UNIVERSITI PUTRA MALAYSIA

ROBUST DETECTION MEASURES AND ROBUST PARAMETER ESTIMATION METHODS IN CIRCULAR UNIVARIATE AND SIMPLE CIRCULAR REGRESSION MODEL

EHAB ABDULSALAM MAHMOOD

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By

EHAB ABDULSALAM MAHMOOD

Thesis Submitted to the School of Graduated Studies, Universiti Putra Malaysia, in Fulfillment of the Requirements for the Degree of Doctor of Philosophy

December 2017

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DEDICATIONS

- *To my parents who teach me so many original principles and support me to obtain the highest certificate by trust of God and self-confidence.*
- *My wife, who was away from her family to realize our common dream of obtaining PhD. She always prays to satisify this dream.*
- *To the best gift of God to me, my kids, Hawraa, Zahraa, Hussein and Afnan, who are all my life … I can not imagine the life without seeing their innocent eyes.*
- *To my sister and brother, who are partners of my childhood and toys. They always encourage and support me to do the best.*

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Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfillment of the requirements for the degree of Doctor of Philosophy

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By

EHAB ABDULSALAM MAHMOOD

December 2017

Chairman : Professor Habshah Midi, PhD
Faculty : Science

The univariate and the simple circular regression model can be used in many scientific fields. There is evidence that the classical methods to estimate the parameters are adversely affected by outliers. Hence, it is very crucial to detect outliers in circular data. Some existing methods such as *Mardia*, *M*, *A*, and *Chord* are developed in this regard. Unfortunately, these methods are formulated to identify only a single outlier. Hence, we propose robust circular distance (*RCD*u) statistic to identify a single and multiple outliers in the univariate circular data. The results of the study indicate that the *RCD*u statistic is successful in detecting outliers with smaller masking and swamping rates.

Not much research is focused on the robust estimation of univariate circular distribution when the circular data have outliers. Thus, robust methods are proposed to estimate the circular location parameter, circular variance and mean resultant length of von Mises distribution. The findings signify that the two proposed methods have done a credible job compared to other methods in this study.

This thesis also addresses the issue of existing outliers in the simple circular regression model. Not much consideration is given to investigate the identification methods of outliers in such model. Hence, we propose robust circular distance (*RCD*y) statistic to detect outliers in the response variable of the simple circular regression model. The results of the study indicate that the *RCDy* has the highest proportion of detection outliers with the lowest rate of masking.

To the best for our knowledge, no research is focused on the detection of outliers in the response and the explanatory variables of a simple circular regression model.

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Hence, robust circular distance (*RCD*xy) statistic is formulated to detect outliers in the response and the explanatory variables. The results show that the *RCD*xy statistic is very successful to detect outliers with low rates of masking and swamping.

The maximum likelihood estimator (MLE) is the commonly used method to estimate model parameters of the simple circular regression model. However, the MLE is inefficient if the circular data have outliers. To the best of our knowledge, no work has been done to propose robust method to estimate parameters of the simple circular regression model when the response variable has outliers. Therefore, the robust MWLE 1 and MWLE 2 are developed. The findings indicate that the MWLE2 and the MWLE1 are more efficient than the MLE.

To date, there is no robust parameters estimation method of a simple circular regression model is developed when outliers are present in the response and the explanatory variables. Therefore, two robust estimators namely MWLE1 and MWLE2 are established. The results show that the performance of the MWLE2 and the MWLE1 are more efficient than the MLE when outliers are present in both X and Y directions.

