UNIVERSITI PUTRA MALAYSIA

ON THE DIOPHANTINE EQUATION $x^2+2^a .zb= y^3n$

NUR HIDAYAH BINTI AMALUL HAIR

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ON THE DIOPHANTINE EQUATION $x^2 + 2^a \cdot z^b = y^{3n}$

By

NUR HIDAYAH BINTI AMALUL HAIR

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirements for the Degree of Master of Science

December 2020

DEDICATIONS

To
my lovely parents,
Amalul Hair bin Johari ,
my mother,
Wan Mahani Bt Mokhtar,
my husband,
Muhammad Firdaus B. Ahmad Nawi,
Gadis-Gadis INSPEM,
families and friends
who always support me . . .

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Master of Science

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NUR HIDAYAH BINTI AMALUL HAIR

December 2020

Chairman: Siti Hasana binti Sapar, PhD
Institute:Mathematical Research

Diophantine equation is a polynomial equation with two or more unknowns for which only integral solutions are sought. Exponential Diophantine equation is a Diophantine equation that has additional variable or variable occuring as exponentents polynomial.

Let α_1 and α_2 be algebraic numbers with $|\alpha_1| \geqslant 1$ and $|\alpha_2| \geqslant 1$, we will consider the Diophantine equation $x^2 + 2^a \cdot z^b = y^{3n}$ for $z = 7$ in the form of

$$\Lambda = b_2 \log \alpha_2 - b_1 \log \alpha_1$$

where α_1, α_2, b_1 and b_2 are positive integers. In order to find upper bound for value of n in Diophantine equation $x^2 + 2^a \cdot z^b = y^{3n}$ for for $z = 7$, we will use:

$$h(\alpha) = \frac{1}{d}\left(\log|a_0| + \sum_{i=1}^{d} \log\,[1, |\alpha^{(i)}|]\right)$$

where $\alpha \in \mathbb{Z}$.

This research concentrates on finding an integral solution to the exponential Diophantine equation on $x^2 + 2^a \cdot z^b = y^{3n}$ for a, b, x, y, n and $z = 7$ are positive integers. By focused $n = 1, 2$, $n = 3$, and $2 \leqslant a \leqslant 8$ with any values of b, the integral solution of x and y are determined. Limitation of the value of $x, y \leqslant 50,000$, an integral solution to the Diophantine equation for x and y will be obtained. By considering the parity of x and y and also by using substitution method, simple parametrization, quadratic residue modulo, cubic residue modulo, Baker's method and local method, integral solution of x and y will be determined. In order to derive effective bounds of the Diophantine equation, Baker's method are used in proving Diophantine equation in this research.

This research found that there is no pattern of solution obtained. Therefore, the upper bound technique method is used to get the integral solution of Diophantine equation which is more precise and effective. By combining Baker and local methods, an upper bound for the values of $n \geqslant 3$ is obtained. By considering two cases that is either $y > 2$ or $y = 2$, then the upper bound for $y = 2$ is $n < 15,000$ and for $y > 2$, is $n < 11,000$.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Master Sains

PERSAMAAN DIOFANTUS $x^2 + 2^a \cdot z^b = y^{3n}$

Oleh

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Persamaan Diophantus adalah persamaan polinomial dengan dua atau lebih pemboleh ubah dengan penyelesaian integer sahaja yang dicari. Persamaan Diophantus eksponen ialah persamaan yang mempunyai penambahan pemboleh ubah yang bertindak sebagai polinomial eksponen.

Misalkan α_1 dan α_2 nombor algebra dengan $|\alpha_1| \geqslant 1$ dan $|\alpha_2| \geqslant 1$, dipertimbangkan persamaan Diophantus $x^2 + 2^a \cdot z^b = y^{3n}$ untuk $z = 7$ dalam bentuk

$$\Lambda = b_2 \log \alpha_2 - b_1 \log \alpha_1$$

di mana α_1, α_2, b_1 dan b_2 adalah integer positif. Untuk mencari batas atas nilai n dalam persamaan Diophantus $x^2 + 2^a \cdot z^b = y^{3n}$ untuk $z = 7$, akan digunakan:

$$h(\alpha) = \frac{1}{d}\left(\log|a_0| + \sum_{i=1}^{d} \log\,[1, |\alpha^{(i)}|]\right)$$

dengan $\alpha \in \mathbb{Z}$.

