UNIVERSITI PUTRA MALAYSIA

RATIONAL METHODS FOR SOLVING FIRST ORDER ORDINARY DIFFERENTIAL EQUATION

A'IN NAZIFA BINTI FAIRUZ

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RATIONAL METHODS FOR SOLVING FIRST ORDER ORDINARY DIFFERENTIAL EQUATION

By

A'IN NAZIFA BINTI FAIRUZ

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfillment of the Requirements for the Degree of Master of Science

November 2019

DEDICATIONS

To:

My strength and backbone;

My beloved parents:
Fairuz Leman and Roszilah Bassin,

...

My wonderful sisters:
Ain Zahirah Fairuz,
Ain Malihah Fairuz,
Ain Waheedah Fairuz.

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfillment of the requirement for the degree of Master of Science.

RATIONAL METHODS FOR SOLVING FIRST ORDER ORDINARY DIFFERENTIAL EQUATION

By

A'IN NAZIFA BINTI FAIRUZ

November 2019

Chairman: Zanariah binti Abdul Majid, PhD
Institute: Institute for Mathematical Research

In this study, two classes of rational methods of second to fourth order of accuracy are proposed. The formulation of the methods are based on two distinct rational functions that are proposed in thesis, where the first class of methods are derived based on rational function with denominator of degree one, as the degree of the numerator increases. Meanwhile, the second class uses a rational function with the numerator of degree one, as the degree of its denominator increases. The derivation and implementation techniques are adapted from an existing study mentioned in the thesis. The concept of the closest points of approximation is applied on the Taylor series expansion in the derivation of the methods to increase the accuracy of the proposed methods.

The stability regions of the proposed rational methods are illustrated. The second order methods from the first class is found to be A-stable, while third and fourth order methods are found to be absolutely stable. On the other hand, the methods from the second class are all A-stable. Besides that, the algorithm for the proposed methods are developed with constant step size strategy, in which the strategy to compute the starting values by an existing methods is also included.

Both classes of methods are tested in solving initial value problems of different nature which are singular, stiff and singular perturbation. Based on the numerical results, it is observed that the proposed methods are capable to give comparable or more accurate solutions compared to some of the existing methods in solving the tested problems. The application of closest points of approximation concept have shown the capability of the proposed methods in solving problem with integer singular point compared to the existing rational multistep methods. Nevertheless, as the proposed methods are compared to the existing methods which apply self-starting

mechanism in its formula, it is found that the accuracy of the proposed methods is comparable or outperformed by the existing methods. In terms of efficiency, the proposed methods require comparable or lesser time of execution compared to the existing methods of the same order. Besides that, the proposed methods also require lesser number of total function evaluation compared to the existing methods, except for the second order methods, where the number is found to be similar to the existing methods. In conclusion, the proposed methods are suitable in solving problems with singularity, stiff and singularly perturbed problems.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Master Sains

KAEDAH NISBAH BAGI MENYELESAIKAN PERSAMAAN PEMBEZAAN BIASA PERINGKAT PERTAMA

Oleh

A'IN NAZIFA BINTI FAIRUZ

November 2019

Pengerusi: Zanariah binti Abdul Majid, PhD
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Dalam kajian ini, dua kelas kaedah nisbah bagi peringkat kejituan kedua hingga keempat dicadangkan. Perumusan kaedah-kaedah ini adalah berdasarkan dua fungsi nisbah yang berlainan, iaitu dalam kelas pertama, fungsi nisbah yang digunakan dalam penyebut adalah berdarjah satu, sementara darjah bagi pengangkanya yang semakin meningkat. Dalam kelas kedua pula, fungsi nisbah menggunakan pengangka berdarjah satu, sementara darjah bagi penyebutnya semakin meningkat. Kaedah perumusan dan penggunaan telah dirujuk daripada satu kajian sedia ada yang dibincangkan dalam tesis. Konsep titik penghampiran terdekat juga digunakan pada kembangan siri Taylor dalam perumusan semua kaedah tersebut untuk meningkatkan kejituan kaedah yang dicadangkan.

