UNIVERSITI PUTRA MALAYSIA

BOUNDARY LAYER FLOW AND HEAT TRANSFER OF HYBRID Cu-Al_2O_3/WATER NANOFLUID PAST A PERMEABLE SURFACE

NAJIYAH SAFWA BINTI KHASHI'IE

IPM 2021 10

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By

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Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfillment of the Requirements for the Doctor of Philosophy

October 2020

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DEDICATIONS

To The Love of My Life
Mohd Zaki bin Abdul Manaf

To My Amazing Children
Nur Afrina Batrisya Binti Mohd Zaki
Muhammad Adam Haris Bin Mohd Zaki
Muhammad Ammar Hazim Bin Mohd Zaki
Aneesa Binti Mohd Zaki
Areef Irfan Bin Mohd Zaki

To My Mum & Beloved Siblings

To My In-Laws

. . .
Thank you for being a part of my journey
. . .

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfillment of the requirement for the degree of Doctor of Philosophy

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NAJIYAH SAFWA BINTI KHASHI'IE

October 2020

Chairman : **Norihan Md Arifin, PhD**
Institute : **Mathematical Research**

Hybrid nanofluid is invented to improve the heat transfer performance of traditional working fluids in many engineering and industrial applications. This thesis presents the numerical solutions and stability analysis of five problems related to the boundary layer flow with heat transfer in Cu-Al_2O_3/water hybrid nanofluid over different permeable surfaces. The five considered problems are (1) mixed convective stagnation point flow towards a vertical Riga plate, (2) magnetohydrodynamics (MHD) flow past a stretching/shrinking disc with Joule heating, (3) magnetohydrodynamics (MHD) flow past a stretching/shrinking cylinder with Joule heating, (4) three-dimensional flow past a stretching/shrinking sheet with velocity slip and convective boundary condition and (5) three-dimensional flow past a nonlinear stretching/shrinking sheet with orthogonal surface shear. The combination of copper (Cu) and alumina (Al_2O_3) nanoparticles with water as the base fluid is modeled using the single phase model and modified thermophysical properties of nanofluid. A set of similarity transformation is opted to reduce the complexity of the governing model and then, computed using the bvp4c solver in the Matlab software. For all the problems, the validation of model are conducted by comparing the numerical values of present and previously published report in a specific case. The surfaces are permeable to allow the usage of suction parameter and generate the possible solutions. Dual solutions exist in all problems within a specified range of parameters, but it is found that only the first problem has dual solutions without the utilization of suction parameter. However, higher values of suction parameter can affect the performance of hybrid Cu-Al_2O_3/water nanofluid in augmenting the heat transfer rate as reported in second to fifth problems. Among all the parameters discussed in this thesis, copper volumetric concentration, electromagnetohydrodynamics (EMHD), magnetic, velocity slip and suction parameters can delay the boundary layer separation. Meanwhile, Biot number (convective condition), EMHD, suction, magnetic, velocity slip and nonlinear parameters have potential to increase the heat transfer rate of the hybrid nanofluid. Stability analysis proves that the first solution is more realistic than the second solution.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

ALIRAN LAPISAN SEMPADAN DAN PEMINDAHAN HABA BAGI NANOBENDALIR HIBRID Cu-Al_2O_3/AIR TERHADAP PERMUKAAN TELAP

Oleh

NAJIYAH SAFWA BINTI KHASHI'IE

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Nanobendalir hibrid dicipta untuk meningkatkan prestasi pemindahan haba cecair tradisional dalam kebanyakan aplikasi kejuruteraan dan perindustrian. Tesis ini membentangkan penyelesaian berangka dan analisis kestabilan bagi lima masalah yang berkaitan dengan aliran lapisan sempadan dan pemindahan haba dalam hibrid nanobendalir Cu-Al_2O_3/air ke atas permukaan telap yang berlainan. Lima masalah yang dipertimbangkan adalah (1) aliran titik genangan dengan olakan campuran ke arah plat menegak Riga, (2) aliran magnetohidrodinamik (MHD) terhadap cakera meregang/mengecut dengan pemanasan Joule, (3) aliran magnetohidrodinamik (MHD) terhadap silinder meregang/mengecut dengan pemanasan Joule, (4) aliran tiga dimensi terhadap permukaan meregang/mengecut dengan slip halaju dan syarat sempadan olakan dan (5) aliran tiga dimensi terhadap permukaan meregang/mengecut tak linear dengan permukaan ricih ortogon. Gabungan nanopartikel tembaga (Cu) dan alumina (Al_2O_3) dengan air sebagai cecair asas dimodelkan dengan menggunakan model fasa tunggal nanobendalir dan sifat-sifat termofizikal yang diubahsuai. Satu set penjelmaan keserupaan dipilih untuk menurunkan kerumitan model dan kemudian, dikira menggunakan penyelesaian bvp4c dalam perisian Matlab. Untuk kesemua masalah, pengesahan model dijalankan dengan membandingkan nilai-nilai berangka semasa dengan laporan yang telah diterbitkan dalam kes tertentu. Permukaan adalah telap untuk membenarkan penggunaan parameter sedutan dan menjana penyelesaian yang berkemungkinan. Penyelesaian dual wujud dalam kesemua masalah dalam julat parameter tertentu, tetapi didapati hanya masalah pertama mempunyai penyelesaian dwi tanpa penggunaan parameter sedutan. Walau bagaimanapun, nilai parameter sedutan yang tinggi boleh menjejaskan prestasi nanobendalir hibrid Cu-Al_2O_3/air dalam menambah kadar pemindahan haba seperti yang dilaporkan dalam masalah kedua hingga kelima. Di antara semua parameter yang dibincangkan dalam tesis ini, kepekatan volumetrik tembaga, parameter EMHD, parameter magnet, parameter slip halaju dan parameter sedutan dapat melambatkan pemisahan lapisan sempadan. Sementara itu, nombor Biot (syarat sempadan olakan), parameter EMHD, parameter sedutan, parameter magnet, parameter halaju slip dan parameter tak linear berpotensi untuk meningkatkan kadar pemindahan haba bagi nanobendalir hibrid. Analisis kestabilan membuktikan bahawa penyelesaian pertama adalah lebih realistik daripada penyelesaian kedua.

