigh di sekitar satu sel perolakan berjaya didapati.


Perubahan kepulan haba di dalam ceair bukan Newtonian pekat didapati berlainan sedikit dengan perubahan di dalam ceair Newtonian yang cair. \( Nu_{\text{max}} \) untuk simulasi keadaan tidak mantap ceair Newtonian dan ceair bukan Newtonian didapati berada dalam lingkungan \( 3.8 \pm 2.0 \) untuk kes-kes FST dan \( 2.7 \pm 1.8 \) untuk kes-kes CHF.

Kesan keadaan sempadan di permukaan dwihala pada permulaan perolakan telah diselidiki. Keadaan sempadan halaju yang tegar pada permukaan atas didapati paling baik dikaji dengan simulasi penyejukan dari atas. Simulasi pemanasan dari bawah dalam ceair berlanjutan mencadangkan permukaan dwihala atas mempunyai sifat di antara tegar dan tidak tegar.
ACKNOWLEDGEMENTS

The author wishes to express his deepest gratitude to his supervisors, Associate Professor Dr. Tan Ka Kheng of Chemical and Environmental Engineering Department, Dr. Thomas Choong of Chemical and Environmental Engineering Department, Mr. Hishammuddin bin Jamaluddin of Biological and Agricultural Engineering Department and Dr. Nor Mariah Adam of Mechanical and Manufacturing Engineering Department for their invaluable advice and guidance throughout the course of this subject.

The provision of a scholarship by the Ministry of Science, Technology and Environment under National Science Fellowship scheme is gratefully acknowledged.

The spiritual support and encouragement given by family members and friends have carried the author through the struggles of materializing this thesis.
I certify that an Examination Committee met on 12th November 2001 to conduct the final examination of Ting Kee Chien on his Master of Science thesis entitled "Simulations of Onset of Convection in Non-Newtonian Liquids Induced by Unsteady-State Heat Conduction" in accordance with Universiti Pertanian Malaysia (Higher Degree) Act 1980 and Universiti Pertanian Malaysia (Higher Degree) Regulations 1981. The Committee recommends that the candidate be awarded the relevant degree. Members of the Examination Committee are as follows:

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Date: 14 MAR 2002
DECLARATION

I hereby declare that the thesis is based on my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously or concurrently submitted for any other degree at UPM or other institutions.

Ting Kee Chien

Date: 17th January 2002
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<td>Dimensionless wave number</td>
</tr>
<tr>
<td>$B$</td>
<td>Constant rate of surface temperature variation (K/s)</td>
</tr>
<tr>
<td>$C$</td>
<td>Heating current (A)</td>
</tr>
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</tr>
<tr>
<td>$D$</td>
<td>Diameter (m)</td>
</tr>
<tr>
<td>$d$</td>
<td>Total depth of liquid layer for steady-state heat conduction (m)</td>
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<tr>
<td>$g$</td>
<td>Gravitational acceleration (m/s$^2$)</td>
</tr>
<tr>
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<td>$n$</td>
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<tr>
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<td>Nusselt number</td>
</tr>
<tr>
<td>$q$</td>
<td>Heat flux (W/m$^2$)</td>
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<tr>
<td>$q^0$</td>
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</tr>
<tr>
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<tr>
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<td>$Pe$</td>
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<tr>
<td>$t$</td>
<td>Time (s)</td>
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<tr>
<td>$T$</td>
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<td>$\Delta T$</td>
<td>Temperature difference between top and bottom surfaces (K)</td>
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\( u \) Fluid velocity (m/s)

\( X \) Horizontal length of computational domain (m)

\( \Delta x \) Computational grid size in horizontal direction (m)

\( Y \) Vertical height of computational domain (m)

\( \Delta y \) Computational grid size in vertical direction (m)

\( z \) Vertical distance in liquid measured from the bounding surface (m)

**Greek Symbols**

\( \alpha \) Volumetric coefficient of thermal expansion (K\(^{-1}\))

\( \beta \) (Constant) temperature gradient (K/m); \( \beta = \Delta T / d \)

\( \delta \) Thickness of effective thermal layer (m)

\( \Gamma \) ratio of thermal conductivity to heat capacity (kg/s.m); \( \Gamma = k / c_p \)

\( \dot{\gamma} \) Strain rate (s\(^{-1}\))

\( \kappa \) Thermal diffusivity (m\(^2\)/s)

\( \lambda \) Wavelength (m)

\( \mu \) Viscosity (Pa.s)

\( \mu_{\text{app}} \) Apparent Viscosity (Pa.s)

\( \nu \) Kinematic Viscosity (m\(^2\)/s)

\( \rho \) Density (kg/m\(^3\))

\( \sigma \) Under relaxation factor
Subscripts

\[ c \] Critical

\[ 0 \] Initial state

\[ s \] Surface

\[ T \] temperature dependent

\[ \text{app} \] apparent

\[ \text{max} \] Maximum

Abbreviation

CFD Computational Fluid Dynamics

CHF Constant Heat Flux

CMC Carboxy Methyl Celulose

FST Fixed surface temperature

NN Non-Newtonian liquid
CHAPTER 1

INTRODUCTION

Studies of natural convection phenomena have been done in general for many years in natural sciences like astrophysics, geology, oceanography, climatology and meteorology. In convection, the hotter and lighter fluid rises while the colder and heavier fluid sinks. Natural convection, or free convection, seems to have been first described by Thomson (1882), but the first quantitative experiment was done by Benard (1900).

Lord Rayleigh (1916) studied the onset of buoyancy convection in a horizontal liquid layer bounded by two free surfaces based on an adverse linear temperature gradient. A dimensionless stability parameter was defined after him, the Rayleigh number. Convection occurs when Rayleigh number exceeded its critical value.

For natural convection induced by a time-dependent and non-linear temperature profile, Tan and Thorpe (1996) developed a new transient Rayleigh number for the deep fluid under various boundary conditions. They proposed that the correct way to begin any stability analysis is to identify the Biot number. They analyzed previous researchers’ experimental data by first determine the Biot number for each case (Tan & Thorpe, 1996; 1999a). Critical Rayleigh number were re-calculated and found to be consistent within a range of identified Biot number and wave number.