Kajian ini tertumpu kepada mencari penyelesaian integer kepada persamaan eksponen Diophantus $x^2 + 2^a \cdot 7^b = y^{3n}$ dengan a, b, x, y, n dan $z = 7$ adalah integer positif. Dengan menumpukan $n = 1, 2, 3$, dan $2 \leqslant a \leqslant 8$ dengan sebarang nilai b, penyelesaian integer x dan y akan diperoleh. Penyelesaian x dan y untuk persamaan Diophantus ini diperoleh dengan nilai had $x, y \leqslant 50,000$. Dengan mempertimbangkan pariti x and y dan juga kaedah penggantian, kaedah parameterasasi, modulo residu kuadratik, modulo residu kubik, kaedah Baker dan kaedah tempatan, penyelesaian integer x dan y akan ditentukan. Untuk mendapatkan batas persamaan Diophantus yang berkesan, kaedah Baker digunakan dalam membuktikan persamaan Diophantus dalam kajian ini.

Penyelidikan ini mendapati bahawa tidak ada pola penyelesaian yang diperoleh. Oleh itu, kaedah teknik batas atas digunakan untuk mendapatkan penyelesaian integral bagi persamaan Diophantus yang lebih tepat dan berkesan. Dengan menggabungkan kaedah Baker dan kaedah tempatan, nilai batas atas untuk nilai $n \geqslant 3$ diperoleh. Dengan mempertimbangkan dua kes iaitu $y > 2$ atau $y = 2$, kemudian batas atas untuk $y = 2$ ialah $n < 15,000$ dan untuk $y > 2$, adalah $n < 11,000$.

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This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfilment of the requirement for the degree of Master of Science. The members of the Supervisory Committee were as follows:

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LIST OF ABBREVIATIONS

$\mathbb{Z}$	Integer
$\mathbb{N}$	Natural number
mod	Modulo
$\left(-\frac{1}{p}\right)$	Lagendre symbol
RHS	Right hand side
LHS	Left hand side

CHAPTER 1

INTRODUCTION

1.1 Preliminary

This research will be focused on finding an integral solution to the Diophantine equation $x^2 + 2^a \cdot z^b = y^n$ for $z = 7$. In this chapter, we will brief some background of Diophantine equation. Then, we state the problem statement, objectives and methodology of this research followed by the literature reviews from the previous researchers. Lastly, the organization of the thesis according to each chapter will be given.

1.2 Mathematical Background

In this section, we will give some background of Number Theory and Diophantine equation. James and Erica (2010) stated that Number theory is the study of natural numbers and called "the queen of mathematics" by Carl Friedrich Gauss. The beautiful patterns and theorems that emerge have fascinated many of the greatest mathematical minds throughout the centuries. Yet, give challenges to the mathematicians to solve the problems.

An equation with the restriction that only integer solutions are sought is called Diophantine equation. The main focus of this research is to solve exponential Diophantine equation. Exponential Diophantine equation is an equation that has additional variable or variables occuring as exponents. The simple expression of exponential Diophantine equation is $x^a + y^b = z^c$ where all the unknowns must be natural numbers.

There is no general method for solving Diophantine equation. We also obtain a general theorem about bounds for solutions of diophantine equations with a finite numbers of solutions. Laurent (2008) stated the following lemma:

Lemma 1.1 *For an algebraic number α of degree d over $\mathbb{Q}$, we define that absolute logarithmic height of α by the following formula:*

$$h(\alpha) = \frac{1}{d}\left(\log|a_0| + \sum_{i=1}^{d} \log\,[1, |\alpha^{(i)}|]\right)$$

where a_0 is the leading coefficient of polynomial of α over $\mathbb{Z}$.

Lemma 1.2 *Let α_1 and α_2 be multiplicatively algebraic numbers with $|\alpha_1| \geqslant 1$ and*

$|\alpha_2| \geqslant 1$. *We will consider the diophantine equation of the form*

$$\Lambda = b_2 \; log \; \alpha_2 - b_1 \; log \; \alpha_1$$

where log α_1*, log* α_2*,* α_2 *and* b_1 *and* b_2 *are positive integers.*

First, by using above lemma to get upper bound for n in equation $x^2 + 2^a \cdot z^b = y^n$ for $z = 7$. Put $t = 2^a \cdot 7^b$. Then, we have

$$x^2 + t = y^n.$$

Since $a \leqslant 8$ and $b < 8$, then we have $t \leqslant 2^8 \cdot 7^7 = 210827008$. Then, we will have the following theorem.

