ANALYTICAL SOLUTION OF BOUNDARY LAYER FLOW AND HEAT TRANSFER FOR NON-NEWTONIAN FLUIDS AND NANOFLUID

NUR SYAHIRAH BINTI WAHID

FS 2020 27

ANALYTICAL SOLUTION OF BOUNDARY LAYER FLOW AND HEAT TRANSFER FOR NON-NEWTONIAN FLUIDS AND NANOFLUID

By

NUR SYAHIRAH BINTI WAHID

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirements for the Degree of Master of Science.

June 2020

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Master of Science

ANALYTICAL SOLUTION OF BOUNDARY LAYER FLOW AND HEAT TRANSFER FOR NON-NEWTONION FLUIDS AND NANOFLUID

By

NUR SYAHIRAH BINTI WAHID

June 2020

Chair: Norihan Md Arifin, PhD
Faculty: Science

In this thesis, the boundary layer flow and heat transfer of non-Newtonian fluids and nanofluid are studied. The involved non-Newtonian fluids are viscoelastic fluid, Jeffrey fluid and Casson fluid. Meanwhile, for the nanofluid, we consider the hybrid nanofluid, as it is a recent interesting topic among researchers nowadays. The models are studied over a stretching surface. New additional parameters are being added to the model as to modify, improve, and extend the previous studies. The velocity slip parameter is the main parameter that is being added to all of the fluid models instead of the porosity and thermal radiation parameters. The governing partial differential equations are being reduced to the ordinary differential equations by applying the appropriate similarity transformation. The ordinary differential equations are then being solved by using analytical method with the aid of Maple program. The velocity and temperature profiles are then being plotted. The skin friction and heat transfer coefficients are being tabulated. We found out that the present results show a good agreement as to be compared with the previous studies, which then verify the method we used.

From the results obtained, it can be concluded that the effect of the main parameter of the study which is the velocity slip parameter towards both of these fluids are the same for the profiles and the main physical quantities of interest. In which, the increment of the velocity slip parameter has decreased the velocity profile and the local Nusselt number which is the heat transfer coefficient, however, it also has enhanced the temperature profile and the skin friction coefficient for both of the considered fluids that are the non-Newtonian fluids and also hybrid nanofluid.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia
sebagai memenuhi keperluan untuk ijazah Master Sains

PENYELESAIAN BERANALITIK BAGI ALIRAN LAPISAN SEMPADAN DAN PEMINDAHAN HABA TERHADAP BENDALIR BUKAN NEWTONIAN DAN BENDALIR NANO

Oleh

NUR SYAHIRAH BINTI WAHID

Jun 2020

Pengerusi: Norihan Md Arifin, PhD
Fakulti: Sains

Dalam tesis ini, aliran lapisan sempadan dan pemindahan haba bendalir bukan Newtonian dan bendalir nano telah dikaji. Bendalir bukan Newtonian yang terlibat adalah bendalir viskoelastik, bendalir Jeffrey dan bendalir Casson. Sementara itu, untuk bendalir nano, kami mengambil kira bendalir nano hibrid, kerana hal ini merupakan topik terbaru yang menarik dalam kalangan para penyelidik pada masa kini. Model-model ini dikaji ke atas permukaan regangan. Parameter tambahan yang baharu telah ditambah kepada model untuk mengubah, memperbaiki dan melanjutkan pengajian terdahulu. Parameter gelincir halaju adalah parameter utama yang ditambah pada semua model bendalir selain daripada parameter keliangan dan parameter sinaran terma. Persamaan pembezaan separa menakluk diturunkan kepada persamaan pembezaan biasa dengan menggunakan penjelmaan keserupaan yang sesuai. Persamaan pembezaan biasa kemudiannya diselesaikan dengan menggunakan kaedah analitikal dengan bantuan program Maple. Profil halaju dan suhu kemudiannya diplotkan. Pekali geseran kulit dan pemindahan haba ditabulasi. Kami mendapati bahawa keputusan menunjukkan persetujuan yang baik apabila dibandingkan dengan kajian terdahulu, yang kemudian mengesahkan kaedah yang telah kami gunakan.

