UNIVERSITI PUTRA MALAYSIA

STAGNATION POINT FLOW OVER A STRETCHING/SHRINKING SHEET IN A CARBON NANOTUBES WITH SLIP EFFECTS

NUR HAZIRAH ADILLA BT NORZAWARY

IPM 2021 13

STAGNATION POINT FLOW OVER A STRETCHING/SHRINKING SHEET IN A CARBON NANOTUBES WITH SLIP EFFECTS

By

NUR HAZIRAH ADILLA BT NORZAWARY

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirements for the Degree of Master of Science

October 2020

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Master of Science

STAGNATION POINT FLOW OVER A STRETCHING/SHRINKING SHEET IN A CARBON NANOTUBES WITH SLIP EFFECTS

By

NUR HAZIRAH ADILLA BT NORZAWARY

October 2020

Chair: Norfifah Bachok, PhD
Faculty: Institute for Mathematical Research

The consideration of the stagnation point flow over a stretching/shrinking sheet in carbon nanotubes (CNTs) is investigated. Mathematical models are derived for three boundary layer flow problems over a linearly, nonlinearly and exponentially stretching/shrinking sheet in CNTs with the presence of slip at the surface. Both single- and multi-wall CNTs are used along with two base fluids which are water and kerosene. Similarity transformation are used to transform the partial differential equations into a nonlinear ordinary differential equations. The stability analysis is derived for linear case by introducing the partial differential equations in unsteady case. These equations are then solved by using bvp4c solver in Matlab.

Numerical results of skin friction coefficient and local Nusselt number are exhibited in forms of table and graph and also for profiles of velocity and temperature for a range of numerous parameters such as Prandtl number Pr, CNTs volume fraction φ, velocity slip parameter σ, thermal slip parameter σ_t, heat generation parameter Q, suction/injection parameter S, stretching/shrinking parameter ϵ and nonlinear parameter β. These parameters are observed to have a major influence on coefficient of skin friction and the local Nusselt number which illustrates the rate of heat transfer at the surface. The results show that solutions for shrinking sheet are dual solutions while unique solutions for stretching sheet. It is noticed that slip parameter, suction/injection parameter and nonlinear parameter widens the range in which the dual solutions exist. Furthermore, the first solution is found stable meanwhile the second solution is unstable and it is obtained by performing a stability analysis.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Master Sains

ALIRAN TITIK GENANGAN PADA PERMUKAAN MEREGANG ATAU MENGECUT DALAM KARBON NANOTIUB DENGAN KESAN GELINCIRAN

Oleh

NUR HAZIRAH ADILLA BINTI NORZAWARY

Oktober 2020

Pengerusi: Norfifah Bachok, PhD
Fakulti: Institut Penyelidikan Matematik

Pertimbangan aliran titik gengangan pada permukaan meregang atau mengecut dalam karbon nanotiub (KNT) disiasat. Model matematik diterbitkan bagi tiga masalah dalam aliran putaran lapisan sempadan terhadap permukaan meregang atau mengecut dalam KNT secara linear, tak linear dan eksponen dengan kehadiran kesan slip pada permukaan. Kedua-dua satu- dan pelbagai-dinding KNT digunakan bersama dua bendalir asas iaitu air dan minyak tanah. Penjelmaan keserupaan digunakan untuk menjelmakan persamaan perbezaan separa kepada persamaan perbezaan biasa tak linear. Analisis kestabilan diperoleh dengan memperkenalkan persamaan pembezaan separa dalam kes aliran tak mantap. Persamaan ini seterusnya diselesaikan dengan menggunakan kaedah bvp4c dalam perisian Matlab.

Keputusan berangka bagi pekali geseran kulit dan nombor Nusselt setempat dipamerkan dalam bentul jadual dan graf dan juga untuk profil halaju dan suhu untuk julat tertentu pelbagai parameter seperti nombor Prandtl Pr, pecahan isipadu KNT φ, parameter halaju gelinciran σ, parameter termal gelinciran σ_t, parameter penjanaan haba Q, parameter sedutan/suntikan S, parameter regangan/kecutan ϵ dan parameter tak linear β. Didapati bahawa parameter tersebut mempengaruhi pekali geseran kulit dan nombor Nusselt setempat yang mewakili kadar pemindahan haba pada permukaan. Keputusan menunjukkan bahawa penyelesaian untuk permukaan mengecut adalah dual penyelesaian manakala penyelesaian tak unik untuk permukaan menegang. Diperhatikan bahawa parameter gelinciran, parameter sedutan/suntikan dan parameter tak linear menambah julat bagi penyeselaian dual wujud. Tambahan pula, penyelesaian pertama didapati stabil manakala penyelesaian kedua tidak stabil dan diperhatikan dengan mempersembahkan analisis kestabilan.

ACKNOWLEDGEMENTS

First of all, all thanks to Allah S.W.T for granting me with His blessing and giving me a functioning mind as finally I was able to complete my Master's studies. I also would like to enunciate my sincere thanks to my supervisor Assoc. Prof. Dr. Norfifah binti Bachok @ Lati for her full support, expert guidance, excellent encouragement and endless support throughout my research and thesis writing. Not to forget, my co-supervisor, Dr. Fadzilah binti Md Ali for her guidance too. Without their time, guidance, and assistance, my whole project will not be complete as it is now.

I really thank my friends who have helped me in these tough years. They helped me to complete my project with their encouragement and treatment. I have deeply enjoyed their friendship and I sincerely admire their trust in me. Most notably, none of this would have been possible without my family's love and patience. I would like to share my appreciation as well to my father, Norzawary bin Isa and my mother Azimah binti Roslan who never stop to give me support, advice and continuous encouragement throughout my study and completing this project. Without them, this achievement would not have been feasible. Thank you.

This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfilment of the requirement for the degree of Master of Science. The members of the Supervisory Committee were as follows:

Norfifah Bachok, PhD
Associate Professor
Faculty of Science
Universiti Putra Malaysia
(Chairman)

Fadzilah Md Ali, PhD
Senior Lecturer
Faculty of Science
Universiti Putra Malaysia
(Member)

ZALILAH MOHD SHARIFF, PhD
Professor and Dean
School of Graduate Studies
Universiti Putra Malaysia

Date: 11 February 2021

Declaration of graduate student

I hereby confirm that

- this thesis is my original work;
- quotations, illustrations and citations have been duly referenced;
- this thesis has not been submitted previously or concurrently for any other degree at any other institutions;
- intellectual property from the thesis and copyright of thesis are fully-owned by Universiti Putra Malaysia, as according to the Universiti Putra Malaysia (Research) Rules 2012;
- written permission must be obtained from supervisor and the office of Deputy Vice-Chancellor (Research and Innovation) before thesis is published (in the form of written, printed or in electronic form) including books, journals, modules, proceedings, popular writings, seminar papers, manuscripts, posters, reports, lecture notes, learning modules or any other materials as stated in the Universiti Putra Malaysia (Research) Rules 2012;
- there is no plagiarism or data falsification/fabrication in the thesis, and scholarly integrity is upheld as according to the Universiti Putra Malaysia (Graduate Studies) Rules 2003 (Revision 2012-2013) and the Universiti Putra Malaysia (Research) Rules 2012. The thesis has undergone plagiarism detection software.