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Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

SUKATAN PENGESANAN TEGUH DAN KAEDAH PENGANGGARAN PARAMETER TEGUH DALAM UNIVARIAT SIRKULER DAN MODEL REGRESI SIRKULAR MUDAH

Oleh

EHAB ABDULSALAM MAHMOOD

Disember 2017

Pengerusi : Profesor Habshah Midi, PhD
Fakulti : Sains

Univariat dan model regresi sirkuler mudah boleh digunakan dalam pelbagai bidang saintifik. Terdapat bukti bahawa kaedah klasik untuk menganggar parameter model sirkuler terjejas teruk oleh pencilan. Oleh itu, adalah sangat penting untuk mengesan pencilan dalam data sirkuler. Beberapa kaedah sedia ada seperti Mardia, M, A, dan Chord telah dibangunkan dalam hal ini. Malangnya, semua kaedah ini dirumuskan hanya untuk mengesan pencilan tunggal. Oleh itu, kami mencadangkan statistik sirkuler jarak teguh (RCDu) untuk mengesan pencilan tunggal dan berganda dalam data sirkuler univariat. Hasil kajian menunjukkan bahawa statistik RCDu berhasil dalam mengesan pencilan dengan kadar litupan dan limpahan yang lebih kecil.

Tidak banyak kajian difokuskan ke atas penganggaran teguh taburan sirkuler univariat bila data sirkuler mempunyai pencilan. Oleh yang demikian, kaedah teguh dicadangkan untuk menganggar parameter lokasi sirkuler, varians sirkuler dan purata hasil panjang taburan von Mises. Penemuan menunjukkan bahawa kedua kaedah yang dicadangkan telah menghasilkan keputusan yang dipercayai berbanding kaedah lain.

Tesis ini juga mengetengahkan isu kewujudan pencilan dalam model regresi sirkuler mudah. Tidak banyak pertimbangan yang diberikan untuk menyelidik kaedah pengesanan pencilan dalam model tersebut. Oleh yang demikian, kami mencadangkan statistik jarak sirkuler teguh (RCDy) untuk mengesan pencilan dalam pembolehubah sambutan model regresi sirkuler mudah. Keputusan kajian menunjukkan bahawa RCDy mempunyai kadar pengesanan pencilan paling tinggi dengan kadar litupan paling rendah.

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Sepanjang pengetahuan kami, belum ada kajian difokuskan ke atas pengesanan pencilan dalam pembolehubah sambutan dan penerang bagi model regresi sirkuler mudah. Oleh yang demikian, statistik sirkuler jarak teguh (RCDxy) diformulasi untuk mengesan pencilan dalam pembolehubah sambutan dan penerang. Keputusan menunjukkan bahawa statistik RCDxy sangat berhasil mengesan pencilan dengan kadar litupan dan limpahan yang rendah.

Penganggar kebolehjadian maksimum (MLE) merupakan kaedah umum digunapakai untuk menganggar parameter model regresi sirkuler mudah. Namun begitu, penganggar MLE tidak cekap sekiranya data sirkuler mempunyai pencilan. Sepanjang pengetahuan kami, belum ada kajian dibuat bagi mencadangkan kaedah teguh untuk menganggar parameter bagi model regresi sirkuler mudah bila pembolehubah sambutan mempunyai pencilan. Oleh yang demikian, penganggar teguh MWLE 1 dan MWLE 2 dibangunkan. Penemuan menunjukkan bahawa MWLE 1 dan MWLE 2 lebih cekap berbanding MLE.

Sehingga kini, belum ada kaedah pengganggaran parameter teguh bagi model regresi sirkuler mudah dibangunkan bila pencilan wujud dalam pembolehubah sambutan dan penerang. Oleh yang demikian, dua penganggar teguh iaitu MWLE 1 dan MWLE 2 diwujudkan. Keputusan menunjukkan bahawa prestasi MWLE 1 dan MWLE 2 lebih cekap berbanding MLE bila pencilan wujud dalam kedua arah X dan Y.