ACKNOWLEDGEMENTS

Alhamdulillah, all praises to Allah S.W.T for the blessing, good health and enormous strength in completing this thesis entitled *Boundary layer flow and heat transfer of hybrid* $Cu\text{-}Al_2O_3$*/water nanofluid past a permeable surface*. The success of this research is impossible without the encouragement, support and help from the people in this PhD journey.

First and foremost, my deepest gratitude to my main supervisor, Professor Dr. Norihan Md Arifin who have supported me in any manner, who have shared the knowledge in order to make this research a reality and who have granted me the opportunity for academic conferences throughout this PhD study. Under her supervision, I published more than 10 papers in Science Citation Index/WOS-ISI and 5 papers in Scopus (selected from the international conferences). Also, I would like to thank and gratefully acknowledge Emeritus Professor Dr. Ioan Pop from Babeş-Bolyai University, Romania who gave me many ideas in defining the research problems and have recognized my capabilities in this research field. Many thanks for having faith on me by collaborating through the papers publication.Secondly, my sincere appreciation to my supervisory committee, Dr. Nadihah Wahi and Dr. Ezad Hafidz Hafidzuddin for their assistance, knowledge and motivation in making this research a success with good quality. Furthermore, I also would like to thank Professor Dr. Roslinda Mohd Nazar (UKM) and Professor Mohammad Mehdi Rashidi (Tongji University, Shanghai) for their invaluable and thoughtful comments and participation in the papers publication.

Special thanks and gratitudes to my family for their continuos support and motivation especially to my husband and children. Without their faith and patience, PhD is only a lonely journey. Additionally, million of thanks to all my friends specifically in the field of fluid dynamics who have assisted and motivated me to be in the right path. I dedicate this to Iskandar Waini (UTeM), Rusya Iryanti (UPM), Nurul Amira (UTeM), Khairum Hamzah (UTeM), Alwani (UPM) and Abdul Rahman (UMP). Besides, I want to acknowledge the Ministry of Higher Education (Malaysia) and Universiti Teknikal Malaysia Melaka through the SLAB-UTeM scholarship. Also, many thanks to all the staffs of Institute for Mathematical Research and School of Graduate Studies in UPM who have directly or indirectly supported my research work and helped with administrative matters.

…

Strength grows in the moments when you think you can't go on but you keep going anyway

…

This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfilment of the requirement for the degree of Doctor of Philosophy. The members of the Supervisory Committee were as follows:

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Date: 14 Januari 2021

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LIST OF ABBREVIATIONS

B_0	strength of the magnetic field
Bi	Biot number
C_p	specific heat capacity at a constant temperature
C_f	skin friction coefficient
d	dimensionless parameter related to the electrodes and magnets width (Chapter 4)
Ec	Eckert number
g	gravitational acceleration
Gr	Grashof number
h_f	heat transfer coefficient
j_0	density of the current in electrodes (Chapter 4)
K	curvature parameter
L	characteristic length of the surface
M_0	magnetization of the magnets (Chapter 4)
M	magnetic parameter
n	nonlinear parameter
$N-S, \quad S-N$	polarity of the magnet (Chapter 4)
Nu_x	Nusselt number
p	magnets and electrodes width (Chapter 4)
Re_x	Reynolds number
τ_w	wall shear stress
τ	dimensionless time variable
q_w	surface heat flux
k	thermal conductivity
s_1	first nanoparticle (alumina)
s_1	second nanoparticle (copper)
S	suction parameter
t	time
T	fluid temperature
T_f	temperature of the heated surface (Chapter 7)
T_w	wall/surface temperature
T_∞	ambient temperature
T_0	characteristic temperature (Chapter 4,5,6)
u_e	free stream velocity
u_w	stretching/shrinking velocity
u, v, w	velocity components
v_w	term for surface mass flux velocity (Chapter 4)
W_w	term for surface mass flux velocity (Chapter 5,6,7,8)
Z	modified Hartmann number or EMHD parameter

