UNIVERSITI PUTRA MALAYSIA

BOUNDARY LAYER FLOW AND HEAT TRANSFER OF CARBON NANOTUBE OVER MOVING PLATE AND STRETCHING/SHRINKING SHEET WITH STABILITY ANALYSIS

NUR SYAZANA BINTI ANUAR

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BOUNDARY LAYER FLOW AND HEAT TRANSFER OF CARBON NANOTUBE OVER MOVING PLATE AND STRETCHING/SHRINKING SHEET WITH STABILITY ANALYSIS

By

NUR SYAZANA BINTI ANUAR

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfillment of the Requirements for the Degree of Doctor of Philosophy

January 2021

DEDICATION

To my beloved parents;

Anuar Bin Mat Noh
&
Norizan Binti Wahab

and to all of my love;

siblings, lecturers & friends

-Thank you for everything-

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Doctor of Philosophy

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January 2021

Chairman : **Norfifah binti Bachok @ Lati, PhD**
Faculty : **Science**

This study has been undertaken to solve numerically the boundary layer flow and heat transfer over various geometric surfaces such as horizontal and vertical moving plate, stretching/shrinking sheet and stretching/shrinking cylinder of carbon nanotubes subjected to different effects (slip, suction, magnetohydrodynamic, chemical reaction). These problems took into account two types of carbon nanotubes, namely single–wall carbon nanotubes (SWCNT) and multi–wall carbon nanotubes (MWCNT) that were dispersed into base fluids (water and kerosene). The governing partial differential equations were transformed into a system of nonlinear ordinary differential equations using a similarity transformation which was then solved numerically using a `bvp4c` function in `MATLAB` software. Numerical results for the local skin friction and local Nusselt number, which represents the heat transfer rate at the surface as well as velocity, temperature and concentration profiles were presented graphically and discussed in detail. The results show that all of the problems possessed dual solutions for a certain range of parameter, hence a stability analysis was performed to verify the stability of the solutions. The local skin friction, the local Nusselt number and concentration are significantly influenced by all the parameters studied, such as the nanoparticle volume fraction, moving parameter, slip parameter, suction parameter, mixed convection parameter, stretching/shrinking parameter, homogeneous parameter, heterogeneous parameter, nonlinear parameter, magnetic parameter, curvature parameter, Schmidt number and chemical reaction parameter. It was noticed that the nanoparticle volume fraction can increase the heat transfer rate and accelerates the cooling process. Furthermore, the kerosene–SWCNT offers a higher heat transfer efficiency compared to other carbon nanotubes. From the stability analysis, it was found that the first solution is stable, while the second solution is unstable.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

ALIRAN LAPISAN SEMPADAN DAN PEMINDAHAN HABA DALAM NANOTIUB KARBON TERHADAP PLAT BERGERAK DAN PERMUKAAN MEREGANG/MENGECUT DENGAN ANALISIS KESTABILAN

Oleh

NUR SYAZANA BINTI ANUAR

Januari 2021

Pengerusi : **Norfifah binti Bachok @ Lati, PhD**
Fakulti : **Sains**

Kajian ini telah dilakukan untuk menyelesaikan secara berangka aliran lapisan sempadan dan pemindahan haba ke atas pelbagai permukaan geometri seperti plat bergerak secara mendatar dan menegak, permukaan meregang/mengecut dan silinder meregang/mengecut dalam nanotiub karbon tertakluk kepada kesan yang berbeza (gelinciran, sedutan, magnet-hidrodinamik, tindak balas kimia). Masalah ini mengambil kira dua jenis nanotiub karbon, iaitu dinding tunggal nanotiub karbon (SWCNT) dan dinding pelbagai nanotiub karbon (MWCNT) yang diserakkan ke dalam cecair asas (air dan kerosin). Persamaan pembezaan separa menakluk dijelmakan menjadi sistem persamaan pembezaan biasa yang tidak linear menggunakan penjelmaan keserupaan yang kemudian diselesaikan secara berangka menggunakan fungsi `bvp4c` dalam perisian `MATLAB`. Keputusan berangka bagi geseran kulit setempat dan nombor Nusselt setempat yang mewakili kadar pemindahan haba pada permukaan serta profil halaju, suhu dan kepekatan dipersembahkan dalam bentuk graf dan dibincangkan dengan terperinci. Keputusan kajian menunjukkan bahawa semua masalah mempunyai penyelesaian dual bagi sesetengah julat parameter, oleh itu analisis kestabilan dilakukan untuk mengesahkan kestabilan penyelesaian. Geseran kulit setempat, nombor Nusselt setempat dan kepekatan dipengaruhi oleh semua parameter yang dikaji seperti pecahan isipadu nanozarah, parameter bergerak, parameter gelinciran, parameter sedutan, parameter olakan campuran, parameter meregang/mengecut, parameter heterogen, parameter homogen, parameter tidak linear, parameter magnet, parameter kelengkungan, nombor Schmidt dan parameter tindak balas kimia. Didapati bahawa pecahan isipadu nanozarah dapat meningkatkan kadar pemindahan haba dan mempercepatkan proses penyejukan. Tambahan lagi, kerosin-SWCNT menawarkan

kecekapan pemindahan haba yang lebih tinggi berbanding dengan nanotiub karbon yang lain. Dari analisis kestabilan, didapati bahawa penyelesaian pertama stabil manakala penyelesaian kedua tidak stabil.

ACKNOWLEDGEMENTS

بِسْمِ اللَّهِ الرَّحْمَٰنِ الرَّحِيمِ

In the name of Allah, the most Gracious and the most Merciful

Alhamdulillah, all praise be to Allah S.W.T for granting me the courage, strength, patience and perseverance to overcome all the obstacles that I faced throughout my PhD journey.

