UNIVERSITI PUTRA MALAYSIA

ANALYSIS OF STABILITY OF SOME POPULATION MODELS WITH HARVESTING

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By

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IN THE NAME OF ALLAH SWT

MOST BENEFICENT MOST MERCIFUL

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Master of Science

ANALYSIS OF STABILITY OF SOME POPULATION MODELS WITH HARVESTING

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September 2000

Chairman : Associate Professor Dr. Harun Bin Budin

Faculty : Science and Environmental Studies

Applied mathematics, which means application of mathematics to problems, is a wonderful and exciting subject. It is the essence of the theoretical approach to science and engineering. It could refer to the use of mathematics in many varied areas. Mathematical model is applied to predict the behaviour of the system. This behaviour is then interpreted in terms of the word model so that we know the behaviour of the real situation.

We can apply mathematical languages to transform ecology's phenomena into mathematical model, including changes of populations and how the populations of one system can affect the population of another. The model is expected to give us more information about the real situation and as a tool to make a decision.

Some models that constitute autonomous differential equations are presented; Malthusian and logistic model for single population; two independent populations, competing model, and prey-predator model for two populations; and extension of prey-predator model involving three populations. In this thesis we will study the effect of harvesting on models.

The models are based on Lotka-Volterra model. All models involve harvesting problem and some stable equilibrium points related to maximum profit or maximum sustainable yield problem. The objectives of this thesis are to analyse, to investigate the stability of equilibrium point of the models and to control the exploitation efforts such that the population will not vanish forever although being exploited. The methods used are linearization method, eigenvalues method, qualitative stability test and Hurwitz stability test. Some assumptions are made to avoid complexity. Maple V software release 4 is used to determine the equilibrium points of the model and also to plot the trajectories and draw the surface. The single population model is solved analytically.

We found that in single population model, the existence of population depends on the initial population and harvesting rate. In model that involves two and three populations, the populations can live in coexistence although harvesting is applied. The level of harvesting, however, must be strictly controlled.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia
sebagai memenuhi keperluan untuk ijazah Master Sains

ANALISIS KESTABILAN BAGI BEBERAPA MODEL POPULASI DENGAN TUAIAN

Oleh

SYAMSUDDIN TOAHA

September 2000

Pengerusi : Professor Madya Dr. Harun Bin Budin

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Matematik gunaan, yang bererti penggunaan matematik kepada penyelesaian masalah, adalah suatu perkara yang sangat bagus dan menarik. Ia adalah inti pati bagi pendekatan berasaskan teori kepada sains dan kejuruteraan. Ia boleh merujuk kepada kegunaan matematik dalam berbagai bidang. Model matematik digunakan untuk meramalkan kelakuan sistem. Kelakuan ini kemudian ditafsirkan sehingga kita boleh mengetahui kelakuan bagi situasi yang sebenarnya.

Kita dapat menggunakan bahasa matematik untuk menjelmakan fenomena ekologi kepada model matematik, termasuk perubahan populasi dan bagaimana populasi dalam satu sistem dapat mempengaruhi populasi yang lain. Model itu diharap memberi kita banyak maklumat tentang situasi yang sebenarnya dan sebagai alat untuk membuat keputusan.

Beberapa model berupa persamaan pembezaan berautonomi dipersembahkan, model Mathusian dan model logistik untuk satu populasi, dua populasi yang bebas, model persaingan, dan model mangsa-pemangsa untuk dua populasi, dan perluasan bagi model mangsa-pemangsa yang meliputi tiga populasi Dalam tesis ini kita akan mengkaji kesan tuaian ke atas model

