UNIVERSITI PUTRA MALAYSIA

SOLVING CRACK PROBLEMS IN BONDED DISSIMILAR MATERIALS USING HYPERSINGULAR INTEGRAL EQUATIONS

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IPM 2020 5

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By

KHAIRUM BIN HAMZAH

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirements for the Degree of Doctor of Philosophy

November 2019

DEDICATIONS

To
The Blessings of My Late Parents:
Dad: Hamzah Bin Said,
Mum: Hasnah Binti Yamat,
...
My Beloved Siblings:
Hamizon Binti Hamzah,
Hamzani Binti Hamzah,
Hasni Bin Hamzah,
Hazalina Binti Hamzah,
Mohd Hazlin Bin Hamzah,
Hamka Bin Hamzah,
Hanifah Bin Hamzah,
Hasyira Binti Hamzah,
Hazrien Syam Bin Hamzah,
...
All My Amazing Friends.

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Doctor of Philosophy

SOLVING CRACK PROBLEMS IN BONDED DISSIMILAR MATERIALS USING HYPERSINGULAR INTEGRAL EQUATIONS

By

KHAIRUM BIN HAMZAH

November 2019

Chairman : **Nik Mohd Asri Bin Nik Long, PhD**
Faculty : **Institute for Mathematical Research**

Inclined or circular arc cracks problems and thermally insulated inclined or circular arc cracks problems subjected to remote stress in bonded dissimilar materials are formulated. The modified complex variable function method with the continuity conditions of the resultant force and displacement function are used to formulate the hypersingular integral equations (HSIEs) for these problems. Whereas, the continuity condition of heat conduction function is utilized to formulate the HSIEs for the thermally insulated cracks problems. The unknown crack opening displacement (COD) function is mapped into the square root singularity function using the curved length coordinate method. Then the appropriate quadrature formulas are used to solve the obtained equations numerically, with the traction along the crack as the right hand term. The obtained COD function is then used to compute the stress intensity factors (SIF) in order to determine the stability behavior of bodies or materials containing cracks or flaws. Numerical results of the nondimensional SIF at all the cracks tips are presented. Our results are totally in good agreements with those of the previous works. It is observed that the nondimensional SIF at the cracks tips depend on the remote stress, the elastic constants ratio, the crack geometries, the distance between each cracks and the distance between the crack and the boundary. Whereas for thermally insulated cracks, the nondimensional SIF at the cracks tips depend on the heat conductivity ratio and the thermal expansion coefficients ratio.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

PENYELESAIAN MASALAH RETAKAN DI DALAM DUA BAHAN BERBEZA YANG TERCANTUM MENGGUNAKAN PERSAMAAN KAMIRAN HIPERSINGULAR

Oleh

KHAIRUM BIN HAMZAH

November 2019

Pengerusi : **Nik Mohd Asri Bin Nik Long, PhD**
Fakulti : **Institut Penyelidikan Matematik**

Masalah retakan condong atau retakan tembereng membulat dan masalah haba teraruh bagi retakan condong atau retakan tembereng membulat yang dibatasi oleh regangan jauh di dalam dua bahan berbeza yang tercantum diformulasikan. Kaedah fungsi pembolehubah kompleks terubah dengan syarat keselanjaran daya terhasil dan fungsi anjakan digunakan untuk memformulasikan persamaan kamiran hipersingular (PKH) untuk masalah ini. Manakala syarat keselanjaran fungsi pengaliran haba digunakan bagi merumuskan PKH untuk masalah retakan haba teraruh. Anu fungsi anjakan bukaan retakan (ABR) dipetakan kepada fungsi singular punca kuasa dua mengunakan kaedah koordinat panjang terlengkung. Kemudian rumus kuadratur yang sesuai digunakan untuk menyelesaikan secara berangka persamaan terhasil dan regangan di sepanjang retakan sebagai sebutan di sebelah kanan PKH. Fungsi ABR yang diperoleh kemudian digunakan untuk mengira faktor keamatan regangan (FKR) dalam menentukan tingkah laku kestabilan bahan yang mengandungi retakan atau cacatan. Keputusan berangka terhadap FKR tak berdimensi di setiap hujung retakan dibentangkan. Keputusan yang kami perolehi adalah selari dengan kerja sebelumnya. Dapat diperhatikan bahawa FKR tak berdimensi pada hujung retakan bersandar kepada regangan jauh, nisbah pemalar elastik, kedudukan retakan, jarak antara retakan dan jarak antara retakan dan sempadan. Manakala, untuk retakan haba teraruh, FKR tak berdimensi pada hujung retakan bergantung kepada nisbah kekonduksian haba dan nisbah pemalar pengembangan haba.