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Last, but definitely not the least, I would like to express my deepest appreciation to my beloved parents for their unconditional love, sincere encouragement and support especially during the hard times. Many thanks and appreciation also go to my colleague and people who have willingly helped me out with their abilities. This accomplishment would not have been possible without them. Thank you.

This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfilment of the requirement for the degree of Doctor of Philosophy. The members of the Supervisory Committee were as follows:

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Universiti Putra Malaysia

Date:

Declaration by graduate student

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This is to confirm that:

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a, b	concentration of species A and B respectively, $\left[kgm^{-3}\right]$
A, B	chemical species A and B respectively, $[-]$
a_0	constant concentrations of species A, $\left[kgm^{-3}\right]$
B	strength of magnetic field, $[T]$
b_0, c, d	constant, $\left[s^{-1}\right]$
C	concentration, $\left[kgm^{-3}\right]$
C_p	specific heat at constant pressure, $\left[J\ kg^{-1}K^{-1}\right]$
C_f	skin friction coefficient, $[-]$
C_w	surface concentration, $\left[kgm^{-3}\right]$
C_∞	free stream concentration, $\left[kgm^{-3}\right]$
D	mass diffusivity, $\left[m^2s^{-1}\right]$
D_A, D_B	diffusion coefficient of chemical species A and B, $\left[m^2s^{-1}\right]$
f	dimensionless stream function, $[-]$
$\mathbf{g}$	gravitational acceleration, $\left[ms^{-2}\right]$
g	dimensionless concentration of chemical species, $[-]$
Gr	local Grashof number, $[-]$
h	dimensionless concentration of species B, $[-]$
k	thermal conductivity, $\left[Wm^{-1}K^{-1}\right]$
k_1, k_s, k_{hm}, k_{ht}	reaction rate constant, $\left[s^{-1}\right]$
K	homogeneous reaction parameter, $[-]$
K_s	heterogeneous reaction parameter, $[-]$
L	characteristic length of a sheet, $[m]$
L_1	slip factor, $[m]$
L_s	initial length of slip factor, $[m]$
M	magnetic parameter, $[-]$
MWCNT	Multi–wall carbon nanotubes, $[-]$
n	nonlinear parameter, $[-]$
Nu_x	Nusselt number, $[-]$
p	pressure, $\left[kgm^{-1}s^{-2}\right]$
Pr	Prandtl number, $[-]$
q_w	surface heat flux, $\left[Wm^{-2}\right]$
r	cylindrical coordinates along the cylinder axis, $[m]$
R	radius of the cylinder, $[m]$
R_r	reaction rate parameter, $\left[s^{-1}\right]$
Re_x	Reynolds number, $[-]$
s	suction parameter, $[-]$
Sc	Schmidt number, $[-]$
Sh_x	Sherwood number, $[-]$
SWCNT	Single–wall carbon nanotubes, $[-]$
t	time, $[s]$
T	fluid temperature, $[K]$
T_0	characteristic temperature of the sheet, $[K]$

T_w	wall temperature, $[K]$
T_∞	ambient temperature, $[K]$
u, v	velocity component along x-axis and y-axis, $\left[ms^{-1}\right]$
U	composite velocity, $\left[ms^{-1}\right]$
U_w	velocity at the boundary, $\left[ms^{-1}\right]$
U_∞	free stream velocity, $\left[ms^{-1}\right]$
$\vec{V}$	velocity vector, $\left[ms^{-1}\right]$
v_w	suction/mass transfer velocity, $\left[ms^{-1}\right]$
x, y	cartesian coordinate, $[m]$

Greek Symbols

α	thermal diffusivity, $\left[m^2s^{-1}\right]$
ν	kinematic viscosity, $\left[m^2s^{-1}\right]$
η	similarity variable, $[-]$
ψ	stream function, $\left[m^2s^{-1}\right]$
β	thermal expansion coefficient, $\left[K^{-1}\right]$
σ	electrical conductivity, $\left[Wm^{-1}K^{-1}\right]$
ϕ	dimensionless concentration, $[-]$
θ	dimensionless temperature, $[-]$
λ	moving parameter, $[-]$
Λ	mixed convection/buoyancy parameter, $[-]$
ε	stretching or shrinking parameter, $[-]$
τ	dimensionless time variable, $[-]$
τ_w	surface shear stress, $\left[kgm^{-1}s^{-2}\right]$
δ	ratio of the diffusion coefficient, $[-]$
$\bar{\delta}$	boundary layer thickness, $[-]$
γ	unknown eigenvalues, $[-]$
Ω	slip parameter, $[-]$
ρ	density, $\left[kgm^{-3}\right]$
μ	dynamic viscosity, $\left[kgm^{-1}s^{-1}\right]$
ρC_p	volumetric heat capacity, $\left[JK^{-1}m^{-3}\right]$
φ	nanoparticle volume fraction parameter, $[-]$
χ	curvature parameter, $[-]$
ζ	chemical reaction parameter, $[-]$
∇	del operator, $[-]$
∇^2	Laplacian operator, $[-]$

Subscripts

c	critical value, $[-]$
cnt	carbon nanotube, $[-]$
f	fluid, $[-]$
nf	nanofluid, $[-]$
s	solid nanoparticles, $[-]$
w	condition at the wall, $[-]$
∞	condition at infinity, $[-]$

Superscript

$'$	differentiation with respect to η, $[-]$

CHAPTER 1

INTRODUCTION

1.1 Preface

Fluid dynamics involves the study of a fluid in motion (White (2011)). Understanding fluid dynamics has been one of the major advances of the last hundred years in physics, engineering and applied mathematics. This is also the key for researchers to understand some of the most important applications such as artificial hearts, breathing machines, dialysis systems, design and analysis of aircraft and biomedical devices (Çengel and Cimbala (2006)). The essential part of the study of fluid dynamics is to transform the physical problems into mathematical models so that mathematical techniques/software can be used to solve them.