Theorem 1.1 *Let x, t be positive integers with* $x, y \leqslant 50,000$ *and* $t \leqslant 2^8 \cdot 7^7$*. Then y and n be a solution of the Diophantine equation with* $n \geqslant 3$*. There exist integral solution of Diophantine equation for* $n < 11,000$ *if* $y > 2$ *and* $n < 15,000$ *if* $y = 2$*.*

Besides the above theorems, we also consider the following definitions, propositions and theorems to find the integral solution to the Diophantine equation. From Kumanduri and Romero (1998) we have:

Definition 1.1 : *(Divisibility). If a and b are integers, we say that a divides b (denoted as* $a \mid b$*) if there exists an integer c such that* $b = ac$*. If no such c exists, then a does not divide b (denoted by* $a \nmid b$*). If a divides b, we say that a is a divisor of b and b is divisible by a.*

Theorem 1.2 *There are infinitely many prime numbers.*

Proposition 1.1 : *(Primality Test). A number p is prime if and only if it is not divisible by any prime q.*

Proposition 1.2 : *Let a and b be integers. If p is a prime number such that* $p \mid ab$*, then* $p \mid a$ *or* $p \mid b$*.*

Definition 1.2 : *(Greatest Common Divisor). The greatest common divisor (gcd) of two numbers a and b, not both zero, is the largest integer dividing both a and b. It will be denoted by gcd(a,b) or* (a,b)*.*

Definition 1.3 : *Let a, b, m be integers, we say that a is congruent to b modulo m denoted by $a \equiv b \pmod{m}$ if $m \mid a - b$. If $m \nmid a - b$, we write $a \not\equiv b \pmod{m}$ and say that a is not congruent or incongruent to b modulo m.*

Definition 1.4 : *(Quadratic Residue Modulo). Let a, m be integers such that $(p, q) = 1$. Then, congruence $x^2 \equiv a \pmod{m}$ has an integer solution, then a is a quadratic residue modulo m. Otherwise, it is a quadratic nonresidue modulo m.*

Definition 1.5 : *(Cubic Residue Modulo). Let p, q be integers such that $(a, m) = 1$. Then, congruence $x^3 \equiv p \pmod{q}$ is solvable if and only if $x^3 \equiv p \pmod{q}$.*

Definition 1.6 : *(Euler's criterion). Let p be an odd prime and a an integer such that $(a, p) = 1$, then*

$$a^{\frac{p-1}{2}} \equiv \left(\frac{a}{p}\right) \pmod{p}.$$

Proposition 1.3 : *Let p be an odd prime, then*

$$\left(\frac{-1}{p}\right) = \begin{cases} 1 \text{ if } p \equiv 1 \pmod{4}, \\ -1 \text{ if } p \equiv 3 \pmod{4}. \end{cases}$$

From Cook (2014) we have:

Theorem 1.3 : *(Quadratic Congruence Powers of 2). Let a be an odd integer. Lets $x^2 \equiv a \pmod{2}$, it has a solution if $x \equiv 1 \pmod{2}$ only.*

Theorem 1.4 : *(Quadratic Congruence Powers of 2). Let a be an odd integer. Lets $x^2 \equiv a \pmod{4}$, it has a solution if $x \equiv 1 \pmod{4}$ only.*

Theorem 1.5 : *(Quadratic Congruence Powers of 2). Let a be an odd integer. Lets $n \geqslant 3$, $x^2 \equiv a \pmod{2^n}$ has four solutions if $a \equiv 1 \pmod{8}$ and no solutions otherwise.*

1.3 Problem Statement

This research is extended from Yow and Atan (2013), where he considered the Diophantine equation of the form $x^2 + 2^a \cdot 7^b = y^r$ where r is even. It has difficulties higher degree of r. Thus, to extend this problem, we consider the Diophantine equation $x^2 + 2^a \cdot z^b = y^{3n}$ for $z = 7$, where $1 \leqslant a \leqslant 8$ for $n \leqslant 3$. By finding the higher degree of exponential Diophantine equation in more general, we can obtain in general forms by using upper bound as more precise and effective.

1.4 Objective and Methodology

In this section, we will state the objective and methodology of this research. The main objectives of this research are:

(i) to find integral solutions a,b,x,y,n to the Diophantine equation

$$x^2+2^a\cdot z^b=y^{3n} \tag{1.1}$$

for $z=7$ and $n=1$.