Greek Symbols

θ	dimensionless temperature
β_0	slip length as the proportional constant of the slip velocity
β	velocity slip parameter
λ	mixed convection or buoyancy parameter
ν	kinematic viscosity of the fluid
μ	dynamic viscosity of the fluid
ρ	density of the fluid
$\rho\beta_T$	thermal expansion of the fluid
ρC_p	heat capacitance of the fluid
ψ	stream function
η	similarity variable
ε	stretching/shrinking parameter
σ	electrical conductivity of the fluid
ϕ_1	volumetric concentration of alumina
ϕ_2	volumetric concentration of copper
γ	unknown eigenvalue
γ_1	smallest eigenvalue

CHAPTER 1

INTRODUCTION

1.1 Introduction

In the real industrial processes, there exist situations of continuous moving surfaces in a moving or quiescent ambient environment. For example, the hot steel extrusion, the lamination and the melt-spinning process in the polymer's extrusion and the heat treatment for the material moves between a wind-up roll or conveyor belts and a feed roll (Moutsoglou and Bhattacharya, 1982). The importance of the final product quality which depends on the heat transfer and cooling fluid performance attract many researchers to further the study of the flow field and heat transfer. The problems of boundary layer flow induced by a moving and deformable surfaces have drawn an extensive attention among the researchers after first attempt made by Blasius in 1907 and Sakiadis in 1961, respectively (Ahmad et al., 2011).

1.2 Boundary Layer Theory

1.2.1 Velocity and Thermal Boundary Layer

In 1904, Ludwig Prandtl introduced the concept of fluid viscosity and contributed to the discovery of the boundary layer theory (Acheson, 1990; Anderson, 2005). Before Prandtl published the report, the viscosity effect was neglected in the ideal flow solution, therefore the equations regarding viscosity became complicated. The Navier-Stokes equations were used to give exact solutions for flows with small Reynolds number before the concept of boundary layer flow was introduced. In contrast, the Navier-Stokes equations gave insignificant solutions for flows with high Reynolds number. Therefore, the concept by Prandtl stated that the viscosity has a large impact at the solid boundary and this effect is insignificant in areas further away from the solid boundary. The boundary layer is a region between the wall or surface (below) and the inviscid free-stream (above) as shown in Figure 1.1. The flow past a solid boundary can be divided into two regions. The first region is thin and near to the solid boundary which is termed as the boundary layer. In the boundary layer or first region, fluid viscosity has a great and significant effect on the flow. Meanwhile, the fluid viscosity has very low effect in the second region. Referring to the concept presented by Prandtl, there are various terms that can be neglected in the

Navier-Stokes equations through the assumption of a thin boundary layer.

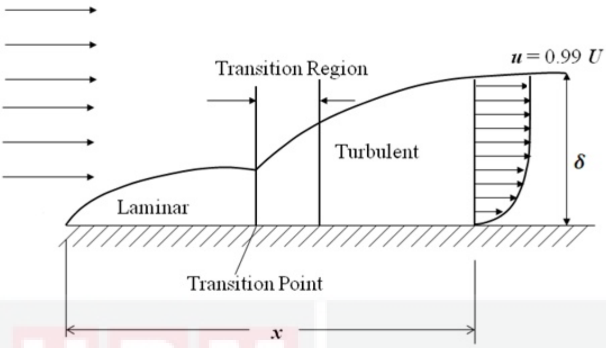

Figure 1.1: An illustration of boundary layer flow

The Cartesian coordinates (x,y) are taken such that the $x-$axis is measured along the sheet oriented in the horizontal direction and the $y-$axis is perpendicular to it. At any given coordinate, the velocity distribution can be drawn as a function of y. This is the most common way to illustrate a boundary layer. Referring to Figure 1.1, there are two points of velocity cross sections in the boundary layer. The first is cross section for a laminar boundary layer while the second is after transition and represents a turbulent boundary layer. In a boundary layer, the velocity is always zero at the wall, and asymptotically approaches the free-steam velocity.

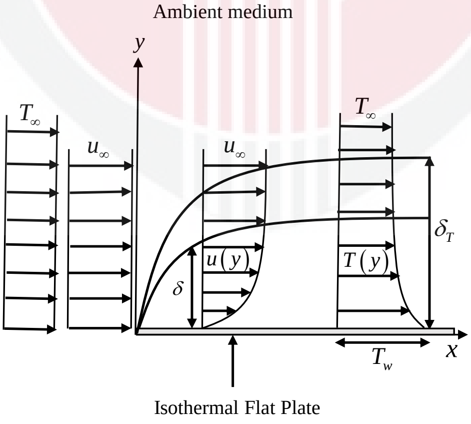

Figure 1.2: Velocity and thermal boundary layer

Similar to the velocity boundary layer, a thermal boundary layer develops if there is a difference between the ambient and surface temperatures. Consider a fluid flow over a flat plate (isothermal) with constant temperature T_w as shown in Figure 1.2. At the leading edge, the fluid temperature profile is uniform with the ambient temperature T_∞. However, when the fluid particles contact the surface, the thermal equilibrium is achieved between the fluid particles and the wall temperature. At this point, energy flow occurs at the surface where the fluid particles transfer the energy with those in the adjoining fluid layer (by conduction and diffusion) and temperature gradients will develop in the fluid. The region of the fluid where the temperature gradient exists is known as the thermal boundary layer. The thermal boundary layer thickness δ_T, is defined as the distance from the surface where the temperature is 99% of the temperature from an inviscid solution or mathematically written as $(T - T_w)/(T_\infty - T_w) = 0.99$. As the distance x from the leading edge increases, the thermal boundary layer thickens while the effect of heat transfer penetrates farther into the free stream.

