



**UNIVERSITI PUTRA MALAYSIA**

***STAGNATION POINT FLOW AND HEAT TRANSFER OVER A  
STRETCHING OR SHRINKING SHEET IN A POROUS MEDIUM***

**NIRWANA BINTI JAPILI**

**FS 2021 36**



**STAGNATION POINT FLOW AND HEAT TRANSFER OVER A  
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By

**NIRWANA BINTI JAPILI**

**Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia,  
in Fulfillment of the Requirements for the Degree of Master of Science**

**June 2021**

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Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfillment  
of the requirement for the degree of Master of Science

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**June 2021**

**Chairman: Haliza Bt Rosali, PhD**

**Faculty: Science**

The steady stagnation point flow towards a stretching/shrinking sheet in a porous medium is investigated. Mathematical models are derived for stagnation point flow and heat transfer problems towards a linearly and exponentially stretching or shrinking sheet in a porous medium in the presence of magnetohydrodynamics, suction and slip effect at the surface. Similarity transformations are used to transform the partial differential equations into a nonlinear ordinary differential equation. These equations are numerically solved by using a shooting method in Maple software.

Results for the skin friction coefficients, local Nusselt number, velocities and temperature profiles are presented using graphs with respect to the involved parameters of interest, including stretching or shrinking parameter  $c$ , suction parameter  $s$ , magnetic parameter  $M$ , permeability parameter  $K$ , velocity slip parameter  $A$  and thermal slip parameter  $B$ . The findings of the present work reveal that the magnitude of the skin friction coefficients and the local Nusselt number which reflects the rate of heat transfer at the surface are strongly dependent on these parameters.

Besides, for a certain range of the shrinking parameter, dual solutions exist. It is also observed that in the exponentially shrinking sheet case, the solutions' domain is greater than in the linear case.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Master Sains

**ALIRAN TITIK GENANGAN DAN PEMINDAHAN HABA TERHADAP  
PERMUKAAN MEREKANG ATAU MENGECEUT DI DALAM  
BAHANTARA BERLIANG**

Oleh

**NIRWANA BINTI JAPILI**

**Jun 2021**

**Pengerusi: Haliza Bt Rosali, PhD**

**Fakulti: Sains**

Aliran titik genangan mantap terhadap permukaan merekang atau mengecut di dalam bahantara berliang disiasat. Model matematik diterbitkan bagi masalah dalam aliran titik genangan dan pemindahan haba terhadap permukaan merekang atau mengecut secara linear dan eksponen dalam bahantara berliang dengan kehadiran magnetohidrodinamik, kesan sedutan dan kesan gelincir pada permukaan. Penjelmaan keserupaan digunakan untuk menjelmakan persamaan perbezaan separa kepada persamaan perbezaan biasa tak linear. Persamaan ini diselesaikan secara berangka menggunakan kaedah tembakan dalam perisian Maple.

Keputusan untuk pekali geseran kulit, nombor Nusselt setempat, profil halaju dan suhu dipamerkan melalui graf bagi parameter yang terlibat, iaitu, parameter regangan atau kecutan  $c$ , parameter sedutan  $s$ , parameter magnetik  $M$ , parameter ketelapan  $K$ , parameter gelincir halaju  $A$  dan parameter gelincir haba  $B$ . Kajian ini juga menunjukkan pekali geseran kulit dan nombor Nusselt setempat yang mewakili kadar pemindahan haba pada permukaan adalah bergantung sepenuhnya kepada parameter tersebut.

Selain itu, penyelesaian dual wujud untuk julat tertentu bagi parameter mengecut. Juga didapati bahawa julat untuk nilai parameter merekang atau mengecut dimana penyelesaian dual wujud adalah lebih besar dalam masalah eksponen berbanding masalah linear.