Dari hasil yang diperoleh, dapat disimpulkan bahawa pengaruh parameter utama bagi kajian ini iaitu parameter gelincir halaju terhadap kedua-dua bendalir ini adalah sama untuk profil dan kuantiti tumpuan fizikal utama. Iaitu, kenaikan parameter gelincir halaju telah menurunkan profil halaju dan nombor Nusselt tempatan yang merupakan pekali pemindahan haba, namun ia juga telah meningkatkan profil suhu dan pekali geseran kulit untuk kedua-dua bendalir iaitu bendalir bukan Newtonian dan juga bendalir nano.

ACKNOWLEDGEMENTS

In the name of Allah, the Most Gracious and the Most Merciful, Alhamdulillah and infinite praises to Allah for the strengths and His blessing in completing this thesis as a fulfillment of the Master of Science in Applied Mathematics. Without His willingness and guidance, I would not have been able to complete this thesis on time.

I owe a deep debt of gratitude to Universiti Putra Malaysia for giving us an opportunity to complete our postgraduate studies. Also to Department of Mathematics, Faculty of Science Universiti Putra Malaysia.

Deepest appreciation goes to my supervisor Prof. Dr. Norihan Md Arifin for her awesome supervision and constant support through the journey of completing this research project. Not forgotten, my appreciation to my co-supervisors for this project, Dr. Ezad Hafidz Hafidzuddin and Mrs. Nor Aliza Rahmin for their support and knowledge regarding this topic of research.

Special gratitude goes to my beloved parents Mr. Wahid Jonid and Mrs. Siti Fatimah Mansor, my dearest brother and sister as well as my whole family members for their endless love, prayers, and encouragement.

Lastly, I would also like to express my thanks to all my friends who have supported me, helping me directly in the progress of this research and constantly providing me with moral support all along. I am really grateful to have you all by my side all these while.

This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfilment of the requirement for the degree of Master of Science. The members of the Supervisory Committee were as follows:

Norihan Md Arifin, PhD
Professor
Faculty of Science
Universiti Putra Malaysia
(Chairman)

Mohd Ezad Hafidz Hafidzuddin, PhD
Senior lecturer
Pusat Asasi Sains Pertanian
Universiti Putra Malaysia
(Member)

Nor Aliza Ab Rahmin, MSc
Lecturer
Faculty of Science
Universiti Putra Malaysia
(Member)

ZALILAH MOHD SHARIFF, PhD
Professor and Dean
School of Graduate Studies
Universiti Putra Malaysia

Date: 13 August 2020

Declaration of graduate student

I hereby confirm that

- this thesis is my original work;
- quotations, illustrations and citations have been duly referenced;
- this thesis has not been submitted previously or concurrently for any other degree at any other institutions;
- intellectual property from the thesis and copyright of thesis are fully-owned by Universiti Putra Malaysia, as according to the Universiti Putra Malaysia (Research) Rules 2012;
- written permission must be obtained from supervisor and the office of Deputy Vice-Chancellor (Research and Innovation) before thesis is published (in the form of written, printed or in electronic form) including books, journals, modules, proceedings, popular writings, seminar papers, manuscripts, posters, reports, lecture notes, learning modules or any other materials as stated in the Universiti Putra Malaysia (Research) Rules 2012;
- there is no plagiarism or data falsification/fabrication in the thesis, and scholarly integrity is upheld as according to the Universiti Putra Malaysia (Graduate Studies) Rules 2003 (Revision 2012-2013) and the Universiti Putra Malaysia (Research) Rules 2012. The thesis has undergone plagiarism detection software.