ACKNOWLEDGEMENTS

Bismillahirrahmanirrahim, Alhamdulillah, all praise be to Allah Subhanahu Wa Ta'ala for the blessing and giving me the strength, health and intelligence for doing the research and making this PhD thesis entitled "Solving crack problems in bonded dissimilar materials using hypersingular integral equations" a success. The completion of this thesis would not have been possible without the support and encouragement of several special people.

First and foremost, I would like to express my deepest gratitude to the chairman of the supervisory committee, Assoc. Prof. Dr. Nik Mohd Asri bin Nik Long for giving me the golden opportunity to embark on this wonderful journey of research. Without his continuous invaluable guidance, inspiration, thoughtful comments and his cooperation from the beginning this thesis would not have been possible.

Secondly my sincere appreciation goes to the members of the supervisory committee, Assoc. Prof. Dr. Norazak bin Senu and Assoc. Prof. Dr. Zainidin Eshkuvatov for their direct or indirect assistance throughout the period my study.

Special thanks to my family for their encouragement, valuable support, understanding and helping me reach this stage in my life. To my late parents, Mr. Hamzah bin Said and Mrs. Hasnah binti Yamat, thank you for your shower of love, and life lessons who always thought me to be grateful and thankful in life. Million of thanks to all my friends who have supported me on this journey and assist me to be on the right path to achieve my goal in this research.

Last but not least, I would like to thank Ministry of Education Malaysia and Universiti Teknikal Malaysia Melaka for giving me the scholarship to upgrade my knowledge. Also, I would like to express my heartfelt thanks to all the staff of the Institute for Mathematical Research and Department of Mathematics, UPM for their help and support. Thank you all.

This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfilment of the requirement for the degree of Doctor of Philosophy. The members of the Supervisory Committee were as follows:

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LIST OF ABBREVIATIONS

LEFM	Linear Elastic Fracture Mechanics
EPFM	Elastic Plastic Fracture Mechanics
SIF	Stress Intensity Factor
BIE	Boundary Integral Equations
WSIE	Weakly Singular Integral Equations
CSIE	Cauchy Singular Integral Equation
HSIE	Hypersingular Integral Equations
FIE	Fredholm Integral Equations
COD	Crack Opening Displacement
F_1	Mode I Nondimensional SIF
F_2	Mode II Nondimensional SIF
σ_x	Stress in the x-direction

CHAPTER 1

INTRODUCTION

1.1 Overview

Fracture mechanics is one of the engineering field of mechanics describe on the behavior of solids or structures containing geometric discontinuity such as crack propagation in materials. The force on the crack and those of the experimental solid mechanics characterize the resistance of materials to fracture can be calculated by using the methods of analytical solid mechanics. The geometric discontinuity features may be in the form of line discontinuities for two dimensional plane and surface discontinuities for three dimensional plane.

The investigation on the behavior and life cycle of the crack components is one of the most important tasks in the engineering fracture mechanics. The fracture mechanics plays an important tool in improving the mechanical performance of mechanical structures in terms of stability and safety of the materials. In order to predict the mechanical failure of the structures, the stress and strain to the materials was applied based on the theories of elasticity and plasticity. Fracture mechanics can be divided into two main categories which is Linear Elastic Fracture Mechanics (LEFM) and Elastic Plastic Fracture Mechanics (EPFM).