Signature: ____________________ Date:____________________

Name and Matric No.: Nur Hazirah Adilla Norzawary (GS53440)

Declaration by Members of Supervisory Committee

This is to confirm that:
- the research conducted and the writing of this thesis was under our supervision;
- supervision responsibilities as stated in the Universiti Putra Malaysia (Graduate Studies) Rules 200 (Revision 2012-2013) are adhered to

Signature: ________________
Name of Chairman of Supervisory Committee: Norfifah Bachok

Signature: ________________
Name of Member of Supervisory Committee Fadzilah Md Ali

TABLE OF CONTENTS

			Page
ABSTRACT			i
ABSTRAK			ii
ACKNOWLEDGEMENTS			iii
APPROVAL			iv
DECLARATION			vi
LIST OF TABLES			x
LIST OF FIGURES			xi
LIST OF ABBREVIATIONS			xiii

CHAPTER

1	**INTRODUCTION**		1
	1.1	Introduction	1
	1.2	Research Background	2
		1.2.1 Stagnation Point Flow	2
		1.2.2 Stretching/shrinking Sheet	3
		1.2.3 Carbon Nanotubes (CNTs)	3
		1.2.4 Slip Effects	4
		1.2.5 Heat Generation Effects	5
		1.2.6 Suction/injection Effects	5
	1.3	Problem Statement	5
	1.4	Objective and Scope	6
	1.5	Significant of Study	7
	1.6	Outline of Thesis	7
2	**LITERATURE REVIEW**		9
	2.1	Introduction	9
	2.2	Stagnation Point Flow via Linearly Stretching/Shrinking Sheet	9
	2.3	Stagnation Point Flow via Nonlinearly Stretching/Shrinking Sheet	12
	2.4	Stagnation Point Flow via Exponentially Stretching/Shrinking Sheet	13
	2.5	Boundary Layer Stagnation Point Flow with Slip Effects	16
	2.6	Boundary Layer Stagnation Point Flow with Heat Generation Effects	19
	2.7	Boundary Layer Stagnation Point Flow with Suction/Injection Effects	21
	2.8	Boundary Layer Stagnation Point Flow in Carbon Nanotubes	22
3	**METHODOLOGY**		25
	3.1	Introduction	25
	3.2	The Governing Equation	25

	3.3 Similarity Transformation	28
	3.3.1 Derivation of Continuity Equation	29
	3.3.2 Derivation of Momentum Equation	30
	3.3.3 Derivation of Energy Equation	32
	3.3.4 Derivation of Boundary Conditions	34
	3.3.5 Derivation of Physical Quantities	36
	3.4 Numerical Computation: bvp4c Solver in MATLAB	38
4	**STAGNATION POINT FLOW OVER A STRETCHING/SHRINKING SHEET IN A CARBON NANOTUBES WITH SLIP EFFECTS AND STABILITY ANALYSIS**	40
	4.1 Introduction	40
	4.2 Mathematical Formulation	40
	4.3 Stability Analysis	40
	4.4 Results and Discussions	42
	4.5 Conclusions	44
5	**STAGNATION POINT FLOW OVER A NONLINEARLY STRETCHING/SHRINKING SHEET IN A CARBON NANOTUBES WITH SLIP AND SUCTION/INJECTION EFFECTS**	55
	5.1 Introduction	55
	5.2 Mathematical Formulation	55
	5.3 Results and Discussions	56
	5.4 Conclusions	59
6	**STAGNATION POINT FLOW OVER A EXPONENTIALLY STRETCHING/SHRINKING SHEET IN A CARBON NANOTUBES WITH EFFECTS OF SLIP AND HEAT GENERATION**	75
	6.1 Introduction	75
	6.2 Mathematical Formulation	75
	6.3 Results and Discussions	76
	6.4 Conclusions	78
7	**CONCLUSION**	89
	4.1 Research Summary	89
	4.2 Future Works	90
REFERENCES		91
APPENDICES		102
BIODATA OF STUDENT		153
LIST OF PUBLICATIONS		154

LIST OF TABLES

Table		Page
3.1	Thermophysical properties of different base fluids and CNTs (see Khan et al. (2014))	25
4.1	$C_f {Re_x}^{1/2}$ for certain values of ε and φ	45
4.2	$Nu_x {Re_x}^{-1/2}$ for certain values of ε and φ	45
4.3	Smallest eigenvalues γ at selected values of ε for different σ when $\varphi = 0.1$ for water-SWCNTs	46
5.1	$C_f {Re_x}^{1/2}$ for certain values of ε and φ	60
5.2	$Nu_x {Re_x}^{-1/2}$ for certain values of ε and φ	60
6.1	$C_f {Re_x}^{1/2}$ for certain values of ε and φ	79
6.2	$Nu_x {Re_x}^{-1/2}$ for certain values of ε and φ	79
6.3	$Nu_x {Re_x}^{-1/2}$ for certain values of Q and φ	80

LIST OF FIGURES

Figure		Page
1.1	Physical model of stagnation point (https://www.transtutors.com/questions/potential-flow-near-a-stagnation-point-fig-4b-6-a-show-375083.htm)	2
1.2	Physical model of stretching or shrinking (see Awaludin et al. (2016))	3
1.3	Physical model of carbon nanotubes (https://steemit.com/technology/@eng.ramy/carbon-nanotubes-what-is-it)	4
1.4	Physical model of SWCNT and MWCNT (see Ribeiro et al. (2017))	4
1.5	Physical model of slip (see Lauga et al. (2005))	5
3.1	Physical model and coordinate system (see Bachok et al. (2011))	26
4.1	$f''(0)$ with ε and φ for water-SWCNTs	46
4.2	$-\theta'(0)$ with ε and φ for water-SWCNTs	47
4.3	$f''(0)$ with ε and σ for water-SWCNTs	47
4.4	$-\theta'(0)$ with ε and σ for water-SWCNTs	48
4.5	Variation of $C_f(Re)_x^{1/2}$ with φ and σ for water base fluid	48
4.6	Variation of $Nu_x(Re)_x^{-1/2}$ with φ and σ for water base fluid	49
4.7	Variation of $C_f(Re)_x^{1/2}$ with φ for various base fluids	49
4.8	Variation of $Nu_x(Re)_x^{-1/2}$ with φ for various base fluids	50
4.9	Velocity profiles for φ and water-SWCNTs	50
4.10	Temperature profiles for φ and water-SWCNTs	51
4.11	Velocity profiles for σ and water-MWCNTs	51
4.12	Temperature profiles for σ and water-MWCNTs	52
4.13	Velocity profiles for various base fluids for SWCNTs	52
4.14	Temperature profiles for various base fluids for SWCNTs	53
4.15	Velocity profiles for various carbon nanotubes for water base fluid	53
4.16	Temperature profiles for various carbon nanotubes for water base fluid	54
4.17	γ at selected ε for $\sigma = 0.2$ and $\varphi = 0.1$ for water-SWCNTs	54
5.1	$f''(0)$ with ε and φ for water-SWCNTs	61
5.2	$-\theta'(0)$ with ε and φ for water-SWCNTs	61
5.3	$f''(0)$ with ε and σ for water-SWCNTs	62
5.4	$-\theta'(0)$ with ε and σ for water-SWCNTs	62
5.5	$-\theta'(0)$ with ε and σ_t for water-SWCNTs	63
5.6	$f''(0)$ with ε and β for water-SWCNTs	63
5.7	$-\theta'(0)$ with ε and β for water-SWCNTs	64
5.8	$f''(0)$ with ε and S for water-SWCNTs	64
5.9	$-\theta'(0)$ with ε and S for water-SWCNTs	65
5.10	Variation of $C_f(Re)_x^{1/2}$ with φ and σ for water base fluid	65
5.11	Variation of $Nu_x(Re)_x^{-1/2}$ with φ and σ for water base fluid	66

5.12 Variation of $C_f(Re)_x^{1/2}$ with φ and β for water base fluid 66
5.13 Variation of $Nu_x(Re)_x^{-1/2}$ with φ and β for water base fluid 67
5.14 Variation of $C_f(Re)_x^{1/2}$ with φ for various base fluids 67
5.15 Variation of $Nu_x(Re)_x^{-1/2}$ with φ for various base fluids 68
5.16 Velocity profiles for φ and water-SWCNTs 68
5.17 Temperature profiles for φ and water-SWCNTs 69
5.18 Velocity profiles for σ and water-MWCNTs 69
5.19 Temperature profiles for σ and water-MWCNTs 70
5.20 Temperature profiles for σ_t and water-MWCNTs 70
5.21 Velocity profiles for β and water-SWCNTs 71
5.22 Temperature profiles for β and water-SWCNTs 71
5.23 Velocity profiles for S and water-SWCNTs 72
5.24 Temperature profiles for S and water-SWCNTs 72
5.25 Velocity profiles for various base fluids for MWCNTs 73
5.26 Temperature profiles for various base fluids for MWCNTs 73
5.27 Velocity profiles for various carbon nanotubes for water base fluid 74
5.28 Temperature profiles for various carbon nanotubes for water base fluid 74