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This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfillment of the requirement for the degree of Doctor of Philosophy. The members of the Supervisory Committee were as follows:

Habshah Bt Midi, PhD
Professor
Faculty of Science
Universiti Putra Malaysia
(Chairman)

Jayanthi A/P Arasan, PhD
Associate Professor
Faculty of Science
Universiti Putra Malaysia
(Member)

Abdul Ghapor Hussin, PhD
Professor
Faculty of Defence Science and Technology
Universiti Pertahanan Nasional Malaysia
(Member)

ROBIAH BINTI YUNUS, PhD
Professor and Dean
School of Graduate Studies
Universiti Putra Malaysia

Date:

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Declaration by graduate student

I hereby confirm that:

- this thesis is my original work.
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Declaration by Members of Supervisory Committee

This is to confirm that:

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Signature : ________________________________
Name of Chairman of Supervisory Committee : Professor Dr. Habshah Midi

Signature : ________________________________
Name of Member of Supervisory Committee : Associate Professor Dr. Jayanthi A/P Arasan

Signature : ________________________________
Name of Member of Supervisory Committee : Professor Dr. Abdul Ghapor Hussin

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LIST OF ABBREVIATIONS

TMD	Trimmed Mean Direction
(P-P) Plot	(Probability-Probability) Plot
dist	Circular Distance
$\hat{\mu}$	Mean Direction
disp	Dispersion Measure
R	Resultant Length
$\bar{R}$	Mean Resultant Length
CV	Circular Variance
k	Concentration Parameter
I_0	Modified Bessel function of the first kind and order zero
MLE	Maximum Likelihood Estimator
r_c	Circular Correlation Coefficient
RCD	Robust Circular Distance Statistic
Mar	Mardia Statistic
M	M statistic
A	A Statistic
Chord	Chord Statistic
med	Median Direction
TCV	Trimmed Circular Variance
TMRL	Trimmed Mean Resultant Length
CHL	Circular Hodge-Lehmann
CMD	Circular Mean Deviation
MCE	Mean Circular Error
$mean_t$	Trimmed Mean
MWLE	Maximum Weighted Likelihood Estimator
var	Variance
MSE	Mean Square Error

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CHAPTER 1

INTRODUCTION

1.1 Introduction and Background of the Study

Statistical data are classified according to their distributional topologies into two sets : Linear data, which they can be represented on the straight line. Second, Circular data, which they can be represented on the circumference of unit circle.

Circular data can be measured by degree and distributed within [0^o - 360^o). However, it is sometimes useful to measure by radians within [0 - 2π). There are two main ways to represent typical circular observations: (i) The Compass, which is used to measure typical circular observations such as wind directions and the directions of migrating birds; and (ii) The Clock, which is used to measure typical circular observations such as arrival times (on a 24-hour clock) (Mardia and Jupp, 2000).

Circular data are used in contrastive scientific fields such as :

i. Meteorology

The circular data are used in the meteorological studies such as the wind directions (Gatto and Jammalamadaka, 2007; Johnson and Wehrly, 1977). The circular data also include the times of day at which thunderstorms occur and the frequencies of heavy rain in a year (Mardia and Jupp, 2000).

ii. Biology

The circular data are applied for animal navigation. For example, the direction of birds migpercentagen (Batschelet, 1981; Schmidt-Koenig, 1963, 1965). The circular data may measure the spawning times of a particular fish (Lund, 1999).

iii. Physics

In physics, the circular motion is defined as a movement of an object along the circumference of a circle, it is measured by angles (Knudsen and Hjorth, 2002). Another example is the source of signals in the case of airplane crashes (Lenth, 1981).

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iv. Psychology

The circular data may be used in the studies of mental maps to represent the surroundings (Gordon et al. 1989; Rustler, 2012).

v. Medicine

The medical professionals are shown that many fields of medicine may be used the circular data such as chronobiology, chronotherapy and the study of biological clock (Mahesh, 2011). The angle of knee flexion is used as a measure of recovery of orthopaedic patients (Jammalamadaka et al. 1986).

vi. Geology

The circular data may be used in several fields of geology. For example, Geologists interest is to find out the direction of flow of rivers in the past (Sengupta and Rao, 1966) and to find out the direction of earthquake displacement in terms of the direction steepest decent (Rivest, 1997).

vii. Political Science

Gill and Hangartner (2010) used circular regression model in the study of domestic terrorism analysis.

viii. Demography

The circular data are used in the demography studies such as geographic marital patterns (Coleman and Haskey, 1986).