LEFM is the basic theory of fracture which deals with the cracks in elastic plane. It is applicable to any materials as long as the material is elastic except in a vanishingly small region at the crack tip, brittle or quasibrittle fracture, stable or unstable crack growth. The stress field near the crack tip can be evaluated using the theory of elasticity. If the plastic crack tip zone is too large, the stress and strain fields from LEFM are not valid any more. This is also the case when the material behavior is nonlinear elastic such as in polymers and composites. Crack growth criteria are no longer be formulated with the stress intensity factor (SIF). In order to overcome this limitation, EPFM will be used if large zones of plastic deformation develop before the crack grows. EPFM is the theory of ductile fracture, usually characterized by stable crack growth such as ductile metals, the fracture process is accompanied by formation of large plastic zone at the crack tip.

The investigation on the effect of surface scratches on the mechanical strength of solids and to understand about fracture mechanics based on linear elasticity was developed from the pioneer researchers by Inglis (1913), Griffith (1920), Westergaard (1939) and Muskhelishvili (1953) (Roylance, 2001; Pommier, 2017). Inglis investigated the stress for an elliptical hole in an infinite linear elastic plate and modeled the crack discontinuity by making the minor axis very much less than the major axis but the solution is limited to a perfectly sharp crack only. Griffith extended the previous work by Inglis to overcome the existence problem by employed the energy balance approach compared to focus on the crack tip stresses directly. Westergaard used the complex stress functions to develop the asymptotic solution for a stationary crack loaded dynamically. His method provides a powerful technique for solving the infinite linear elastic plane containing a crack or array of cracks. Whereas, Muskhelishvili developed the complex potentials method for solving all major problems of two dimensional linear elasticity by reducing the plane problem to finite systems of linear algebraic equations singular kernels.

1.2 Stress function

The stress distribution, σ_{ij} near crack tip in polar coordinate system for an isotropic linear elastic material shown in Figure 1.1 with origin at the crack tips is defined by (Anderson, 1991)

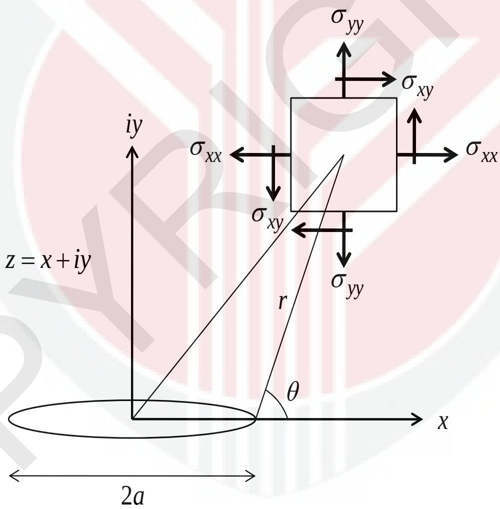

Figure 1.1: Stress distribution for a crack in an isotropic linear elastic material.

$$\sigma_{ij} = \frac{k}{\sqrt{r}} f_{ij}(\theta) + \sum_{m=0}^{\infty} A_m r^{m/2} g_{ij}^{(m)}(\theta), i, j = x, y \tag{1.1}$$

where σ_{ij} is the stress tensor, r and θ are defined in Figure 1.1, k is constant, f_{ij} is dimensionless function of θ in the leading term, A_m is the amplitude and $g_{ij}^{(m)}$ is a dimensionless function of θ for the higher-order terms.

The equilibrium, compatibility and boundary conditions must be satisfied in order to find the solution for two dimensional plane stress problem based on the theory of elasticity. The equations of elasticity is reduced to two dimensional forms in three special cases as follows

- For the case of plane strain the displacement component u_z is identically equal to zero, and none of the physical quantities depends on z.
- In a state of plane stress parallel to the xy-plane, the stress components σ_{xz}, σ_{yz} and σ_{zz} all vanish but the components of the displacement vector are not independent of z.
- Generalized plane stress is a state of stress in a thin plate $-h \leq z \leq h$ when $\sigma_{zz} = 0$ throughout the plate but $\sigma_{xz} = \sigma_{yz} = 0$ only on the surfaces $z = \pm h$ of the plate.