6.1 $f''(0)$ with ε and φ for water-SWCNTs 80
6.2 $-\theta'(0)$ with ε and φ for water-SWCNTs 80
6.3 $f''(0)$ with ε and σ for water-SWCNTs 81
6.4 $-\theta'(0)$ with ε and σ for water-SWCNTs 81
6.5 $-\theta'(0)$ with ε and Q for water-SWCNTs 82
6.6 Variation of $C_f(Re)_x^{1/2}$ with φ and σ for water base fluid 82
6.7 Variation of $Nu_x(Re)_x^{-1/2}$ with φ and σ for water base fluid 83
6.8 Variation of $C_f(Re)_x^{1/2}$ with φ for various base fluids 83
6.9 Variation of $Nu_x(Re)_x^{-1/2}$ with φ for various base fluids 84
6.10 Velocity profiles for φ and water-SWCNTs 84
6.11 Temperature profiles for φ and water-SWCNTs 85
6.12 Velocity profiles for σ and water-MWCNTs 85
6.13 Temperature profiles for σ and water-MWCNTs 86
6.14 Temperature profiles for Q and water-SWCNTs 86
6.15 Velocity profiles for various base fluids and SWCNTs 87
6.16 Temperature profiles for various base fluids and SWCNTs 87
6.17 Velocity profiles for various carbon nanotubes and water base fluid 88
6.18 Temperature profiles for various carbon nanotubes and water base fluid 88

LIST OF ABBREVIATIONS

a, b constants

C_f	skin-friction coefficient
C_p	specific heat at constant pressure
f	dimensionless stream function
k	thermal conductivity
Nu_x	local Nusselt number
Pr	Prandtl number
q_w	surface heat flux
Re_x	local Reynolds number
T	fluid temperature
T_w	plate temperature
T_∞	ambient temperature
u, v	velocity components along the $x-$ and $y-$ direction
U_w	stretching/shrinking velocity
U_∞	ambient fluid velocity
x, y	Cartesian coordinates measured along the plate and normal to it
S	suction/injection parameter
Q	heat generation parameter

Greek symbols

α	thermal diffusivity
φ	carbon nanotube volume fraction
η	similarity variable
ν	kinematic viscosity
μ	dynamic viscosity
θ	dimensionless temperature
ε	stretching/shrinking parameter
σ	velocity slip parameter
σ_t	thermal slip parameter
ρ_f	fluid density
ρ_{CNT}	carbon nanotube density
$(\rho C_p)_f$	heat capacity of the fluid
$(\rho C_p)_{CNT}$	heat capacity of the carbon nanotube
τ_w	shear stress
ψ	stream function

Subscripts

c	critical value
w	condition at the surface of the plate
∞	ambient condition
nf	nanofluid
f	fluid
CNT	carbon nanotube

CHAPTER 1

INTRODUCTION

1.1 Introduction

Throughout physics and engineering, fluid dynamics is a subdiscipline of fluid mechanics that explains the flow of fluids liquids and gases. They have several subdisciplines, including aerodynamics (moving air and other gases) and hydrodynamics (moving liquids). Fluid dynamics has a broad variety of applications, including measuring aircraft forces and moments, assessing petroleum mass flow rate across pipelines, predicting patterns of weather, understanding nebulae in outer space and modeling detonation of fission bombs.

Boundary layer refers to the fluid layer in the surrounding neighborhoods of a bounding surface where the viscosity effects are important. In 1908, it was Blasius who solved the boundary layer problem for a free stream past a fixed flat plate using a similarity transformation technique, while the boundary layer on a continuous moving surface was first examined by Sakiadis (1961).

Heat transfer is a method of transferring heat from an object of higher temperature to a subject of lower temperature. Following the First Law of Thermodynamics, heat transfer affects the internal energy of all systems involved. There are four basic heat transfer models, where the first is that advection is a method of transportation of a fluid from one position to another, which relies on the movement which momentum of that fluid. Second is conduction/diffusion. Conduction is a transfer of energy between objects which in physical contact. Third, convection is a transfer of energy between an object, due to fluid motion, and its environment. And lastly, radiation which a transfer of energy by electromagnetic radiation emission. This research, however, only took into consideration convection heat transfer.

Prandtl number, Pr can be referred to as a dimensionless parameter used to measure the transfer of heat between a moving fluid and a solid object. The Prandtl number is also used in heat transfer and the measurement of free and forced convection. These values depend on the fluid properties. This Prandtl number was proposed by the German physicist, Ludwig Prandtl in 1904 and it can be stated as

$$Pr = \frac{\text{viscous diffusion rate}}{\text{thermal diffusion rate}} = \frac{v}{\alpha} = \frac{\mu C\rho}{k},$$

where μ is the dynamic viscosity, C is the specific heat, ρ is the density of fluid, k is the thermal conductivity, v is the kinematic viscosity and α is thermal diffusivity.

1.2 Research Background

Fluid dynamics apply to many science and engineering fields and have taken into consideration many aspects of our daily lives. Some important keywords in the thesis will be introduced in this subsection.

1.2.1 Stagnation Point Flow

A stagnation point is a point in a flow region where the fluid's local velocity is zero. Stagnation points occur in the flow field at the object's surface, where the object brings the fluid to rest. At the point of stagnation, fluid does not accumulate; it flows away one way or the other. It runs very slowly at the stagnation point and the nearer you go the slower it runs. Streamlines can terminate at a stagnation point. Due to their significance in many engineering disciplines, e.g. cooling of electronic devices or nuclear reactors, and other hydrodynamics processes, the analysis of stagnation point flows has received considerable attention.

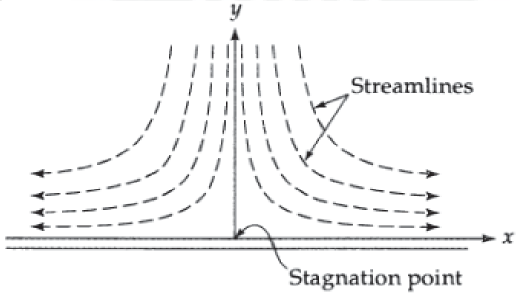

Figure 1.1: Physical model of stagnation point (https://www.transtutors.com/questions/potential-flow-near-a-stagnation-point-fig-4b-6-a-show-375083.htm)

1.2.2 Stretching/shrinking Sheet

Stretching surface is a surface that is extended in its own plane, or that happens when the boundary velocity moves away from a fixed point. Several examples of this flow in industrial processes and engineering are plastic and rubber sheet aerodynamic extrusion, hot rolling, wire drawing, and development of glass-fibre. While, shrinking surface is a surface that shrunk where the boundary velocity is going towards a fixed point. Some common applications regarding the shrinking surface, including the shrinking film for packing of bulk products and effects of capillary in small pores.

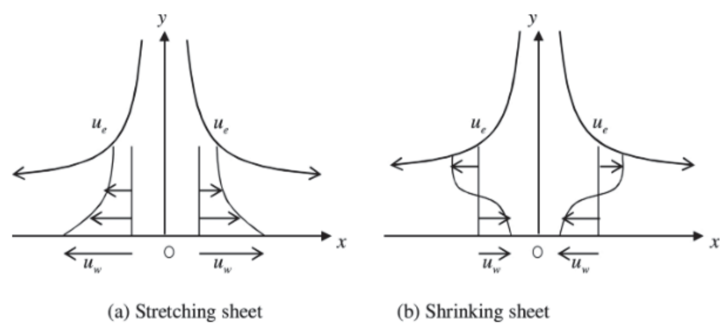

Figure 1.2: Physical model of stretching or shrinking (see Awaludin et al. (2016))

1.2.3 Carbon Nanotubes (CNTs)

Iijima (1991) was credited because he is the first person who did the first observation about the multi-walled CNTs. He discovered multi-walled CNTs by the arc-discharge method. Many scientists in the area have credited him with the first visual experience of the tubes of atoms rolling up and being covered with fullerene molecules. Iijima and Ichihashi (1993) and Bethune et al. (1993) discovered buckytubes known as single-walled nanotubes and they were produced using metal catalyst in the arc-discharge method. Continuing work has shown that three specific forms of nanotubes exist (zigzag, armchair, and chiral), as well as single-walled and multi-walled CNTs. CNTs is an carbon allotrope, a tube-shaped form made of carbon with a nanometer-scale measurement of diameter. Nanotubes are part of the structured family of fullerenes. Their name derives from their long, hollow structure, with carbon sheets of one atom thick, called graphene, forming the walls.