Signature: ____________________ Date: ____________________

Name and Matric No.: Nur Syahirah Wahid (GS52178)

Declaration by Members of Supervisory Committee

This is to confirm that:

- the research conducted and the writing of this thesis was under our supervision;
- supervision responsibilities as stated in the Universiti Putra Malaysia (Graduate Studies) Rules 200 (Revision 2012-2013) are adhered to

Signature:	
Name of Chairman of Supervisory Committee:	Norihan Md Arifin

Signature:	
Name of Member of Supervisory Committee	Mohd Ezad Hafidz Hafidzuddin

Signature:	
Name of Member of Supervisory Committee:	Nor Aliza Ab Rahmin

TABLE OF CONTENTS

LIST OF TABLES

Table **Page**

LIST OF FIGURES

LIST OF ABBREVIATIONS

A, B	constants
Al	alumina
B_0	uniform magnetic field
c	positive stretching rate constant
C	relation of slip condition
C_f	skin friction coefficient
Cu	copper
Ec	Eckert number
k	thermal conductivity
k_0	modulus of viscoelastic fluid
k_1^*	viscoelastic parameter
K	porosity parameter
$K*$	permeability of porous media
f	non-dimentional stream function
l, L	velocity slip parameter
M	magnetic parameter
Nu_x	local Nusselt number coefficient
Pr	Prandtl number
q_r	radiative heat flux
R_d	thermal radiation parameter
Re_x	local Reynolds number
s	suction parameter
T	temperature
T_∞	ambient fluid temperature
T_w	surface temperature
u	velocity component along x - axis
v	velocity component along y - axis
x_L	characteristic length
w	spherical nanoparticles
x	cartesian coordinates
y	cartesian coordinates
MHD	magnetohydrodynamic
HAM	homotopy analysis method

Greek Symbols

α	thermal diffusivity
β	Deborah number
β_c	Casson parameter
η	similarity variables
μ	fluid viscosity
ν	kinematic viscosity
ρ	fluid density
C_p	heat capacity
θ	non-dimentional temperature
σ	fluid electrical conductivity

σ^*	Stefan - Boltzman constant
k^*	absorption coefficient
λ_1	ratio of relaxation and retardation times
λ_2	relaxation time
ϕ_1	solid volume fraction for nanoparticle alumina
ϕ_2	solid volume fraction for nanoparticle copper

Subscripts

f	fluid
nf	nanofluid
hnf	hybrid nanofluid
s_1	solid nanoparticles of alumina
s_2	solid nanoparticles of copper
w	condition at the surface
∞	free stream condition

Superscripts

$'$	differentiation with respect to η

CHAPTER 1

INTRODUCTION

1.1 Introduction

In general, fluid is either liquid or gas or any substances that cannot resist shear stress or any external forces. Meanwhile, dynamic is related to the motion when a force is being applied. Therefore, the combination of these two words which is fluid dynamic is the study about the movement of fluids and their behaviour when dealing with forces. This branch of studies in applied science has helped many researchers and scientists to have a better understanding about the behaviour of fluids which in consequence giving them the idea to solve real life problems that deal with fluids.

In order to solve the real life problems that relate to fluid dynamics, mathematical modeling is one of the ways for us to translate, abstract, and simplify the real phenomenon problems into a mathematical description and solved them mathematically. Mathematical modeling helps us to simulate, manipulate and predict the real life events that have occurred or may occur. For example, in the context of fluid dynamics, we may construct the mathematical model of the problem and manipulating the model by adding some effects or parameters to predict the impacts of those effects towards the problem before handling them in real life.

Boundary layer is one of the prominent concepts that involves in the science of fluid dynamics which was first discovered by a physicist named Ludwig Prandtl in 1904. In the absence of slip condition, he theorized that the frictional effects towards the fluid flow occurred only in a thin layer of a flowing fluid that is very close to the surface (Anderson, 2005). His discovery on this boundary layer concept has made an enormous contribution to the area of physical and engineering sciences. By using some systematic calculations, the boundary layer flow variables can be obtained.

Besides, heat transfer is also a phenomenon that is so near to us in real life. The concept of boundary layer theory is also applicable to explain the heat transfer phenomena. In the boundary layer theory, we concern about the flow velocity, meanwhile, in heat transfer, we concern about the temperature difference. There are three mechanisms that involved in heat transfer. The first one is conduction, which is the transfer of heat through a solid or fluid at rest. Second is convection, which the heat is transferred between a surface and a moving fluid either gas or liquid. And the third is radiation, which the heat is transferred by electromagnetic waves through a vacuum or transmitting medium (Stölting, 1980).

Thus, in this thesis, we will discuss the science of fluid dynamics which emphasizes on boundary layer flow and also the heat transfer of the fluid.