1.2.2 Stagnation Point Flow

Fluid stagnation is a phenomenon where the fluid is immovable at a region where the local velocity is zero. The pressure, heat transfer and mass deposition have maximum value at this region which known as the stagnation point (Wang, 2008). In 1752, D' Alembert pioneered the fluidic stagnation point notion and investigated the drag flow on solid boundaries (Brimmo and Qasaimeh, 2017). During the time, fluid stagnation was only limited to liquid-solid interfaces and refered as a disturbance. After Prandtl proposed the boundary layer theory, he concluded that the frictional force is the reason of the attached fluidic thin layer (stagnation point) to a rigid boundary (see Figure 1.3). Since then, the concept of fluid stagnation towards a static or moving body has captivated many researchers from various backgrounds (mechanical, mathematics, physics) due to its numerous industrial and engineering applications such as counterflow jet, aerodynamics and heat transfer.

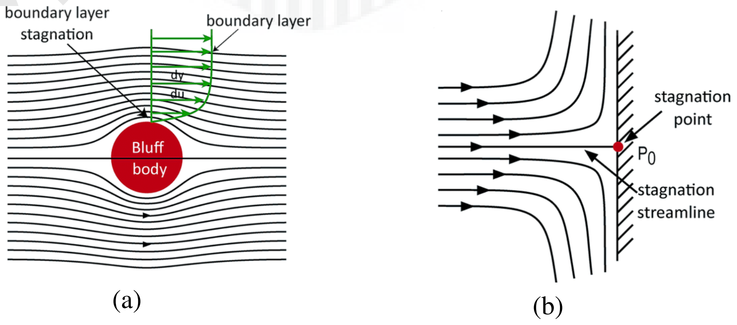

Figure 1.3: (a) Stagnation flow schematics, (b) Stagnation point flow on a vertical plate (Brimmo and Qasaimeh, 2017)

1.2.3 Boundary Layer Separation

In general, there are three stages of boundary layer, namely laminar boundary layer, transition phase and turbulent boundary layer. The transition from laminar boundary layer flow to the turbulent boundary layer flow is known as the boundary layer separation. The fluid velocity theoretically decreases when the fluid passes a surface as a result of the skin friction between the fluid and the surface which simultaneously, forms a boundary layer. The laminar boundary layer flow is characterized by a smooth flow while the turbulent flow contains swirls or vortices. In addition, the laminar flow creates less skin friction forces than the turbulent flow. Separation occurs in the flow with increasing pressure (adverse pressure gradient). As shown in Figure 1.4, the fluid motion (illustrated by the arrow) starts to change from laminar flow (left) to turbulent flow (right).The boundary layer separates when it has travelled far enough in an adverse pressure gradient where the velocity boundary layer relative to the surface has stopped and reversed the direction as illustrated in Figure 1.4.

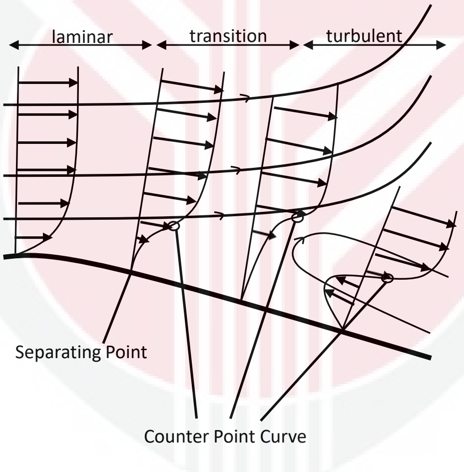

Figure 1.4: Boundary layer separation

The flow becomes detached from the surface, and may take the forms of eddies and vortices. Besides, the boundary layer solution only exist up to the boundary layer separation point. From mathematical view, no solution can be generated for the flow beyond this separation point because the boundary layer equation is invalid for a turbulent flow. A full Navier-Stokes with energy equations are necessary to observe the flow and heat transfer characteristics beyond this separation point. Therefore, it is important to identify the possible factors which can decelerate the boundary layer separation.

1.3 Regular and Hybrid Nanofluids

Fluid is a substance that can continuously flow and change its shape under applied shear stress or external force. Fluid can be classified into two categories; Newtonian and non-Newtonian fluids. Newtonian fluid refers to the fluid that obeys the Newton's law of viscosity (direct proportion between the shear stress of the fluids viscosity and shear rate). Non-Newtonian fluid is represented either by shear thickening (fluid viscosity enhances due to the reduction of shear rate) or by shear thinning (fluid viscosity decreases due to the increment of shear rate).