For two dimensional problems, the first two requirements can be automatically satisfied by choosing an Airy stress function, Φ such that (Timoshenko et al., 1970)

$$\sigma_x = \frac{\partial^2 \Phi}{\partial y^2}, \; \sigma_y = \frac{\partial^2 \Phi}{\partial x^2}, \; \sigma_{xy} = -\frac{\partial^2 \Phi}{\partial x \partial y} \tag{1.2}$$

where the stress function is bi-harmonic,

$$\frac{\partial^4 \Phi}{\partial x^4} + 2\frac{\partial^4 \Phi}{\partial x^2 \partial y^2} + \frac{\partial^4 \Phi}{\partial y^4} = 0. \tag{1.3}$$

1.3 Stress intensity factors

There are three types of modes in fracture mechanics on SIF for a crack in the materials such as tensile opening (Mode I), in plane shear (Mode II) and out of plane tearing (Mode III) as shows in Figure 1.2. The Mode I SIF represented as K_I, whereas K_{II} and K_{III} represented the SIF for Mode II and Mode III, respectively. The Mode I SIF corresponds to normal separation of the crack faces under the action of tensile stresses, which is by far the most widely encountered in practice. The difference between Mode II and Mode III is that the shearing action in the former case is normal to the crack front in the plane of the crack whereas the shearing action in Mode III is parallel to the crack front. A cracked body in reality can be loaded in any one of these three modes or a combination of these three modes. For an isotropic linear elastic material the stress fields ahead of a crack tip can be defined as follows

$$\lim_{r \to 0} \sigma_{ij}^{(I,II,III)} = \frac{K_{(I,II,III)}}{\sqrt{2\pi r}} f_{ij}^{(I,II,III)}(\theta), i, j = x, y \tag{1.4}$$

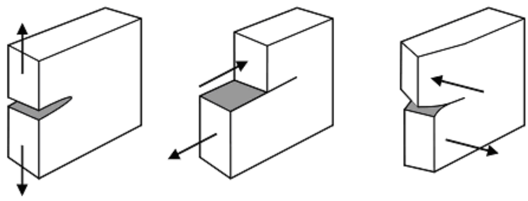

Figure 1.2: Three types of modes in fracture mechanics.

where the three SIF can be defined by

$$K_I = \lim_{r\to 0} \sqrt{2\pi r}\sigma_{yy}(r,0),$$
$$K_{II} = \lim_{r\to 0} \sqrt{2\pi r}\sigma_{xy}(r,0),$$
$$K_{III} = \lim_{r\to 0} \sqrt{2\pi r}\sigma_{yz}(r,0).$$

Once the value of SIFs (K_I, K_{II}, K_{III}) are obtained, we can determine the crack stability by comparing K_I, K_{II} and K_{III} with critical SIF K_{IC}, K_{IIC} and K_{IIIC} which depend on the type of materials, respectively, called the fracture toughness of the material. If the value of SIFs less than value of critical SIFs then the crack will not propagate. Whereas the crack will propagate if the value of SIFs greater than or equals to value of critical SIFs (Petersen, 2013). According to Wang (2003) the strength of the materials more stronger when the value of SIF at crack tip approaches to zero.

1.4 Integral equation in elasticity

The boundary integral equations (BIE) are used to solve the crack problems in elasticity such as an infinite plane, half plane or bonded dissimilar materials, and these BIE may be generally expressed as

$$\int_L K(t,t_0)f(t)dt = p(t_0), \text{ (or } p(t_0)+c, t_0 \in L) \tag{1.5}$$

where $K(t,t_0)$ is the kernel, $f(t)$ is the unknown function, $p(t_0)$ is the right hand term for the known function and L is the configuration of a single or multiple cracks

(Chen et al., 2003). The displacement jump or the dislocation distribution can be chosen as the unknown function in the Equation (1.5) since the displacements are discontinuous along the crack L. Therefore, two possibilities exist to choose the known function at the right hand term which are traction and the resultant force functions along the crack.

Table 1.1: The classification of the BIE for crack problems in elasticity.

