APPLICATION OF KRYLOV SUBSPACE METHODS FOR SOLVING CONTINUOUS POWER FLOW PROBLEM IN VOLTAGE STABILITY ANALYSIS OF POWER SYSTEM

JASRONITA JASNI

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JASRONITA JASNI

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By

JASRONITA JASNI

January 2010

Chair: Hashim Hizam, PhD

Faculty: Engineering

Continuation Power Flow (CPF) analysis is developed to overcome singularity problem of Jacobian matrix of power flow analysis. This analysis is done by reformulating the power flow equations so that they remain well-conditioned at all possible loading conditions. This allows the solution of the power flow problem for both stable and unstable equilibrium points. However, its effectiveness and efficiency are still in question as it needs many continuation steps to solve each problem. This situation will delay the process of corrector in the system. The CPF algorithm has also been found to fail for a system which has a very sharp turning point for the solution curve which can drag the system to have convergence problem. The step cutting technique that is used to improve convergence can lead to slightly incorrect results in the case of sharp turning point.

In order to provide continuity of the power flow in both stable and unstable situations, the numerical method chosen in the analysis should be able to provide predictor and corrector values with minimal computational effort. Therefore, the aim of this work is to introduce new algorithms that can ensure the continuous power flow eliminate the convergence problem for all power systems regardless of the size of the system and improve the existing CPF. This research will focus on static voltage stability analysis where voltage collapse is explained as static bifurcation phenomenon. Three algorithms, which are based on Krylov Subspace method, have been developed in order to overcome the drawbacks of the existing CPF. These developed algorithms are tested on 14, 118 and 300 IEEE bus systems. Furthermore, the real data with 293 buses and 595 lines is used as a practical system for verification of the new algorithms.

The results show that these new algorithms are able to eliminate the convergence problem faced by the existing CPF algorithm. For IEEE 300 bus system, the iteration has been reduced from 36 to 34 iterations. The CPU time ratio in performing the analysis has also been reduced between three to twenty percent. These new algorithms are also able to produce more reliable results compared to the existing CPF method.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan ijazah Doktor Falsafah

PENGGUNAAN KAEDAH-KAEDAH SUB-RUANG KRYLOV BAGI MENYELESAIKAN MASAALAHALIRAN KUASA BERTERUSAN DI DALAM ANALISA KESTABILAN VOLTAN BAGI SISTEM KUASA

Oleh

JASRONITA JASNI

Januari, 2010

Chair: Hashim Hizam, PhD

Faculty: Faculty of Engineering

Analisis aliran kuasa berterusan telah dibangunkan untuk mengatasi masalah keadaan tunggal yang dihadapi oleh matriks Jacobian di dalam analisa aliran kuasa. Analisa tersebut dilakukan dengan perumusan semula persamaan aliran kuasa supaya aliran kuasa tersebut berada dalam keadaan baik pada semua keadaan bebanan yang mungkin. Ini membenarkan penyelesaian masalah aliran kuasa untuk kedua-dua titik keseimbangan iaitu stabil dan tidak stabil. Walaubagaimanapun, keberkesanan dan kecekapan CPF ini masih diragui kerana memerlukan langkah berterusan yang banyak untuk menyelesaikan setiap masalah. Situasi ini akan melengahkan proses pembetul di dalam sistem tersebut. Algoritma CPF juga didapati gagal memberikan penyelesaian yang lengkap untuk sistem yang mempunyai titik lengkuk yang tajam pada lengkungan penyelesaian di mana boleh menghasilkan masalah penumpuan. Teknik pemotongan langkah yang digunakan untuk membaiki penumpuan juga boleh menjurus kepada keputusan

yang tidak tepat di dalam kes titik lengkuk tajam.

Untuk menyediakan keterusan pada aliran kuasa dalam keadaan stabil dan tidak stabil, kaedah berangka yang dipilih di dalam analisa hendaklah berupaya menyediakan nilai peramal dan pembetul dengan kaedah pengiraan yang minimal. Oleh yang demikian, matlamat utama kerja ini adalah untuk memperkenalkan algoritma baru yang dapat memastikan aliran kuasa berterusan yang dapat menghapuskan masalah pencapahan bagi semua sistem kuasa tidak kira kecil atau besar dan memperbaiki kaedah sedia ada. Kajian ini akan memfokus dalam analisa kestabilan voltan statik di mana kejatuhan voltan diterangkan sebagai fenomena statik dwiwujudan. Tiga algoritma berdasarkan kaedah Sub-ruang Krylov telah dibangunkan untuk menyelesaikan permasalahan CPF sedia ada. Ketiga-tiga algoritma ini telah diuji ke atas sistem IEEE 14, 118 dan 300 bas. Selanjutnya, data sebenar yang mempunyai 295 bas dan 595 talian digunakan sebagai sistem paktikal untuk verifikasi algoritma-algoritma yang telah dibangunkan.