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This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfilment of the requirement for the degree of Master of Science. The members of the Supervisory Committee were as follows:

**Haliza Bt Rosali, PhD**

Senior Lecturer  
Faculty of Science  
Universiti Putra Malaysia  
(Chairperson)

**Norfifah binti Bachok @ Lati, PhD**

Associate Professor  
Faculty of Science  
Universiti Putra Malaysia  
(Member)

---

**ZALILAH MOHD SHARIFF, PhD**

Professor and Dean  
School of Graduate Studies  
Universiti Putra Malaysia

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Signature: \_\_\_\_\_

Name of

Chairman of

Supervisory

Committee: Dr. Haliza Bt Rosali

Signature: \_\_\_\_\_

Name of

Member of

Supervisory

Committee: Assoc. Prof. Dr. Norfifah Bachok @ Lati

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## LIST OF ABBREVIATIONS

$a, b$	constants
$A$	velocity slip parameter
$B$	thermal slip parameter
$B_0$	magnetic field
$c$	stretching or shrinking parameter
$C_f$	skin friction coefficient
$f$	dimensionless component of velocity
$k$	thermal conductivity
$K$	permeability parameter
$K_1$	permeability of the porous medium
$L$	slip length
$M$	magnetic parameter
$Nu_x$	local Nusselt number
$Pe_x$	local Péclet number
$Pr$	Prandtl number
$q_w$	surface heat flux
$Re_x$	local Reynolds number
$S$	proportionality constant
$s$	suction parameter
$T$	fluid temperature
$T_w$	wall temperature
$T_\infty$	ambient temperature
$U_w$	velocity of the stretching/shrinking surface
$U_\infty$	velocity of the external flow
$u$	velocity component along $x$ -axis
$v$	velocity component along $y$ -axis
$V_w$	constant mass flux
$x$	Cartesian coordinate
$y$	Cartesian coordinate

### **Greek Symbols**

$\alpha$	thermal diffusivity
$\beta$	thermal expansion coefficient
$\eta$	similarity variables
$\theta$	dimensionless temperature
$\mu$	dynamic viscosity
$\nu$	kinematic viscosity
$\psi$	stream function
$\rho$	fluid density
$\sigma$	surface tension
$\tau_w$	surface shear stress

### **Subscripts**

$c$	critical value
$f$	fluid
$w$	condition at the wall
$\infty$	condition at infinity

### **Superscript**

$'$	differentiation with respect to $\eta$
-----	--

# CHAPTER 1

## INTRODUCTION

This thesis concerned on the mathematical model describing the actual application of fluid dynamics. This model attempts to explain the behavior of the real problems in mathematical way and may often deviate from its assumptions. Changing a better model is, thus, the way to solve this situation. A few general introductions in fluid dynamics are defined in this chapter including the boundary layer concept, stagnation point flow, magnetohydrodynamics (MHD) fluid flow, heat transfer, porous medium, types of boundary conditions which are stretching or shrinking sheet, suction and injection, slip and no slip as well as dimensionless parameter. Other than the basic introduction to fluid dynamics, the objectives and scopes of the study, the significance of the research and thesis outline for the whole research are also discussed.