(ii) to find integral solutions a,b,x,y,n to the Diophantine equation

$$x^2+2^a\cdot z^b=y^{3n} \tag{1.2}$$

for $z=7$ and $n=2,3$.

(iii) to provide upper bound for integral solutions to the Diophantine equation

$$x^2+2^a\cdot z^b=y^n, z=7 \tag{1.3}$$

for $n\geqslant 3$.

Now, we will present the methodology to determine the integral solution for the Diophantine equation. In order to find the integral solution for $x^2+2^a\cdot z^b=y^{3n}$ for $z=7$ when $n\leqslant 3$, we will consider the parity of x and y. By using substitution method, quadratic residue modulo and cubic residue modulo, the integral solutions of a,b,x,y will be obtained. Lastly, as we do not get any pattern of solution, hence we could not form the general form of solution. Then, we use Baker's and local method to find out upper bound for n.

1.5 Organization of Thesis

This thesis covers eight chapters as follows:

Chapter 2 provides a literature review related to the research. The previous research give an idea and more effective techniques on this research by finding the integral of Diophantine equation $x^2+2^a\cdot z^b=y^{3n}$ for $z=7$.

Chapter 3 focus on finding the integral solution to the Diophantine equation $x^2+2^a\cdot 7^b=y^{3n}$ for $n=1$. We focused only for $a=2$ for any values of b. In order to solve the equation, we consider two cases either x and y are even or x and y are odd.

Chapters 4 and 5 study on the integral solution to the Diophantine equation $x^2+2^a\cdot 7^b=y^{3n}$ for $n=1$. This chapter focus on $a\leqslant 8$ and for any values of b. In order to solve the equation, we consider the case either x and y are even or x and y

are odd. We used used simple substitution method, and quadratic residue modulo in our results.

Chapter 6, we investigate on the intgeral solution to the Diophantine equation $x^2 + 2^a \cdot 7^b = y^{3n}$ for $n = 2,3$. This chapter study on $a \leqslant 8$ for any values of b. In order to solve the equation, we consider two case either x and y are even or x and y are odd. We also used simple substitution method, quadratic residue modulo and cubic residue modulo to prove our results.

Then, followed by finding bound for n in Chapter 7 for the integral solutions of the Diophantine equation $x^2 + 2^a \cdot 7^b = y^n$ where a, b, x, y are positive integers. We used local argument combining with Baker's method to find bound for $n \geqslant 3$.

Chapter 8, provide a summary of the research and the upper bound for all the integral solutions for case $y = 2$ and $y > 2$. Also, some future works will be discused in the last section of this chapter.

REFERENCES

Astana, S. and Singh, M. M. (2017). On the diophantine equation $8^x + 113^y = z^2$. *International Journal of Algebra*, 11(5):225–230.

Bakar, H. S., S. S. H. and M.A.M. (2019). On the diophantine equation $5^x + p^m n^y = z^2$. *Malaysian Journal Of Mathematical Sciences*, 13(S):41–50.

Bennett, M. A. (2017). The polynomial-exponential equation $1 + 2^a + 6^b = y^q$. *Period Math Hungary*, 75:387–397.

Berczes, A., H. L. M. T. and Pink, I. (2016). On the diophantine equation $1 + x^a + z^b = y^n$. *Journal of Combination and Number Theory*, 8(2):145–154.

Cangul, 1.N., D. M. I. I. L. F. and S, G. (2013). On the diophantine equation $x^2 + 2^a \cdot 3^b \cdot 11^c = y^n$. *Math. Slovaca*, 63(3):647–659.

Chotchaisthit, S. (2013). On the diophantine equation $2^x + 11^y = z^2$. *Maeojo International Journal Science Technology*, 7(2):291–293.

Cook, J. (2014). On the solving quadratic congruence of diophantine equation. *http://www.johndcook.com/blog/quadratic congruences.*

Demirci, M. (2017). On the diophantine equation $x^2 + 5^a \cdot p^b = y^n$. *Faculty of Sciences and Mathematics, University of Ni ˇs, Serbia*, 31(16):5263–5269.

Deng, M. (2015). A note on the diophantine equation $x^2 + q^m = c^{2n}$. *Proceeding Japan Academy*, 91(2):15–18.

Gou, S., W. T. and Xian (2010). The exponential diophantine equation $(x)^2 + 2^a \cdot 17^b = y^n$. *Czechoslovak Mathematical Journal*, 62(137):645–654.