Meanwhile, nanofluids are a special class of fluids with great thermophysical properties, are expected to improve the heat transfer performance of applications related to nuclear cooling systems, lubrication, biomedical applications, solar water heating, thermal storage, coolant in automobile radiator, refrigeration and many others. The nanofluids are prepared by dispersing single nanoparticles into a base fluid. The frequently used nanoparticles are classified into these groups, (i) metals (copper/Cu, silver/Ag, Nickel/Ni), metal oxides (aluminum oxide/Al_2O_3, ferric oxide/Fe_2O_3, cupric oxide/CuO, silicon dioxide/SiO_2), carbon materials (carbon nanotubes/CNTs, multi-walled carbon nanotubes/MWCNTs, diamond, graphite), metal nitride (aluminium nitride/AIN) and metal carbide (silicon carbide/SiC). On the other hand, water, ethylene glycol and oil are commonly used as the base fluid in the formation of nanofluids.

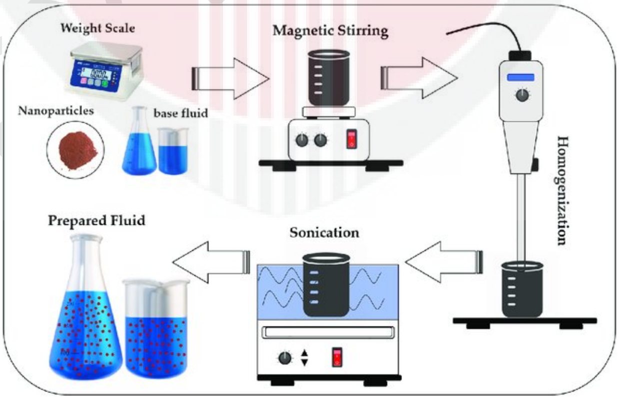

Figure 1.5: An illustration of nanofluid's preparation (Babar and Ali, 2019)

The nanofluids are not simply made by adding the nanoparticles into the base fluid, but involves specific physical and chemical procedures for extensive period stability and large-scale applications. The process of stabilizing nanofluid is a difficult task due to the presence of static electricity and Van der Waals force (Sun et al., 2015). There are two general methods for the preparation of nanofluids; single- or two-step method. The two-step method is a low cost method and widely used in laboratories. This method involve preparing and dispersing of solid particles in the base fluid, separately. First, the raw material (solid particles) are transformed into the powder form using physical or chemical procedures and then, dispersed in the base fluid with pH adjustment, ultrasonic agitation, surfactant addition, magnetic stirring or homogenizing (Yu and Xie, 2012) until the stabilized nanofluid is obtained as shown in Figure 1.5. Meanwhile, in the single-step method, the agglomeration of nanoparticles is depreciated by combining the mixing and synthesizing process of nanoparticles at one time. This method is comparatively expensive and only convenient for small scale production.

The invention of a stable hybrid nanofluid as a promising heat transfer fluid with better heat transfer performance can fulfil the industrial demand. There are two ways to prepare hybrid nanofluids which are (i) by suspending different types of nanoparticles in a base fluid (water/oil) or (ii) by suspending hybrid form of nanoparticles in the base fluid. The hybrid nanofluids combine different composite materials such as metal matrix nanocomposites (Al_2O_3/Cu, Al_2O_3/Ni, Mg/CNT, MgO/Fe), ceramic matrix nanocomposites (Al_2O_3/SiO_2, Al_2O_3/TiO_2, CNT/Fe_3O_4) and polymer matrix nanocomposites (polymer/CNT, polyester/TiO_2) with traditional base fluid. According to Sajid and Ali (2018), Turcu et al. (2006) being the first to report the synthesis of MWCNTs/Fe_2O_3 hybrid nanoparticles. The aggregation of nanoparticles will cause sedimentation or clogging, which simultaneously leads to the reduction in nanofluids' thermal conductivity. Hence, it is crucial and important to have a stable hybrid nanofluids.

Not all the combination of the nanoparticles are suitable for the hybrid nanofluids. Jana et al. (2007) and Baghbanzadeh et al. (2012) reported that the thermal conductivity of hybrid nanofluids is less than regular nanofluid due to the compatibility issues of nanoparticles. Baghbanzadeh et al. (2012) synthesized and investigated hybrid nanofluid with SiO_2-MWCNTs nanoparticles in two set of ratios (80:20 and 50:50). The hybrid nanofluid has lower thermal conductivity than MWCNTs nanofluid because of poor thermal conductivity of SiO_2. The ascending order of thermal conductivity for nanofluids was $SiO_2<$Hybrid nanofluid (80:20)$<$Hybrid nanofluid (50:50)$<$MWCNTs nanofluids. Jana et al. (2007) compared the thermal conductivity of regular nanofluids (Au-water, Cu-water, CNTs-water) and hybrid nanofluids (CNTs-Cu/water, CNTs-Au/water). Hybrid nanofluids showed less enhancement in thermal conductivity compared to mono nanofluids. However, there are many successful experimental works reported for the hybrid nanofluid with Cu and Al_2O_3 nanoparticles which can be found in Suresh et al. (2011, 2012) and Parsian and Akbari (2018).