1.2 Research Background

Fluid can be categorized into two which are Newtonian and non-Newtonian. Non-Newtonian fluid is one kind of characteristic of fluid that unobey the Newton's law of viscosity, meaning that this type of fluid viscosity depends on the applied force acting on it that is the deformation history or the shear rate. In addition, this type of fluid is viscoelastic, which they have the combined properties of elastic and viscous. Meanwhile, Newtonian fluid is vice versa to that non-Newtonian fluid.

1.2.1 Non-Newtonian Fluid

In this research work, we will focus and emphasize more on non-Newtonian fluid. The study of this fluid is important as it involves in many applications in real life, especially in the engineering field. For example, in the manufacturing of plastic sheets, paper, and glass. All of these industrial processes requiring us to have a better knowledge about the non-Newtonian fluid flow phenomena. Since a few decades, many scientists have been researching about the boundary layer flow of this fluid, even to this day. There have been many developments, including the invention of new models and advanced methods for solving the problem (Bush, 1989).

We will study three different types of non-Newtonian fluids in this thesis, which are the viscoelastic fluid (the common form of non-Newtonian fluid), Jeffrey fluid, and also Casson fluid. Several relevant effects or parameters will be considered to investigate the impacts towards the non-Newtonian fluids and nanofluid boundary layer flow and heat transfer past a stretching sheet.

1.2.2 Nanofluid

Nanofluid is a new kind of heat transfer fluid that includes a small amount of nanosized particles, typically smaller than 100 nanometers, that are stored in a liquid evenly and stably (Sheikholeslami and Ganji, 2015). Recently, many investigations also have been done towards the hybrid nanofluid, which engineered by suspending different nanoparticles either in a mixture or composite form (Sarkar et al., 2015).

1.2.3 Types of Effects

These effects or parameters can help us understand a particular phenomenon's physical significance. In this section, we will present several related parameters that we used in the problem as a brief introduction.

1.2.3.1 Slip condition

There are two types of slip condition which are the velocity slip condition and thermal slip condition. So, in order to define the slip condition for viscous fluids, we first need to know about the no slip condition. A condition that requires no tangential motion is known as the no slip condition (Day, 1990). In other words, the relative velocity between the solid boundary and the adjacent fluid is zero. Thus, the definition for the slip condition is vice versa.

1.2.3.2 Magnetohydrodynamic (MHD)

As the name implies, magnetohydrodynamic comprised of three words which are magneto, hydro and dynamic in which also can be defined as magnetic, water or liquid, and movement of an object by forces, respectively. Magnetohydrodynamic is the mathematical framework that is concerned with the movement of an electrically conducting fluid in a magnetic field. There are also another alternative names that give the same meaning as MHD which are magnetofluiddynamics, hydromagnetics, and magnetogasdynamics. MHD has been used to describe a variety of configurations, for instance, incompressible and compressible flows, liquid or gaseous state, dynamic or static configurations, particle atomic-physics analyses or continuum fluid analyses (Streeter, 1961). The first to explore the magnetohydrodynamic non-Newtonian fluid flow was Sarpkaya (1961), and then preceded by many others.

1.2.3.3 Porous medium

A porous medium can be simply described as a solid that has holes in it. Darcy's law is the fundamental law that describes the fluid flow through the porous matrix. The investigation towards the fluid flow through the porous matrix is much interested by the reservoir engineer, hydrologist, and soil scientist in order to help them in dealing with real life phenomena.

1.2.3.4 Thermal radiation

Thermal radiation can be described as the electromagnetic radiation emitted from a substance induced by the material's heat (Enderle and Bronzino, 2012). It is a very common energy transport phenomenon in our daily life and also very important in many engineering applications. For instance, it significantly contributes to the transfer of energy in furnaces, combustion chambers, fires, and heat exchangers at high temperatures. The calculations regarding radiative transfer need to be precise with the use of accurate radiative properties in order for us to improve the design and operation of those mentioned devices.

1.2.3.5 Suction and Injection

Suction is used in chemical processes to eliminate reactants while injection is used to add reactants. Suction and injection are important in real life operations, especially in the engineering field, such as in the recovery of thermal oil, and in the development of a thrust bearing and a radial diffuser (Ishak et al., 2009).