In general, circular data can be found whenever periodic phenomena occur. However, one of vital problems which may occur in the statistical data is the existence of outliers. In real-life applications, samples from any field might include noise, or outliers. The outliers cause a huge interpretative problem, misleading of statistical analysis and incorrect of parameters estimation. This problem is common and there are methods to detect them in the linear data. Researchers are interested in improving the methods of detecting outliers in statistical data. Many researchers have proposed methods to identify the outliers and proposed robust methods to estimate model parameters of linear data. As well as, statistical software packages, such as SPSS and Minitab and R program also provide a variety of methods for identifying outliers and robust methods for univariate and linear regression model. However, there are few methods in the literature that can detect the outliers in univariate circular data. These methods are proposed to identify a single outlier (Abuzaid et al. 2009; Abuzaid, 2010; Collett, 1980; Mardia, 1975).

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In the literature, there is lack of researches that are interested to propose robust methods to estimate circular parameters of circular univariate (Ackermann, 1997; Ducharme and Milasevic, 1987; He and Simpson, 1992; Ko and Guttorp, 1988; Kutil, 2012; Laha et al. 2013; Laha and Mahesh, 2011; Lenth, 1981; Otieno and Anderson-Cook, 2006; Wehrly and Shine, 1981). The word "robust" is loaded with many-sometimes inconsistent-connotations. In statistic science, it is used for the purposes of robustness signifying insensitivity to small deviations from the assumptions, where the robust methods are more resistance than classical methods. The robust methods are used if the assumptions of estimation are not satisfied or the statistical have outliers (Huber and Ronchetti, 2009) .

The problem of the existing outliers in the response variable of the simple circular regression model has not been remedied adequately (Abuzaid et al. 2013; Abuzaid, et al., 2011; Abuzaid, 2010) . Furthermore, no work has been done to propose methods to identify outliers in the response and the explanatory variables of the simple circular regression model.

Another serious problem, there is no robust method that has been proposed to estimate parameters of simple circular regression model when the response variable has outliers. Moreover, no work has been done to propose robust method to estimate model parameters of the simple circular regression model when the response and the explanatory variables have outliers.

1.2 Importance and Motivation of the Study

Circular data are used in many scientific fields. They might be represented by univariate or bivariate. The simple circular regression model is one of the important models to represent the relationship between two circular variables. Efficiency and accuracy of estimation of the model parameters depend on the suitability of data that is fitted to the circular regression model. However, circular data may have some inconsistent observations with the majority of the circular data, which are called outliers. The classical methods that applied to estimate model parameters are successful under some conditions. One of these conditions is that the circular data is free of outliers. Researchers have suggested either to identify outliers and then remedy them or apply some robust methods to estimate model parameters. In the literature, there are many methods to detect outliers in univariate and bivariate linear data , as well as robust methods to estimate model parameters. However, the methods that are suggested for linear data cannot be used for circular data because of the circular geometry theory. To overcome the problem of existing outlier in the univariate circular data, Abuzaid et al. (2009), Abuzaid (2010), Collett (1980) and Mardia (1975) proposed methods to identify outliers. However, these methods can detect a single outlier point but they are not successful to identify multiple outliers. The problem of existing outliers in the univariate circular data has not received enough consideration. This motivated us to propose a statistical method to identify outliers in the univariate circular data in the presence of multiple outliers. The proposed statistic *RCD*u is

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expected to show higher proportion of detection of outliers with smaller masking and swamping rates.