Keputusan menunjukkan bahawa algoritma yang baru itu telah berjaya menghapuskan masalah penumpuan yang dihadapi oleh algoritma CPF sedia ada. Untuk sistem IEEE 300 bas, didapati lelaran berkurang dari 36 kepada 34 lelaran. Nisbah masa CPU yang diperlukan untuk menyelesaikan analisa telah berkurangan di antara tiga hingga dua puluh peratus. Juga, algoritma yang baru ini dapat memberikan keputusan yang bolehharap berbanding dengan kaedah CPF yang sedia ada.

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LIST OF ABBREVIATIONS

AC/DC	Alternating Current/Direct Current
BiCG	BiConjugate Gradient
BiCGSTAB	BiConjugate Gradient Stabilized
CEP	Constraint Exchange Points
CGS	Conjugate Gradient Squared
CPF	Continuation Power Flow
CPF-BiCGSTAB	Continuation Power Flow- BiConjugate Gradient Stabilized
CPF-CGS	Continuation Power Flow- Conjugate Gradient Squared
CPF-GMRES	Continuation Power Flow- Generalized Minimize Residual
CPFLOW	Continuation Power Flow Program
CPU	Central Processing Unit
CRIC	Constraint Implicit Coupling
GMRES	Generalized Minimize Residual
IEEE	Institute of Electrical and Electronics Engineers
ILU	Incomplete LU
MATLAB	The Language Of Technical Computing
MCPF	Modified Continuation Power Flow

P.U	Per Unit
PoC	Point of Collapse
PV curve	Power-Voltage curve
SCADA	Supervisory Control And Data Acquisition
SNB	Saddle Node Bifurcation

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CHAPTER 1

INTRODUCTION

1.1 OVERVIEW

Power system stability is a feature of a power system that allows the system to remain in a state of operating equilibrium under normal operating condition and be at an acceptable state of operating equilibrium after facing disturbances [1]. There are several forms of stability involved in power system stability studies such as rotor angle stability, voltage stability and voltage collapse, as well as mid term and long term stability. However, in this research the focus is on the voltage stability and voltage collapse analysis.

Stability is defined by Taylor [2] as:

1. A power system at a given operating state is small-disturbance voltage stable if, following any small disturbance, voltages near loads are identical or close to the pre-disturbance values.

2. A power system at a given operating state and subject to a given disturbance is voltage stable if voltages near loads approach post-disturbance equilibrium

values. The disturbed state is within the region of attraction of the stable post-disturbance equilibrium.

3. A power system at a given operating state and subject to a given disturbance undergoes voltage collapse if post-disturbance equilibrium voltages are below acceptable limits. Voltage collapse may be total or partial.

Voltage stability is usually known as a steady-state viability problem which is suitable for static analysis [2]. The major aspect in any voltage stability is the ability to transfer power from the power source to consumers during the steady operating condition.

On the other hand, voltage instability is known as a phenomenon when the voltage drops rapidly and the system control fails to improve the voltage level due to the load increase or some other systems change which last from several seconds to several minutes. These cause instability and the result is often the decrease in voltage magnitude.

1.2 VOLTAGE COLLAPSE EXPERIENCE

According to Hill and Hisken [3], voltage collapse is defined as a power system at a given operating state and subject to a given large disturbances undergoes voltage collapse if its voltage is unstable or the post-disturbance equilibrium values are non-viable. During the last decades there have been several large voltage collapses almost every year somewhere in the world including in Malaysia.