1.2.3.6 Prandtl number

The Prandtl number is a dimensionless quantity, which correlates the viscosity of a fluid with the thermal conductivity. It is invented by Ludwig Prandtl and named after him. Moreover, it is used to calculate the ratio of diffusiveness of momentum to the diffusiveness of thermal. The small value of Prandtl number indicates that the heat dissipates faster than the velocity. Therefore, in the context of fluid dynamics, fluids with small Prandtl number are high thermal conductivity free-flowing fluids (Rapp, 2016).

1.2.3.7 Eckert number

In the early 1950s, Ernst R. G. Eckert has discovered Eckert number, which named after himself. The Eckert number is a dimensionless number used to characterize the influence of self-heating of a fluid as a consequence of viscous dissipation effects which is the transformation of kinetic energy to internal energy due to viscosity (Rapp, 2016).

1.2.3.8 Deborah number

In 1964, Marcus Reiner has introduced the Deborah number. It is a dimensionless quantity to describe the viscoelastic behaviour of any material and also can be described as the proportion of material's relaxation time to experimental time (Reiner, 1964). The material is supposed to be more solid when the Deborah number increased.

1.3 Problem Statement

In this study, three types of non-Newtonian fluids named viscoelastic fluid, Jeffrey fluid and Casson fluid, and nanofluid will be focused on and covered. So, the issues regarding these non-Newtonian fluids and nanofluid are as follow:

1. How to formulate the mathematical model of boundary layer flow and heat transfer for non-Newtonian fluids and nanofluid?
2. Does the technique of the analytical method can be used to solve non-Newtonian fluids and nanofluid?
3. How the parameters applied in those three different non-Newtonian fluids and nanofluid affect the velocity and temperature fields as well as the skin friction coefficient and the local Nusselt number?

1.4 Objective and Scope

The aim of the thesis is to analyze and extend the mathematical model of boundary layer flow and heat transfer analytically for four problems:

1. MHD Darcy flow of viscoelastic fluids over a stretching sheet and heat transfer in the presence of velocity slip as the new additional parameter.
2. MHD flow and heat transfer of Jeffrey fluid over a stretching sheet in the presence of velocity slip and thermal radiation as the new additional parameters.
3. Casson fluid flow and heat transfer past a stretching surface in the presence of velocity slip, magnetohydrodynamic, and porosity as the new additional parameters.

4. MHD hybrid Cu-Al_2O_3/water nanofluid flow over a permeable stretching sheet in the presence of velocity slip and thermal radiation as the new additional parameters.

The scope for this research is emphasizing on the non-Newtonian fluids and nanofluid boundary layer flow past a stretchable surface with velocity slip as the main new additional parameter with several other pertinent effects and focusing on the analytical method instead of numerical method in order to obtain better results.

1.5 Significant of Study

The arises in the industry recently require us to understand well on the behaviour of the complex non-Newtonian fluids and nanofluid. Thus, this is a big challenge for the scientist to model, simulate, and simplify this real life phenomena. It is important to know the effects and behaviours of the fluid when engaging with the surrounding environment, and the first step should be taken by simulating the phenomena mathematically before dealing and handling them in real life.

In addition, for the context of hybrid nanofluid, the idea on the hybridization of nanofluid gives an improvement towards the heat transfer by balancing between the pros and cons of individual suspension of nanoparticle, better thermal network, assigned to good nanomaterial aspect ratio and synergistic effect (Minea and Moldoveanu, 2018).

Generally, problem solving involving mathematical modeling somehow providing the solutions towards the problem involved in the industry activities nowadays. The advancement in the technology like the computer aided analysis has made the problem solving becomes more easier, where we can solve the problem numerically by using certain coded program. However, providing the solutions in the form of exact analytical value should lessen the error that might occur in solving the problem, although the analytical method is restricted to certain kind of limitations. Therefore, in this thesis, the solutions presented are in the form of exact solution as the problems are possible to be solved by using the exact analytical method, instead of numerical method.

