UNIVERSITI PUTRA MALAYSIA

CONVECTION BOUNDARY LAYER FLOW PAST A MOVING THIN NEEDLE IN A NANOFLUID WITH STABILITY ANALYSIS

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By

SITI NUR ALWANI BINTI SALLEH

**Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia,
in Fulfillment of the Requirements for the degree of Doctor of Philosophy**

July 2020

DEDICATIONS

To my parents;

Salleh Bin Hussain
&
Sabariah Binti Samsu @ Ismail

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfillment of the requirement for the degree of Doctor of Philosophy

CONVECTION BOUNDARY LAYER FLOW PAST A MOVING THIN NEEDLE IN A NANOFLUID WITH STABILITY ANALYSIS

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SITI NUR ALWANI BINTI SALLEH

July 2020

Chairman: Norfifah binti Bachok @ Lati, PhD
Institute: Institute for Mathematical Research

This study focuses on the problem of steady laminar boundary layer flow past a continuously moving thin needle in a nanofluid. Four different problems are considered by using two types of nanofluid model which are Tiwari and Das (2007) and Boungiorno (2006) models. The Tiwari and Das (2007) model is applied for the first two problems, namely (i) forced convection flow past a moving horizontal thin needle in a nanofluid with slip effect and convective boundary condition and (ii) mixed convection flow past a moving vertical thin needle in a nanofluid. Meanwhile, the Boungiorno (2006) model is considered for the next two problems, namely (iii) free convection flow past a moving horizontal thin needle in a nanofluid with chemical reaction and heat generation and (iv) mixed convection flow past a moving vertical thin needle in a nanofluid with the magnetic field effect. The governing coupled partial differential equations are transformed into nonlinear ordinary differential equations by adopting suitable similarity transformations. The bvp4c solver is used to solve the given system of equations through MATLAB software. The influences of the governing parameters which include the needle thickness, velocity ratio, mixed convection or buoyancy, nanoparticle volume fraction, Brownian motion, thermophoresis, slip, convective or Biot number, chemical reaction, heat generation and magnetic on the characteristics of the flow, heat and mass transfer are analyzed. The physical quantities of interest such as the skin friction coefficient, the heat and mass transfer rate as well as the velocity, temperature and concentration distribution are graphically presented through graphs, and discussed further with the variation of governing parameters. Since all the problems possess dual solutions, the stability analysis is performed to identify which of the solutions are linearly stable. Validation of the present work is done by comparing the current results with those available in the literature and found to be in an excellent agreement. It is noticed in this work that the decrement in the needle thickness increases the skin friction coefficient, heat and mass transfer rate as well as widening the domain of the dual solutions obtained. Also, the study shows that the dual solutions exist when the needle surface and the buoyancy force are against the direction of the fluid motion. In Tiwari and Das problems, it is noted that the addition of nanoparticle volume fraction offers a greater skin friction coefficient. It also increases the heat transfer rate for the thin surface of the needle, and in the meantime decreases the heat transfer rate for the thick surface of the needle. Meanwhile, for the Buongiorno problems, it is found that the higher Brownian motion rate diminishes the heat and mass transfer rate in the flow. Similarly, the heat transfer rate decreases with higher values of the thermophoresis parameter, while the opposite effect is seen for the mass transfer rate. It is noticed in the stability analysis that the solution for the upper branch is always stable. Meanwhile, the solution for the lower branch indicates both stable and unstable solutions for the problem of forced convection flow. Nevertheless, other problems such as free and mixed convection flow, the lower branch solution represents an unstable solution.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

ALIRAN OLAKAN LAPISAN SEMPADAN TERHADAP JARUM NIPIS YANG BERGERAK DI DALAM NANOBENDALIR DENGAN ANALISIS KESTABILAN

Oleh

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Kajian ini memfokuskan masalah aliran lapisan sempadan yang mantap berlamina terhadap jarum nipis yang sentiasa bergerak di dalam nanobendalir. Empat masalah yang berbeza dipertimbangkan dengan menggunakan dua jenis model nanobendalir iaitu, model Tiwari dan Das (2007) dan model Buongiorno (2006). Model Tiwari dan Das (2007) digunakan untuk dua masalah yang pertama, iaitu (i) aliran olakan paksa melepasi jarum nipis mendatar yang bergerak di dalam nanobendalir dengan kesan slip dan keadaan sempadan olakan dan (ii) aliran olakan campuran melepasi jarum nipis menegak yang bergerak di dalam nanobendalir. Sementara itu, model Buongiorno (2006) dipertimbangkan untuk dua masalah yang berikutnya, iaitu (iii) aliran olakan bebas melepasi jarum nipis mendatar yang bergerak di dalam nanobendalir dengan tindak balas kimia dan penjanaan haba dan (iv) aliran olakan campuran melepasi jarum nipis menegak yang bergerak di dalam nanobendalir dengan kesan medan magnet. Persamaan pembezaan separa menakluk terganding telah dijelmakan kepada persamaan pembezaan biasa tak linear dengan menggunakan penjelmaan keserupaan yang bersesuaian. Pakej bvp4c digunakan untuk menyelesaikan sistem persamaan yang diberi melalui perisian MATLAB. Pengaruh parameter menakluk yang merangkumi ketebalan jarum, nisbah halatuju, olakan campuran atau keapungan, pecahan isipadu nanozarah, gerakan Brown, termoforesis, gelincir, olakan atau nombor Biot, tindak balas kimia, penjanaan haba dan magnet ke atas ciri-ciri aliran, pemindahan haba dan jisim dianalisis. Kuantiti fizikal seperti pekali geseran kulit, kadar pemindahan haba dan jisim serta profil halaju, suhu dan kepekatan ditunjukkan secara bergraf melalui rajah, dan dibincangkan dengan lebih lanjut dengan variasi parameter menakluk. Oleh kerana kesemua masalah mempunyai penyelesaian dual, analisis kestabilan dilakukan untuk mengenal pasti penyelesaian mana yang stabil. Pengesahan kajian ini dilakukan dengan membandingkan penyelesaian semasa dengan kajian terdahulu dan didapati hasil perbandingan sangat baik. Diperhatikan dalam kajian ini bahawa pengurangan ketebalan jarum meningkatkan pekali geseran kulit, kadar pemindahan haba dan jisim serta meluaskan domain penyelesaian dual yang diperoleh. Kajian ini juga menunjukkan bahawa penyelesaian dual wujud apabila permukaan jarum dan daya keapungan melawan arah gerakan bendalir. Dalam masalah Tiwari dan Das, didapati bahawa penambahan pecahan isipadu nanozarah menyumbang kepada pekali geseran kulit yang lebih besar. Ia juga meningkatkan kadar pemindahan haba untuk permukaan jarum yang nipis, dan dalam masa yang sama menurunkan kadar pemindahan haba untuk permukaan jarum yang tebal. Sementara itu, untuk masalah Buongiorno, didapati bahawa kadar gerakan Brown yang lebih tinggi mengurangkan kadar pemindahan haba dan jisim di dalam aliran. Begitu juga, kadar pemindahan haba berkurangan dengan nilai parameter termoforesis yang tinggi, manakala kesan sebaliknya dilihat untuk kadar pemindahan jisim. Diperhatikan dalam analisis kestabilan bahawa penyelesaian untuk cabang atas adalah sentiasa stabil. Sementara itu, penyelesaian untuk cabang bawah menunjukkan kedua-dua penyelesaian stabil dan tidak stabil untuk masalah aliran olakan paksa. Walau bagaimanapun, masalah-masalah lain seperti aliran olakan bebas dan campuran, penyelesaian cabang bawah mewakili penyelesaian yang tidak stabil.