### 1.1 Research Background

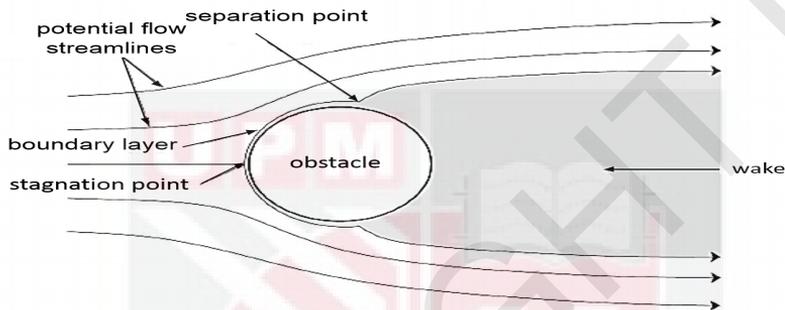
The field of applied science dealing with the flow of liquids and gases is known as fluid dynamics. It is one of the two arms of fluid mechanics that investigate fluids and how they are affected by forces. Fluid statics, on the other hand, is concerned with fluids at rest. Studying fluid dynamics allows scientists to study topics like star evolution, ocean currents, plate movements as well as blood circulation. In addition, fluid dynamics also have important applications in air conditioning, oil pipelines, and wind turbines. Flow is the description of how fluids move and interact with their surroundings, such as how water flows through a pipe or over a surface. Liquid flow studies are referred to as hydrodynamics. There are numerous types of liquid substances such as chemical solutions and oil, but water is the most common one, and a majority of hydrodynamics applications involve liquid flow, including flood control and sewer systems. On the other hand, aerodynamics is the study of the mechanics of bodies moving in relation to gases, especially how they interact with the atmosphere. The flow of gas has a lot in common with the flow of liquid, but there are also significant differences between them. First, gas is compressible, while liquids are typically incompressible. Since air is the most common element, scientists have extensively studied its movement. Aims of applications in this field include streamlining car bodies, aerodynamic research, the investigation of aircraft designs, and the study of the aerodynamics of animals.

#### 1.1.1 Boundary Layer Theory

In view of history, the Navier-Stokes equation was known by the middle of the 1800s, but could not be solved unless for flows of very simple geometrics. Until 1904, the boundary layer approximation was developed by Ludwig Prandtl. It simplified the complex Navier-Stokes and energy equations, making it possible to derive solutions

for different applications.

The concept of Prandtl was to separate the flow into two component regions. To begin, the Euler equations describe an inviscid outer flow field. The second is the boundary layer, which is a thin layer of flow next to the solid wall. This region obeys the Navier-Stokes equation, but can be expressed much more simpler, as boundary layer equations (Çengel and Cimbala, 2006). The interpretation of boundary layer is only true for the areas of the surface where the main flow remains attached, that is un-separated. The boundary layer principle has been the workhorse in fluid mechanics engineering.



**Figure 1.1: Physical model of stagnation point flow**

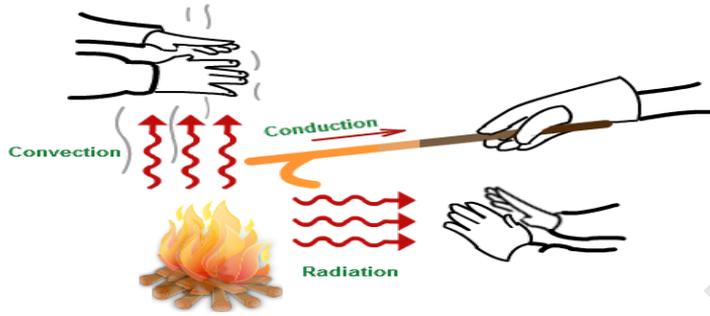
### 1.1.2 Stagnation Point Flow

If the path comes into contact with a solid object and the constant speed at the stagnation point is zero, the stagnation point occurs (see Figure 1.1). Maximum values of heat transfer and mass deposition happen in the stagnation point area. Hiemenz (1911) was the first known person to have discovered the flow pattern near a stagnation level. In general, these courses are commonly used in manufacturing process for product creation.

### 1.1.3 Heat Transfer

Heat transfer is another phenomenon which is extremely significant in fluid dynamics. Heat transfer is the flow of heat energy at different temperatures from one region to another region. The heat transfer path is always from a high to a low temperature zone. The heat transfer rate is highly affected by temperature gradients. The flow of heat will also increase as the temperature gradient increases. Figure 1.2 illustrates three types of heat transfer processes: conduction, convection, and radiation.