Model-model itu berasaskan model Lotka-Volterra Kesemua model meliputi masalah penuaian dan beberapa titik keseimbangan yang stabil dihubungkan kepada masalah keuntungan maksimum dan hasil berterusan yang maksimum Matlamat tesis ini adalah untuk menganalisis, menyiasat kestabilan titik keseimbangan model, dan mengawal usaha eksploitasi supaya populasi tidak akan pupus untuk selama-lamanya meskipun populasi dieksploitasikan Kaedah yang digunakan adalah kaedah pelinearan, kaedah nilai eigen, ujian kestabilan kualitatif dan ujian kestabilan Hurwitz Beberapa anggapan dibuat untuk menghindari kerumitan Perisian Maple V (versi 4) digunakan untuk menentukan titik keseimbangan bagi model dan juga memplot trajektori dan melukis permukaan Bagi model satu populasi, model diselesaikan secara analitik

Kita mendapati bahawa dalam model satu populasi untuk populasi wujud ia mesti bergantung kepada populasi awal dan kadar tuaian Pada model yang melibatkan dua atau tiga populasi, populasi dapat hidup bersama meskipun dikenakan tuaian Tentunya, tahap tuaian dikawal dengan ketat

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I certify that an Examination Committee met on 25 September 2000 to conduct the final examination of Syamsuddin Toaha on his Master of Science thesis entitled "Analysis of Stability of Some Population Models with Harvesting" in accordance with Universiti Pertanian Malaysia (Higher Degree) Act 1980 and Universiti Pertanian Malaysia (Higher Degree) Regulation 1981 The Committee recommends that the candidate be awarded the relevant degree Members of the Examination Committee are as follows

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DECLARATION

I hereby declare that the thesis is based on my original work except for quotations and citations which have been duly acknowledged I also declare that it has not been previously or concurrently submitted for any other degree at UPM or other institutions

SYAMSUDDIN TOAHA

Date 25 September 2000

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CHAPTER I

INTRODUCTION

Many problems in the world usually involve continuously changing quantities such as distance, velocity, acceleration, or force On the other hand, many problems in the life sciences deal with aggregates of individuals that are clearly discrete rather than continuous Since derivatives, and hence differential equations, are meaningful only for quantities that change continuously, one might think that differential equations would arise only in the formulation of physical problems This is not the case However, if the population in a biological problem is sufficiently large, it can usually be approximated, or modelled, by a continuous system in which the rate of change can be expressed as derivatives and the behaviour of the system can be described by a system of differential equations

Population dynamics is the study of changes in the populations of systems and how the population of one system can affect the population of another Population change may have important economic and social consequences For example, the farmer wants to know how large the pest population is when his crop is most vulnerable and what effects pesticide spraying will have, and the fisherman wants to know what effects fishing quotas will have on fishing stock and consequently on fishing catches

There are three main ways in which animals of two different populations can interact. They can help each other's population growth, or can hinder such growth or one can help and the other can hinder. These are respectively known as the symbiotic, the competitive species and the prey-predator systems. In the last mentioned, one species, the predator feeds on the other, the prey; for example, foxes catch and kill rabbits, and sharks consume the small fish in the sea. Hence, the presence of sharks increases the death rate of small fish and the presence of small fish increases the supply, and hence the birth rate of sharks. In the competitive species system, the two populations compete for the same resources, usually food. In the prey-predator system it is not clear how the populations of the species vary and a mathematical model may help us to predict the behaviour of the populations.

This research presents a model of the population behaviour by using deterministic model which is presented as a system of differential equations. The models are Malthusian model, logistic model, while the model that involves two or more populations is based on Lotka-Volterra model. Chapter II reviews the literatures in order to analyse and investigate the stability of the system and also give a brief description pertaining to population and harvesting. Chapter III presents one population behaviour model which follows Malthusian model and logistic growth. The model is extended such that it involves harvesting problem. Chapter IV presents the interaction of the two-population model with harvesting which covers two independent populations model, competing model, and prey-predator model. Chapter V presents how the model is extended to three populations model with harvesting and divided into two cases. The first kind of model presents model with two preys and one predator without harvesting and model with rate of harvesting proportional to the size of predator population. The second one shows model with one prey and two predators. Firstly, this kind of model is investigated without harvesting then the model is modified to incorporate harvesting function. The two kinds of harvesting, that is, constant rate of harvesting and rate which is proportional to the size of population are considered. Chapter V describes conclusions for all kinds of models.