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This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfilment of the requirement for the degree of Doctor of Philosophy. The members of the Supervisory Committee were as follows:

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This is to confirm that:

- the research conducted and the writing of this thesis was under our supervision;
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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF ABBREVIATIONS

a	condition at the surface
b	condition at the free stream
Al_2O_3	Alumina
B	transverse magnetic field
B_0	uniform magnetic field strength
Bi	Biot number
c	needle thickness
C	nanofluid concentration
C_f	skin friction coefficient
C_p	specific heat at constant pressure
C_w	surface concentration
C_∞	ambient concentration
Cu	Copper
d	constant
D_B	Brownian diffusion coefficient
D_T	thermophoresis diffusion coefficient
$\hat{e}_r$	radial unit vector
$\hat{e}_\theta$	tangential unit vector
$\hat{e}_z$	axial unit vector
f	dimensionless components of velocity
g	acceleration due to gravity
Gr_x	local Grashof number
h_f	heat transfer coefficient
k	thermal conductivity
K	chemical reaction parameter
K_0	chemical reaction coefficient
K^*	dimensionless reaction rate
L	characteristic length of the needle
L_0	slip coefficient
L_s	dimensionless slip
M	magnetic parameter
Nb	Brownian motion parameter
Nr	buoyancy ratio parameter
Nt	thermophoresis parameter
Nu_x	local Nusselt number
p	pressure
Pr	Prandtl number
q_w	surface heat flux
q_m	surface mass flux
Q	heat generation parameter
Q_0	heat generation coefficient
Q^*	dimensionless heat generation
R	needle radius
Re_x	local Reynolds number

Sc	Schmidt number
Sh_x	local Sherwood number
t	time
T	nanofluid temperature
T_f	hot fluid temperature
T_w	surface temperature
T_∞	ambient temperature
TiO_2	Titania
u	velocity component along the x-axis
U	composite velocity between the needle and free stream
U_w	velocity of the needle
U_∞	free stream velocity
v	velocity component along the r-axis
$\vec{V}$	velocity vector
x, r	Cylindrical coordinates

Greek Symbol

α	thermal diffusivity
β	thermal expansion coefficient
γ	eigenvalue parameter
δ	thickness of the boundary layer
ε	velocity ratio parameter
η	similarity variable
θ	dimensionless temperature
κ	ratio of effective heat capacity of nanofluid to the base fluid
λ	mixed convection or buoyancy parameter
μ	dynamic viscosity
ν	kinematic viscosity
ρ	fluid density
ρC_p	volumetric heat capacity
σ	slip parameter
τ	dimensionless time variable
τ_w	surface shear stress
φ	nanoparticle volume fraction parameter
ϕ	dimensionless concentration
ψ	stream function
∇	del operator
∇^2	Laplace operator
ω	electrical conductivity

Subscripts

c	critical value
f	fluid
nf	nanofluid
s	solid nanoparticles
w	condition at the wall
∞	condition at infinity

Superscript

$'$	differentiation with respect to η

CHAPTER 1

INTRODUCTION

1.1 Fluid Dynamics

Fluid dynamics is one of the two main branches of fluid mechanics, which is concerned with the motion of fluids such as liquids and gases, and their interactions as two fluids are in contact with each other. The other branch of fluid mechanics is fluid statics, which deals with the study of fluids at rest (White, 2011). However, this branch may be considered a bit less appealing than fluid dynamics. Fluid dynamics is also known as hydrodynamics, and it would be more convenient to say that when the fluid dynamics is applied to liquids in motion. Otherwise, when the fluid dynamics are applied to gases in motion, it is called aerodynamics (Batchelor and Batchelor, 2000; White, 2011). Since two-thirds of the Earth's surface consists of water and the planets are surrounded by the atmosphere's layer, hence, there are many applications involving fluid dynamics. For instance, it provides ways for studying the evolution of stars, ocean currents, plate tectonics, weather patterns, climate trends and even circulation of the blood (Batchelor and Batchelor, 2000).

Fluid dynamics can be studied either experimentally or computationally. In this study, we consider the computational study in which all the problems considered in this thesis would be computed numerically. This numerical approach is successfully applied to our problems with the help of mathematical software. All the physical phenomena for each problem are demonstrated in the mathematical model. By representing such physical problems in the mathematical model, it assists to explain the behavior of the problem in mathematical form.

1.2 Research Background

In this section, all the definitions of the important terms for this research are explained in detail. To understand the problems very well, we need to know what kind of effects or parameters that we want to use, and how these affect the flow system. The terms used for the present study are discussed in the following Subsection 1.2.1 until Subsection 1.2.8.

1.2.1 Boundary Layer Theory

In fluid mechanics, the boundary layer exists everywhere when there is an interaction between the surface and the fluid flowing over it where the influences of the viscosity are important. The concept of the boundary layer was first introduced by a German engineer, Ludwig Prandtl in 1904 at the third International Congress of

Mathematicians (Acheson, 1990). According to Prandtl's theory, the fluid in the boundary layer is subjected to friction forces. The velocity range exists across the boundary layer from maximum to zero gives the fluid is in contact with the solid surface. Typically, the boundary layers are thinner at the leading edge (upstream portion) of the surface and thicker toward the trailing edge (downstream portion). At the leading or upstream portion, the flow in the boundary layer is laminar, meanwhile, at the trailing or downstream portion, the flow is turbulent.