CNTs are peculiar due to the very strong bonding between the atoms, and the tubes may have extreme aspect ratios. CNTs have many different structures, varying in length, thickness and number of layers. The characteristics of nanotubes depend upon how the graphene sheet has rolled up to form the tube. There are several different types of CNTs but usually, they are known as either single-walled carbon nanotubes (SWCNTs) or multi-walled carbon nanotubes (MWCNTs). A SWCNTs is like a normal straw, where they only have one layer or wall. In the meantime, MWCNTs are a series of continuously growing diameters of nested tubes.

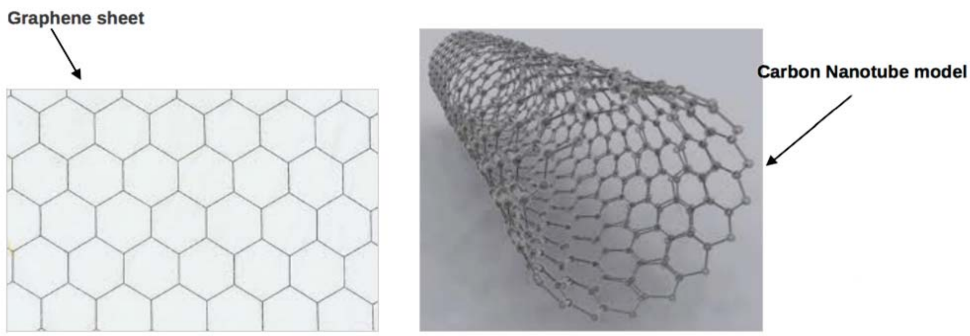

Figure 1.3: Physical model of carbon nanotubes (https://steemit.com/technology/@eng.ramy/carbon-nanotubes-what-is-it)

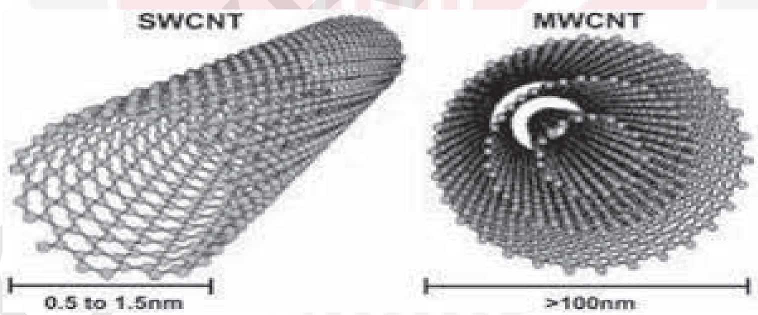

Figure 1.4: Physical model of SWCNT and MWCNT (see Ribeiro et al. (2017))

1.2.4 Slip Effects

Slip occurs when the fluid and the plate cannot stick together due to slippery surface of the plate. Some of the researchers investigated the boundary layer flow with no-slip condition. Sometimes, in certain cases, the no-slip condition can be changed to the partial slip condition, which is given by

$$u(x, y) = L\frac{\partial u}{\partial y}$$

where u is the velocity of the fluid, L (λ as in Figure 1.5) is the length of the slip.

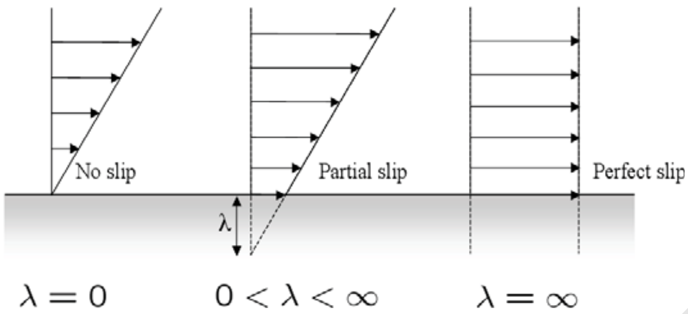

Figure 1.5: Physical model of slip (see Lauga et al. (2005))

1.2.5 Heat generation Effects

Research into the generation/absorption of heat in moving fluids is crucial for problems concerned with reactions of chemical and dissociating fluids. Heat generation is known as the convection of some sort of energy in the medium into sensible heat energy. Examples of energy are electrical, chemical and nuclear energy. Heat generation occurs throughout the medium and exhibits itself as a rise in temperature. Temperature distribution may be modified by possible heat generation effects, which may influence particle deposition and distribution rate.

1.2.6 Suction/injection Effects

The fluid suction/injection through the bounding surface will ultimately affect the flow field and as a result, change the rate of heat transfer from the plate. Generally, suction appears to increase skin friction coefficients and heat transfer , while the injection acts in the opposite manner (see Al-Sanea (2004)). For practical problems such as film cooling, boundary layer control, etc the injection or removal of fluid through a heated or cooled porous bounding wall is of general interest. It can help to boost the system of heating or cooling which can help slow the transition of the laminar flow. (see Chaudhary and Merkin (1993)).

1.3 Problem Statement

The problems regarding the stagnation point flow due to a stretching/shrinking sheet in CNTs have been given attention by many authors. For the present study, the term stagnation point flow over three different problems which are linear, nonlinear and exponential stretching/shrinking sheet with slip effects at the surface are studied. Some of the issues about the stagnation point flow are:

1. What are the differences in the range of the dual solutions exist for stagnation point flow due to linear, nonlinear and exponential stretching/shrinking sheet problems?

2. What happens to the nature of skin friction coefficient and local Nusselt number when considering velocity slip parameter, thermal slip parameter, heat generation parameter, suction/injection parameter, nonlinear parameter and different types of CNTs?

3. How does the presence of CNTs give impact on the flow and heat transfer characteristics over a stretching/shrinking sheet?

4. What will happen to flow characteristics when we used stretching and shrinking surface?

1.4 Objective and Scope

The objectives of the present study are to

1. construct and derive the mathematical model,

2. solve the mathematical model numerically via bvp4c solver in Matlab software,

3. analyze the influence of the considered parameters on the characteristics of the fluid flow and heat transfer,

for the following problems of:

1. stagnation point flow in a carbon nanotubes with slip effects over:

    - a **linearly** stretching/shrinking sheet,
    - a **nonlinearly** stretching/shrinking sheet together with suction/injection effects,
    - an **exponentially** stretching/shrinking sheet together with heat generation effects,

    and also

2. conduct stability analysis for dual solutions exist in first problem (linear) by finding the smallest unknown eigenvalues.

The scope is limited to the two-dimensional boundary layer flow, steady, laminar and incompressible in the region $y > 0$ driven by a stretching/shrinking surface located at $y = 0$ with a fixed stagnation point $x = 0$. Both single-walled and multi-walled CNTs are used together with two base fluids which are water and kerosene.

1.5 Significant of Study

CNTs have been an important new class of technical materials since the discovery of CNTs in 1991 which have various useful properties. These impressive structures have a variety of interesting electrical , magnetic and chemical properties. They are at least 100 times stronger than steel but one-sixth heavier than steel. There are numerous properties and applications of CNTs.

Jorio et al. (2007) stated that CNTs occupy important properties and qualities as structural materials. In textiles, CNTs can produce waterproof and tear-resistant fabrics. Besides, they presented that from sheets of parallel CNTs, loudspeakers can be created which use to generate sound. CNTs fibres are also being used as combat jackets which used to provide protection from bullets in body armor. Next, CNTs can be manufactured as electrical conductors, semiconductors and insulators. Jornet and Akyildiz (2010) declared that by using multi-walled CNTs, a strong magnetic field can be produced. They also declared that CNTs can act as an antenna for radio due to its durability and light in weight.

As in air pollution filters, CNTs are one of the best materials because they occupy high absorption capacity and dominate large specific area. And for water filters, CNTs membranes can benefit in filtration because their tubes are so thin that small particles can move through them, thus blocking larger particles such as chloride ions in salt. In mechanical field, CNTs oscillators have achieved higher speeds than other technologies due to low friction and low wear bearing properties of multi-walled CNTs. In addition, in the biomedical sector, since a large part of the human body is made up of carbon, it is usually a very biocompatible material, though. Cells on CNTs have been shown to expand, so they do not appear to have any toxic effect. Even the cells do not follow the CNTs, possibly giving rise to applications like prosthetic coatings. The ability to functionalize (chemically modify) the sidewalls of CNTs also contributes to biomedical applications including vascular stents, and growth and regeneration of neurons.