The mean direction is used to estimate circular location parameter of univariate circular data. However, this estimation gives incorrect estimation if the circular data have outliers. To overcome this problem, Lenth (1981) proposed M estimator to estimate circular location parameter when the circular data have outliers. Ackermann (1997), Ducharme and Milasevic (1987), He and Simpson (1992), Ko, and Guttorp (1988) and Wehrly and Shine (1981) proposed to use the median direction if the circular data have outliers. Otieno and Anderson-Cook (2006) extended Hodges-Lehmann method to estimate circular location parameter when the circular data have outliers. Kutil (2012) explained that the mean resultant length is bias. Laha et al. (2013) and Laha and Mahesh (2011) explained that the mean direction is not SB-robust but the trimmed mean direction (TMD) is SB-robust whereby SB-robust is called standardized bias robust (Ko and Guttorp, 1988). However, Laha et al. (2013) and Laha and Mahesh (2011) did not propose method for trimming in their algorithm of circular location parameter estimation even though it is now evident that TMD is SB-robust. This inspired us to extend two methods for trimming to estimate circular location parameter, circular variance and the mean resultant length.

This thesis is also concerned in the detection of outliers in the response variable of the simple circular regression model. This issue has been addressed by Abuzaid et al. (2013), Abuzaid et al. (2011) and Abuzaid (2010). However, these methods have a low proportion of detection of outliers and high rate of masking especially if the response variable has a high percentage of contamination. The weakness of these methods has motivated us to propose a new statistic that can identify outliers with higher proportion of detection and lower rate of masking.

Many methods have been proposed to detect outliers in the response and the explanatory variables of linear regression model. To the best of our knowledge, no work has been done to propose method to detect outliers in the response and the explanatory variables of the simple circular regression model. This inspired us to propose a statistic to identify outliers in the response and the explanatory variables.

This thesis also addresses the issue of robust estimation of the parameters of the simple circular regression model. For the linear regression model, this fact is pointed out by many standard books, articles and researchers. However, to date, no robust estimation approach has been proposed to estimate the model parameters of the simple circular regression model. This motivate us to propose two methods by extending maximum weighted likelihood estimator MWLE. First, to estimate parameters when the response variable has outliers. Second, when the response and the explanatory variables have outliers.

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1.3 Research Objectives

The main goal is to investigate the existing of outliers problem in the univariate and bivariate circular data. The simple circular regression model is an important model to represent the relationship of bivariate circular data. The classical methods are used to estimate parameters of circular univariate and the simple circular regression model. There is evidence that the classical methods of estimation are affected by the outliers. Therefore, The foremost objectives of our research can be outlined systematically as follows:

i. To formulate a new robust statistic to detect a single and multi-outliers in the univariate circular data.
ii. To extend robust method to estimate circular location parameter, circular variance and mean resultant length when the circular data have outliers.
iii. To formulate a new robust statistic to detect outliers in the response variable of a simple circular regression model.
iv. To formulate a new robust statistic to detect outliers in the response and the explanatory variables of the simple circular regression model.
v. To extend a robust method to estimate the parameters of the simple circular regression model when the response variable has outliers.
vi. To extend a robust method to estimate the parameters of the simple circular regression model when the response and the explanatory variables have outliers.

1.4 Scope and Limitation of Study

Circular data are widely used in many scientific fields such as meteorology, biology, physics, psychology, medicine, geology, political science and demography.

In linear data, the Normal distribution has many desirable properties so all the classical statistical analysis methods have been applied under Normal distribution. Similarly, in circular data, the von Mises distribution has many desirable properties so it is considered in this study.

The classical methods are used to estimate parameters of the circular univariate such as the mean direction, the circular variance and the mean resultant length. Nonetheless, these methods are affected by the existence of outliers in the dataset. Hence, researchers proposed methods to identify outlier. Nevertheless, these methods are not enough to resolve the problem of presence outliers in the circular data and it still exists. Very few researchers have focused to propose robust methods to estimate the parameters.