In Prandtl's theory, the boundary layer flow can be divided into two regions (Anderson, 2005). First is a tiny region near the solid boundary where the viscous effects and rotation cannot be ignored. In this region, the effect of friction is to cause the fluid immediately adjacent to the surface to stick to the surface or can be assumed as the no-slip condition occurs at the surface. The second is an outer region away from the solid boundary where the viscous effects are small enough and can be ignored (Schlichting, 1979). In this region, the flow is essentially inviscid flow or similar to the upstream flow. In this case, a potential flow must be considered. The boundary layer theory has many practical applications in aerodynamics, including, heat transfer occurs in high-speed flight, skin friction drag on the object and wing stall. The formation of the boundary layer can be seen in Figure 1.1.

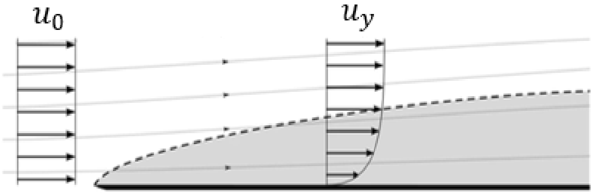

Figure 1.1: Formation of boundary layer
(https://en.wikipedia.org/wiki/Boundary_layer)

1.2.2 Heat Transfer

Heat transfer plays an important role in some kind of application in fluid dynamics. In terms of the thermodynamic system, heat transfer is the transmission of thermal energy across the boundary of the system due to the temperature difference between the system and the surroundings (Ling et al., 2016). The heat will flow from the high-temperature reservoir to the low-temperature reservoir which is a direct consequence of the second law of thermodynamics. Heat transfer also occur within the system due to the difference in temperature at various points inside the system. The difference in temperature is said to be potential that leads the heat flow and the heat itself (known as a flux). Heat transfer can take place through three principal mechanisms which are conduction, radiation and convection as can be seen in Figure 1.2 (see Wong (1977) and Çengel and Ghajar (2011)). These three mechanisms are described as follows:

1. Conduction is the transfer of heat between two bodies that kept in contact with each other (see Ghassemi and Shahidian (2017)). It relies on the difference in temperature of the hot and cold body. In this process, when the body is heated, the molecules will gain more energy and vibrate. As a consequence, these molecules hit with the adjoining molecules and transfer some of their energy to them. An example of conduction heat transfer is heating one end of the metal rod and due to the mechanisms of heat conduction, the other end of the metal rod also gets heated.

2. Radiation heat transfer is a phenomenon of the transmission of energy from one body to another by propagation through a medium (Howell et al., 2010). These two bodies must have different temperatures and separated by distance. All bodies persistently emit energy through electromagnetic radiation (see Geankoplis (2003)). The intensity of such energy flux depends both on the temperature of the body and surface characteristics. For instance, if we place our hands near the campfire, most of the heat that reaches us is called radiant energy. Another example of radiation heat transfer is Sun's energy coming on the earth.

3. Convection heat transfer is a process by which heat is transferred by the movement or circulation of the heated parts of the fluid such as water or air. It involves the combined effect of conduction and fluid motion. Typically, convection takes place in mixtures or soft solids in which the solid particles can move through each other. Convection can be classified into two types; natural or free convection and forced convection by depending on how the motion of the fluid is initiated.

   Natural convection occurs when the motion of the fluid is driven by buoyancy forces resulting from the density variations due to the temperature difference (Pop and Ingham, 2001). In the absence of external source and when the fluid is in contact with a hot surface, its molecules separate and clutter causing the fluid to be less dense. Consequently, the fluid is displaced while the cooler fluid gets denser and the fluid sinks. Hence, the hotter layer of the fluid transfers the heat towards the cooler ones. A familiar example of natural convection is when the pot of water is heated from the bottom. Forced convection or also known as heat advection is the movement of fluid resulting from the external forces like a pump, fan or suction device. This convection is commonly used to enhance the rate of heat exchange. Some examples of the forced convection are heating and cooling of parts of the body by blood circulation and fluid radiator system.

   Interestingly, the combination of forced and free convection will generate mixed convection. It occurs when these two convections act together to transfer heat. In this case, both buoyancy forces and pressure forces interact. The temperature, orientation, flow and geometry are the main components to compute the quantity of each form of convection that contributes to the evacuation of heat. Some applications of the mixed convection are nuclear reactor and electronic cooling. In general, mixed convection can be described in two ways (see Çengel (2007)).

First is an aiding or assisting flow. In this case, free convection aids forced convection. This can be seen when the buoyant motion is in the same direction as the forced motion and consequently, accelerating the boundary layer and increasing the heat transfer (Çengel and Ghajar, 2007). As a result, delay the transition to the turbulent flow (Abedin et al., 2012). An example of this case is a fan blowing upward on a hot surface. Since heat naturally ascends, the air is forced upward over the surface leads to the heat transfer.

Second is an opposing flow. In this case, free convection acts in the opposite way of the forced convection. For instance, a fan forcing air upward over a cold surface (Çengel and Ghajar, 2007). In such a situation, the buoyant force of cold air naturally causes the air to fall, however, the air being forced upward resists the free motion. This case will cause strong shear in the boundary layer and rapidly transitions into a turbulent flow.

Among these three mechanisms of heat transfer, convection is more related to fluid dynamics. Hence, the present study only deals with this kind of heat transfer mechanism.

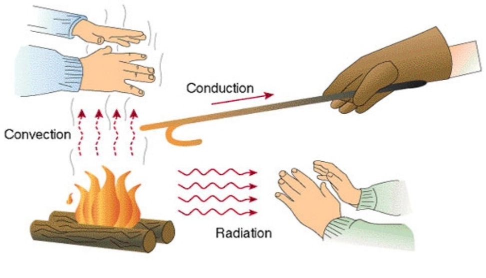

Figure 1.2: Mechanisms of heat transfer
(https://science4fun.info/heat-transfer)

1.2.3 Mass Transfer

In contrast to the heat transfer, mass transfer generally refers to the relative motion of some chemical species in a mixing process driven by concentration gradients as shown in Figure 1.3. During the mass transfer process, molecules or other small particles spontaneously mix and moving from high-concentration regions to low-concentration regions (Çengel and Ghajar, 2011). The mass transfer can take place in a single phase or over the phase boundaries in multiphase systems. It is commonly used in engineering for the physical processes that involve diffusive and convective transport of chemical species within physical systems (Smith et al., 2005).

Furthermore, heat and mass transfer are kinetic processes that may occur jointly or separately (Çengel and Ghajar, 2011). In diffusion and convection cases, it is most appropriate to realize that both processes are modeled by similar mathematical equations. Thus, it is more suitable to consider them jointly. Some common applications of mass transfer processes are the purification of the blood in the liver and kidney, the distillation of alcohol, evaporation of water from a pond to the atmosphere, combustion, absorbers such as stripping or scrubbers and activated carbon beds, separation of the chemical component in distillation columns and liquid-liquid extraction.