In this introductory chapter, we have included some basic definitions and concepts of fluid flow and dimensionless number. This chapter also includes the problem statement, aim, objectives, scopes and outline of the thesis.

1.2 Boundary Layer Theory

In 1904, Ludwig Prandtl established the famous "boundary–layer theory" for investigating a viscous flow over a solid surface and it revolutionized the concept of solving Navier Stokes equations. According to Prandtl theory, when a real fluid flows past a stationary solid boundary, the flow will be divided into two regions as (see Acheson (1990), Schlichting and Gersten (2000)):

1. A thin region near the surface of the object, where viscous effects cannot be ignored, and are as important as inertia.
2. Away from the surface of the object, where the viscous force is very small and can be neglected. The flow behavior is similar to the potential flow.

Although the boundary layer is thin, it plays an essential role in fluid dynamics and becomes an excellent method for analyzing the complex behavior of real fluids. The formation of the boundary layer can be seen in Figure 1.1.

Prandtl (1904) deduced that under some circumstances, a simplified form of the governing equations could be used. The boundary layer equations were then derived under the following two conditions (see Pletcher et al. (2016)):

1. The viscous layer must be thin compared to the characteristic streamwise

dimension of the object immersed in the flow, $\delta/L \ll 1$, where δ is the distance away from the wall at which velocity attains its free–stream value and L is the characteristic length of the wall.

2. The largest viscous term must have the same approximate magnitude as any inertia term.

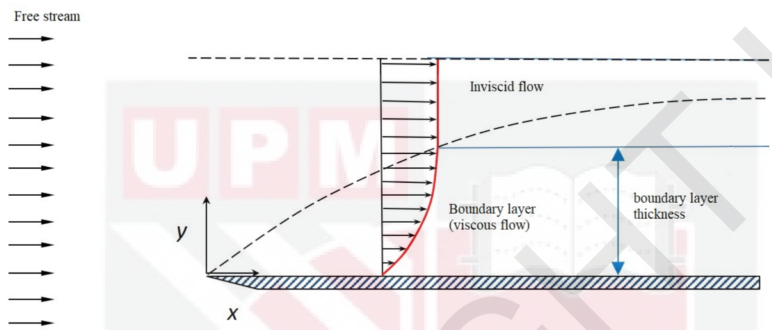

Figure 1.1: The boundary layer concept, (https://en.wikipedia.org/wiki/Boundary_layer_thickness)**.**

In boundary layer flow, there exist two types of flows, namely the laminar flow, which is defined by smooth streamlines and highly–ordered motion, and the turbulent flow, where it is defined by velocity fluctuations and highly–disordered motion (Çengel and Cimbala (2006)). The formation of the boundary layer is initially laminar, but minor flow disruptions tend to intensify at some critical distance from the leading edge and a transition phase takes place until the flow becomes turbulent in the boundary layer, as shown in Figure 1.2.

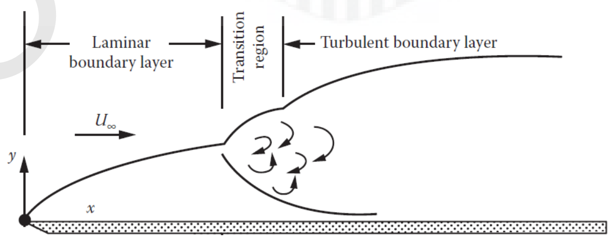

Figure 1.2: Laminar and turbulent boundary layers on a flat plate, (Kakaç et al. (2013))**.**

1.2.1 Stagnation Point Flow

A stagnation point is a point on an object's surface where the fluid velocity is zero, while the flow around a stagnation point is called the stagnation point flow (White (2011)). The stagnation region has the highest pressure, heat and mass decomposition. There are two types of stagnation point flows namely orthogonal and non–orthogonal stagnation point flows. Figure 1.3 shows an example of orthogonal stagnation point in two–dimensional flow. Initially, the study of two–dimensional (2D) stagnation point flow was first investigated by Hiemenz (1911). From his theory, the stagnation point flow illustrates the movement of fluid particles which are adjacent to the stagnation region of a solid surface for both moving and fixed bodies. Stagnation–point flow has been encountered in various applications in engineering and technological processes, including the cooling of nuclear reactors, cooling of electronic devices by fans and many other hydrodynamics processes (Sadiq (2019)). This thesis will explores the stagnation point that focuses only on the orthogonal flow.

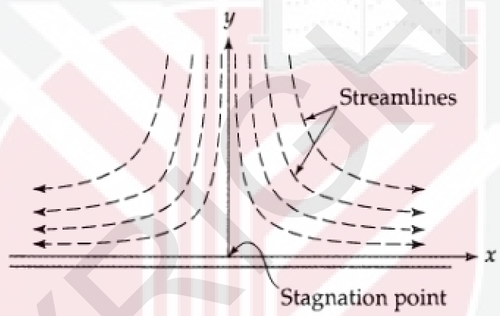

Figure 1.3: Orthogonal stagnation point flow, (https://www.transtutors.com/questions/potential-flow-near-a-stagnation-point-fig-4b-6-a-show-375083.htm)**.**

1.3 Fluid Flow Geometry and Orientation

Geometry and orientation are important factors in the study of fluid flow. When modeling fluid flow, the geometry of the surface will influence the flow of the fluids and other properties. In addition, different geometries present different mathematical expressions. Only the fluid flow on horizontal and vertical moving plate, stretching and shrinking sheet as well as stretching and shrinking cylinder were considered in this thesis.