All the considered models will be analyzed for the stability of their equilibrium point, if any, and we will determine the necessary and sufficient conditions for the existence of the equilibrium point, if possible. For this purpose, some assumptions are considered. The methods used to study the stability of the equilibrium point are linearization methods, eigenvalue method, qualitative stability test, and also Hurwitz stability test. The objectives of this research are to control the model, include the parameters, initial values and level of harvesting so that the populations will not vanish although they are harvested and to determine the kind of stability of positive equilibrium point. Some of the equilibrium points, which are stable, are related to maximum profit problem, known as maximum economic rent (*MER*), and to maximum sustainable yield (*MSY*) problem. In addition, in some cases, the level of harvesting needs to be controlled so that the equilibrium point is stable and gives either *MER* or *MSY*.

The model that involves a single population is solved analytically and the graph of the solution is also described. While for the other models, the stability of their equilibrium point is investigated by the linearization method and plotting the trajectories around the equilibrium point for non-linear systems. Maple V software is used to display the solution or trajectories of the model graphically. Such a graphical display is often much more illuminating and helpful in understanding and interpreting the solution of the model. The related computer programmes are given in the appendix, and the books on Maple V by various authors are listed in the list of references.

CHAPTER II

LITERATURE REVIEW

This chapter reviews the notion of dynamical system and its stability, thus, several definitions and theorems are given Linear system and almost linear system are also considered since they provide many useful properties Since most of dynamical system problems are unsolvable, another approach is presented to give qualitative information of the system Furthermore, population model and its interaction and also harvesting function are described briefly

Dynamical Systems

A continuous, finite-dimensional dynamical system is described by a first-order vector-differential equation

$$x(t) = f\left(x(t), t\right), \tag{2 1}$$

where $x(t)$ is the value at time t of the n-dimensional state vector

$$x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix},$$

n is the order of the system

The solution with the given initial conditions x_0 at t_0 is denoted by $x(t, x_0, t_0)$, or by $x(t)$, if no confusion can arise A solution is also called a *trajectory* or a *motion* or a *path* or an *orbit*, it is represented by a curve in the *n*-dimensional state space

The dynamical system (2 1) is called *stationary* if the vector function f does not depend explicitly on time Such dynamical system is called *autonomous*, and is governed by the equation

$$\dot{x}(t) = f(x(t)) \tag{2 2}$$

The motions of an autonomous system are invariant for a translation of time, that is, if $x(t, x_0, t_0)$ is a motion of (2 2), then

$$x(t; x_0, t_0) = x(t + T; x_0, t_0 + T)$$

for all t, x_0, t_0, and T

A dynamical system is called *linear* if the function f is linear with respect to x Then the system equation is

$$x(t) = J(t)\, x(t),$$

where $J(t)$ is an (n, n) matrix The equation of a linear autonomous system is

$$x(t) = Jx(t)$$

If for all t

$$f(x_e, t) = 0, \tag{2 3}$$

then

$$x(t; x_e, t_0) = x_e$$

for any t_0 A point x_e satisfying (2.3) is called an *equilibrium point* or a *critical point* or an *equilibrium state*. Therefore, a solution which passes through x_e at some time, remains there for all time. The solution is called the *equilibrium solution* or *constant trajectory*, and if $x_e = 0$, it is called the *null solution*.

Definitions of Stability and Limit Cycle

Let x_e be an equilibrium point of the dynamical system

$$\dot{x} = f(x, t),$$

with

$$f(x_e, t) = 0 \text{ for all } t.$$

Definition 2.1 The equilibrium point x_e, or the equilibrium solution $x(t) = x_e$ is said to be *stable* if for any given t_0 and positive ε, there exists a positive number $\delta(\varepsilon, t_0)$ such that

$$\| x_0 - x_e \| < \delta$$

implies

$$\| x(t; x_0, t_0) - x_e \| < \varepsilon$$

for all $t \geq t_0$.