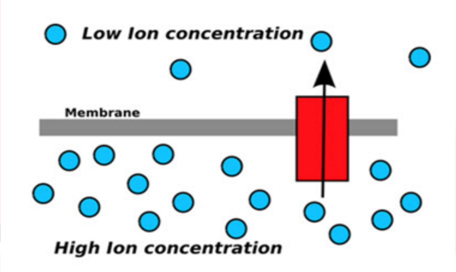

Figure 1.3: Mass transfer processes
(http://ecoursesonline.iasri.res.in/mod/page/view.php?id=2367)

1.2.4 Nanofluid

In fluid dynamics, the thermal conductivity of heat transfer fluid plays a key role in the development of heat transfer applications. Many kinds of research and development processes have been done for enhancing heat transfer properties of conventional heat transfer fluids. The conventional heat transfer fluids such as lubricants, ethylene glycol, kerosene, oil and water have become less favorable and reach their thermal performance limitation in certain applications (Bachok et al., 2010). To overcome such a situation and satisfy the cooling process requirement, new kinds of fluid are needed to reach the thermal efficiency of heat exchangers in the future. Following this, Choi (1995) solved the problem by combining nanometer-sized particles called nanoparticles with the diameter is less than 100 nm (see Figure 1.4) into the conventional fluids. This mixture is known as a nanofluid.

Nanoparticles consist of different materials, for instance, metals, nanotubes, ceramics, alloys, semiconductors and composite particles. Interestingly, nanofluids possess strong suspension stability and the ability to move without clogging the flow system. Nanofluid also has some special behaviors such as it is very stable and does not have any additional problems including erosion, sedimentation, additional pressure drop or non-Newtonian behavior (Khanafer et al., 2003). These good features of nanofluid are due to the tiny dimension of nanoelements in the fluid. In recent years, nanofluid has many practical applications since it has good thermal performance in the heat transfer. Nanofluid has the potential of being a new generation of coolants particularly in the biomedical applications, heat exchangers, electronic cooling and automotive cooling applications (Saidur et al., 2011; Huminic and Huminic, 2012; Colangelo et al., 2017; Selvaraj and Krishnan, 2020).

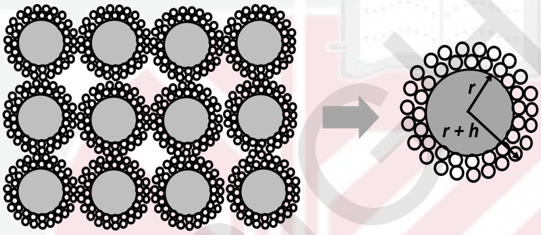

Figure 1.4: Physical model of nanofluid
(Kakaç and Pramuanjaroenkij, 2009)

There are two models that describe the transport behavior in nanofluid; one is a model proposed by Boungiorno (2006) and the second is a model proposed by Tiwari and Das (2007) as given below:

1. Buongiorno model is known as a non-homogeneous or two-component model in which the slip velocity of the base fluid and nanoparticles are non-zero. This model comprises of several slip mechanisms such as Brownian diffusion, inertia, the effect of Magnus, gravity, fluid drainage, diffusiophoresis and thermophoresis. It is worth mentioning that, the thermophoretic diffusion and Brownian movement of nanoparticles are two significant effects that enhance the thermal conductivity of ordinary or base fluids.

    (a) Brownian motion is defined as random motion of particles suspended in a fluid due to collisions with the molecules of the surrounding medium (see Albert (1956)).

    (b) Thermophoresis is defined as a motion of particles suspended in fluid influenced by a temperature gradient (see Duhr and Braun (2006)).

2. In contrast to the Buongiorno model, Tiwari and Das model is a homogeneous or a single component model in which the base fluid and nanoparticles are in thermal equilibrium flowing at uniform velocity and there is no slip condition occurs between them. This model considers the viscosity model proposed by Brinkman (1952) and Maxwell-Garnet thermal conductivity (see Maxwell (1881)) and it takes into consideration the influence of the nanoparticle volume fraction. The thermal conductivity of nanofluid increases significantly with an increase in the nanoparticle volume fraction rate. This consequently increases the performance of heat transfer in a system. In the published work by Jang and Choi (2007), they proved that only a small amount of the solid volume fraction is required to ensure the effectiveness and efficiency of the conventioal heat transfer fluids.

Thereafter, Nield and Kuznetsov (2009) and Kuznetsov and Nield (2013) continued the Buongiorno's work by applying the new boundary condition that has thermophoresis and Brownian movement parameter in the energy and concentration equations. The presence of these two parameters is to generate their effects directly into the equations. Hence, the temperature and concentration are paired in a particular way, and consequently, the thermal and concentration buoyancy effects also being paired in the usual way (Zaimi et al., 2014). This proposed model is called a revised model where the nanoparticle fraction on the boundary is controlled passively rather than actively.

1.2.5 Thin needle

The term thin needle simply means a parabolic revolution about its axes direction in addition to the variable thickness (Lee, 1967). It is considered thin when its thicknesses do not exceed the boundary on it or smaller. The topic of thin needle seems very famous due to the movement of the needle that distracts the free stream flow. This criterion is the main concern of the flow and heat transfer process in order to compute the velocity and temperature distributions in experimental studies. The boundary layer flow over a thin needle is of considerable importance in the medicine and engineering industries. For instance, it is commonly used in hot wire anemometer or protected thermocouple for calculating the wind velocity, transportations, circulatory problems, transportation and coating of wires. The study of the thin needle in the boundary layer flow is first discussed by Lee (1967).

There are two types of thin needle that we have considered in this study, namely horizontal and vertical surfaces. Figures 1.5 and 1.6 present the physical models of the horizontal and vertical thin needle, respectively. It is important to note that, in the case of a vertical thin needle, there exist buoyancy forces due to the pressure exerted on the surface by the fluid. Hence, we can say that there exists mixed convection in the vertical case. Meanwhile, for the horizontal case, free and forced convection occur.