Rantau kestabilan bagi kaedah nisbah yang dicadangkan telah dilakar. Kaedah peringkat kedua bagi kelas pertama adalah A-stabil, manakala kaedah peringkat ketiga dan keempat pula stabil secara mutlak. Bagi kaedah kelas kedua pula, semua kaedah adalah A-stabil. Selain itu, algoritma bagi kaedah yang dicadangkan dicipta dengan strategi saiz langkah malar, yang turut melibatkan strategi bagi mengira nilai-nilai pemula dengan menggunakan kaedah sedia ada.

Kedua-dua kelas kaedah telah diuji dalam menyelesaikan masalah nilai awal yang berlainan iaitu kesingularan, kekakuan dan pengusikan singular. Berdasarkan hasil berangka yang ditunjukkan, kaedah yang dicadangkan didapati berupaya untuk memberikan penyelesaian yang setara atau lebih jitu berbanding beberapa kaedah sedia ada dalam menyelesaikan masalah-masalah yang dinyatakan. Penggunaan konsep titik penghampiran telah membuktikan keupayaan kaedah yang dirumus dalam menyelesaikan masalah yang mempunyai titik singular integer. Walaubagaimanapun, apabila kaedah yang dicadangkan dibandingkan dengan kaedah sedia ada yang

menggunakan pendekatan mula sendiri di dalam formulanya, kejituan bagi kaedah yang dicadangkan didapati setara atau kurang jitu berbanding kaedah sedia ada tersebut. Dari segi kecekapan, kaedah yang dicadangkan mengambil masa pelaksanaan yang setara atau kurang berbanding kaedah sedia ada. Selain itu, kaedah-kaedah yang dicadangkan juga memerlukan jumlah fungsi seluruh yang kurang berbanding kaedah sedia ada, kecuali bagi kaedah peringkat kedua yang memerlukan jumlah fungsi seluruh yang sama dengan kaedah sedia ada. Kesimpulannya, kaedah yang dicadangkan adalah sesuai untuk menyelesaikan masalah kesingularan, kekakuan dan pengusikan singular.

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Last but not least, my deepest gratitude goes to my beloved parents as well as my sisters for their endless love, prayers and encouragement. I am so grateful to every single person who has directly and indrectly contributed in this journey, for their kindness means a lot to me.

Thank you so much.

This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfilment of the requirement for the degree of Master of Science. The members of the Supervisory Committee were as follows:

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LIST OF ABBREVIATIONS

NS	Non-standard method
BDF	Backward Differentiation Formula
IVP	Initial Value Problem
LMM	Linear Multistep Method
ODE	Ordinary Differential Equation
RKM	Runge-Kutta Method
RMM	Rational Multistep Method
TTS	Taylor Truncated Series
BBDF	Block Backward Differentiation Formula
RM(1,p)	p-th Order One-Step Rational Method
RM(1,2)	Second Order One-Step Rational Method
RM(1,3)	Third Order One-Step Rational Method
RM(1,4)	Fourth Order One-Step Rational Method
RM1S(p)	p-th Order A-Stable One-Step Rational Method
RM1S(2)	Second Order A-Stable One-Step Rational Method
RM1S(3)	Third Order A-Stable One-Step Rational Method
RM1S(4)	Fourth Order A-Stable One-Step Rational Method
RMM2(2,p)	2-step p-th Order Rational Multistep Method 2
RMM3(2,p)	2-step p-th Order Rational Multistep Method 3
RMM3(2,2)	2-step Second Order Rational Multistep Method 3
RMM3(2,3)	2-step Third Order Rational Multistep Method 3
RMM3(2,4)	2-step Fourth Order Rational Multistep Method 3
2IBBDF	2-Point Improved Block Backward Differentiation Formula
2OBBDF	2-Point Block Backward Differentiation Formula with Off-Step Points
3BEBDF	3-Point Block Extended Backward Differentiation Formula
2ODISBBDF	2-Point Diagonally Implicit Super Class Backward Differentiation Formula with Off-Step Points

CHAPTER 1

INTRODUCTION

1.1 Background

In the field of differential equations, an Initial Value Problem (IVP) of first order Ordinary Differential Equation (ODE) is an equation that has the form of;

$$y'(x) = f(x,y(x)), y(a) = \eta; \quad y(x), f(x,y(x)) \in R, \ \ x \in [a,b] \subset R \qquad (1.1.1)$$

where $y(a) = \eta$ is a given specified value, called the initial condition of the unknown function at a given point in the domain of the solution.