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The linear relationship between bivariate circular data can be represented by the simple circular regression model. The maximum likelihood estimator is used to estimate the model parameters. However, it has some assumption to apply, one of them that the circular variables are free of outliers. Abuzaid et al. (2013), Abuzaid et al. (2011) and Abuzaid (2010) proposed methods to identify outliers in the response variable of a simple circular regression model. Nonetheless, they have not proposed method to detect outliers in the explanatory variable. In addition, no work has been done to propose method to detect outliers in the response and the explanatory variables of the simple circular regression model. Furthermore, to date, no robust method has been proposed to estimate parameters of the simple circular regression model when outliers are present in a data set.

Since the robust statistic is relatively new technique in the circular data, there is no algorithm and statistical software and less references and data related to this area. Hence, it is so difficult to extend the robust methods that apply in the linear regression model such as least median of squares, least trimmed squares, S-estimator, M-estimator, MM-estimator and GM-estimator.

In the literatures, not many outlying data sets are available. Hence, the same data sets were used repeatedly for different objectives of this study.

1.5 Overview of the Thesis

In accordance with the objectives and the scope of the study, the contents of this thesis are structured in the nine chapters. The thesis chapters are organized so that the study objectives are apparent and are conducted in the sequence outline.

Chapter Two: This chapter briefly presents the literature review of the univariate circular data and circular regression model. The definition of the circular data, circular distance and graphical representation are illustrated. The definition of outliers, masking and swamping are given. Finally, bootstrapping methods are also briefly discussed.

Chapter Three: This chapter discusses the existing of outliers in univariate circular data. The proposed robust statistic to detect a single and multi-outliers is presented. It depends on the circular distance between the observations and the median direction as a measure for detection. Finally, numerical examples and a Monte Carlo simulation study are presented.

Chapter Four: In this chapter**,** a robust method to estimate the mean direction, circular variance and mean resultant length is proposed. The proposed robust method is based on extending the trimmed procedure. Two methods of trimming are proposed. First, it depends on the circular distance between observations and the median

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direction as a measure for trimming. Five circular distances away from the median direction are tested. Second, it depends on the method that is proposed in Chapter 3 as a measure for trimming. A real data analysis and a Monte Carlo simulation study are carried out to assess the performance of our proposed method.

Chapter Five: This chapter deals with the existing of outliers in the response variable of the simple circular regression model. A robust statistic is proposed to identify outliers. The proposed statistic depends on the circular distance between circular residuals and the median of the circular residuals as a measure for detection. A Monte Carlo simulation study and a numerical example are carried out to assess the performance of the proposed method.

Chapter Six: In this chapter, a proposed robust statistic to detect outliers in the response and the explanatory variables of the simple circular regression model is presented. The proposed method depends on the circular distance between observations of the response and the explanatory variables as a measure for detection. A Monte Carlo simulation study and a numerical example are presented to assess the performance of the proposed method.

Chapter Seven: In this chapter, we propose a robust method to estimate the parameters of the simple circular regression model when the response variable has outliers. The proposed method extends the maximum weighted likelihood estimator. Two weight functions are proposed to apply the maximum weighted likelihood estimator. A numerical example and a Monte Carlo simulation study are applied to evaluate the performance of the proposed method.

Chapter Eight: This chapter deals with the existing of outliers in the response and the explanatory variables of the simple circular regression model**.** A robust method to estimate the model parameters is proposed. The maximum weighted likelihood estimator is extended to estimate the parameters. For this method, two weight functions are proposed. A numerical example and a Monte Carlo simulation study are applied to assess the performance of the proposed method.

Chapter Nine: This chapter provides summary and detailed discussions of the thesis conclusions. Areas for future research are also recommended.

The flow of the thesis is summarised by the following chart.