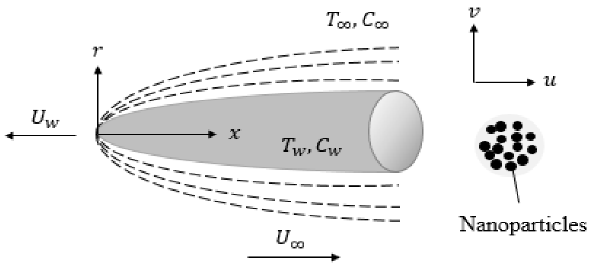

Figure 1.5: Physical model of horizontal thin needle

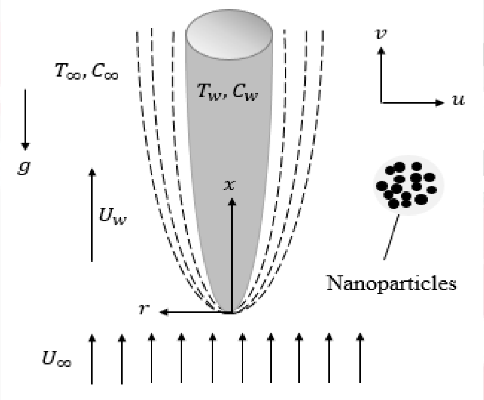

Figure 1.6: Physical model of vertical thin needle

1.2.6 Types of Effects

In this study, several physical effects such as partial slip, convective boundary condition, chemical reaction, heat generation and also magnetohydrodynamics have been taken into consideration in order to analyze their impact on the characteristics of the flow, heat and mass transfer. The detail explanation of the effects are described below:

1. **Partial Slip**

   In recent years, a large number of slip flow models are proposed to illustrate the slip phenomenon occurring at the solid boundaries. Previously, it has been proven that the presumption of flow that adheres no-slip condition on the boundary is no longer appropriate in certain situations, namely rarefied gas flows in micro-scale tools and some coated surfaces (Wang, 2002). It is necessary to be changed by a partial slip condition especially in cases of suspensions, emulsions, polymer solutions and foams (Yoshimura and Prud'homme, 1988).

The study of the fluid flow in restricted geometries is highly affected by slip at the liquid-solid interface. In recent studies, it is shown that the velocity of the fluid near the solid surface is not equal to the velocity of the solid surface. This kind of phenomenon is known as a boundary slip which proposed by Navier (1827). The equation that represents the partial slip condition or first-order slip can be defined as:

$$u(x,y) = L_s \frac{\partial u}{\partial y}, \tag{1.2.1}$$

where u is the velocity of the fluid, L_s is the slip length, $\partial u/\partial y$ is the shear rate at the boundary and y is the coordinate tangential to the surface (see Navier (1827) and Andersson (2002)).

Experimental and theoretical studies suggest that at the liquid-solid interface, the existence of nanobubbles is accountable for boundary slip. A partial or full slip can occur at the liquid-solid interface under certain circumstances. The slip effect can be described by the slip length or the slip ratio. There are some factors that affect the slip length, for example, weak wall fluid attraction, high shear rates and also surface roughness.

2. **Convective Boundary Condition**

The convection or convective boundary condition is also known as the Newton boundary condition in heat transfer processes. It corresponds to the presence of convection heating or cooling at the surface and it is obtained from the surface energy balance (Çengel and Ghajar, 2011). In some practical applications, the convective boundary condition is possibly the most common boundary condition encountered since most heat transfer surfaces are exposed to a convective situation. This condition presumes that the heat conduction at the surface of the substance is equivalent to the heat convection at the surface in a similar direction. Given the fact that since the boundary cannot store the energy, the net heat entering the surface from the convective edge must leave the surface from the conduction edge.

The study of the convective boundary condition in the boundary layer flow was initiated by Aziz (2009). In the study, he assumed that the bottom of the surface is heated by convection from a hot fluid at temperature T_f which gives a heat transfer coefficient h_f. Hence, the boundary condition at the surface $(y=0)$ can be written as:

$$-k\frac{\partial T}{\partial y}(x,0) = h_f\left[T_f - T(x,0)\right], \tag{1.2.2}$$

It is important to mention that the dimensionless quantity involved in this situation is the Biot number (see Makinde and Olanrewaju (2011)).

3. **Chemical Reaction**

The chemical reaction is a reaction that occurs due to the presence of a foreign mass in a fluid (Muthucumaraswamy, 2002). In many chemical reactions, the reaction rate depends on the concentration of the species itself. The chemical reactions which take place between nanoparticles and conventional fluid can be categorized as a homogeneous or heterogeneous reaction (Chambré and Young, 1958). A reaction that occurs consistency in a single phase such as gaseous, liquid, or solid is called a homogeneous reaction. A heterogeneous reaction is a reaction that involves two or more phases like solid and gas, solid and liquid, two immiscible liquids, and it takes place within the boundary of a phase.

A chemical reaction is called a first-order reaction, if the reaction rate is directly proportional to the concentration itself (Muthucumaraswamy, 2002). Thus, the term that represents the chemical reaction can be expressed as follows (see Najib et al. (2017a) and Mabood et al. (2016a)):

$$\text{Chemical reaction} \Rightarrow K^{*}(C - C_{\infty}), \tag{1.2.3}$$

where K^{*} is the dimensionless reaction rate, C is the fluid concentration in the boundary layer and C_{∞} is the ambient concentration.

4. **Heat Generation**

Heat generation is a conversion of one form of energy such as nuclear, electrical or chemical energy into thermal or heats energy inside a solid. Due to the mechanism of heat generation, the heat source or generator produces a hot fluid layer adjacent to a solid surface which may exert strong influence on the heat transfer characteristics (Vajravelu, 1986). As an example, consider a system undergoing an exothermal reaction raises the temperature of the system. The system here is the solid and the thermal energy increase in this solid is known as heat generated.

According to Vajravelu (1986), the analysis of temperature distribution influenced by generation or absorption of heat in moving fluids is significant in some physical problems such as:

(a) Problems dealing with exothermic or endothermic chemical reactions, and

(b) Problems concerned with dissociating fluids.

Nowadays, many investigations are dealing with heat generation in the fluids. Although, exact modeling of internal heat generation or absorption is quite tough, some simple mathematical models can express its average behavior for most physical situations (Hakeem et al., 2014). Due to the importance of energy conservation, the study of heat generation is a priority in scientific instrumentation and nuclear reactor engineering. Heat pumps and heat absorption chillers are essential in the industry due to advantages in renewable utilization and waste heat recovery.

The term that defines the heat generation in the energy equation is taken as (see Vajravelu (1986)):

$$\text{Heat generation} \Rightarrow \frac{Q^*}{\rho C_p}(T - T_\infty), \tag{1.2.4}$$

where Q^* is the dimensionless heat generation, ρC_p is the volumetric heat capacity, T is the fluid temperature and T_∞ is the ambient temperature.