1.6 Outline of Thesis

Chapter 1 represents an introduction, research background where the definitions of the terms are discussed, problem statements, objective and scopes, study's significance and also the thesis's outline. Chapter 2 addresses the literature review of previous studies applicable to our thesis in detail.

In addition, the mathematical formulas in which partial differential equations (PDEs) are reduced using similarity transformation into ordinary differential equations (ODEs) are described in Chapter 3. Often discussed in detail through this section was the numerical method used to solve the current problems.

Chapter 4, 5 and 6, respectively, present the mathematical formulations for stagnation point flow due to a linear, nonlinear and exponential stretching/shrinking sheet in CNTs with slip effects. The influence of the nanoparticle volume fraction, velocity slip, thermal slip, heat generation, suction/injection, nonlinear as well as stretching and shrinking parameters on the flow are found and have discussed in detail here.

Next, the stability analysis is carried out to identify which solutions are stable that is included in Chapter 4. Last but not least, Chapter 7 discusses the results of study and some recommendations for further analysis.

REFERENCES

Ahmad, S., Rohni, A. M., and Pop, I. (2011). Blasius and sakiadis problems in nanofluids. *Acta Mechanica*, 218(3-4):195–204.

Akbar, N. S. and Khan, Z. H. (2016). Effect of variable thermal conductivity and thermal radiation with cnts suspended nanofluid over a stretching sheet with convective slip boundary conditions: Numerical study. *Journal of Molecular Liquids*, 222:279–286.

Akbar, N. S., Khan, Z. H., and Nadeem, S. (2014). The combined effects of slip and convective boundary conditions on stagnation-point flow of cnt suspended nanofluid over a stretching sheet. *Journal of Molecular Liquids*, 196:21–25.

Al-Sanea, S. A. (2004). Mixed convection heat transfer along a continuously moving heated vertical plate with suction or injection. *International Journal of Heat and Mass Transfer*, 47(6-7):1445–1465.

Aladdin, N. A. L., Bachok, N., and Anuar, N. S. (2020). Mhd stagnation point flow in nanofluid over shrinking surface using buongiorno's model: A stability analysis. *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences*, 76(3):12–24.

Aleng, N., Bachok, N., and Arifin, N. (2015). Flow and heat transfer of a nanofluid over an exponentially shrinking sheet. *Indian J. Sci. Technol*, 8(31).

Aman, F., Ishak, A., and Pop, I. (2013). Magnetohydrodynamic stagnation-point flow towards a stretching/shrinking sheet with slip effects. *International Communications in Heat and Mass Transfer*, 47:68–72.

Aman, F. and Ishak, A. M. (2010). Boundary layer flow and heat transfer over a permeable shrinking sheet with partial slip. *Journal of Applied Sciences Research*, 6(8):1111–1115.

Andersson, H. I. (2002). Slip flow past a stretching surface. *Acta Mechanica*, 158(1-2):121–125.

Anuar, N., Bachok, N., and Pop, I. (2018). A stability analysis of solutions in boundary layer flow and heat transfer of carbon nanotubes over a moving plate with slip effect. *Energies*, 11(12):3243.

Anuar, N. S., Bachok, N., Arifin, N. M., and Rosali, H. (2019). Stagnation point flow and heat transfer over an exponentially stretching/shrinking sheet in cnt with homogeneous–heterogeneous reaction: Stability analysis. *Symmetry*, 11(4):522.

Arpaci, V. S. and Larsen, P. S. (1984). *Convection heat transfer*. Prentice Hall.

Awaludin, I., Weidman, P., and Ishak, A. (2016). Stability analysis of stagnation-point flow over a stretching/shrinking sheet. *AIP Advances*, 6(4):045308.

Bachok, N. and Ishak, A. (2011). Similarity solutions for the stagnation-point flow and heat transfer over a nonlinearly stretching/shrinking sheet. *Sains Malaysiana*, 40(11):1297–1300.

Bachok, N., Ishak, A., Nazar, R., and Senu, N. (2013a). Stagnation-point flow over a permeable stretching/shrinking sheet in a copper-water nanofluid. *Boundary Value Problems*, 2013(1):39.

Bachok, N., Ishak, A., and Pop, I. (2011). Stagnation-point flow over a stretching/shrinking sheet in a nanofluid. *Nanoscale research letters*, 6:623.

Bachok, N., Ishak, A., and Pop, I. (2012a). Boundary layer flow over a moving surface in a nanofluid with suction or injection. *Acta Mechanica Sinica*, 28(1):34–40.

Bachok, N., Ishak, A., and Pop, I. (2012b). Boundary layer stagnation-point flow and heat transfer over an exponentially stretching/shrinking sheet in a nanofluid. *International Journal of Heat and Mass Transfer*, 55(25-26):8122–8128.

Bachok, N., Ishak, A., and Pop, I. (2013b). Boundary layer stagnation-point flow toward a stretching/shrinking sheet in a nanofluid. *Journal of heat transfer*, 135(5).

Bachok, N., Ishak, A., and Pop, I. (2013c). Stagnation point flow toward a stretching/shrinking sheet with a convective surface boundary condition. *Journal of the Franklin Institute*, 350(9):2736–2744.

Bachok, N., Najib, N., Arifin, N. M., and Senu, N. (2016). Stability of dual solutions in boundary layer flow and heat transfer on a moving plate in a copper-water nanofluid with slip effect. *WSEAS Transactions on Fluid Mechanics*, 11:151–158.

Bakar, N. A. A., Bachok, N., Arifin, N. M., and Pop, I. (2018). Stability analysis on the flow and heat transfer of nanofluid past a stretching/shrinking cylinder with suction effect. *Results in Physics*, 9:1335–1344.

Bethune, D., Kiang, C. H., De Vries, M., Gorman, G., Savoy, R., Vazquez, J., and Beyers, R. (1993). Cobalt-catalysed growth of carbon nanotubes with single-atomic-layer walls. *Nature*, 363(6430):605–607.

Bhattacharyya, K. (2011). Boundary layer flow and heat transfer over an exponentially shrinking sheet. *Chinese Physics Letters*, 28(7):074701.

Bhattacharyya, K. and Layek, G. (2011). Effects of suction/blowing on steady boundary layer stagnation-point flow and heat transfer towards a shrinking sheet with thermal radiation. *International Journal of Heat and Mass Transfer*, 54(1-3):302–307.

Bhattacharyya, K., Mukhopadhyay, S., and Layek, G. (2011a). Mhd boundary layer slip flow and heat transfer over a flat plate. *Chinese Physics Letters*, 28(2):024701.

Bhattacharyya, K., Mukhopadhyay, S., and Layek, G. (2011b). Slip effects on boundary layer stagnation-point flow and heat transfer towards a shrinking sheet. *International Journal of Heat and Mass Transfer*, 54(1-3):308–313.

Bhattacharyya, K. and Pop, I. (2011). Mhd boundary layer flow due to an exponentially shrinking sheet. *Magnetohydrodynamics*, 47(4):337–344.

Bhattacharyya, K. and Vajravelu, K. (2012). Stagnation-point flow and heat transfer over an exponentially shrinking sheet. *Communications in Nonlinear Science and Numerical Simulation*, 17(7):2728–2734.

Bidin, B. and Nazar, R. (2009). Numerical solution of the boundary layer flow over an exponentially stretching sheet with thermal radiation. *Eur. J. Sci. Res*, 33(4):710–717.

Brinkman, H. (1952). The viscosity of concentrated suspensions and solutions. *The Journal of Chemical Physics*, 20(4):571–571.

Buongiorno, J. (2006). Convective transport in nanofluids. *Journal of heat transfer*, 128(3):240–250.

Chaudhary, M. and Merkin, J. (1993). The effects of blowing and suction on free convection boundary layers on vertical surfaces with prescribed heat flux. *Journal of Engineering Mathematics*, 27(3):265–292.