1.3.1 Moving plate

The problem of boundary layer due to a horizontal moving flat plate is a classical problem that has been examined by Sakiadis (1961). This problem appeared in many industrial applications such as in material handling conveyors, cooling of an infinite metallic plate in a cooling bath, aerodynamic extrusion of plastic sheets and paper production (Mureithi et al. (2013)). In the case of a moving plate, the flat plate is assumed to move in the opposite direction or in the same direction to a parallel free stream with constant velocity (Bachok et al. (2012c)). Figure 1.4 presents the physical model of a horizontal moving plate.

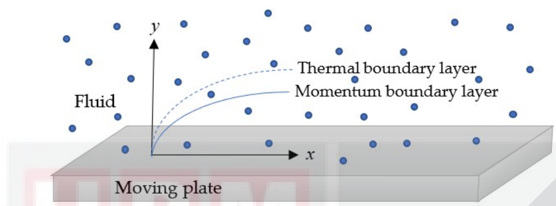

Figure 1.4: Physical model of horizontal moving plate.

Meanwhile in many physical situations, the moving plate may be moved vertically rather than horizontally. Under this circumstance, the flow and heat transfer characteristics are determined by two important mechanisms, namely the motion of the plate and the buoyant force. This situation of flow rising is due to the gravity.

1.3.2 Stretching and Shrinking Surfaces

Crane (1970) was a pioneering researcher who formulated the boundary layer flow caused by stretching sheet by extending the work of Sakiadis (1961). The stretching sheet can be defined as a surface that is stretched in its own plane or occurs when the velocity at the boundary is drawn away from the fixed point due to the application of stress. Flow and heat transfer phenomena over a stretching sheet has various theoretical and technical applications, especially in manufacturing processes including wire drawing, paper production, glass-fiber production, liquid metal, polymer sheet synthesis, artificial fibers and continuous stretching of plastic films (Salleh et al. (2010)). The physical model of stretching sheet is displayed in Figure 1.5.

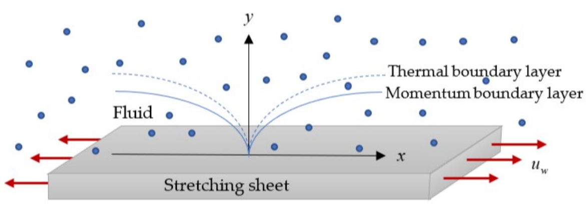

Figure 1.5: Physical model of stretching sheet.

More recently, the boundary layer flow caused by a shrinking sheet has fascinated the interest of researchers for its interesting characteristics. Miklavčič and Wang (2008) were the first to consider this unusual type of flow. For this flow configuration, mass suction is needed to preserve the vorticity generated by the shrinking sheet. An example of application for the shrinking sheet is the thermal shrinkage of thermoplastic sheets (Vdorenko et al. (1982)). When a sheet is reheated, the sheet tends to revert to its original shape when it is cooled. This is typically referred to as shrinkage. Both the kinematics of stretching/shrinking and simultaneous heating or cooling during such processes could be manipulated to the desired specifications on the quality of the final product.

Most researchers restricted their analysis on the flow of stretching or shrinking sheets, but not much analysis have been done on the flow of stretching or shrinking cylinders. The physical model of a stretching cylinder is shown in Figure 1.6. The boundary layer flow over a cylinder have been a field of interest for many theoretical and experimental researchers due to the broad range of applications such as in the coating of wires, hot rolling and polymer fiber spinning, which involve cylindrical geometries. The study of stretching cylinder in the boundary layer flow was first discussed by Wang (1988).

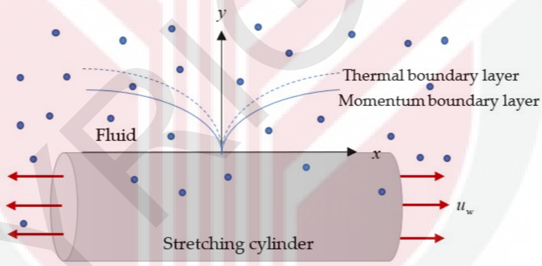

Figure 1.6: Physical model of stretching cylinder.

1.4 Heat transfer

Heat transfer is the study of energy transfer processes between material bodies which take place where a temperature gradient is present within a system or whenever two systems at different temperatures are brought into thermal contact. However, there appears, to be three rather basic and distinct modes of heat transfer. These are conduction, convection, and radiation, which are illustrated in Figure 1.7.

A brief overview of each mode is given below (Cengel (2003), Bergman and Lavine (2017)):

- **Conduction:** A heat transfer mechanism in which energy exchange takes place from a high temperature to a lower temperature region by means of kinetic motion or direct molecular impact and electron drift.

- **Convection:** the mechanism of heat transfer between a solid body and a flowing fluid or gas, and it engages the combined effects of fluid motion and conduction. The convective heat transfer can further be subdivided into various forms such as natural convection and forced convection. In natural convection, or also known as free convection, the flow is caused by natural means without the aid of an external mechanism. It is initiated by a change in the density of fluids produced by heating (buoyancy effect). Whereas, in forced convection, the fluid is forced to flow by an external source such as a compressor, pump, fan, etc. In addition, the flow mechanism which is simultaneously contributed by both force and free convection processes and acting simultaneously are known as mixed convection.

- **Thermal radiation:** or simply known as radiation is energy emitted by matter that is at a nonzero temperature. The energy of the radiation field is transferred through space in the form of electromagnetic waves or photons. In fact, radiation transfer occurs most efficiently in a vacuum.

However, this thesis only takes into consideration the heat transfer by convection because the problem only involves heat transfer between a solid and flowing fluid, which is more related to fluid dynamics.