Some IVPs can be identified to have some special features. According to Lambert (1973), the special properties may be some analytical property of the function $y' = f(x,y(x))$ of the problem, or it may be that the theoretical solution that exhibits special characteristic, such as being periodic, possessing singularity or showing traits of stiffness. Moreover, Gear (1971) added another type of special problem called singularly perturbed IVP which literally exhibits stiffness properties, thus this kind of problem can be similarly solved by the approaches that are proposed to treat stiff equations.

1.1.1 Singular IVP

Singularity is defined by Tawfiq and Hussein (2013) as a point at which a given mathematical object is not defined or fails to be well-behaved, as appeared in many areas of study such as thermodynamics, electrostatics and physics. Hence, singular IVP can be simply described as a problem which fails to be defined at some points on the solution, and these points are called singular points.

Problems with singularity can be recognized by the presence of pole in its solution, or by a discontinuous low order derivative of the solution according to Ramos (2007). Meanwhile, Ikhile (2001) described that IVP with singularities fails to satisfy the requirement of existence and uniqueness theorem which leads to the poor performance of the conventional methods near the points of singularity. Contrarily, Luke et al. (1975) and Fatunla (1986) stated that the theory of ordinary differential equations offers no clue on the detection of singularities in the solutions, thus it must be identified heuristically.

1.1.2 Stiff IVP

Stiff problem is a common type of differential equation as it arises in various field of study such as vibrations, chemical reactions and electrical circuits. There is no universally accepted definition of stiffness as it is difficult to come out with mathematically precised definition to describe the properties of such phenomena, as stated by Lambert (1991).

However, stiff problem is recognisable as equations whose a part of its solution will rapidly decay, while the other part of it does not decay as quick as the first part. Meanwhile, a system of differential equations is said to be stiff if the ratio between the largest and the smallest eigenvalue is large.

This can be simply illustrated by considering the following linear problem;

$$y'(x) = Ay(x), \tag{1.1.2}$$

where A is $n \times n$ matrix which has distinct eigenvalues $\lambda_i (i = 1, 2, \ldots, n)$. The solution of the above equation is of the form;

$$y'(x) = [v_1, v_2, \ldots, v_i] \left[c_1 e^{\lambda_1 x}, c_2 e^{\lambda_2 x}, \ldots, c_i e^{\lambda_i x} \right]^T, \tag{1.1.3}$$

where v_i are the eigenvectors and c_i are constants which depend on the initial conditions.

It can be observed that if $Re(\lambda_i) < 0$ for each of the equations, then we have $c_i e^{\lambda_i x} \to 0$ for the solutions when $x \to \infty$. Considering the case where the eigenvalues are different in the system, for instance, one of the eigenvalue has very negative real part relative to the others. This means that,

$$R = \frac{max\,|Re(\lambda_i)|}{min\,|Re(\lambda_i)|}, \tag{1.1.4}$$

where R in the above equation is the stiffness ratio, and for stiff problems, this value tends to be large. Denoting $max\,|Re(\lambda_i)|$ as λ_0 and $min\,|Re(\lambda_i)|$ as λ_1, it can be seen that the exponential function $e^{\lambda_0 x}$ decays to zero more rapidly than $e^{\lambda_1 x}$. Such behaviour leads to great difficulties in obtaining the accurate numerical approximations to the theoretical solution and is referred as 'stiffness'. A class of methods that is well known for solving these kind of problems is Backward Differentiation Formula (BDF), and some recent studies on such methods are such as, Musa et al. (2012), Musa et al. (2013) and Abasi et al. (2014).

1.1.3 Singularly perturbed IVP

Singularly perturbed IVP can be identified by the presence of a small parameter, usually epsilon, ε that multiplies the first order derivative of the problem, for instance;

$$\varepsilon y'(x) = -y(x)\,(y(x)-1)\cos(x),$$

with the initial condition $y(0) = 0.5$, as mentioned in Ramos (2005). The eigenvalue of such problems tends to become larger as the value of the parameter decreases, thus causing the equation to become stiffer.

According to Ramos et al. (2015), this problem can be characterised by the presence of thin layers near the solution that cause it to vary quickly. However, the solution of such problems behaves regularly and varies slowly as it is away from these layers. Some authors also classify this kind of problem as a subset of stiff problems because of the behaviour exhibited by the equation.