Hajdu, L. and Pink, I. (2014). On the diophantine equation $1 + 2^a + x^b = y^n$. *Journal of Number Theory*, 143:1–13.

James, E. P., T. K. M. and Erica, L. (2010). Number theory a lively introduction with proofs, applications and stories. *Laurie Rosatone, United States of America.*

Jerico, B. and Rabago, J. F. T. (2015). The complete set of the diophantine equation $p^x + q^y = z^2$ for twin prime p abn q. *International Journal of Pure and Applied Mathematics*, 104(4):517–521.

Kumanduri, R. and Romero, C. (1998). Number theory with computer applications. *Pearson College Division*.

Laurent, M. (2008). Linear forms in two logarithms and interpolation determinants ii. *Acta Arithmetica*.

Luca, F. (2000). On the equation $x^2 + 2^a \cdot 3^b = y^n$. *International Journal of Mathematics and Mathematical Sciences*, 29(4):239–244.

Mihăilescu, P. (2004). Primary cyclotomic units and a proof of catalan's conjecture. *J. Reine Angew Math*, 572:167–195.

Muriefah, F., F.L, and Togbe, A. (2008). On the diophantine equation $x^2 + 5^a \cdot 13^b = y^n$. *Glasgow Mathematical Journal Trust*, 50:75–181.

Muriefah, F. S. A. (2006). The diophantine equation $x^2 + c = y^n$. *Revista Colombiana de Matematicas*, 40:31–37.

Pink, I. and Rabai, Z. (2011). On the diophantine equation $x^2 + 5^k 17^l = y^n$. *Communication in Mathematics*, 19:1–9.

Qi, L. and Li, X. (2015). The diophantine equation $8^x + p^q = z^2$. *The Scientific World Journal*, 2015:1–3.

Rabago, J. (2016). On the diophantine equation $2^x + 17^y = z^2$. *J.Indones.Math.Soc.*, 22(2):85–88.

Soydan, G. and Tzanakis, N. (2017). Complete solution of the diophantine equation $x^2 + 5^a \cdot 11^b = y^n$. *arXiv:1703.04950v1[math.NT]*, pages 1–35.

Soydan, G., U. M. and Zhu, L. (2012). A note on the diophantine equation $x^2 + 2^a \cdot 19^b = y^n$. *Indian J. Pure Appl. Math*, 43(3):251–261.

Sroysang, B. (2012a). More on the Diophantine Equation $7^x + 31^y = z^2$. *International Journal of Pure and Applied Mathematics*, 92(1):109–112.

Sroysang, B. (2012b). More on the Diophantine Equation $8^x + 19^y = z^2$. *International Journal of Pure and Applied Mathematics*, 81(4):601–604.

Sroysang, B. (2012c). On the diophantine equation $31^x + 32^y = z^2$. *International Journal of Pure and Applied Mathematics*, 81(4):609–612.

Sroysang, B. (2013a). More on the Diophantine Equation $2^x + 19^y = z^2$. *International Journal of Pure and Applied Mathematics*, 88(1):157–160.

Sroysang, B. (2013b). On the Diophantine Equation $2^x + 3^y = z^2$. *International Journal of Pure and Applied Mathematics*, 84(2):133–137.

Sroysang, B. (2014). On the diophantine equation $5^x + 63^y = z^2$. *International Journal of Pure and Applied Mathematics*, 91(4):541–544.

Survanamani, A. (2014). Solution of the diophantine $p^x + q^y = z^2$. *Internation Journal of Pure and Applied Mathematics*, 94(4):457–460.

W.Ljunggren (1964). On the diophantine equation $cx^2 + d = y^n$. *Pacific Journal of Mathematics*, 14:2.

Yow, K.S., S. S. and Atan, K. (2013). On the diophantine equation $x^2 + 4^a \cdot 7^b = y^v$. *Pertanika Journal Science Technology.*, 21(2):443–458.

Zhu, H., L. M. and Soydan, G. (2015). On the number of solution diophantine equation $x^2 + 2^a \cdot p^b = y^4$. *International Journal of Pure and Applied Mathematics*, 67(17):255–263.

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Nur Hidayah Amalul Hair, Siti Hasana Sapar and Mohamat Aidil Mohamat Johari (2020). On the Diophantine equation $x^2 + 2^a \cdot 7^b = y^n$. *Dicovery Mathematics* 42(2): 67-75.

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