UNIVERSITI PUTRA MALAYSIA

COSMIC CRYSTALLOGRAPHY:
CCP-INDEX OF THURSTON MANIFOLD

TOH SING POH

FSAS 2002 21

COSMIC CRYSTALLOGRAPHY:
CCP-INDEX OF THURSTON MANIFOLD

TOH SING POH

MASTER OF SCIENCE
UNIVERSITI PUTRA MALAYSIA

2002

COSMIC CRYSTALLOGRAPHY:
CCP-INDEX OF THURSTON MANIFOLD

By

TOH SING POH

Thesis Submitted to the Graduate School, Universiti Putra Malaysia, in Fulfilment of the Requirements for the Degree of Master of Science

January 2002

To my understanding wife,

Chong Hooi Sun

Abstract of the thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of requirement for the degree of Master of Science.

COSMIC CRYSTALLOGRAPHY:
CCP-INDEX OF THURSTON MANIFOLD

By

TOH SING POH

January 2002

Chairman : Assoc. Prof. Dr. Hishamuddin Zainuddin

Faculty : Science and Environmental Studies

The universe is assumed to have negative spatial curvature with 3-dimensional hyperbolic Thurston manifold as the fundamental domain. The universal covering space of the universe is tessellated by fundamental domain through holonomy group. Collecting correlated pair method (*CCP*-method) is implemented to this model to compute *CCP*-index which indicates the multi-connectedness of the universe.

Abstrak thesis yang dikemukakan kepada Senat Universiti Putra Malaysia
sebagai memenuhi keperluan untuk ijazah Master Sains.

**KOSMIK KRSYTALLOGRAPI:
INDEKS CCP BAGI THURSTON MANIFOLD**

Oleh

TOH SING POH

Januari 2002

Pengerusi : Prof. Madya. Dr. Hishamuddin Zainuddin

Fakulti : Sains dan Pengajian Alam Sekitar

Alam semesta dianggap mempunyai kelengkungan ruang negatif dan manifold hiperbolik Thurston diambil sebagai domain asasnya. Dengan menggunakan kumpulan holonomi, ruang liputan umum alam semesta diteselasi oleh domain asasnya. Kaedah himpunan pasangan berkorelasi (*CCP*) kemudian digunakan ke atas model sedemikian untuk menghitung indeks *CCP* bagi manifold Thurston yang menunjukkan kaitan berganda alam semasta.

ACKNOWLEDGEMENTS

I would like to thank my dear supervisor, Assoc. Prof. Dr. Hishamuddin Zainuddin, for his introduction of the field of cosmic topology to me. I am very grateful for the freedom he has given to me in doing the research. He is undoubtedly a wise researcher knows well the ways to conduct open and sincere discussions with me, and he masters well the skills in fine tuning guidance from himself and opinions from me. I also truly appreciate with his positive assessment about my research ability that greatly strengthens my endurance to face challenges in doing this research.

I would also like to thank my co-supervisors, Prof. Madya Dr. Zainal Abidin Talib, Dr. Zainul Abidin Hassan and Dr. Zaidan Abdul Wahab for spending their invaluable time in examining my thesis.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF SYMBOLS AND ABBREVIATIONS

□	End of definition, proposition or theorem
END	End of proof
$\mathcal{R}$	*CCP*-index
$\mathcal{A}$	Number of objects in fundamental domain
$\mathcal{B}$	Number of nearest copies of fundamental domain
$\mathcal{M}$	4-dimensional spacetime
$\mathcal{H}^3$	Hyperbolic space
$\mathcal{S}$	Curvature radius
k	Curvature constant
$R(t)$	Scale factor
$\mathcal{T}$	Topology
$\rightleftharpoons$	Equivalence relation
$\sim$	Homotopy between paths, loops or functions
$\approx$	Same homotopy type of spaces X and Y
$H(s,t)$	Homotopy function between paths
$F(s,t)$	Homotopy function between maps
$[\alpha]$	Homotopy class of path α
$\pi_1(X,x_0)$	Fundamental group of space X at base point x_0
$\{e\}$	Fundamental group of a point
$\cong$	Isomorphic to

$\mathbb{M}$	Metric space
$\mathbb{R}^3$	3-dimensional Euclidean plane
$\mathbb{R}^n$	n-dimensional plane
S^n	n-sphere
I	Unit interval
id_X	Identity maps on X
Σ	Fundamental domain
Γ	Holonomy group
N_g	Number of generators
d_{co}	Comoving distance
R_{LSS}	Radius of last scattering surface
p	Covering map
U	Canonical neighborhood
$\hat{X}$	Covering space of X
ϕ	Covering morphism
FL	Friedmann-Lemaître
RW	Robertson-Walker
MCM	Multiply connected model
SCM	Simply connected model
FP	Fundamental Polyhedron
LSS	Last scattering surface
CMB	Cosmic microwave background

PSH	Pair separation histogram
EPSH	Expected pair separation histogram
MPSH	Mean pair separation histogram

CHAPTER 1

INTRODUCTION

1.1 What is Topology?

Any subset of the plane (2 dimension) or the space (3 dimension) is called a figure. Two figures are said to be congruent if they are identical except for position in space. The common properties occupied by the congruent figures are called geometrical properties. Geometry is the study of common geometric properties of congruent figures. In geometry, the movements allowed are translation, rotation and reflections. These movements are referred as rigid motions, in which the distance between any two points of the figure is not changed. Under the rigid motions, the geometric properties are invariant.