Type	$f(t)$	$p(t_0)$	Property of $K(t,t_0)$
WSIE	Dislocations	Resultant force	Weakly singular
CSIE1	Dislocations	Traction	Cauchy singular
CSIE2	Displacement jump (COD)	Resultant force	Cauchy singular
HSIE	Displacement jump (COD)	Traction	Hypersingular
FIE1A	Dislocations	-	Fredholm/Regular
FIE1B	Traction	Traction	Fredholm/Regular
FIE2	Displacement jump (COD)	-	Fredholm/Regular

The classification of the BIE for crack problems in elasticity are listed in Table 1.1. For the type of weakly singular integral equations (WSIE), the dislocation distribution and resultant force function are chosen for the unknown function $f(t)$ and the right hand term $p(t_0)$, respectively. This BIE is named as WSIE since the kernel is a logarithmic function which has a weaker singularity for integration. Cheung and Chen (1987) used the type of WSIE to solve the crack problem in elasticity. For the type of Cauchy singular integral equation (CSIE1), the dislocation distribution and the traction are chosen for $f(t)$ and $p(t_0)$, respectively. This BIE is named as CSIE1 since the integral in the equation is a Cauchy principle value integral. The CSIE1 was developed by researchers to investigate the relevant numerical solution technique in crack problems elasticity (Erdogan et al., 1973; Panasyuk et al., 1977). For the type of CSIE2, the crack opening displacement (COD) and the resultant force are chosen for $f(t)$ and $p(t_0)$, respectively. The CSIE2 posseses a Cauchy principle value integral in the integral equation. Chen (1993) and Chen (1999) used the CSIE2 to analyze the crack problems elasticity. For the hypersingular integral equations (HSIE), the COD and traction are chosen for $f(t)$ and $p(t_0)$, respectively. This BIE is named as HSIE since the kernel in the integral equation is hypersingular. This type of BIE have particular advantage since the COD function can be obtained directly from the solution. The HSIE get more attention from the researchers to solve cracks problems in elasticity and it was used by Nied (1987) and Ioakimidis (1988). For the type of Fredholm integral equations (FIE1A), the dislocation distribution is chosen for $f(t)$ and it is obtained from a regularization of the singular integral equation of type CSIE1. This BIE is named as FIE1A since the kernel in the integral equation is regular. Chen and Hasebe (1992a) used the type of FIE1A to solve the crack problem in elasticity. For the type of FIE1B, the traction applied on the crack and the traction in the actual problem are chosen for $f(t)$ and $p(t_0)$, respectively. Chen (1984) used the type of FIE1B to solve the crack problem in elasticity. Whereas for type of FIE2, it is obtained from generalized of the singular integral equation of type CSIE2 (Chen, 1993).

1.5 Research problem

Fracture mechanics is very important in the field of engineering structures and one of the subarea is crack problems. The study on crack geometry make more interesting due to the existence of the cracks may jeopardise the materials strength, stability and safety. This situation makes more worsen when the structures are exposed to the thermal. This phenomenon has lead to many research works in order to investigate the behavior of SIF at the crack tips in order to identify the strength and stability behavior of engineering structures containing cracks or flaws and predict life cycles of the structures. To this end, an efficient and accurate method is needed to evaluate the SIF at the crack tips. Thus, the main idea of this research is to analyze the behavior of SIF for the crack problems and thermally insulated crack problems in bonded dissimilar materials. The previous works were solved for these problems by using singular integral equation, Fredholm integral equation, continuous distributions of the body force method, boundary integral method and the other methods as reviews in Chapter 2. Therefore, this research analyze these problems using the new system of hypersingular integral equations (HSIEs) by applying the modified complex variable function method with the help of the continuity conditions of the resultant force and displacement function. Whereas, the continuity condition of heat conduction function is utilized for the thermally insulated cracks problems.

1.6 Research objectives

The main objectives of this research are:

1. To analyze the systems of HSIEs for a crack and two cracks in the upper part of bonded dissimilar materials.

2. To analyze a system of HSIEs for two cracks in both upper and lower parts of bonded dissimilar materials.

3. To analyze the systems of HSIEs for a thermally insulated crack and two thermally insulated cracks in the upper part of bonded dissimilar materials.