1.6 Thesis Outline

This thesis contains eight chapters that are structured as follows. The introduction of the thesis and a short brief necessary research background that are involved in this project as well as the objectives and scopes are discussed in Chapter 1. Some literature reviews that related to this research project on the specific scopes are discussed and taken into consideration in Chapter 2. The details of mathematical formulation and the exact analytical method used to solve the different problems which displayed in Chapter 4-7, are shown in Chapter 3.

The study about the magnetohydrodynamic slip Darcy flow of viscoelastic fluids over a stretching sheet and heat transfer with thermal radiation and viscous dissipation is presented in Chapter 4. Chapter 5 discusses magnetohydrodynamic flow and heat transfer of a Jeffrey fluid over a stretching sheet with velocity slip and thermal radiation. Then, Chapter 6 presents the magnetohydrodynamic Casson fluid flow and heat transfer past a stretching surface in porous medium with slip condition. And, in Chapter 7, we investigate about the magnetohydrodynamic hybrid Cu-Al_2O_3/water nanofluid flow over a permeable stretching sheet with velocity slip and thermal radiation.

Finally, Chapter 8 provides the conclusion including the summary for the whole thesis.

REFERENCES

Ahmad, K. and Ishak, A. (2016). MHD flow and heat transfer of a Jeffrey fluid over a stretching sheet with viscous dissipation. *Malaysian Journal of Mathematical Sciences*, 10:311–323.

Ahmad, K. and Ishak, A. (2017). Magnetohydrodynamic (MHD) Jeffrey fluid over a stretching vertical surface in a porous medium. *Propulsion and Power Research*, 6(4):269–276.

Ahmad, K., Wahid, Z., and Hanouf, Z. (2019). Heat transfer analysis for Casson fluid flow over stretching sheet with Newtonian heating and viscous dissipation. 1127(1):012028.

Alsaedi, A., Iqbal, Z., Mustafa, M., and Hayat, T. (2012). Exact solutions for the magnetohydrodynamic flow of a Jeffrey fluid with convective boundary conditions and chemical reaction. *Zeitschrift für Naturforschung A*, 67(8-9):517–524.

Aly, E. H. and Ebaid, A. (2013). New exact solutions for boundary-layer flow of a nanofluid past a stretching sheet. *Journal of Computational and Theoretical Nanoscience*, 10(11):2591–2594.

Aly, E. H. and Ebaid, A. (2016). Exact analysis for the effect of heat transfer on MHD and radiation Marangoni boundary layer nanofluid flow past a surface embedded in a porous medium. *Journal of Molecular Liquids*, 215:625–639.

Anderson, J. D. (2005). Ludwig Prandtl's boundary layer. *Physics Today*, 58(12):42–48.

Andersson, H. (1992). MHD flow of a viscoelastic fluid past a stretching surface. *Acta Mechanica*, 95(1-4):227–230.

Ariel, P. (1994). MHD flow of a viscoelastic fluid past a stretching sheet with suction. *Acta mechanica*, 105(1-4):49–56.

Ariel, P., Hayat, T., and Asghar, S. (2006). The flow of an elastico-viscous fluid past a stretching sheet with partial slip. *Acta Mechanica*, 187(1-4):29–35.

Bhattacharyya, K., Hayat, T., and Alsaedi, A. (2014). Exact solution for boundary layer flow of Casson fluid over a permeable stretching/shrinking sheet. *ZAMM-Journal of Applied Mathematics and Mechanics/Zeitschrift für Angewandte Mathematik und Mechanik*, 94(6):522–528.

Brewster, M. Q. (1992). *Thermal radiative transfer and properties*. John Wiley & Sons.

Bush, M. (1989). *Applications in Non-Newtonian Fluid Mechanics*. Springer.

Chen, C. H. (2010). On the analytic solution of MHD flow and heat transfer for two types of viscoelastic fluid over a stretching sheet with energy dissipation, internal heat source and thermal radiation. *International Journal of Heat and Mass Transfer*, 53(19-20):4264–4273.

Crane, L. J. (1970). Flow past a stretching plate. *Zeitschrift für angewandte Mathematik und Physik ZAMP*, 21(4):645–647.