Due to the costly experimental works, many researchers preferred to further investigate the regular and hybrid nanofluids through numerical simulation (CFD) and classical boundary layer analysis (Sheremet et al., 2020; Ghalambaz et al., 2019; Sheikholeslami et al., 2019a; Izadi et al., 2018). The numerical investigations are conducted using the established mathematical model of nanofluids, (i) single phase model by Tiwari and Das (2007), and (ii) two phase model by Buongiorno (2006). Moreover, for this theoretical analysis, the nanofluids are considered as a stable form of the base fluid and nanoparticles including the exclusion of the aggregation effect. Further explanation including the thermophysical properties and the theoretical model of nanofluid used in this research can be found in Chapter 3.

1.4 Heat Transfer

Heat transfer is the process of exchanging thermal energy between physical systems through the dissipation of heat. The process of transferring heat is induced by pressure and temperature difference that occurs within the physical systems. Generally heat transfer can be divided into three types; convection, conduction and radiation as illustrated in Figure 1.6. This research will focus on the convective heat transfer as illustrated in Figure 1.6. Convection describes that the heat transfer from one place to another through the mass motion of fluids. It occurs when the heated fluid moves away from the source of heat and carries the energy acquired. The ideal gas law describes that convection on a hot surface occurs when heated air (temperature increases) is expanded (volume increases), becomes less dense and then rises. Convection can further be subdivided into various forms that include natural convection, forced convection and mixed convection. Natural convection occurs when the fluid has density differences while forced convection describes fluid flow that is induced by external forces caused by a suction device, pump or fan. Mixed convection occurs when forced and natural convections simultaneously occur.

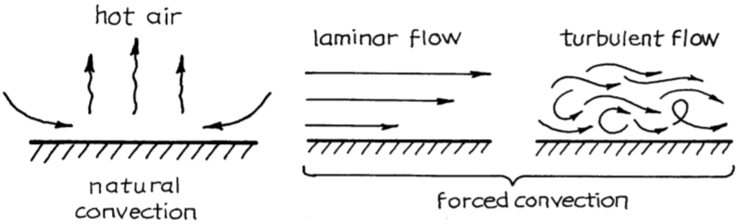

Figure 1.6: Convective heat transfer (Levenspiel, 2014)

1.5 Permeable surface

A permeable (porous) surface is used to allow the wall fluid suction or injection (fluid removal) in the boundary layer. Suction is one of the boundary layer control method, which is traditionally used in drag reduction of bodies in an external flow or energy losses in channels (Gad, 1990). An application of suitable wall mass suction through the permeable surface can effectively be used to stabilize the vorticity within the boundary layer and, subsequently, delay the boundary layer separation.

1.6 Dimensionless Numbers in Fluid Mechanics

In fluid mechanics, the dimensionless numbers are the ratio of involving quantities and widely used to reduce the variables involved in the physical system. The ratio of involving quantities which are used in this research are:

1.6.1 Prandtl number

Prandtl number (Pr) can be defined as the ratio of viscous diffusion rate (momentum diffusivity) to thermal diffusion rate (thermal diffusivity) or mathematically written as

$$\mathrm{Pr} = \frac{\nu_f}{\alpha_f} = \frac{\mu_f/\rho_f}{k_f/\left(\rho C_p\right)_f} = \frac{\left(\mu C_p\right)_f}{k_f}, \tag{1.6.1}$$

where ν_f is the kinematic viscosity (momentum diffusivity) and α_f is the thermal diffusivity of the fluid. Meanwhile μ_f, ρ_f, k_f and $\left(\rho C_p\right)_f$ are the dynamic viscosity, density, thermal conductivity and heat capacitance of the respective fluid. Generally, for $\mathrm{Pr} > 1$, the momentum diffusivity is higher (dominant) than the thermal diffusivity, and consequently, augments the heat transfer process and diminishes the thermal boundary layer thickness. The value of Prandtl number for few of fluids are presented in Table 1.1.

Table 1.1: Prandtl number for different fluids.

Fluids	Prandtl number
Air	0.71
Water (depends on the temperature)	1-10
Gases	0.7-1
Oil	50-2000
Methanol	7.38
Kerosene	21

1.6.2 Reynolds number

The Reynolds number (Re) is used to predict the patterns of the flow in different situation where a laminar flow is identified through the low Reynolds number while at high Reynolds number, the flow is turbulent. The Reynolds number is defined as the ratio of inertial forces to viscous forces within a fluid and mathematically written as

$$Re = \frac{u_\infty L}{\nu_f}, \tag{1.6.2}$$

where u_∞ is the free stream velocity, L is the characteristic length of the surface and ν_f is the fluid kinematic viscosity.

1.6.3 Skin friction coefficient

Skin friction coefficient $\left(C_f\right)$ can be defined as

$$C_f = \frac{2\tau_w}{\rho_f {u_\infty}^2}, \tag{1.6.3}$$

where u_∞ is the free stream velocity, ρ_f is the fluid density and τ_w is the wall shear stress. The wall shear stress τ_w or also known as friction force per unit area is important to drag the fluid motion along the surface and mathematically expressed as

$$\tau_w = \mu_f \left(\frac{\partial u}{\partial y}\right)_{y=0}, \tag{1.6.4}$$

where $\dfrac{\partial u}{\partial y}$ is the velocity gradient and μ_f is the dynamic viscosity of the fluid.