Heat conduction happens when two objects are in direct contact with each other. As



**Figure 1.2: Physical model for type of heat transfer**

an object is heated, particles absorb more energy and then begin to vibrate and collide with other particles, passing some of their energy to other molecules. Convection is a physical movement of liquid or gas containing heat energy at various temperatures. It happens when there is a large temperature differential between hot and cold objects. In contrast to conduction and convection, radiation is a form of heat transfer that does not require any interaction between the heat source and the heated object. This thermal radiation can transport the heat into a space. For example, the Sun's energy is transferred to Earth as an electromagnetic waves.

Nevertheless, this research only concerned with free convection heat transfer. Fluid motion in free convection is caused mainly by density variations in the fluid and is not caused by external sources (Al-Khoury, 2012).

#### **1.1.4 Porous Medium**

Porous media is a substance containing a solid matrix interspersed with the pores. The matrix is presumed to be either rigid or to undergo small deformed. The interconnectedness of the pores structures allows one or more fluids to flow through the material. The pore would be filled by a single fluid in the most basic case that is single-phase flow. In contrast, gas and liquid occupy the same space in a two phase flow (Nield and Bejan, 2013). In order to qualify as a porous medium, a material or structure must have the following two properties:

1. It must be constructed of solid or semi-solid components separated by spaces or pores. The pores will usually contain air, water, as well as various kind of fluids.
2. It must be permeable to a variety of fluids, which means that the fluids must be able to flow through one side and emerge on the other side of the material.

There are certain natural substances, such as soil, biological tissues, and artificial

ones such as ceramics and plastics, which can be categorised as porous media. Furthermore, the important properties of these substances cannot be completely explained until they are known to be porous media.

Darcy's law is a theoretical equation that deals with the flow of a fluid through a porous medium. Henry Darcy discovered it experimentally, but it has since been derived from the Navier–Stokes equations using homogenization techniques (Whitaker, 1986). The law was formulated based on the experiment's results on the flow of water through sand beds, and it serves as the foundation for hydrogeology, a branch of earth sciences. Darcy's law, however, is only true for slow viscous flow. Thus, a generalised Darcy's law is being used in this study, which includes the convective acceleration (Yamamoto and Iwamura, 1976).

Many applied science and engineering fields use the idea of porous media such as filtration, geosciences, biology and material science. In recent years, the subject of fluid flow through porous media has gained greater attention from many researchers in various subfield.

### **1.1.5 Types of Boundary Conditions**

#### **1.1.5.1 Stretching and Shrinking Sheet**

When the speed at the boundary goes away from the fixed point, this is when the stretching sheet happens. The process of constriction, such as polymer constriction and other industrial processes are generated by the stretching sheet. Whereas, when the speed reaches a fixed point at the boundary, it produces a shrinking sheet. In order for a shrinking sheet's flow to occur, two conditions must be met: if the boundary layer is subjected to stagnation point flow (Wang, 2008) or is subjected to significantly stronger suction (Miklavcic and Wang, 2006). Shrinking problems applicable to the research of environmental management strategies that are relevant for agricultural development, such as research on the hydraulic properties of agricultural clay soils.

#### **1.1.5.2 Suction and Injection**

Suction and injection are the most common forms of operations on porous media. For the suction side operation, low pressure is applied at the suction face known as the inlet, allowing fluid to enter the medium through the inlet easily. Similarly, for the injection side operation, high pressure is exerted at the injection face known as the outlet, allowing fluid to exit to the medium via force. The injection can improve a system's heating or cooling and help to delay the transition from laminar to turbulent flow (Chaudhary and Merkin, 1993).

On the other hand, suction is one of the methods of boundary layer control, which is designed to minimize drag on bodies in an external flow or to reduce energy losses in channels. This approach has been suggested by L. Prandtl in 1904 as one of the methods that was used to prevent or delay boundary layer separations. For suction, the surface should have holes and these holes are used for sucking the portion of the boundary layer that is closest to the wall and moves at lowest velocity. As a consequence, this leads to a higher boundary layer velocity profile, which offers a greater degree of stability when it comes to separation.