Choi, S., Zhang, Z., Yu, W., Lockwood, F., and Grulke, E. (2001). Anomalous thermal conductivity enhancement in nanotube suspensions. *Applied physics letters*, 79(14):2252–2254.

Choi, S. U. and Eastman, J. A. (1995). Enhancing thermal conductivity of fluids with nanoparticles. Technical report, Argonne National Lab., IL (United States).

Cortell, R. (2007). Viscous flow and heat transfer over a nonlinearly stretching sheet. *Applied Mathematics and Computation*, 184(2):864–873.

Das, K. (2015). Nanofluid flow over a non-linear permeable stretching sheet with partial slip. *Journal of the Egyptian Mathematical Society*, 23(2):451–456.

Elbashbeshy, E. and Bazid, M. (2004). Heat transfer in a porous medium over a stretching surface with internal heat generation and suction or injection. *Applied mathematics and computation*, 158(3):799–807.

Elbashbeshy, E., Emam, T., and Abdel-Wahed, M. (2014). An exact solution of boundary layer flow over a moving surface embedded into a nanofluid in the presence of magnetic field and suction/injection. *Heat and Mass Transfer*, 50(1):57–64.

Elbashbeshy, E. M. and Bazid, M. A. (2003). Heat transfer over a stretching surface with internal heat generation. *Canadian journal of physics*, 81(4):699–703.

Fang, T., Zhang, J., and Yao, S. (2009). Slip mhd viscous flow over a stretching sheet–an exact solution. *Communications in Nonlinear Science and Numerical Simulation*, 14(11):3731–3737.

Fauzi, N., Ahmad, S., and Pop, I. (2015). Stagnation point flow and heat transfer over a nonlinear shrinking sheet with slip effects. *Alexandria Engineering Journal*, 54(4):929–934.

Ghosh, S. and Mukhopadhyay, S. (2016). Nanofluid flow past an exponentially porous stretching sheet with heat and mass fluxes. *Acta Technica*, 61:17–29.

Ghosh, S. and Mukhopadhyay, S. (2018). Flow and heat transfer of nanofluid over an exponentially shrinking porous sheet with heat and mass fluxes. *Propulsion and Power Research*, 7(3):268–275.

Ghosh, S., Mukhopadhyay, S., and Vajravelu, K. (2016). Dual solutions of slip flow past a nonlinearly shrinking permeable sheet. *Alexandria Engineering Journal*, 55(2):1835–1840.

Hafidzuddin, E., Nazar, R., Arifin, N., and Pop, I. (2016). Boundary layer flow and heat transfer over a permeable exponentially stretching/shrinking sheet with generalized slip velocity. *Journal of Applied Fluid Mechanics*, 9(4):2025–2036.

Haq, R. U., Khan, Z. H., and Khan, W. A. (2014). Thermophysical effects of carbon nanotubes on mhd flow over a stretching surface. *Physica E: Low-dimensional Systems and Nanostructures*, 63:215–222.

Haq, R. U., Nadeem, S., Khan, Z. H., and Noor, N. F. M. (2015). Convective heat transfer in mhd slip flow over a stretching surface in the presence of carbon nanotubes. *Physica B: condensed matter*, 457:40–47.

Haque, M. R., Alam, M. M., Ali, M. M., and Karim, R. (2015). Effects of viscous dissipation on natural convection flow over a sphere with temperature dependent thermal conductivity in presence of heat generation. *Procedia Engineering*, 105:215–224.

Harris, S., Ingham, D., and Pop, I. (2009). Mixed convection boundary-layer flow near the stagnation point on a vertical surface in a porous medium: Brinkman model with slip. *Transport in Porous Media*, 77(2):267–285.

Hayat, T., Farooq, M., and Alsaedi, A. (2015a). Homogeneous-heterogeneous reactions in the stagnation point flow of carbon nanotubes with newtonian heating. *AIP advances*, 5(2):027130.

Hayat, T., Farooq, M., and Alsaedi, A. (2015b). Stagnation point flow of carbon nanotubes over stretching cylinder with slip conditions. *Open Physics*, 13(1).

Hayat, T., Hussain, Z., Alsaedi, A., and Asghar, S. (2016a). Carbon nanotubes effects in the stagnation point flow towards a nonlinear stretching sheet with variable thickness. *Advanced Powder Technology*, 27(4):1677–1688.

Hayat, T., Muhammad, K., Farooq, M., and Alsaedi, A. (2016b). Melting heat transfer in stagnation point flow of carbon nanotubes towards variable thickness surface. *AIP advances*, 6(1):015214.

Hayat, T., Rashid, M., Alsaedi, A., and Ahmad, B. (2018). Flow of nanofluid by nonlinear stretching velocity. *Results in physics*, 8:1104–1109.

Hayat, T., Saif, R. S., Ellahi, R., Muhammad, T., and Ahmad, B. (2017). Numerical study of boundary-layer flow due to a nonlinear curved stretching sheet with convective heat and mass conditions. *Results in physics*, 7:2601–2606.

Huang, C.-J. (2018). Effects of internal heat generation and soret/dufour on natural convection of non-newtonian fluids over a vertical permeable cone in a porous medium. *Journal of King Saud University-Science*, 30(1):106–111.

Hussanan, A., Khan, I., Gorji, M. R., and Khan, W. A. (2019). Cnt s-water–based nanofluid over a stretching sheet. *BioNanoScience*, 9(1):21–29.

Ibrahim, S., Kumar, P., Lorenzini, G., Lorenzini, E., and Mabood, F. (2017). Numerical study of the onset of chemical reaction and heat source on dissipative mhd stagnation point flow of casson nanofluid over a nonlinear stretching sheet with velocity slip and convective boundary conditions. *Journal of Engineering Thermophysics*, 26(2):256–271.

Ibrahim, W. (2017). Magnetohydrodynamic (mhd) boundary layer stagnation point flow and heat transfer of a nanofluid past a stretching sheet with melting. *Propulsion and Power Research*, 6(3):214–222.

Iijima, S. (1991). Helical microtubules of graphitic carbon. *nature*, 354(6348):56–58.

Iijima, S. and Ichihashi, T. (1993). Single-shell carbon nanotubes of 1-nm diameter. *nature*, 363(6430):603–605.

Ishak, A. (2011). Mhd boundary layer flow due to an exponentially stretching sheet with radiation effect. *Sains Malaysiana*, 40(4):391–395.

Ishak, A., Nazar, R., Amin, N., Filip, D., and Pop, I. (2007). Mixed convection of the stagnation-point flow towards a stretching vertical permeable sheet. *Malaysian Journal of Mathematical Sciences*, 1(2):217–226.

Ishak, A., Nazar, R., and Pop, I. (2008). Uniform suction/blowing effect on flow and heat transfer due to a stretching cylinder. *Applied Mathematical Modelling*, 32(10):2059–2066.

Ismail, N., Ariffin, N., Nazar, R., and Bachok, N. (2019). Stability analysis of stagnation-point flow and heat transfer over an exponentially shrinking sheet with heat generation. *Malaysian Journal of Mathematical Sciences*, 13(2):107–122.

Jahan, S., Sakidin, H., Nazar, R., and Pop, I. (2017). Flow and heat transfer past a permeable nonlinearly stretching/shrinking sheet in a nanofluid: A revised model with stability analysis. *Journal of Molecular Liquids*, 233:211–221.

Jamaludin, A. and Nazar, R. (2019). Dual solutions of stagnation-point flow over an exponentially stretching/shrinking sheet in a porous medium with suction and velocity slip: A stability analysis. In *Journal of Physics: Conference Series*, volume 1212, page 012026. IOP Publishing.

Jorio, A., Kauppinen, E., and Hassanien, A. (2007). Carbon-nanotube metrology. In *Carbon Nanotubes*, pages 63–100. Springer.

Jornet, J. M. and Akyildiz, I. F. (2010). Graphene-based nano-antennas for electromagnetic nanocommunications in the terahertz band. In *Proceedings of the Fourth European Conference on Antennas and Propagation*, pages 1–5. IEEE.

Kamal, F., Zaimi, K., Ishak, A., and Pop, I. (2019). Stability analysis of mhd stagnation-point flow towards a permeable stretching/shrinking sheet in a nanofluid with chemical reactions effect. *Sains Malaysiana*, 48(1):243–250.