Some of the voltage collapse events around the world are:

1. September 12, 2005, a blackout in Los Angeles affected millions of consumer in California [4].
2. December 1, 2006, a blackout in Ontario and continued into December 2, 2006 [5].
3. June 27, 2007, a power failure in New York City and affected Manhattan and Bronx for one hour [6].
4. On July 23, 2007, the city of Barcelona suffered a near-total blackout for more than 78 hours due to a massive electrical substation chain failure [7]
5. On February 20 in 2008, coal supplies to some power plants in Java have been stopped resulting electricity shortage affecting Jakarta [8]

Meanwhile for Malaysia there have been incidences as follows:

1. Blackout in northern states of Malaysia in Penang and Kedah on November 2007 [9]
2. Blackout in the southern Peninsular Malaysia on September 1, 2003, due to power failure affecting 5 states including Kuala Lumpur for 5 hours [10]
3. In Jan 13, 2005, a blackout hits several parts of Malaysia including Petaling Jaya, Putrajaya, Cyberjaya, Melaka, Negeri Sembilan and Johor about 12.30 pm due to a problem at Kapar switchyard at the Port Klang Power Station [11].
4. In August 3 1996, the blackout affected the whole Peninsular Malaysia including Kuala Lumpur. It occurred at 5:17p.m. [12]

Events listed show that voltage collapse is a real problem and needs to be considered as a serious condition. A thorough analysis of each event is essential so that any voltage collapse event could be identified and corrective action can be taken appropriately. Hence, avoidance and fixing ways could be carefully planned to minimize the occurance of voltage collapse.

1.3 STATIC OR DYNAMIC ANALYSIS

A power system may collapse due to some transmission lines reaching the maximum power that can be transmitted or due to the lack of local reactive power support. This voltage collapse is known as a static voltage problem and will be the main focus or scope of this thesis.

Voltage stability analysis is normally done using the power flow simulation. Kundur [1] has classified voltage stability into two subclasses which are:

1. Large disturbance voltage stability which concerns a system's ability to control voltages following large disturbances such as system faults, loss of generation or circuit contingencies. Therefore, this subclass requires a dynamic analysis. A criteria for this voltage stability is to follow a given disturbance and system-control actions. Also, the voltages at all buses must reach acceptable steady-state levels.

2. Small-disturbance voltage stability concerns a system's ability to control voltages following small perturbations such as due to incremental changes in system load. This concept is useful to determine how the system voltage will respond to small system changes. Therefore, a static analysis can be effectively used to determine stability margins and identify factors influencing stability, and to examine a wide range of system conditions and a large number of post-contingency scenarios. A criterion for small-disturbance voltage stability is that, at a given operating condition for every bus in the system, the bus voltage magnitude increases as the reactive power injection

> at the same bus increases. A system is voltage unstable if, for at least one bus in the system, the bus voltage magnitude, V decreases as the reactive power injection Q at the same bus is increased.

Voltage stability is usually not much influenced by system dynamics . Therefore, using static methods, more issues regarding voltage stability can be effectively analyzed. One of the examples is examining the feasibility of the equilibrium point for a specified operating condition of the power system [13, 14]. The static analysis can be used to analyze more system conditions and provide the solution more efficiently [15, 16]. Meanwhile the dynamic analysis is used to study the specific voltage collapse situations, protection and controls.

Dynamic simulation or time domain simulation gives the events and the chronology leading to instability but this simulation is time consuming and does not provide sensitivity information and the degree of stability [17, 18]. The dynamic analysis also does not have the same spectrum as static analysis due to the time frame involved; the static voltage stability analysis may require minutes to hours of analysis but the dynamic voltage stability analysis may require milliseconds to seconds [19, 20]. The use of steady state or static analysis method is permitted in many cases in which load flow equations are used to represent the system conditions. In comparison to the dynamic study, investigation on long-term voltage stability determined by steady-state studies usually offers optimistic results [21].

This work focuses on the small-disturbance voltage stability where the researcher studies the ability of the system to determine the response of the system due to changes in the load system. The static analysis is the best choice to study this issue because this study will concentrate on loading margin which relate to the load flow analysis.

1.4 RESEARCH MOTIVATION AND PROBLEM STATEMENT

Nowadays, the usage of electricity has become more and more important as technologies of electrical apparatus and equipment have rapidly grown. This phenomenon is also due to the modern living style and the needs from industries. The analysis method may help the utilities to be ready with any changes or any contingencies that may happen in order to fulfill the high usage of electricity demand. Therefore, an appropriate analysis method is needed to deal with this dramatic growth of electricity demand. This is to ensure that the demand can be delivered without any interruption.