1.6.4 Nusselt number

The local Nusselt number (Nu) is important in the heat transfer field which indicates the ratio of convective heat transfer to conductive heat transfer. It is mathematically written as

$$Nu = \frac{h_f L}{k_f} = \frac{h_f L \Delta T}{k_f \Delta T}, \tag{1.6.5}$$

where h_f is the heat transfer coefficient of the fluid, L is the characteristic length of the surface, k_f is the fluid thermal conductivity and ΔT is the temperature difference. The Nusselt number $Nu = 1$ represents a similar magnitude between convection and conduction processes.

1.6.5 Grashof number

The Grashof number Gr refers to the ratio of the buoyancy force to the viscous force which acting on the fluid. It is mathematically expressed as

$$Gr = \frac{g\left(\beta_T\right)_f \left(T_w - T_\infty\right) L^3}{{\nu_f}^2} = \frac{g\left(\beta_T\right)_f \Delta T L^3}{{\nu_f}^2}, \tag{1.6.6}$$

where g is the gravitational acceleration, $\left(\beta_T\right)_f$ is the coefficient of volume expansion, T_w and T_∞ are the surface and ambient (far-field) temperatures, respectively, and ν_f is the kinematic viscosity of the fluid.

1.6.6 Eckert number

In the field of convective heat transfer, the Eckert number (Ec) is used to characterize the heat transfer dissipation. The Eckert number is the ratio of flow's kinetic energy to the enthalpy difference in boundary layer and expressed as

$$\mathrm{Ec} = \frac{u^2}{\left(C_p\right)_f \left(T_w - T_\infty\right)}, \tag{1.6.7}$$

where u is the fluid velocity, $\left(C_p\right)_f$ is the specific heat of the fluid at a constant pressure, T_w and T_∞ are the surface and ambient temperatures, respectively. In this study, the Eckert number is used to measure the effect of Joule (Ohmic) heating. Joule heating is a process when an electric current flows through any conducting material and simultaneously, produces heat. The joule heating effect is widely and practically used in most of the electrical and electronic devices.

1.6.7 Biot number

The Biot number (Bi) is a dimensionless number used to measure the heat transfer process. It describes the ratio of the heat transfer resistance inside and at the surface of a solid object (body) and expressed as

$$\mathrm{Bi} = \frac{h_f L}{k_s}. \tag{1.6.8}$$

The ratio determines if the temperature inside a body will vary significantly in space, while the body heats or cools over time, from a thermal gradient applied to its surface. From the mathematical expression of the Biot number and the Nusselt number, both have the same group of physical parameters $\frac{h_f L}{k}$ where L is the characteristic length scale and h_f is the heat transfer coefficient. The Nusselt Number is used to characterize the heat flux from a solid surface to a fluid, hence the thermal conductivity is measured from the fluid. Meanwhile, the Biot number is used to characterize the heat transfer resistance inside a solid body, hence k_s is the thermal conductivity of the body and h_f is the heat transfer coefficient that describes the heat transfer from the surface of the solid body to the surrounding fluid.

1.6.8 Hartmann number

The Hartmann number (Ha) is the ratio of electromagnetic force to the viscous force and frequently encountered in the fluid flow through magnetic field or magnetohydrodynamics (MHD). Magnetohydrodynamics (MHD) is a branch of physical studies that focus on the magnetic properties and characteristics of an electrically conducting fluids such as plasmas, electrolytes, liquid metals and salt water. MHD are widely embedded in many devices such as heat exchangers, power pumps, generators and electrostatic filters. The Hartmann number is mathematically written as

$$\mathrm{Ha} = B_0 L \sqrt{\frac{\sigma_f}{\mu_f}}, \tag{1.6.9}$$

where B_0 is the strength of the magnetic field, σ_f is the electrical conductivity of the fluid and μ_f is the dynamic viscosity of the fluid.

1.7 Stability Analysis

The boundary layer problem are categorized as nonlinear differential equations which is possible to generate non-unique solution (Schlichting and Gersten, 2017). The solution of the boundary layer equations can be zero, unique or multiple solutions with the application of suitable physical parameter such as suction (Miklavčič and Wang, 2006). Generally, for non-unique solutions, the first (upper branch) solution which satisfies the boundary conditions is denoted as the physical and stable solution. Meanwhile, the lower branch solution refers to the second solution which asymptotically fulfills the boundary conditions. Hence, it is important to identify all the possible solutions in the boundary layer problem to avoid misinterpretation of the fluid motion. In certain cases, the second solution may exhibit the same pattern of the real flow characteristics based on the velocity and temperature profiles. Therefore, it is necessary to validate the real solution through a proper analysis. The execution of the stability analysis is mathematically performed to verify the physical or real solution among all the solutions.

Wilks and Bramley (1981) being the first to perform stability analysis for the convection boundary layer flow problem past an impermeable vertical surface with variable surface temperature. They found the existence of dual solutions in the opposing buoyancy stream and perform the stability analysis to determine the stability of the particular dual solutions. They found that the smallest eigenvalue of upper branch solution was positive. However, the lower branch solution has both positive and negative values for the smallest eigenvalues. With such results, Wilks and Bramley (1981) concluded that the upper branch solution was a stable solution while the lower branch solution was unstable.