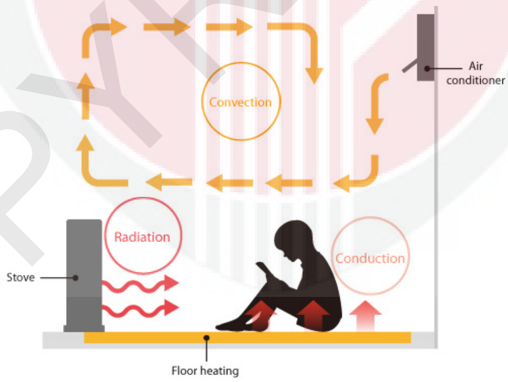

Figure 1.7: Modes of heat transfer, (https://www.cradle-cfd.com /media/column/ a74)**.**

1.5 Nanofluid

Nanofluid is a new kind of heat transfer medium, containing nanoparticles with sizes ranging below 100 nm that are uniformly and stably distributed in conventional heat transfer fluids such as water, kerosene oil, and ethylene glycol (Choi (1995)). These distributed nanoparticles, are generally made of metals (Cu, Al), metal oxides (Al_2O_3, CuO), carbides (TiC, SiC) and non-metals (e.g., graphite and carbon nanotubes). The cross section of the nanofluid structure consisting of the base fluid, nanoparticles and nanolayer in the solid/liquid interface is shown in the Figure 1.8. A very small amount of nanoparticles that are dispersed in a base fluid can greatly enhance the thermal conductivity of the nanofluid, hence allowing for more heat transfer (Eastman et al. (2001)). Nanofluids are used to improve energy efficiency and heat transfer in many thermal control systems, industrial applications, biomedicine, electronics and nuclear reactors (Wong and Leon (2010)). The applications of nanofluids are described in Das et al. (2007). Hence, nanofluid is not only of academic interest but also has many industrial applications.

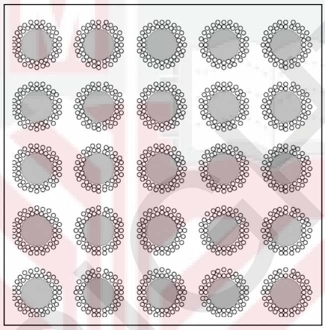

Figure 1.8: Schematic cross–section of nanofluid, (Yu and Choi (2003))**.**

Interestingly, theoretical research has also been performed to estimate the effective thermal conductivity of nanofluids by designing a suitable model such as a model suggested by Khanafer et al. (2003), Buongiorno (2006), Tiwari and Das (2007) and Kuznetsov and Nield (2013). However, this thesis only implemented the model proposed by Tiwari and Das (2007), also known as single–phase models. This model deals with the influence of nanoparticle volume fractions where the nanoparticles and base fluid are assumed to be in thermal equilibrium, no–slip condition and the local flow velocity is equal. The Tiwari and Das model was selected because this model has been successfully implemented in several nanofluid studies and the scientific consensus is that the effective thermal conductivity of nanofluid increases with increasing volume fraction of nanoparticles (Sarif et al. (2016)).

1.5.1 Carbon Nanotubes

Carbon is the most significant nanorevolutionary element. Carbon nanotubes (CNTs) are the smallest cylindrical molecules (10,000 times smaller than a human hair) that consist of rolled–up sheets of graphene (2–dimensional ultra–thin atomic layer). Both CNTs and graphene have exceptionally high mechanical resistance, elastic, thermal and electrical properties, ultra-smooth hydrophobic surface and high aspect ratio (Rafii-Tabar (2008)). CNTs may have a single outer carbon wall (single–wall carbon nanotubes), or they may consist of multiple walls (multi–wall carbon nanotubes) (see Figure 1.9).

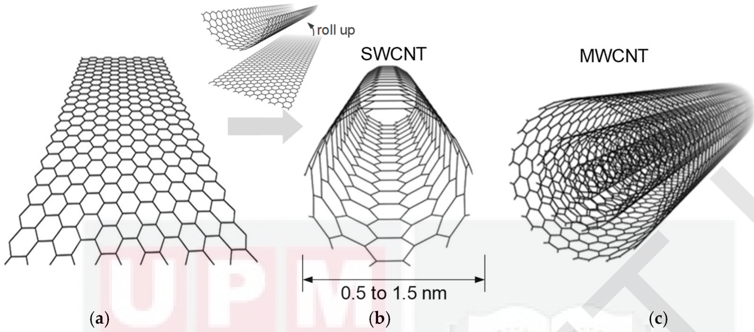

Figure 1.9: Schematics of (a) monolayer graphene (b) single wall carbon nanotube and (c) multi wall carbon nanotube, (Zhao et al. (2019))**.**

The multi–wall carbon nanotube (MWCNT) was first discovered by Iijima (1991), while the single–wall carbon nanotube (SWCNT) was first reported in 1993 by Iijima and Ichihashi (1993). Most SWCNTs have a diameter of approximately 1 nanometer with a tube length up to a million times longer. In contrast to SWCNTs, MWCNTs are nanotubes with more than one graphene cylinder and have an inner diameter ranging from 1 to 3 nanometer, while an outer diameter of approximately 10 nanometer (Kumar and Kumbhat (2016)). Some of the comparison between SWCNTs and MWCNTs are summarized in Table 1.1.