### 1.1.5.3 Slip and No Slip

When the slip condition is applied to a fluid flow, it indicates that there is a relative velocity between the fluid and the solid boundary. The no-slip condition, on the other hand, indicates that the fluid and the solid boundary are moving at the same velocity as the fluid is in direct contact with the boundary, and therefore there is no relative movement between them. This assumption proved true for viscous fluids. The existence of slip on the solid boundary plays an essential role in biological applications such as the polishing of synthetic heart valves (Nandal et al., 2019).

### 1.1.6 Magnetohydrodynamics (MHD) Fluid Flow

MHD is the combinations of hydrodynamics and electromagnetism. The combination of the Navier-Stokes fluid dynamical and Maxwell equations on electromagnetic equations are therefore the set of equations which describe MHD. In 1970 a physicist Hannes Alfvén introduced and developed the MHD. MHD is based on the idea of a magnetic field driving current through a flowing conductive fluid, exerting force and altering the magnetic field on the fluid (Medin, 2011). The fluid with MHD characteristics is considered to be capable of controlling the separation flow, manipulating the flow of the fluid as well as optimizing the transfer of heat from the electrically conductive fluid. Hence, the MHD flow is important in the application of engineering and industry such as power generator and nuclear reactor cooler.

### 1.1.7 Dimensionless Parameters

In convection studies, it is common to transform the governing equations into a dimensionless parameters to reduce the number of variables (Çengel, 2003). These dimensionless parameters have physical definitions related to the flow conditions, not just for boundary layers but also for other types of flow (Bergman et al., 2011). The parameters that are used in this study are:

1. **Prandtl number,  $Pr$**  : Proposed in 1904 by Ludwig Prandtl, a German scientist. It is the ratio of relative momentum diffusivity to thermal diffusivity, where momentum diffusivity refers to the motion of the fluid and thermal dif-

fusivity to the influence of the fluid property on the conductivity process. The Prandtl number is depending only on the fluid property and is unaffected by the surroundings. Prandtl values for fluids vary from below 0.01 in the case of liquid metals to over 100,000 in the case of heavy oils. If Pr is greater than 1, momentum diffusivity dominating over thermal diffusivity. Whereas, if Pr is less than 1, thermal diffusivity dominates, resulting in a thicker thermal boundary layer than a momentum boundary layer. In comparison, the value of Pr for gases is around 1, indicating that the heat and momentum disappear at almost the same rate in the fluid. Hence, Pr is used to determine the momentum and thermal boundary layers thickness.

2. **Nusselt number, Nu** : Proposed by Wilhem Nusselt, who contributed significantly to convective heat transfer in the early twentieth century. Nusselt number quantifies convective heat flow to the boundary of the object. The greater the value of the Nusselt number, the more effective the process of convection that occurs (Çengel and Ghajar, 2011).
3. **Reynolds number, Re** : Osborn Reynolds discovered that the flow regime is mainly determined by the ratio of inertia forces to viscous forces in the fluid (Çengel and Ghajar, 2011). Reynolds number is also expected to affect the velocity boundary layer thickness. As Reynolds number rises at a fixed location on a surface, viscous forces become less influential in comparison to inertia forces. Thus, viscosity's effects are not as far-penetrate in the free stream (Bergman et al., 2011).
4. **Péclet number, Pe** : Representing the ratio of convective transport to diffusion transport. It correlates with both the Reynolds and Prandtl numbers. The Péclet number depends on the system's characteristic length as well as the flow field's velocity (Rapp, 2017). If the Peclet number is much smaller than 1, diffusion is dominant over convective transport. Therefore, by getting this information, transport can be modelled on the basis of diffusion only in that particular study.
5. **Coefficient of friction** : It is a numerical number that indicates the correlation between the resistance and the normal reaction of two distinct objects. The coefficient of friction is dependent on the objects that are producing friction. 0 value indicates that no friction exists between the objects. A value of one indicates that the frictional and the normal forces are equal.