The study by Merkin (1986) further became the main reference to the other researchers regarding the stability analysis. Later, Merrill et al. (2006), Weidman et al. (2006) and Harris et al. (2009) have used and improved the stability analysis method introduced by Merkin (1986).

1.8 Problem Statement

- The separation of boundary layer flow mostly occurs in the shrinking region or opposing buoyancy region. Theoretically, the fluid motion past a shrinking sheet is restricted due to the unconfined vorticity within the boundary layer. However, dual/multiple solutions are usually detected in this region with the imposition of wall mass suction parameter (Miklavčič and Wang, 2006) or the use of stagnation point flow (Wang, 2008).

- The solutions usually exist up to a meeting point or also known as critical or turning point. This turning point signifies the occurence of boundary layer separation from laminar to turbulent. Beyond this point, the usual boundary layer and energy equations are invalid to analyze the fluid flow and heat transfer characteristics. It is crucial to maintain the laminar flow from the separation process.

- On the other hand, the application of hybrid nanofluids in the research of boundary layer flow is still new. There are problems where the hybrid nanofluids are not useful in the heat transfer enhancement as reported by Jana et al. (2007) and Baghbanzadeh et al. (2012). Hence, it is beneficial to examine if the hybrid nanofluid including the governing parameters are capable to delay the separation process and increase the heat transfer rate.

The research questions associated with the problem statement are

- Does the dual similarity solutions possible for all research problems (Chapters 4-8) if no suction is imposed?

- Does the dual similarity solutions exist for both assisting buoyancy flow and opposing buoyancy flow (Chapter 4)?

- Does the power law velocity is better than the linear velocity in delaying the separation and enhancing the heat transfer rate (Chapter 8)?

- Which parameters are potential to delay the boundary layer separation and increase the heat transfer rate?

1.9 Objectives and Scope of Study

The objectives are

- construct and derive the mathematical model,
- solve the mathematical model numerically using bvp4c solver
- conduct the stability analysis for the dual solutions to determine which of the solutions represent a stable flow
- analyze the influence of the considered parameters on the characteristics of the fluid flow and heat transfer

for the following problems

1. Mixed convective stagnation point flow of Cu-Al_2O_3/water hybrid nanofluid towards a permeable vertical Riga plate.
2. MHD flow and heat transfer of Cu-Al_2O_3/water hybrid nanofluid past a permeable stretching/shrinking disc with Joule heating.
3. MHD flow and heat transfer of Cu-Al_2O_3/water hybrid nanofluid past a permeable stretching/shrinking cylinder with Joule heating.
4. Three-dimensional flow and heat transfer of Cu-Al_2O_3/water hybrid nanofluid past a permeable stretching/shrinking sheet with velocity slip and convective boundary condition.
5. Three-dimensional flow and heat transfer of Cu-Al_2O_3/water hybrid nanofluid past a permeable nonlinear stretching/shrinking sheet with orthogonal surface shear.

Meanwhile, the scope of the study is only decisive to

1. Fluid : Hybrid Cu-Al_2O_3/water nanofluid.

2. Type of Flow : Boundary layer and stagnation point flow with heat transfer; two-dimensional flow (Chapter 4-6) and three-dimensional flow (Chapter 7 and 8).

3. Surface : Riga plate (Chapter 4), disc (Chapter 5), circular cylinder (Chapter 6) and flat plate (Chapter 7 and 8).

4. Physical parameters : Suction, mixed convection, EMHD (Riga plate), MHD, Joule heating, velocity slip and convective condition.

5. Model: Single phase nanofluid model by Tiwari and Das (2007) and thermophysical properties of hybrid nanofluid by Devi and Devi (2016a,b).

1.10 Thesis Framework

There are nine chapters in this thesis. Chapter 1 is the introduction and basic description of research background which are boundary layer theory, heat transfer, single and hybrid nanofluids and the dimensionless numbers in fluid mechanics. Besides, the research objectives, scopes and framework are also comprised in this chapter.

The review of the previous published literatures which are relevant to the research objectives and scopes are discussed in Chapter 2. The pioneer works on the boundary layer flow, stagnation point flow, mixed convective flow, nanofluids and stability analysis are also highlighted in this chapter.

Chapter 3 is the methodology of the research work which are divided into 5 parts; introduction, boundary layer and energy equations, similarity transformation and equations, numerical method (bvp4c) and stability analysis. In this chapter, the derivation of the reduced ordinary differential equations with boundary condition using the similarity transformation and linearized eigenvalue problem for the stability analysis are shown for the first problem.

Chapters 4 to 8 present the five research problems as stated in the Section 1.9. Each chapter is divided into 5 parts; introduction, problem formulation, temporal stability analysis, results and discussion, and conclusion. In the results and discussion section, the reduced skin friction coefficient, local Nusselt number, velocity and temperature profiles are presented in the graphs and tables form. The comparison of numerical values between present and previous studies in limiting case is also conducted to validate the present model and method. The derivation of the reduced ordinary (similarity) differential equations for each problem is presented in the Appendix A (Chapter 4), Appendix B (Chapter 5), Appendix C (Chapter 6), Appendix D (Chapter 7) and Appendix E (Chapter 8).

The conclusion for all the problems are summarized in Chapter 9. Besides, the recommendation for the future studies is also proposed in this chapter.

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