Table 1.1: Comparison between SWCNT and MWCNT, (Iijima and Ichihashi (1993), Eatemadi et al. (2014))**.**

SWCNT	MWCNT
• Single layer of graphene • A chance of defect is more during functionalization • Catalyst is required for synthesis • It can be easily twisted and is more pliable • Poor purity • Less accumulation in the body • Characterization and evaluation is easy	• Multiple layers of graphene • A chance of defect is less but once occurred it is difficult to improve • Can be produced without catalyst • It cannot be easily twisted • High purity • More accumulation in the body • It has very complex structure

CNTs have drawn a lot of interest due to their potential use as next generation electrical and structural materials. Thus, CNTs have been suggested for several nanoelectronics applications in the last decade, such as diodes and transistors, nano–antennas, nano–interconnects, lumped passives, plastic and transparent devices (see Hanson (2005), Li et al. (2009) and Valitova et al. (2013)). In addition, CNTs have been successfully used in pharmaceutical products and medicines due to their high surface area that can adsorb or combine with a broad range of therapeutic and diagnostic agents such as drugs, vaccines, genes, biosensors and antibodies (He et al. (2013)). Zhang et al. (2010) and Singh et al. (2012) showed that these molecules are delivered more efficiently and safely in cells when bonded to CNTs than by traditional methods. Hence, this is the main reason behind the choice of CNTs in the current study.

1.6 Type of effects

1.6.1 Partial slip

When fluid flows on a surface and it sticks to the surface, this is commonly referred to as the no slip condition. Meanwhile, in the situation where the fluid is not sticking to the solid boundary, the velocity slip arises. The idea of a slip boundary condition was first proposed by Navier (1827), which relates the tangential slip velocity, u_s, to the shear rate at the interface

$$u_s = b\left.\frac{\partial u}{\partial x}\right|_s \tag{1.6.1}$$

where x is the normal from the surface pointing into the liquid and b refers to the slip length. The subscript s denotes the value of the variable at the surface.

A schematic illustration of the definition of slip length is presented in Figure 4.2.1. Under no slip boundary conditions, the relative velocity between the fluid and the solid wall is zero at the wall. When the slip occurs, the extent of the slip is characterized by the slip length b.

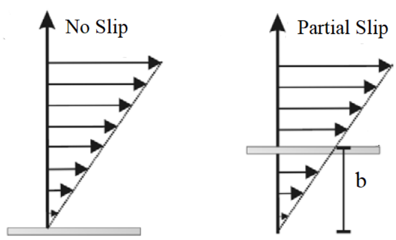

Figure 1.10: Schematic representation of no slip and partial slip boundary condition, (Neto et al. (2003))**.**

1.6.2 Suction

Suction is one of the boundary layer control methods which aim to reduce the drag on bodies in an external flow or reduce energy losses in channels. This method was suggested by Ludwig Prandtl in 1904 as one of the means to prevent or delay the boundary layer separation. Suction of fluid through the bounding surfaces can significantly change the flow field and accordingly will affect the rate of heat transfer from the bounding surfaces. The process of suction is important in many engineering activities, for instance, thermal oil recovery and design of thrust bearing and radial diffusers. Suction is also employed in chemical processes to remove reactants (Mukhopadhyay (2013)).

1.6.3 Magnetohydrodynamics (MHD)

Magnetohydrodynamics (MHD) is the branch of continuum mechanics that compromises with the movement of an electrically conducting fluid in the existence of a magnetic field. Examples of those fluids include liquid metals, plasmas, salt water, and electrolytes. The first work reported on MHD flows was initiated by Alfvén (1942). The set of equations that describe MHD are a combination of the Maxwells electromagnetism equations and Navier-Stokes equations of fluid dynamics. These differential equations must be solved simultaneously, either numerically or analytically.

When a conducting fluid moves through a magnetic field, an electric field and current may be induced. This effect polarizes the fluid and as a result the magnetic field is changed and the action of the magnetic field on these currents escalate the mechanical forces, consequently changing the fluid motion. Numerous applications of MHD are significant in the field of meteorology, aeronautics, cosmic fluid dynamics, solar physics, chemical engineering, geophysics and electronics.

1.6.4 Chemical reaction

In many chemical reactive processes such as combustion, biochemical systems and catalysis, mass transfer takes place by diffusive operations which involve the molecular diffusion of species in the presence of two types of chemical reactions, namely homogeneous and heterogeneous reactions (Chaudhary and Merkin (1994)). There are certain chemical reactions which have the ability to either progress gradually or do not progress at all, depending on whether they occur in the bulk of the fluid (homogeneous) or occur on some catalytic surfaces (heterogeneous). At several rates, the interaction between the homogeneous and heterogeneous reactions comprising the production and consumption both within the fluid and catalytic surfaces is overly complex. Notable applications of such reactions can be found in industry such as in hydrometallurgical, polymer production and manufacturing of ceramics and fog dispersion and formation (Imtiaz et al. (2019)).

1.7 Dimensionless Number

In the field of fluid mechanics, dimensionless numbers represent the ratio of different forces or the transport phenomenon involved in the fluid flow. These dimensionless numbers are useful to investigate the effect of different flow properties cumulatively on the mathematical results. Some of the significant dimensionless numbers utilized in fluid mechanics are given below:

1. **Prandtl Number**

   The Prandtl number, Pr was named after the German physicist Ludwig Prandtl and described as the ratio of momentum diffusivity (kinematic viscosity) to thermal diffusivity. The Prandtl number gives an idea about the sort of liquid as presented in Table 1.2. Numerically, we can formulate it as (Cengel (2003)):

$$\mathrm{Pr} = \frac{\text{momentum diffusivity}}{\text{thermal diffusivity}} = \frac{\mu C_p}{k} = \frac{\nu}{\alpha} \tag{1.7.1}$$

   where μ denotes the dynamic viscosity, C_p is the specific heat at constant pressure, k is the thermal conductivity, ν is the kinematic viscosity and α is the thermal diffusivity.