## 1.2 Problem Statement

Studies of stagnation point flow and MHD flow in porous media play an important role in various applications, including petroleum engineering, hydrology and biomedical application such as magnetic resonance imaging (MRI). Over the year, there will be an emergence of new applications which require, probably a newer insight into the way we model the porous media problems, hence creating a new research opportunity. There's a theory behind all these applications but in all cases,

it should be supported by experimental data. Therefore, the results obtained in this research will help them to enhance the development of their final product. Numerous researchers have considered flow over a stretching/shrinking sheet in recent years, but these studies have focused primarily on fluid flow and heat transfer without taking into account the effect of slip and suction on the boundary conditions, particularly in porous media. Therefore, more research is necessary, as certain numerical analysis demonstrate that these parameters may improve heat exchange in the fluid and also increase the fluid's velocity. Furthermore, some research indicates that the suction effect might help in minimizing drag and energy loss in channels. Thus, numerical studies should be undertaken to validate all of these possibilities in order to achieve more precise findings. Besides, the use of the porous media model proposed by Yamamoto and Iwamura (1976), which is generalized Darcy's law, is still limited in the literature. Therefore, the investigations of several physical parameters on two-dimensional stagnation point flow of an incompressible fluid over a linearly and exponentially stretching/shrinking sheet in a porous medium in the presence of MHD, suction and slip effects will be conducted in this study.

### **1.3 Research Question**

Many researchers have discussed the stagnation point flow theoretically over a stretching or shrinking surface in a porous medium. The concept of stagnation point flow is analysed in this study over three separate problems: linear, exponential, and MHD stagnation point flow, as well as the results of certain physical parameters. Hence, the research questions are as follows:

1. What are the effects of physical parameters on the skin friction coefficients and local Nusselt number?
2. What are the differences in suction values for the dual solutions exist for linear and exponential problems?
3. What are the parameters that contribute to the widening or narrowing of the range of solutions?
4. What will happen to the flow characteristics when we use the stretching and shrinking surface in a porous medium?

### **1.4 Objective and Scopes**

The aim of this thesis is to investigate the fluid flow and heat transfer characteristics over a stretching or shrinking sheet in porous medium and subjected to the boundary conditions. The following three problems are:

1. Stagnation point flow over a stretching or shrinking sheet in a porous medium in the presence of slip and suction effect.

2. Stagnation point flow and heat transfer over an exponentially stretching or shrinking sheet in a porous medium in the presence of suction effect.
3. Stagnation point flow over a stretching or shrinking sheet in a porous medium in the presence of MHD and velocity slip effect.

While, the objectives of this present study are to:

1. formulate and derive the mathematical model for the three problems,
2. develop an algorithm, solve the mathematical model numerically using shooting method in Maple software and conduct the validation tests for the current research in comparison with the numerical results in the literature, and
3. study the effect of various parameters on the flow and heat transfer characteristics over a stretching or shrinking sheet in a porous medium.

The scope of the research is limited to the problems of stagnation point flow for steady, incompressible and two dimensional towards stretching/shrinking sheets in a porous medium. Besides, slip and suction effect at the boundary condition is also considered in this study.

### **1.5 Significant of the Study**

Over the past few decades, there has been sustained research activity in the study of fluid and heat flow within porous media as it brings various importance in many other fields of science and engineering. Porous materials are found practically everywhere in everyday life. Nearly all products, including solid or semi-solid, have a degree of porosity, except for some metals. There are several examples of porous media playing a significant role in technology and many different technologies that rely on porous media, on the other hand.

Hydrology, which is concerned with the movement of water in earth and sand structures, such as water flow from water-bearing formations, and petroleum engineering, which is primarily concerned with the discovery and development of petroleum and natural gas, are two of the most important technologies that rely on porous media properties. The petroleum engineer is concerned with the amount of fluid material in the rocks, the transmission of fluids through the rocks, and other characteristics linked to the rocks. Such characteristics are determined by the rock and, in certain cases, by the distribution of fluid character inside the rock. As a result, understanding and evaluating the efficiency of a particular reservoir requires knowledge of the physical characteristics of the rock as well as the interaction between the fluid and

the structure. Therefore, the permeability properties studied in this research is a characteristic of the porous medium that evaluates the ability and capacity of fluid transmission. The permeability of the rock is an important rock characteristic because it governs the lateral movement and rate of flow of reservoir fluids. Hence, this study will help to explain how porosity affects the fluid flow in a porous material.