Khan, W., Khan, Z., and Rahi, M. (2014). Fluid flow and heat transfer of carbon nanotubes along a flat plate with navier slip boundary. *Applied Nanoscience*, 4(5):633–641.

Khan, W. and Pop, I. (2010). Boundary layer flow of a nanofluid past a stretching sheet. *International journal of heat and mass transfer*, 53(11-12):2477–2483.

Khanafer, K., Vafai, K., and Lightstone, M. (2003). Buoyancy-driven heat transfer enhancement in a two-dimensional enclosure utilizing nanofluids. *International journal of heat and mass transfer*, 46(19):3639–3653.

Lauga, E., Brenner, M. P., and Stone, H. A. (2005). Microfluidics: the no-slip boundary condition. *arXiv preprint cond-mat/0501557*.

Liu, M.-S., Lin, M. C.-C., Huang, I.-T., and Wang, C.-C. (2005). Enhancement of thermal conductivity with carbon nanotube for nanofluids. *International communications in heat and mass transfer*, 32(9):1202–1210.

Lok, Y., Ishak, A., and Pop, I. (2011). Mhd stagnation-point flow towards a shrinking sheet. *International Journal of Numerical Methods for Heat & Fluid Flow*, 21(1):61–72.

Mabood, F., Khan, W., and Ismail, A. M. (2015). Mhd boundary layer flow and heat transfer of nanofluids over a nonlinear stretching sheet: a numerical study. *Journal of Magnetism and Magnetic Materials*, 374:569–576.

Mahapatra, T. R. and Gupta, A. (2002). Heat transfer in stagnation-point flow towards a stretching sheet. *Heat and Mass transfer*, 38(6):517–521.

Mahapatra, T. R. and Gupta, A. S. (2003). Stagnation-point flow towards a stretching surface. *The Canadian Journal of Chemical Engineering*, 81(2):258–263.

Makinde, O. D. and Aziz, A. (2011). Boundary layer flow of a nanofluid past a stretching sheet with a convective boundary condition. *International Journal of Thermal Sciences*, 50(7):1326–1332.

Malvandi, A., Hedayati, F., and Domairry, G. (2013). Stagnation point flow of a nanofluid toward an exponentially stretching sheet with nonuniform heat generation/absorption. *Journal of Thermodynamics*, 2013.

Malvandi, A., Hedayati, F., and Ganji, D. (2017). Nanofluid flow on the stagnation point of a permeable non-linearly stretching/shrinking sheet. *Alexandria engineering journal*.

Malvandi, A., Hedayati, F., and Nobari, M. (2014). An analytical study on boundary layer flow and heat transfer of nanofluid induced by a non-linearly stretching sheet. *Journal of Applied Fluid Mechanics*, 7(2):375–384.

Mandal, I. C. and Mukhopadhyay, S. (2013). Heat transfer analysis for fluid flow over an exponentially stretching porous sheet with surface heat flux in porous medium. *Ain Shams Engineering Journal*, 4(1):103–110.

Mansur, S., Ishak, A., and Pop, I. (2015). The magnetohydrodynamic stagnation point flow of a nanofluid over a stretching/shrinking sheet with suction. *PLoS one*, 10(3):e0117733.

Maré, T., Halelfadl, S., Sow, O., Estellé, P., Duret, S., and Bazantay, F. (2011). Comparison of the thermal performances of two nanofluids at low temperature in a plate heat exchanger. *Experimental Thermal and Fluid Science*, 35(8):1535–1543.

Merkin, J. (1986). On dual solutions occurring in mixed convection in a porous medium. *Journal of engineering Mathematics*, 20(2):171–179.

Miklavvcivc, M. and Wang, C. (2006). Viscous flow due to a shrinking sheet. *Quarterly of Applied Mathematics*, 64(2):283–290.

Misra, S. and Kamatam, G. (2020). Effect of magnetic field, heat generation and absorption on nanofluid flow over a nonlinear stretching sheet. *Beilstein Journal of Nanotechnology*, 11(1):976–990.

Mohamed, R., Abo-Dahab, S., and Nofal, T. (2010). Thermal radiation and mhd effects on free convective flow of a polar fluid through a porous medium in the presence of internal heat generation and chemical reaction. *Mathematical problems in Engineering*, 2010.

Molla, M. M., Hossain, M. A., and Paul, M. C. (2006). Natural convection flow from an isothermal horizontal circular cylinder in presence of heat generation. *International Journal of Engineering Science*, 44(13-14):949–958.

Molla, M. M., Paul, S. C., and Hossain, M. A. (2009). Natural convection flow from a horizontal circular cylinder with uniform heat flux in presence of heat generation. *Applied Mathematical Modelling*, 33(7):3226–3236.

Mukhopadhyay, S. (2013a). Analysis of boundary layer flow over a porous nonlinearly stretching sheet with partial slip at the boundary. *Alexandria Engineering Journal*, 52(4):563–569.

Mukhopadhyay, S. (2013b). Mhd boundary layer flow and heat transfer over an exponentially stretching sheet embedded in a thermally stratified medium. *Alexandria Engineering Journal*, 52(3):259–265.

Mukhopadhyay, S. (2013c). Slip effects on mhd boundary layer flow over an exponentially stretching sheet with suction/blowing and thermal radiation. *Ain Shams Engineering Journal*, 4(3):485–491.

Mustafa, M., Hayat, T., Pop, I., Asghar, S., and Obaidat, S. (2011). Stagnation-point flow of a nanofluid towards a stretching sheet. *International Journal of Heat and Mass Transfer*, 54(25-26):5588–5594.

Nadeem, S., Hayat, T., Malika, M. Y., and Rajputa, S. A. (2010). Thermal radiation effects on the flow by an exponentially stretching surface: a series solution. *Zeitschrift für Naturforschung A*, 65(6-7):495–503.

Nadeem, S. and Lee, C. (2012). Boundary layer flow of nanofluid over an exponentially stretching surface. *Nanoscale research letters*, 7(1):94.

Najib, N., Bachok, N., Arifin, N., Ali, F., and Pop, I. (2017). Stability solutions on stagnation point flow in cu-water nanofluid on stretching/shrinking cylinder with chemical reaction and slip effect. In *Journal of Physics: Conference Series*, volume 890, page 012030. IOP Publishing.

Najib, N., Bachok, N., and Arifin, N. M. (2014). Stagnation point flow and heat transfer over an exponentially shrinking cylinder with suction/injection. *Computers and Mathematics in Automation and Materials Science*.

Nandeppanavar, M. M., Kemparaju, M., and Shakunthala, S. (2019). Mhd stagnation point slip flow due to a non-linearly moving surface with effect of non-uniform heat source. *Nonlinear Engineering*, 8(1):270–282.

Nandy, S. K. (2013). Analytical solution of mhd stagnation-point flow and heat transfer of casson fluid over a stretching sheet with partial slip. *International Scholarly Research Notices*, 2013.

Nandy, S. K. and Mahapatra, T. R. (2013). Effects of slip and heat generation/absorption on mhd stagnation flow of nanofluid past a stretching/shrinking surface with convective boundary conditions. *International Journal of Heat and Mass Transfer*, 64:1091–1100.

Naramgari, S. and Sulochana, C. (2016). Mhd flow over a permeable stretching/shrinking sheet of a nanofluid with suction/injection. *Alexandria Engineering Journal*, 55(2):819–827.

Nazar, R., Jaradat, M., Arifin, N., and Pop, I. (2011). Stagnation-point flow past a shrinking sheet in a nanofluid. *Open Physics*, 9(5):1195–1202.

Nazar, R., Noor, A., Jafar, K., and Pop, I. (2014). Stability analysis of three-dimensional flow and heat transfer over a permeable shrinking surface in a cu-water nanofluid. *Int. J. Math. Comput. Stat. Nat. Phys. Eng*, 8(5):782–788.

Othman, N. A., Yacob, N. A., Bachok, N., Ishak, A., and Pop, I. (2017). Mixed convection boundary-layer stagnation point flow past a vertical stretching/shrinking surface in a nanofluid. *Applied Thermal Engineering*, 115:1412–1417.