Table 1.2: Standard ranges of Prandtl numbers for different fluids, (Cengel (2003))**.**

Fluid	Prandtl number
Glycerin	$2000 - 100000$
Oils	$50 - 10000$
Light organic fluids	$5 - 50$
Water	$1.7 - 13.7$
Gases	$0.7 - 1.0$
Liquid metals	$0.004 - 0.03$

   When the Prandtl number is equal to 1, it indicates that both momentum and heat disperse through the fluid at about the same rate. However, as the value of Prandtl number becomes smaller ($\mathrm{Pr} \ll 1$), it means that heat disperses very quickly in liquid metals compared to the velocity (momentum) and consequently cause the thermal boundary layer to be thicker relative to the velocity boundary layer (Bergman and Lavine (2017)). On the contrary, a larger value of Prandtl number ($\mathrm{Pr} \gg 1$) indicates that the heat diffuses very slow in oils and causes a diminished thermal boundary layer thickness (Bergman and Lavine (2017)).

2. **Reynold number**

The Reynolds number, Re is specified as the ratio of inertial forces to viscous forces and accordingly measures the relative significance of these two types of forces for given flow conditions. The Reynolds number is defined as (Schlichting and Gersten (2000)):

$$\mathrm{Re} = \frac{\text{Inertia forces}}{\text{Viscous}} = \frac{UL}{\nu} = \frac{\rho UL}{\mu} \tag{1.7.2}$$

where U is the reference velocity, L is the characteristic length and $\nu = \mu/\rho$ is the kinematic viscosity. Equation (1.7.2) was introduced by Osborne Reynolds through his publication on the classic pipe experiment in 1883 and he developed the dimensionless number (Reynold number) named after him (White (2011)). The Reynolds number is highly significant to solve the Navier Stokes equations. It indicates whether the fluid flow is turbulent or laminar. According to Bergman and Lavine (2017), when Re is small (i.e., viscous forces are superior) then the flow is likely to be laminar. On the other hand, when Re is large (i.e., inertial forces are superior) then the flow tends to be turbulent.

3. **Nusselt Number**

Nusselt number, Nu was named after the German engineer, Wilhelm Nusselt. It is the ratio of total heat transfer in a system to the heat transfer by conduction and denoted as (Cengel (2003)):

$$Nu = \frac{\text{convection heat transfer}}{\text{conduction heat transfer}} = \frac{h\,(T_w - T_\infty)}{k\,(T_w - T_\infty)\,/L} = \frac{hL}{k} \tag{1.7.3}$$

where k is the thermal conductivity, L is the characteristic length, T_w and T_∞ are the temperature of the surface and far from the surface, respectively, and h denotes the heat transfer coefficient. For a fluid layer, a Nusselt number that equals to one describes the heat transfer across the layer by pure conduction. Therefore, the higher the Nusselt number, the more effective the convection (Cengel (2003)).

4. **Skin Friction Coefficient**

Skin friction coefficient, C_f is the ratio between the fluid and the solid surface which measures the retardation of the fluid due to friction and can be written as (Kundu and Cohen (2001)),

$$C_f = \frac{\tau_w}{\rho U^2}$$

where ρ is density, τ_w is the wall shear stress and U is the reference velocity.

5. **Grashof Number**

The Grashof number, Gr is a measure of buoyancy forces relative to viscous forces acting on a fluid and often occurs in the study of situations involving natural convection. It was named after Franz Grashof and can be expressed as (Cengel (2003)):

$$\mathrm{Gr} = \frac{\text{buoyancy force}}{\text{viscous force}} = \frac{g\beta\left(T_w - T_\infty\right)L^3}{\nu^2} \tag{1.7.4}$$

where g refers to gravitational acceleration, β is the coefficient of volume expansion, T_w and T_∞ are the temperature of the surface and far from surface, respectively, and ν is the kinematic viscosity. The Grashof number is the key parameter for deciding whether the flow of fluid is turbulent or laminar in natural convection. For example, the critical Grashof number in vertical plate is known to be about 10^9. Accordingly, the flow regime on a vertical plate becomes turbulent as $\mathrm{Gr} > 10^9$ (Cengel (2003)).

6. **Schmidt Number**

The Schmidt number is a non–dimensional number, it was named after the German engineer Ernst Heinrich Wilhelm Schmidt. It is defined as (Bergman and Lavine (2017)):

$$Sc = \frac{\text{momentum diffusivity}}{\text{mass diffusivity}} = \frac{\nu}{D} \tag{1.7.5}$$

where ν and D are the kinematic viscosity and mass diffusivity, respectively. A Schmidt number of near unity ($Sc \approx 1$) implies that mass transfer by diffusion and momentum is equal, and velocity and concentration boundary layers almost coincide with each other (Cengel (2003)).

1.8 Stability Analysis

The study regarding stability analysis in fluid dynamics problems is an important topic since non–unique solutions exist in the numerical computation. It is worth mentioning that there might be one, zero, or multiple solutions to a boundary layer problem. A further question that will arise later is which solutions are physically acceptable solutions and have physical meanings. In addition, if there happens to exist non–unique solutions in boundary layer problems but only one solution can be found by the researchers, that solution could be the unstable solution. Thus, it may prompt distortions of heat transfer attributes and flow. Hence, the stability analysis of solutions is a significant analysis to check the consistency of the obtained results. This analysis was first initiated by Wilks and Bramley (1981) as they observed the presence of dual solutions when solving the problem of mixed convection in boundary layer flows.

This thesis implemented the stability analysis that was done by Merkin (1986) by considering some small perturbation to the solutions. The stability of the flow solutions is dictated by the smallest eigenvalue γ. If the result obtained is a positive eigenvalue, it can be concluded that the solution is stable (i.e., there exist an initial decay that does not interrupt the flow) and has a physical meaning. However, if the result obtained is a negative eigenvalue, it can be concluded that the solution is unstable (i.e., there exist an early growth or disruption in the flow) and has no physical meaning.