Besides, another important application of porous media can be seen in biomedical studies such as magnetic resonance imaging (MRI) and drug delivery, where it demonstrates the role of transport theory in porous media in the advancement of biomedical progress as reported by (Khanafer et al., 2003). In many therapies, such as delivering drugs to the brain, the diffusion process is considered significant. Therefore, porosity has a significant impact on diffusion transport. Hence the knowledge of the effect of porosity on mass transport will help them better understand the mechanisms that might affect the process. In addition, substantial progress has been made in the application of porous media theory in the modelling of biomedical applications such as porous tissue engineering scaffolds and efficient tissue replacement to mitigate organ shortages and transport in biological tissues (Yang and Vafai, 2006).

Furthermore, in many fundamental heat transfer studies, convective flows in porous media have occupied the central stage and gained significant attention from many previous researchers. This interest is due to their use in high-performance structures including chemical reactor designs and other engineering fields. Moreover, porous materials are also of interest concerning an underground spread of toxins, solar power collectors and geothermal energy, as they can aid in the heat transfer process from the surfaces. The literature concerning convective flows in porous media may be found in the book by (Oosthuizen and Naylor, 1998).

Moreover, boundary layer control method such as suction or injection played an essential role in many areas, including aerodynamics and space sciences. By monitoring the flow in this manner, fuel consumption may be reduced by 30%, pollutant emissions are reduced significantly, and commercial aeroplane running costs are reduced by at least 8% (Braslow, 1999). Shojaefard et al. (2005), in particular, conducted research on the subsonic aircraft surface by applying suction or injection to regulate the fluid movement. Besides, the addition of a fluid suction or injection through the confining surface will greatly alter the flow field, affecting the plate's heat transfer rate, particularly in mass transfer cooling as stated in Ishak et al. (2008) paper. As a result of its many engineering uses, the study of the flow of heat and mass transfer with suction or injection has sparked a great deal of interest by many researchers.

Concerning all these applications, this provides the opportunity to explore and to extend prior work in the field with the introduction of some other parameters that might affect the transfer of heat and the flow of the fluid in a porous medium.

## 1.6 Outline of Thesis

The thesis outline is split into seven chapters. The first chapter is a general overview of the research where some significant comprehensions described include boundary layer theory, stagnation point flow, heat transfer, porous medium, type of boundary conditions, MHD fluid flow and also the dimensionless parameters involved in this study. Apart from that, this chapter also includes the research questions, the objectives of the research, the significance of this particular research and the thesis outline.

The summaries of the previous studies performed by various researchers related to the study's scopes were discussed in the literature review in Chapter 2. This chapter has been grouped into four parts, beginning with the introductory section. In this research, three problems were discussed, so the literature review in this chapter was split into three sections relating to the first to the third problem.

The methodology and numerical methods, separated into four parts, will be discussed in Chapter 3. Firstly, initiated by the chapter's introduction, followed by governing equations, then the mathematical formulations where partial differential equations (PDEs) are reduced into ordinary differential equations (ODEs) using the similarity transformation. The last section of this chapter is the numerical method for obtaining numerical solutions to the problems mentioned in this study.

Next, all three problems in this study are fully clarified in Chapters 4 to 6, in which each chapter is subdivided into four sections. It starts with the introduction of the study. Then, the problem's mathematical formulation discussed in the second part and the results and discussions obtained from this study are presented in the third part. The concluding section is the last part of the chapter.

Lastly, in Chapter 7, the study's general conclusion is given, along with recommendations for further research.

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