1.7 Scope of the study

This research will focus on formulation the HSIEs for the crack problems in bonded dissimilar materials and thermally insulated crack problems in bonded dissimilar materials subjected to remote stress using the modified complex variable function method with the continuity conditions of the resultant force and displacement function, and the continuity condition of the heat conduction function for the thermally insulated cracks problems. The system of HSIEs for these problems are solved using the appropriate quadrature formulas, then we use FORTRAN software to find the numerical solution for these problems.

1.8 Structure of the thesis

This thesis contains seven chapters which are structured as follows:

Chapter 1 describes the general information on fracture mechanics, crack problems and stress intensity factors at the crack tips. The research objectives, motivation and scope of the study are also covered in this chapter.

In Chapter 2, we review the complex variable function method to solve crack problems in elasticity. The previous work done by many researchers on the crack problems in bonded dissimilar materials and thermoelastic fields are discussed.

In Chapter 3, we discuss the methodology used in this research based on the modified complex variable function methods. The general formulation for crack problems in bonded dissimilar materials and thermally insulated crack problems in bonded dissimilar materials in term of HSIEs is also described. The basic quadrature formulas are included in this chapter in order to solve HSIEs.

The main problems in this thesis are discussed in Chapters 4, 5 and 6. For Chapter 4, we analyze the behavior of SIF for an inclined crack and a circular arc crack in the upper part of bonded dissimilar materials subjected to various stresses. Then we analyze the interaction between two inclined cracks, two circular arc cracks and an inclined and a circular arc cracks in the upper part of bonded dissimilar materials subjected to remote stress.

Chapter 5, is focused on the interaction between two cracks in both upper and lower parts of bonded dissimilar materials subjected to remote stress. We analyze the interaction between two inclined cracks, two circular arc cracks and an inclined and a circular arc cracks.

In Chapter 6, we analyze the behavior of SIF for a thermally insulated circular arc crack and a thermally insulated inclined crack in the upper part of bonded dissimilar materials. Then we analyze the interaction between two thermally insulated cracks in the upper part of bonded dissimilar materials subjected to remote stress. The comparison of nondimensional SIF for cracks with and without thermal is also illustrated.

Finally, Chapter 7 presents the summary of this thesis and some recommendations for future works.

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LIST OF PUBLICATIONS

The following are the list of publications that arise from this study.

Hamzah, K.B., Nik Long, N.M.A., Senu, N. and Eshkuvatov, Z.K. (2020). Stress Intensity Factor for Bonded Dissimilar Materials Weakened by Multiple Cracks. *Applied Mathematical Modelling*, 77, 585–601. **Q1 IF:2.841** (Doi: 10.1016/j.apm.2019.07.063).

Hamzah, K.B., Nik Long, N.M.A., Senu, N. and Eshkuvatov, Z.K. (2019). Stress Intensity Factors for Multiple Cracks in Bonded Dissimilar Materials using Hypersingular Integral Equation. *Applied Mathematical Modelling*, 73, 95–108. **Q1 IF:2.841** (Doi: 10.1016/j.apm.2019.04.002).

Hamzah, K.B., Nik Long, N.M.A., Senu, N., Eshkuvatov, Z.K. and Ilias, M.R. (2019). Stress Intensity Factors for a Crack in Bonded Dissimilar Materials Subjected to Various Stresses. *Universal Journal of Mechanical Engineering*, 7(4), 172–182. (Doi: 10.13189/ujme.2019.070405).

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Hamzah, K.B., Nik Long, N.M.A., Senu, N. and Eshkuvatov, Z.K. (2019). Stress Intensity Factors for Multiple Cracks Problems in Bonded Dissimilar Materials using Hypersingular Integral Equations. *International Conference on Communication and Intelligent Systems 2019*.

Hamzah, K.B., Nik Long, N.M.A., Senu, N. and Eshkuvatov, Z.K. (2020). Interaction between Two Inclined Cracks in Bonded Dissimilar Materials subjected to Various Stresses. *International Conference On Green Engineering and Technology 2020*.

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