5. **Magnetohydrodynamics**

Magnetohydrodynamics or MHD is the study of the behavior of electrically conducting fluids such as liquid metals, plasmas, electrolytes and saltwater acted on by the magnetic fields. The term "magnetohydrodynamics" is derived from the words "magneto" which means the magnetic field, "hydro" is the water and "dynamics" is a movement. The field of magnetohydrodynamics was proposed by Alfvén (1942), a Swedish electrical engineer and plasma physicist. Generally, the important concept of magnetohydrodynamics is that the magnetic fields can induce electrical currents in a moving conductive fluid which later polarizes the fluid particle and gradually changes the magnetic field itself.

Physically, the applied magnetic field plays an essential role in controlling momentum and heat transfer in the boundary layer flow past certain surfaces in various fluids. It is worth mentioning that, the intensity and the orientation of the applied magnetic field are two main factors that influenced the characteristics of the flow. The exerted magnetic field has strongly changed the heat transfer performance in the flow by manipulating the suspended particles and also rearranged their concentration in the fluid (Hakeem et al., 2015).

The term for the magnetohydrodynamics in the flow can be defined as (see Chakrabarti and Gupta (1979) and Hakeem et al. (2015)):

$$\text{Magnetohydrodynamics} \Rightarrow \frac{\omega B_0^2}{\rho}u, \tag{1.2.5}$$

in which ω is the electrical conductivity, ρ is the density, B_0 is the uniform magnetic field imposed along y-axis and u is the velocity component along x-axis.

1.2.7 Stability Analysis

Stability analysis is an analysis for identifying the stability of the solutions obtained. This analysis was initiated by Wilks and Bramley (1981) since, in their problem, they obtained dual solutions for the mixed convection in the boundary layer flow. According to their study, it is noticed that the bifurcation point is found to be different from the point of vanishing skin friction in comparison with Falkner-Skan solutions. In the Falkner-Skan case, the numerical evidence in the work by Chen and Libby (1968) had distinguished the upper branch solutions as stable, whilst the lower branch solutions are unstable in a specific range of pressure gradient parameter.

The results displayed in the published paper by Wilks and Bramley (1981) indicate that the minimum eigenvalue for the upper branch solution can be identified by the absence of zeroes in the eigenfunction which show the positive value indicating all higher eigenvalues must then necessarily be positive. In contrast to that, the lower branch solutions obtain both negative and positive eigenvalues. Thus, these solutions must be considered asymptotically unstable. Four years later, Merkin (1985) continued to study the stability analysis on the dual solutions for the mixed convection flow in a porous medium by considering a simple time-dependent problem (see Merkin (1985) for the detail). It is concluded from the study that for a general time-dependent problem, the upper and lower branches of possible steady states are stable and unstable, respectively.

Nevertheless, some cases considering moving surfaces preclude the simple eigenvalue problem obtained by Merkin (1985). To overcome suah a situation, Weidman et al. (2006) suggested that the practical aspect can be determined by initial growth or decay. In their work, they checked the stability of the steady flow solution $f = f_0(\eta)$ satisfying the given boundary-value problem by adapting the method discussed by Merkin (1985) as follows:

$$f(\eta,\tau) = f_0(\eta) + e^{-\gamma\tau}F(\eta,\tau), \tag{1.2.6}$$

where $F(\eta,\tau)$ is a small relative to $f_0(\eta)$ and γ is the eigenvalue. According to Weidman et al. (2006), the initial growth or decay (at $t = 0$) of the above equation can be identified by assuming $\tau = 0$ and hence, F can be written as $F_0(\eta)$. It is worth mentioning that the stability of solutions is significant in physical problems because if there is a slight deviation from the mathematical model caused by the unavoidable error in measurement, the mathematical equations describing the problem will not be able to accurately predict the future outcome.

1.2.8 Dimensionless Parameters

In fluid dynamics, there are several important dimensionless quantities used to predict the fluid behavior patterns in some cases of fluid flow problems. The followings are the dimensionless parameters that we are going to use in the present study:

1. **Prandtl Number**

   Prandtl number, Pr is a dimensionless parameter approximating the ratio of momentum diffusivity or kinematic viscosity to thermal diffusivity (Coulson and Richardson, 1999). The Prandtl number is named after the German physicist Ludwig Prandtl in 1904. The Prandtl number is important because it can be used to compute the thermal conductivity of gases at high temperatures, in which it is difficult to measure experimentally.

   It is important to note that when Pr is small ($\mathrm{Pr} << 1$), it means that the heat diffuses quickly compared to the velocity. This causes the thermal boundary layer is much thicker than the velocity boundary layer for the liquid metals. In contrast to that, when Pr is large ($\mathrm{Pr} >> 1$), it means that the momentum diffusivity dominates the fluid behavior (Kothandaraman, 2006). For gaseous water (steam) with a relatively low thermal conductivity, both velocity and thermal boundary layers are of the same order of magnitude. In this case, the Prandtl number is about 1 (Schlichting and Gersten, 2017).

   The Prandtl number can be expressed as:

   $$\mathrm{Pr} = \frac{\nu}{\alpha} = \frac{\mu C\rho}{k} = \frac{\text{viscous diffusion rate}}{\text{thermal diffusion rate}}, \tag{1.2.7}$$

   where ν is the kinematic viscosity or momentum diffusivity, α is thermal diffusivity, μ is the dynamic viscosity, C_ρ is the specific heat at the constant pressure and k is the thermal conductivity.

2. **Reynolds number**

   The Reynolds number, Re is defined as the ratio of inertial forces to viscous forces and is a suitable parameter used to determine whether the fluid flow is laminar or turbulent (Schlichting and Gersten, 2017). The concept of Reynolds number is proposed by Anglo-Irish physicist and mathematician, Stokes (1851). In 1883, Osborne Reynolds has popularized its use, and this concept is named after Arnold Sommerfeld in 1908 (Rott, 1990).

   The Reynolds number is one of the main controlling parameters in all viscous flows where it can be interpreted that when the Reynolds number is small, the viscous forces are dominant. Thus, the flow will be laminar. Otherwise, if the Re is larger, the inertial forces will dominate over the viscous forces. Hence, the flow will be turbulent. In the laminar case, the flow is characterized by smooth and constant fluid, while in a turbulent case, the flow tends to produce vortices, chaotic eddies and other flow instabilities (Acheson, 1990).

The Reynolds number can be defined as:

$$\mathrm{Re} = \frac{\rho u L}{\mu} = \frac{uL}{\nu} = \frac{\text{inertial force}}{\text{viscous force}}, \tag{1.2.8}$$

where ρ is the density, u is the velocity of the fluid, L is a characteristic linear dimension, μ is the dynamic viscosity and ν is the kinematic viscosity.