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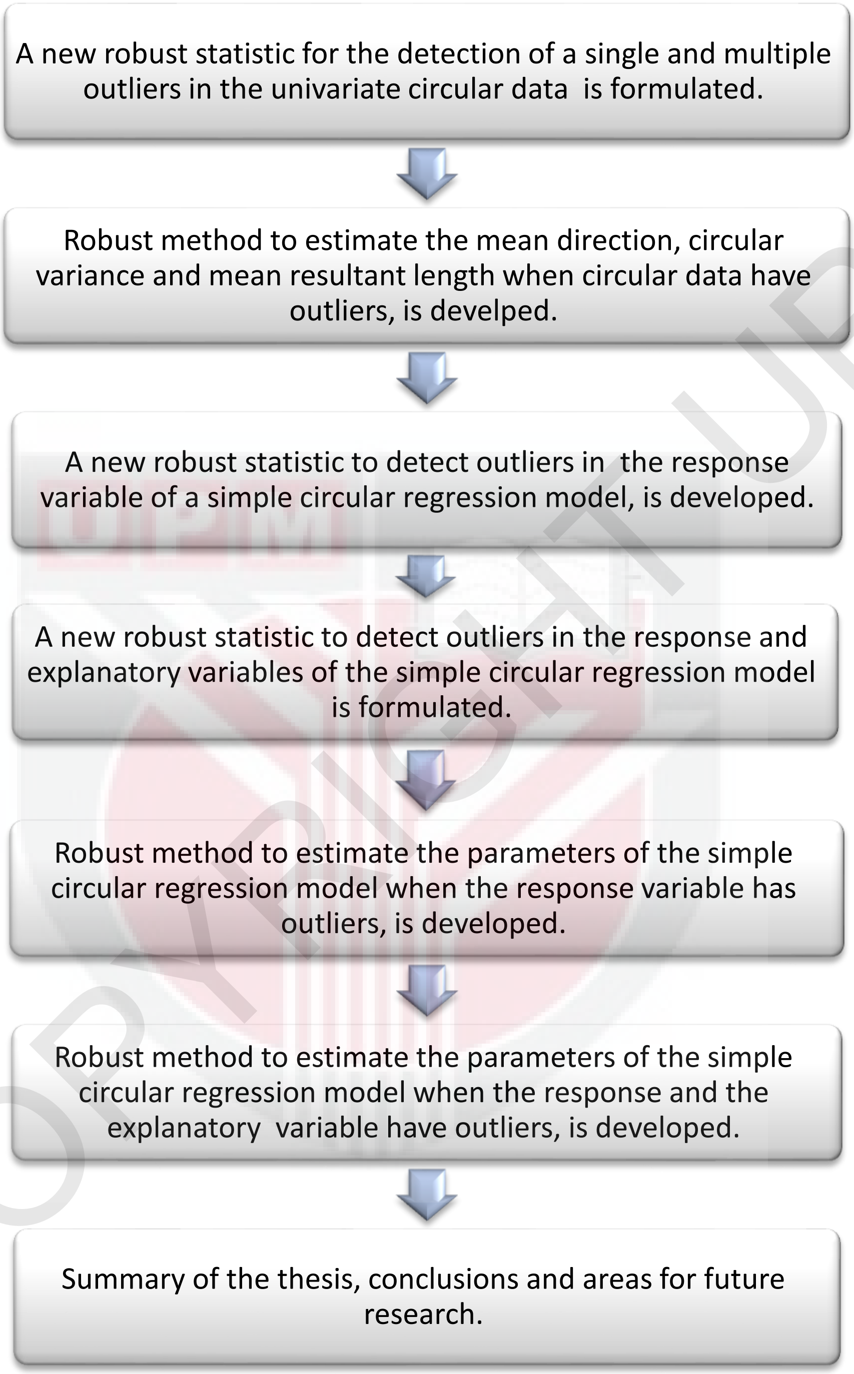
Chart to show the flow of the thesis



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