In topology, contrasted in geometry, the movements allowed is elastic motion, in which the distance between two points could be changed. In moving a figure, we can stretch, shrink, twist, pull and bend the figure. We can even cut the figure, but then sew the cut exactly as it was before, to make sure the points close together before cut is still close together after the cut is sewed up. At this point, it is worth to point out that there is a way to define topology as a study of continuity. The elastic motions that preserve the continuity of the figure are referred as continuous deformation. On the contrary, it is forbidden in topology to force two different points to coalesce into just one point. Two figures are topologically equivalent or homeomorphism if and only if one figure can be transformed into the other by a continuous deformation. For instance, it is easily imagined that a circle shaped rubber band can be continuously stretched to become it's topological equivalent unfilled square shape. On the other hand, a disc with a hole in the center is topologically different from a filled square because one cannot create or destroy holes by continuous deformations. The topological properties of a figure are those that are invariant under elastic motions and so enjoyed by all topologically equivalent figures. In the previous rubber band example, a red spot on the rubber band before stretched will remain on the rubber band after the geometrical shape of the rubber band is changed, so "a spot on the rubber band" is a topological property here.

Any topological property of a figure is also a geometric property of that figure, but many geometric properties are not topological properties. Thus, topology can be thought of as a kind of generalization of geometry. Although by using topological methods one does not expect to be able to identify a geometrical figure as being a doughnut or a coffee cup, one does expect to be able to detect the presence of gross features such as holes etc. [1]

1.2 A Brief History of Cosmological Modeling

One of the fundamental tasks of cosmologist is to determine the structure or the shape of the universe. Regarding to the physical extension of space, there is an oldest question about it: is the space finite or infinite?

Newtonian physical space was constructed in an absolute reference frame and is mathematically identified with the infinite Euclidean space $\mathbb{R}^3$. In his gedanken experiment of a bucket containing water to show the existence of absolute reference frame, Newton reasoned that both the rest bucket and the fixed stars make the water surface flat. In the other case, the fixed stars will cause the concave shape of water surface. According to Newton, the latter case shows the absolute rotation of the bucket with respect to the absolute reference frame. Mach challenged the reasoning from Newton and stated that a rotating body in a non-rotating universe or a non-rotating body in a rotating universe should give the same result: the concave shape of the water surface. By inference of Mach, the concave shape of the water surface is not due to the absolute rotation of the bucket, but is a consequence of the interaction from the mass of universe upon the bucket, which is rotating with respect to them. Thus, Mach concluded that the inertial mass of a body should result from the contributions of all the masses in the universe. In a homogeneous Newtonian universe with non-zero density, these masses summed to infinity, this gives rise to the inertial problem and thus Mach supported the idea of a finite universe in order for it to have a finite local inertia.

To solve the inertia problem, Einstein (1917) assumed in his static cosmological solution that space was a positively curved hypersphere without boundary. Einstein was convinced that the hypersphere provided not only the metric of cosmic space, but also its global structure. Indeed Einstein's general relativity deals only with local geometrical properties of the universe, such as its spacetime curvature (which is determined by the density of matter-energy), but not with its global characteristics, namely its topology. The global shape of space is not merely dependent on the metric. On the other hand, de

Sitter noticed that the Einstein's solution admitted a different space, the three dimensional projective space, constructed by identifying antipodal points of the hypersphere. While Einstein proved that elliptical space is the only variant of spherical space, he preferred --- based on aesthetical consideration rather than physical reasoning --- the latter due to its property of simply connectedness. Indeed Einstein's conclusion is true only in the case of dimension two; In dimension three, there are an infinite number closed topological variants of the spherical space, not known by anyone in 1920.

Friedmann and Lemaître are generally considered as the discoverers of the big bang concept that serves as non-static solutions for relativistic cosmology. They stated that the homogeneous isotropic universe models (FL models) admit spherical, Euclidean or hyperbolic spacelike section according to the sign of their constant curvature. Even in that time, Friedmann had already pointed out that several topological spaces could be used to describe the same solution of Einstein's equations and he also predicted the possible existence of "ghost" images of astronomical sources arising from the multi-connectedness property of the space. While the cosmological solution derived by Einstein, de Sitter and Friedmann has a positive spatial curvature and thus obviously has a finite volume, Friedmann with the lack of knowledge about the hyperbolic space, emphasized the possibility of compactifying space by suitable identifications of points.

On the other hand, Lemaître assumed positive space curvature, and he preferred the projective space. He also noticed the possibility of hyperbolic and Euclidean spaces with finite volumes for describing the physical universe.