3. **Nusselt number**

Nusselt number, Nu is defined as the ratio of convective to conductive heat transfer at a boundary in a fluid (Çengel and Ghajar, 2011). It is named after Wilhelm Nusselt, who made important contributions to the science of convective heat transfer (Çengel, 2002). Convection involves both fluid motion (advection) and diffusion (conduction). In hypothetically motionless fluid, the conductive component is measured under the same conditions as the convective. Noteworthy, the greater the value of Nusselt number, the more effective the convection compared to conduction.

The Nusselt number can be defined as:

$$\mathrm{Nu} = \frac{h}{k/L} = \frac{\text{convective heat transfer}}{\text{conductive heat transfer}}, \tag{1.2.9}$$

where h is the convective heat transfer coefficient of the flow, k is the thermal conductivity of the fluid and L is the characteristic length. When $\mathrm{Re} = 1$, it represents heat transfer by pure conduction (Çengel, 2002).

4. **Schmidt number**

Schmidt number, Sc is a dimensionless number representing the ratio of momentum diffusivity (kinematic viscosity) to mass diffusivity (Bergman and Incropera, 2011). The Schmidt number is named after German engineer Ernst Heinrich Wilhelm Schmidt (1892–1975). It works to characterize fluid flows in which there are simultaneous momentum and mass diffusion convection processes. The Schmidt number physically relates the relative thickness of the hydrodynamic layer and mass transfer boundary layer.

The Schmidt number are defined as (Bergman and Incropera, 2011):

$$\mathrm{Sc} = \frac{\nu}{D} = \frac{\text{viscous diffusion rate}}{\text{mass diffusion rate}}, \tag{1.2.10}$$

where ν is the kinematic viscosity and D is the mass diffusivity.

5. **Grashof Number**

Grashof number, Gr is defined as the ratio between the buoyancy forces and viscous forces acting on a fluid in the momentum or velocity boundary layer (Çengel, 2002). It is named after Franz Grashof. The Grashof number is essential in some cases of fluid flow due to natural convection. It is analogous to the Reynolds number in forced convection.

The Grashof number can be expressed as (Çengel, 2002):

$$\mathrm{Gr} = \frac{g\beta(T_w - T_\infty)L^3}{\nu^2} = \frac{\text{buoyant force}}{\text{viscous force}}, \tag{1.2.11}$$

where g is acceleration due to gravity, β is the thermal expansion coefficient, T_w is the wall temperature, T_∞ is the bulk temperature, L is the vertical length and ν is the kinematic viscosity. It is important to note that when $\mathrm{Gr} << 1$, the viscous force is negligible compared to the buoyancy and inertial forces (Çengel, 2002).

6. **Sherwood Number**

Sherwood number is defined as the ratio of the convective mass transfer to the rate of diffusive mass transport (Heldman, 2003). It is applied in the analysis of mass transfer systems such as liquid-liquid extraction. The Sherwood number is named in honor of Thomas Kilgore Sherwood and represents the effectiveness of mass convection at the surface.

The Sherwood number is given as follows (Heldman, 2003):

$$\mathrm{Sh} = \frac{h}{D/L} = \frac{\text{convective mass transfer rate}}{\text{diffusion rate}}, \tag{1.2.12}$$

where h is the convective mass transfer, D is the mass diffusivity and L is a characteristic length.

7. **Biot Number**

Biot number is a ratio of the heat transfer resistances inside of a body and at the surface of a body (Incropera et al., 2007). It is used to examine whether or not the temperatures inside a body will vary significantly in space, while the body heats or cools over time from a thermal gradient applied to its surface. The Biot number is named after a French physicist, Jean-Baptiste Biot (1774–1862). It is widely used in heat transfer processes in order to calculate the performance of the heat transfer rate.

Generally, for a smaller Biot number ($\mathrm{Bi} << 1$), the problems may be treated as thermally simple due to uniform temperature fields inside the body. Meanwhile, for a larger Biot number ($\mathrm{Bi} >> 1$), it represents more difficult problems due to the non-uniformity of temperature fields within the object.

The Biot number can be defined as (Incropera et al., 2007):

$$\mathrm{Bi} = \frac{L_c h}{k} = \frac{\text{convection at the surface of the body}}{\text{conduction within the body}}, \tag{1.2.13}$$

where h is convective heat transfer coefficient, L_c is the characteristic length and k is the thermal conductivity of the body.

1.3 Problem Statement

In many research and development processes, the axisymmetric boundary layer flow and heat transfer analysis of the slender needle is significant in fluid dynamics because of its industrial and technological purposes. Such kind of flow problems finds application in microscale cooling devices for heat elimination application, shielded thermocouple for determining the velocity of the wind (hot wire anemometer), and in the biomedical area for blood flow problems and cancer treatment. The optimization in equipment and manufacturing processes is a normal routine for the company to enhance the quality of their product (or application). Sometimes, there are incomplete applications that require improvement. Nevertheless, this improvement will cost a lot of money and takes a long time. Thus, the involved companies are encouraged to invent new applications or modify them to meet the desired requirements. Since these tasks are difficult to achieve, hence, more investigations using the mathematical model are performed nowadays. These investigations are able to help engineers for designing their applications involving the thin needle. Motivated by these problems, our intention here to propose several flow models that may give advantages to overcome such problems in certain applications.

Some of the questions that arise in this research are as follows:

1. How does the mathematical model of the boundary layer flow past a moving thin needle is formulated?
2. What happens to the skin friction coefficient, heat and mass transfer rate when a moving horizontal or vertical thin needle is considered?
3. What happens to the fluid flow and heat transfer characteristic when nanoparticles are imposed in the system?
4. What are the effects of Brownian motion and thermophoresis on the fluid flow, heat and mass transfer characteristics?
5. How does the presence of slip, Biot number, chemical reaction, heat generation and magnetic field affect the characteristics of the flow, heat and mass transfer rate?
6. Which of the upper or lower branch solution is linearly stable?

1.4 Objectives and Scopes

The objectives of the present study are to

1. construct and derive the mathematical model,
2. solve the mathematical model numerically via bvp4c solver in MATLAB software,
3. provide the formulation and conduct the stability analysis for the dual solutions obtained to determine which of the solutions represent a stable flow, and
4. analyze the influence of the considered parameters on the characteristics of the fluid flow, heat and mass transfer,

for the following problems:

1. forced convection flow past a moving horizontal thin needle in a nanofluid with slip effect, convective boundary condition and stability analysis using the Tiwari and Das (2007) model,
2. mixed convection flow past a moving vertical thin needle in a nanofluid with stability analysis using the Tiwari and Das (2007) model,
3. free convection flow past a moving horizontal thin needle in a nanofluid with chemical reaction, heat generation and stability analysis using the Buongiorno (2006) model, and
4. mixed convection flow past a moving vertical thin needle in a nanofluid with the magnetic field and stability analysis using the Buongiorno (2006) model.