1.2 Problem Statement

Numerical methods that have been conventionally applied in solving the IVP of first order ODE are from the class of Linear Multistep Method (LMM) and Runge-Kutta Method (RKM), which are derived based on polynomial functions. However, the conventional methods are found to be inefficient in solving problems with special characteristic as described by some authors, which are singular, stiffness and singular perturbation.

Therefore, rational function is chosen to replace polynomial function in developing methods due to its smooth behaviour in the neighbourhood of singularity. One of the most recent studies on Rational Multistep Method (RMM) is proposed by Teh and Yaacob (2013a) in the form of;

$$y_{n+2} = y_n + \frac{2h\,(y'_n)^2}{y'_n - hy''_n}. \tag{1.2.1}$$

where it can be observed that the distance between the approximated point and the point that is taken into consideration for the approximation process is two step length or $2h$. Thus, in this research we intend to develop rational schemes with better accuracy by implementing closest points of approximation concept in deriving the formula.

1.3 Objectives of the Study

The main objective of the study is to develop two classes of one-step rational methods which consider the closest points of approximation in its formulas, and these methods are to be tested for its capability and efficiency in solving different kind of problems which are singular, stiff and singular perturbation. The objective can be achieved by;

(i) deriving explicit one-step rational methods of different order based on two distinct rational functions proposed in the study.

(ii) analysing the stability regions of the proposed rational methods.

(iii) developing the algorithms of the derived methods with constant step size strategy.

(iv) analysing the performance of the derived methods in term accuracy, function evaluation and execution time.

1.4 Scopes of the Research

This study focuses on developing numerical approaches from the class of rational methods of second, third and fourth order of accuracy, and the schemes are derived based on two distinct rational functions suggested in the study. The first rational function is a function with denominator of degree one, whereas the second rational function consists of numerator of degree one as the degree of its denominator increases. The formulated schemes are one step methods and the algorithm are developed with contant step size strategy. The proposed methods are expected to be suitable and capable in solving singular, stiff and singular perturbation IVP of first order ODE.

1.5 Outline of the Thesis

The thesis is divided into five chapters. In Chapter 1, a brief description on IVP and some examples of special problems that are considered by the study are presented. Besides that, the problem statements, objectives and the scopes of the study are as well covered in this chapter.

Chapter 2 describes the justification of choosing rational functions in developing numerical methods. The chapter also includes the literature review of the rational functions and rational methods that have been proposed for solving problems with singularity and stiffness properties.

Chapter 3 focuses on the development of the p-th order absolutely stable one-step rational methods (RM(1,p)). Brief explanation on the derivation and implementation of the methods are provided. The local truncation error and region of absolute stability of this class of methods are also shown and described in the chapter. The formulated methods are tested on singular, stiff and singular perturbed problems and the numerical results are presented.

Chapter 4 on the other hand discusses the formulation of the p-th order A-stable one-step rational methods (RM1S(p)) along with its local truncation error, region of absolute stability and implementation. Similar problems tested in Chapter 3 are solved by the methods formulated in the chapter, and the comparisons of performance of the proposed methods and the existing methods are discussed.

Finally, the conclusion of the study is presented in Chapter 5. Recommendations and the suggestions for further study in this area is also provided in the chapter.

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By the end of 2012, she started her foundation studies at Centre of Foundation Studies for Agricultural Science, Universiti Putra Malaysia for a duration of eight months before enrolling in the program of Bachelor of Science (Honour) Major in Mathematics under Faculty of Science, Universiti Putra Malaysia in September 2013. She obtained her first degree for the same program in 2017.

Soon after graduating for her bachelor degree, she registered for the postgraduate program under the Institute for Mathematical Research, Universiti Putra Malaysia. She has been doing the research in the numerical analysis field, and was supported by Graduate Research Fund (GRF) and Putra Grant from Universiti Putra Malaysia.

LIST OF PUBLICATIONS

The following are the list of publications that arise from this study.

A'in Nazifa Fairuz, Zanariah Abdul Majid and Zarina Bibi Ibrahim, "Numerical Solution of First Order Initial-Value Problem with Singularities and Stiffness Properties by A Rational Scheme", *ASM Science Journal, Special Issue 7, 2019*. (Published)

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