Power flow analysis involves calculation of power flows and voltages of transmission network at specified terminal or bus condition [1]. This calculation is required for analysis of steady state as well as dynamic performance of the power system. Therefore, it is the most important procedure in power system planning and operation.

Traditionally, the Newton-Raphson method has been used for solving the power flow problem. However, researchers found that the Jacobian matrix of power flow analysis for this Newton-Raphson method becomes singular at the voltage stability limit [1, 22, 23]. This means that this power flow algorithm will face convergence problems at operating condition which is near to the stability limit. The Continuation Power Flow(CPF) analysis is meant to overcome this problem by reformulating the power flow equations so that they remain well-conditioned at all possible loading conditions. This allows the solution of the power flow problem for both stable and unstable equilibrium points. Based on previous studies, it has been found that this continuation method can handle the singularity problem faced by Newton-Raphson method [24, 25, 23, 26]. However, its effectiveness and efficiency still require improvement as it needs many continuation steps to solve each problem. This is due to the step-length that needs to be considered in the corrector step. Therefore, computational efforts become heavy with more continuation steps and this situation will delay the process of corrector in the system. The Continuation Power Flow algorithm has also been found to fail for a system which has very sharp turning point for the solution curve which can drag the system to have convergence problem [25]. The step cutting technique is used to improve convergence which can lead to slightly incorrect results in cases of sharp turning points[27, 28, 29, 30].

In order to provide continuity of the power flow in both stable and unstable situations, a numerical method chosen in the analysis is needed to be able to provide predictor and corrector values with a minimal computational effort. Therefore, this work is aimed at tackling this issue using the proposed analysis method.

1.5 AIM AND OBJECTIVES

The aim of this work is to introduce new algorithms that can provide continuous power flow inclusive of upper and lower equilibrium points for any scale of power system as well as improving the existing method to solve the problem faced by Continuation Power Flow.

The objectives of this work are:

1. To prove the convergence problem faced by existing Continuation Power Flow method.
2. To develop new algorithms to address the singularity and convergence problem of Continuation Power Flow.
3. To develop a software tool based on the new algorithms for voltage stability analysis.

1.6 SCOPE OF RESEARCH

This research will focus on static voltage stability analysis where voltage collapse is explained as static bifurcation phenomenon. Three new developed algorithms are tested on 14, 118 and 300 IEEE bus systems which are used to verify these new algorithms. Furthermore, the practical system with 293 buses and 595 lines, provided by Tenaga Nasional Berhad, is used as the real system to test and verify the new algorithms. The contingency analysis for IEEE 118 bus systems is carried out to verify the new algorithms.

1.7 OUTLINE OF THE THESIS

The structure of the thesis is as follows:

Chapter 1 provides the overview of the research where the definition of the voltage stability is briefly described. This follows the discussion on voltage collapse phenomenon with several cases of blackouts around the world including Malaysia. The difference between dynamic and static analysis are discussed. Research motivation and problem statement, aim and objectives, and scope of the research are set out as the guideline of completing this research.

Chapter 2 presents a discussion on voltage stability which details on the basic concept of voltage stability. Voltage collapse and its analysis method is reviewed.

Chapter 3 highlights the Continuation Power Flow starting with the discussion of bifurcation and direct methods and later on lead to Continuation method. This follows with in-depth review of the previous works by previous researchers.

Chapter 4 discusses the development of proposed new algorithms which are based on Krylov Subspace Methods. These algorithms are developed using Generalized Minimized Residual Method (GMRES), Conjugate Gradient Squared (CGS) and BiConjugate Gradient Stabilized (BiCGSTAB) where the fundamental of all the three algorithms come from Krylov Subspace. The review of power flow using Krylov Subspace are also discussed. The methodology and the new algorithm are also explained.

Chapter 5 shows the results of Continuation Power Flow and the proposed algorithms when applied to IEEE bus systems. The results are compared and used as a verification of the new algorithms.

Chapter 6 reports the results of the new algorithms when applied to the practical system. The results include the contingency analysis of the practical system followed by the application of Continuation Power Flow on the system. Later the application of the new algorithms on the system are presented.

Chapter 7 concludes the research work. The capabilities and limitations of the proposed algorithms are presented. Some perspectives for future work are also suggested.

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