In this study, the scope is restricted to the steady two-dimensional laminar boundary layer flow of an incompressible nanofluid for both Tiwari and Das (2007) and Boungiorno (2006) models. For Tiwari and Das model, three different types of nanoparticles namely, copper (Cu), titania (TiO_2) and alumina (Al_2O_3) are used. Meanwhile, Brownian motion and thermophoresis are two main effects that we considered in the Buongiorno model.

1.5 Significant of Study

In recent years, the boundary layer flow past a thin needle becomes one of the crucial subjects in view of its applications in many areas of biological sciences and engineering. This subject provides the ways of modeling and capability for designing some applications including hot wire anemometer for determining the velocity of the wind, metal spinning, aerodynamics, coating of wires, small measuring equipment, paper production, cancer treatment and blood flow problems (Ahmad et al., 2008a; Sulochana et al., 2017a).

In industrial and engineering areas, conventional heat transfer fluids such as water, ethylene glycol and oil are essential in the cooling and heating processes, chemical processes and geothermal power generation. Since these fluids have low thermal conductivity, this tends to decelerate the heat transfer processes. Hence, a new class of high-efficiency heat exchange media has been proposed by dispersing nanoparticles in the base fluid to enhance the thermal conductivity of such fluids. This fluid is known as a nanofluid (Choi, 1995). Nanofluid is important because it can be applied in numerous applications involving heat transfer, automotive, electronic devices and biomedical.

The existence of nanofluid has the potential of being a new generation of coolants in automotive applications due to their higher thermal conductivities than the base fluids (Wong and Leon, 2010; Saidur et al., 2011). Ethylene glycol and water mixture are two types of automotive coolant which has poor heat transfer fluid. The addition of the nanoparticles in the standard engine coolant has the tendency to enhance the automotive and heavy-duty engine cooling rates. Such improvement helps to remove engine heat with a reduced size coolant system. The smaller coolant systems result in smaller and lighter radiators, which in turn benefit almost every aspect of the car and the economy. This situation may reduce the coefficient of drag and thus resulting in less fuel consumption. Alternatively, improved the cooling rates for automotive and truck engines capable of removing more heat from higher horsepower engines with the same size of the coolant system (Saidur et al., 2011).

Furthermore, in electronic devices, nanofluid tend to cool down the system effectively by removing the high heat flux, including liquid cooling, air cooling and two-phase cooling (Saidur et al., 2011; Colangelo et al., 2017). Some examples of electronic applications are the cooling of microchips in computers and microfluidic applications (Wong and Leon, 2010). The usage of nanofluid in this field is necessary due to the rapid development of modern technology where electronic devices produce a large quantity of thermal energy. The production of more heat will change the normal performance of the devices, reduces reliability and expected life. Therefore, in designing the electronic components, the efficient cooling system is one of the important criteria.

Nowadays, nanofluid is very helpful in cancer imaging and drug delivery for cancer therapeutics in biomedical industries. In cancer therapeutics, the use of the iron-based nanoparticles is as the delivery transports for drugs or radiation without damaging nearby healthy tissue in cancer patients, which is a significant side effect of traditional cancer treatment methods. In addition, the magnetic nanofluid can guide such particles in the bloodstream to a tumor using magnets. Besides, nanofluid can be used for safer surgery by providing effective cooling around the surgical region and enhance the patient's chance of survival and reduces the risk of organ damage (Wong and Leon, 2010). Other than that, the consideration of the thin needle in the biomedical area is important because it can assist in blood flow problems and cancer detection or treatment. A thin needle biopsy is a procedure to obtain a sample of cells from the body for laboratory testing (Frable, 1976). The common thin needle biopsy procedures include fine-needle aspiration and core needle biopsy. The thin needle biopsy may be used to take tissue or fluid samples from muscles, bones, and other organs, such as the liver or lungs (Cianci et al., 1987). The function of the thin needle biopsy is to help diagnose a medical condition and assess the progress of treatment.

1.6 Thesis Outline

This thesis consists of eight chapters. Chapter 1 starts with the introduction and research background which includes the definitions and explanation of the important terms used in the present study. Next, problem statements, objectives which are the direction of the study, scopes, significant of study as well as thesis outline are also included in this chapter. Besides, Chapter 2 presents the comprehensive literature of the previous works related to the present study.

Chapter 3 discusses the formulation of the mathematical models for Chapter 4 until Chapter 7 by using two different models of nanofluid which are Tiwari and Das and Buongiorno model. These mathematical models comprise of the basic equations which are derived from the conversation laws of mass, momentum and energy, boundary layer approximation and similarity transformations. Using the appropriate similarity transformations, partial differential equations (PDEs) are transformed into ordinary differential equations (ODEs). Besides, this chapter also presents the formulation of the mathematical models for the stability analysis along with the numerical method used to solve the four problems considered in this thesis.

Chapter 4 explains the mathematical formulation and stability analysis of the boundary layer flow and heat transfer over a continuously moving thin needle in nanofluid under the influences of partial slip and convective boundary conditions at the surface. This chapter considers the nanofluid model proposed by Tiwari and Das (2007). Different from Chapter 4, in Chapter 5 the position of the needle from the horizontal surface is changed to the vertical surface where the mixed convection flow must be considered. In these two chapters, the equations that governed the flow and the effects of the embedded parameters on the characteristics of the flow and heat transfer are presented and have been discussed in detail.

Furthermore, Chapters 6 and 7 discuss the stability analysis and mathematical formulation of the boundary layer flow, heat and mass transfer past a moving thin needle in nanofluid using the Boungiorno (2006) model. In particular, Chapter 6 presents the influence of chemical reaction and heat generation using the horizontal surface of the needle. Meanwhile, Chapter 7 considers the effect of the magnetic field using the vertical surface of the needle with a revised boundary condition of the Buongiorno model. These chapters also present the governing equations of the system as well as the effects of the physical parameters on the characteristics of the flow, heat and mass transfer.

Finally, the conclusions for all problems will be summarized in Chapter 8. This chapter also provides the research suggestions that can be done in the future related to the thin needle.

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LIST OF PUBLICATIONS

Journal Articles:

Salleh, S. N. A., Bachok, N., Arifin, N. M., Ali, F. M. and Pop, I. Stability Analysis of Mixed Convection Flow towards a Moving Thin Needle in Nanofluid. In *Applied Sciences*, 8:842 (2018), Q3.

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