1.9 Significance of the Study

Nowadays, mathematical modelling has been increasingly used to solve complex problems, especially in the field of engineering. Engineering equipment or processes can be investigated either experimentally (testing and measuring) or analytically (by calculation or analysis). The experimental method has the benefit that it interacts with the real physical environment and that the desired quantity is determined by measurement in the limits of the experimental error. However, this technique is time–consuming, expensive and sometimes impractical. On the other hand, analytical and numerical methods have the advantage of being fast and cheap, but the results produced are subject to the precision of the assumptions, approximations and idealizations made in the analysis (Çengel and Cimbala (2006)).

For several years now, the use of nanometer–sized solid particles as additives suspended in the base fluid, which is also known as nanofluids has been well recognized. Since its first introduction in actual engineering applications, nanofluids have been effectively used to enhance heat transfer in many engineering applications. In addition to its very fascinating values in electrical and thermal conductivity, CNTs have attracted intensive attention and interest over the last few decades due to their unique mechanical properties compared to other nanoparticles. (Sanginario et al. (2017)).

CNTs have many fascinating applications such as electrical conductors, semiconductors and insulators. They can be used as heat sinks for chipboards, as LCD panel backlights, and also as electrical protection in Faraday cages because of the light weight and exceptional electrical conductivity of CNTs (Wu et al. (2004)). Furthermore, CNTs also have interesting applications in biotechnology and biomedicine due to its dimensional and chemical compatibility with biomolecules, such as DNA and proteins. Recent studies have shown the ability of hyperthermia to destroy selected cancer cells due to the thermal conductivity of CNTs (Zhang et al. (2015)). This application is very practical because of the invaluable importance of cancer detection at the early stages.

As decribed earlier, it can be seen that CNT research can grow rapidly in various engineering and medical applications. Thus, the nanofluid theory is important in the future because the formulation of nanofluids can be designed to optimize their use in certain applications. With increasing research by many researchers, nanotechnology can have a strong impact on a wide range of engineering and biomedical applications of nanofluids.

1.10 Problem Statement

Interest in studying CNT flow has grown considerably over the past decades due to its existence in many technological and industrial applications (see Section 1.9). In addition, the boundary layer characteristics on moving surface and stretching/shrinking surface are important in the field of fluid dynamics and have a great practical importance to engineers and scientists as it occurs in many industrial and technological processes (see Section 1.3). Hence, this research has been conducted to investigate the boundary layer flow of CNT due to a moving plate as well as stretching or shrinking surface with various effects.

This study therefore addresses the following research questions:

1. What are the parameters that contribute to the existence of the dual solutions?
2. Which parameters contribute to the widening/narrowing of the range of solutions?
3. How does the presence of nanoparticles volume fraction (carbon nanotube) give impact on the flow and heat transfer characteristics at the surface?
4. What are the effects of different types of carbon nanotube on the local skin friction and Nusselt number?

5. How would the skin friction and local Nusselt number be influenced due to the changes in physical parameters (slip, suction and magnetic parameters)?
6. How does the presence of chemical reaction affect the concentration?
7. Which of the solutions obtained is a stable solution?

1.11 Aim and Objectives

The aim of this thesis is to investigate the fluid flow and heat transfer characteristics of CNT in different areas configurations and subjected to various source terms and boundary conditions together with the stability analysis. The following five problems are:

1. Boundary layer flow over a moving plate in the presence of slip effect.
2. Boundary layer flow over a vertical moving plate in the presence of suction.
3. Stagnation point flow over an exponentially stretching or shrinking sheet in the presence of homogeneous and heterogeneous reaction.
4. Stagnation point flow over a nonlinear stretching or shrinking sheet in the presence of magnetohydrodynamics.
5. Stagnation point flow over a stretching or shrinking cylinder in the presence of chemical reaction effect.

While, the objectives of this present study are to:

1. formulate and derive the mathematical model for the various non-linear problems of carbon nanotube flows,
2. develop an algorithm, solve the mathematical model numerically using `bvp4c` solver in `MATLAB` software and conduct the validation tests for the current research in comparison with the numerical results in the literature,
3. provide the formulation and perform the stability analysis on dual solutions obtained, and
4. study the effect of various parameters on the flow and heat transfer characteristics of carbon nanotubes.

1.12 Scope of the Thesis

This study is limited to the problem of steady, incompressible and two–dimensional convective heat transfer boundary layer flow and stagnation point flow towards a moving plate and stretching or shrinking surface of CNTs. Two types of CNTs are used as nanoparticles, namely the single–wall carbon nanotube (SWCNT) and multi–wall carbon nanotube (MWCNT), whilst water and kerosene are used as the base fluid. In addition, the model used for the nanofluid is a model that incorporates the effect of solid volume fraction proposed by Tiwari and Das (2007).

1.13 Thesis Outline

This thesis is divided into nine chapters and organized as follows:

Chapter 1 of this thesis provides some basic definitions, problem statement, aim, objective and scope of the present research.

Chapter 2 elaborate on the previously published work and highlights the areas that relevant to the thesis.

Chapter 3 contains the detailed methodologies that were used to solve the nonlinear system of equations as well as the derivation for the stability analysis. By utilizing similarity transformation, the set of governing nonlinear partial differential equations (PDEs) were transformed into the nonlinear ordinary differential equations (ODEs). In the later part of the chapter, the `MATLAB`'s `bvp4c` solver method is described in detail for solving the boundary value problem.

Chapter 4 to 8 discusses the five main problems mentioned in Section 1.11. These chapters will be divided into five main sections. All the chapters begin with an introduction, followed by mathematical formulation and stability analysis. Next, an analysis of the obtained results is reported with the graphical illustrations. Lastly, the conclusion obtained from the present study is discussed in each chapter.

Chapter 9 summarizes the research work and gives the main conclusion arising from the whole research and recommendations for the future work.

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