Oztop, H. F. and Abu-Nada, E. (2008). Numerical study of natural convection in partially heated rectangular enclosures filled with nanofluids. *International journal of heat and fluid flow*, 29(5):1326–1336.

Pandey, A. K. and Kumar, M. (2016). Effect of viscous dissipation and suction/injection on mhd nanofluid flow over a wedge with porous medium and slip. *Alexandria Engineering Journal*, 55(4):3115–3123.

Ramya, D., Raju, R. S., Rao, J. A., and Chamkha, A. (2018). Effects of velocity and thermal wall slip on magnetohydrodynamics (mhd) boundary layer viscous flow and heat transfer of a nanofluid over a non-linearly-stretching sheet: a numerical study. *Propulsion and Power Research*, 7(2):182–195.

Rana, P. and Bhargava, R. (2012). Flow and heat transfer of a nanofluid over a nonlinearly stretching sheet: a numerical study. *Communications in Nonlinear Science and Numerical Simulation*, 17(1):212–226.

Reddy, S., Reddy, P. B. A., and Chamkha, A. J. (2018). Mhd flow analysis with water-based cnt nanofluid over a non-linear inclined stretching/shrinking sheet considering heat generation. *Chemical Engineering Transactions*, 71:1003–1008.

Ribeiro, B., Botelho, E. C., Costa, M. L., and Bandeira, C. F. (2017). Carbon nanotube buckypaper reinforced polymer composites: a review. *Polímeros*, 27(3):247–255.

Rohni, A. M., Ahmad, S., Ismail, A. I. M., and Pop, I. (2013). Boundary layer flow and heat transfer over an exponentially shrinking vertical sheet with suction. *International Journal of Thermal Sciences*, 64:264–272.

Roşca, N. C. and Pop, I. (2017). Axisymmetric rotational stagnation point flow impinging radially a permeable stretching/shrinking surface in a nanofluid using tiwari and das model. *Scientific reports*, 7(1):1–11.

Sakiadis, B. C. (1961). Boundary-layer behavior on continuous solid surfaces: I. boundary-layer equations for two-dimensional and axisymmetric flow. *AIChE Journal*, 7(1):26–28.

Samuel Raj, J. W. and Anjali Devi, S. (2019). Internal heat generation/absorption effects on mhd radiative flow over a non-isothermal stretching/shrinking surface embedded in a porous medium.

Sanni, K., Asghar, S., Jalil, M., and Okechi, N. (2017). Flow of viscous fluid along a nonlinearly stretching curved surface. *Results in Physics*, 7:1–4.

Seth, G. S., Sharma, R., Kumbhakar, B., and Tripathi, R. (2017). Mhd stagnation point flow over exponentially stretching sheet with exponentially moving free-stream, viscous dissipation, thermal radiation and non-uniform heat source/sink. In *Diffusion Foundations*, volume 11, pages 182–190. Trans Tech Publ.

Sharma, R., Ishak, A., Nazar, R., and Pop, I. (2014). Boundary layer flow and heat transfer over a permeable exponentially shrinking sheet in the presence of thermal radiation and partial slip. *Journal of Applied Fluid Mechanics*, 7(1).

Soid, S., Kechil, S. A., and Ishak, A. (2016). Axisymmetric stagnation-point flow over a stretching/shrinking plate with second-order velocity slip. *Propulsion and Power Research*, 5(3):194–201.

Soid, S. K., Ishak, A., and Pop, I. (2018). Mhd stagnation-point flow over a stretching/shrinking sheet in a micropolar fluid with a slip boundary. *Sains Malaysiana*, 47(11):2907–2916.

Sultana, T., Saha, S., Rahman, M. M., and Saha, G. (2009). Heat transfer in a porous medium over a stretching surface with internal heat generation and suction or injection in the presence of radiation. *Journal of Mechanical Engineering*, 40(1):22–28.

Suriyakumar, P. and Devi, S. (2015). Effects of suction and internal heat generation on hydromagnetic mixed convective nanofluid flow over an inclined stretching plate. *European Journal of Advances in Engineering and Technology*, 2(3):51–58.

Tiwari, R. K. and Das, M. K. (2007). Heat transfer augmentation in a two-sided lid-driven differentially heated square cavity utilizing nanofluids. *International Journal of heat and Mass transfer*, 50(9-10):2002–2018.

Vajravelu, K. (2001). Viscous flow over a nonlinearly stretching sheet. *Applied mathematics and computation*, 124(3):281–288.

Wang, C. (2002). Flow due to a stretching boundary with partial slip—an exact solution of the navier–stokes equations. *Chemical Engineering Science*, 57(17):3745–3747.

Wang, C. (2006). Stagnation slip flow and heat transfer on a moving plate. *Chemical Engineering Science*, 61(23):7668–7672.

Wang, C. (2008). Stagnation flow towards a shrinking sheet. *International Journal of Non-Linear Mechanics*, 43(5):377–382.

Weidman, P., Kubitschek, D., and Davis, A. (2006). The effect of transpiration on self-similar boundary layer flow over moving surfaces. *International journal of engineering science*, 44(11-12):730–737.

Xue, Q. (2005). Model for thermal conductivity of carbon nanotube-based composites. *Physica B: Condensed Matter*, 368(1-4):302–307.

Yacob, N. A., Ishak, A., Pop, I., and Vajravelu, K. (2011). Boundary layer flow past a stretching/shrinking surface beneath an external uniform shear flow with a convective surface boundary condition in a nanofluid. *Nanoscale research letters*, 6(1):314.

Yashkun, U., Zaimi, K., Bakar, N. A. A., and Ferdows, M. (2020). Nanofluid stagnation-point flow using tiwari and das model over a stretching/shrinking sheet with suction and slip effects. *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences*, 70(1):62–76.

Yian, L. Y., Ishak, A., and Pop, I. (2011). Mhd stagnation point flow with suction towards a shrinking sheet. *Sains Malaysiana*, 40(10):1179–1186.

Zaimi, K., Ishak, A., and Pop, I. (2014a). Boundary layer flow and heat transfer over a nonlinearly permeable stretching/shrinking sheet in a nanofluid. *Scientific Reports*, 4:4404.

Zaimi, K., Ishak, A., and Pop, I. (2014b). Flow past a permeable stretching/shrinking sheet in a nanofluid using two-phase model. *Plos one*, 9(11):e111743.

BIODATA OF STUDENT

The student's name is Nur Hazirah Adilla bt Norzawary and she was born on 29th of October 1995. She lives in Serendah, Selangor. In 2002, she began her primary school at Sekolah Kebangsaan Bandar Baru Batang Kali, Selangor and attended her secondary school at Sekolah Menengah Kebangsaan Bandar Baru Batang Kali, Selangor until she finished her Sijil Penilaian Menengah (SPM). After that, she pursued her study in Pulau Pinang Matriculation College in Physical Science. After finished her matriculation level, she furthered her study in Bachelor of Science (Hons.) Major in Mathematics at Universiti Putra Malaysia (UPM) Serdang, Selangor for four years. After the graduation, she directly continued her study for the degree of Master of Science in Fluid Dynamics at UPM, Serdang.

The student can be reached through her supervisor, Assoc. Prof. Dr. Norfifah binti Bachok @ Lati, by address:
Department of Mathematics,
Faculty of Science,
Universiti Putra Malaysia,
43400 Serdang,
Selangor, Malaysia.

Email: norfifah@upm.edu.my
Telephone: 03-89466849

LIST OF PUBLICATIONS

Norzawary, N.H.A., Bachok, N. & Ali, M.F. (2019). Boundary Layer Flow and Heat Transfer on a Moving Plate in a Carbon Nanotubes. *Journal of Multidisciplinary Engineering Science and Technology (JMEST)*, Vol 6(12): 62-67.

Norzawary, N.H.A., Bachok, N. & Ali, M.F. (2020). Stagnation Point Flow Over a Stretching/shrinking Sheet in a Carbon Nanotubes with Suction/Injection Effects. *CFD Letters*, Vol 12(2): 106-114.

Norzawary, N.H.A., Bachok, N. & Ali, M.F. (2020). Effects of Suction/Injection on Stagnation Point Flow Over a Nonlinearly Stretching/shrinking Sheet in a Carbon Nanotubes . *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences*, Vol 76(1): 30-38.