It is frequently implied that the (closed) spherical (S^3) model has a finite volume whereas the (open) Euclidean ($\mathbb{R}^3$) and hyperbolic ($\mathcal{H}^3$) models have infinite volumes. These correspondences between space curvature radius and the volume is true only in the very special case of a simply connected topology and zero cosmological constant. According to Friedmann, in order to know if a space is finite or infinite, it is not sufficient to determine the sign of its spatial curvature, additional consideration arising from topology is necessary.

While the possibility of the multiply connected model seems to disobey the Occam's razor principle, quantum cosmology provided another context of "simple model" by suggesting that the smallest closed hyperbolic manifolds are favored. On the other hand, astrophysical observations (e.g. [25]) suggest that we live in a negatively curved F-L universe (unless the cosmological constant is positive and large enough). Combining these two suggestions imply the hyperbolic space have a finite volume, and in that case it must be multi-connected. [2]

Following the topological consideration, the fundamental question of cosmology regarding to the structure of the universe is then extended to: Is space finite or infinite, oriented or not, made of one piece or not, has it holes or handles, what is its global shape?

1.3 On the Consideration of Multiply Connected Model (MCM)

1.3.1 Direct Implications of the MCM

There exist many multiply connected three-dimensional spaces of constant curvature ($k = -1, 0$, or 1) that can each be represented by a fundamental polyhedron (FP) with the faces of the polyhedron be identified in pairs in some way [see section 3.1]. The FP is embedded in the simply connected space of the same curvature ($\mathcal{H}^3$, $\mathbb{R}^3$ or S^3). The simply connected space is then the covering space, which is tiled by copies of the fundamental polyhedron. If the fundamental polyhedron of the Universe was smaller than the sphere with horizon radius in the universal covering space, then the apparent observable Universe as a part of the covering space would contain multiple apparent copies of the single physical Universe. A single object located in the physical Universe could then be seen as multiple images in different sky directions and at different distances. The existence of these topological images, called ghost images is the key evidence of the multi-connectedness of the Universe.

1.3.2 Compatibility between Multiply Connected Model and Simply Connected Model (SCM)

(a) The Friedmann-Lemaître model (the hot big bang model) with constant curvature is previously confined to the simply connected cases with compact spatial volume for hypersphere and infinite for both Euclidean and hyperbolic cases. In multiply connected model, Euclidean manifolds with flat curvature and the hyperbolic manifolds with negative curvature could be finite or infinite, while the model with positive curvature is still spatially compact. For instance, among 18 Euclidean space-forms with different topologies [see section 3.1], six of them are compact.

(b) Cosmological principle stated that at any given cosmic time, the universe is homogeneous and isotropic. Cosmological principle is assumed in deriving Robertson-Walker metric [Appendix H] and it implies the constancy of the space curvature and the space is spherically symmetric about each point.. However, as the space curvature is a local property, local homogeneity and isotropy of the Universe does not necessarily imply global homogeneity and isotropy and in fact only locally homogeneity and isotropy are required by Friedmann-Lemaître model. It has been shown that [3], in the 2-dimensional simulated universe (i) locally homogeneous and isotropic 2-torus model appears to be globally homogeneous but anisotropic, (ii) locally homogeneous and isotropic 2 dimensional Klein bottle model is globally both inhomogeneous and anisotropic. These results can be extrapolated to the three-dimensional cases. Only projective space is locally and globally both homogeneous and isotropic, even though it is multiply connected. All of these show the richness of the possibilities and we have to recheck the assumption of cosmological principle whenever adopting a particular model. However, it is stated [4] that globally anisotropy models do not contradict observations, since the homogeneity of space and the local isotropy ensure the complete isotropy of the Cosmic Microwave Background. However, the global anisotropy can influence the spectrum of density fluctuations. On switching from SCM to MCM, we have to re-examine every single result hold previously.

(c) In the twin paradox, the best-known thought experiment of special relativity, the twin departs from a point and turns back to the point will be younger than the sedentary twin. That is because of the asymmetry of the reference frame arising from the local acceleration unavoidable by the traveler twin for him to turn back. This version of standard twin paradox is however considered in the context of simply connected space where any two points within the space has only single geodesic to connect them. In multiply connected space, there are more than one geodesic connecting any two points, and so the traveler twin could return to the departing point without encountering any acceleration or direction change. In this case, the traveler twin avoided jumping from one inertial frame into another, the twin paradox thus seems to reemerge at first glance. To solve the paradox, the asymmetry due to a non-trivial topology (that is multiply connected topology) has to be considered and here we have to use the homotopy theory [see section 3.2]. Two loops (a loop is a path with starting point and ending point coincide) are said to be homotopic if they can be continuously deformed into one another. Any loops that can be continuously deformed into one another are said in the same homotopy class. In addition, any loops continuously deformed into a point is said to be homotopic to class $\{0\}$. To solve the twin paradox in non-trivial topology,