Dash, R., Mehta, K., and Jayaraman, G. (1996). Casson fluid flow in a pipe filled with a homogeneous porous medium. *International Journal of Engineering Science*, 34(10):1145–1156.

Datti, P., Prasad, K., Abel, M. S., and Joshi, A. (2004). MHD visco-elastic fluid flow over a non-isothermal stretching sheet. *International Journal of Engineering Science*, 42(8-9):935–946.

Day, M. A. (1990). The no-slip condition of fluid dynamics. *Erkenntnis*, 33(3):285–296.

Devi, S. A. and Devi, S. S. U. (2016). Numerical investigation of hydromagnetic hybrid Cu–Al_2O_3/water nanofluid flow over a permeable stretching sheet with suction. *International Journal of Nonlinear Sciences and Numerical Simulation*, 17(5):249–257.

Ebaid, A. and Alhawiti, H. S. (2015). New application for the generalized incomplete gamma function in the heat transfer of nanofluids via two transformations. *Journal of Computational Engineering*, 2015:1–6.

Elbashbeshy, E., Emam, T., and Abdel-Wahed, M. (2014). An exact solution of boundary layer flow over a moving surface embedded into a nanofluid in the presence of magnetic field and suction/injection. *Heat and Mass Transfer*, 50(1):57–64.

Enderle, J. and Bronzino, J. (2012). *Introduction to biomedical engineering*. Academic press.

Fagbade, A., Falodun, B., and Omowaye, A. (2018). MHD natural convection flow of viscoelastic fluid over an accelerating permeable surface with thermal radiation and heat source or sink: spectral homotopy analysis approach. *Ain Shams Engineering Journal*, 9:1029–1041.

Hassani, M., Tabar, M. M., Nemati, H., Domairry, G., and Noori, F. (2011). An analytical solution for boundary layer flow of a nanofluid past a stretching sheet. *International Journal of Thermal Sciences*, 50(11):2256–2263.

Hayat, T., Asad, S., Qasim, M., and Hendi, A. A. (2012). Boundary layer flow of a Jeffrey fluid with convective boundary conditions. *International Journal for Numerical Methods in Fluids*, 69(8):1350–1362.

Hayat, T., Nadeem, S., and Khan, A. (2018). Numerical analysis of Ag–CuO/water rotating hybrid nanofluid with heat generation and absorption. *Canadian Journal of Physics*, 97(6):644–650.

Hussanan, A., Salleh, M. Z., Khan, I., and Shafie, S. (2018). Analytical solution for suction and injection flow of a viscoplastic Casson fluid past a stretching surface in the presence of viscous dissipation. *Neural Computing and Applications*, 29(12):1507–1515.

Ishak, A., Nazar, R., and Pop, I. (2009). The effects of transpiration on the flow and heat transfer over a moving permeable surface in a parallel stream. *Chemical Engineering Journal*, 148(1):63–67.

Kamran, A., Hussain, S., Sagheer, M., and Akmal, N. (2017). A numerical study of magnetohydrodynamics flow in Casson nanofluid combined with Joule heating and slip boundary conditions. *Results in physics*, 7:3037–3048.

Liu, I.-C. (2005). Flow and heat transfer of an electrically conducting fluid of second grade in a porous medium over a stretching sheet subject to a transverse magnetic field. *International Journal of Non-Linear Mechanics*, 40(4):465–474.

Mahmood, K., Sadiq, M. N., Sajid, M., and Ali, N. (2019). Heat transfer in stagnation-point flow of a Jeffrey fluid past a lubricated surface. *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, 41(2):65.

Minea, A. and Moldoveanu, M. (2018). Overview of Hybrid Nanofluids Development and Benefits. *Journal of Engineering Thermophysics*, 27(4):507–514.

Mukhopadhyay, S., Vajravelu, K., and Van Gorder, R. A. (2013). Casson fluid flow and heat transfer at an exponentially stretching permeable surface. *Journal of Applied Mechanics*, 80(5):054502.

Nadeem, S., Ahmad, S., and Muhammad, N. (2017). Cattaneo-Christov flux in the flow of a viscoelastic fluid in the presence of Newtonian heating. *Journal of Molecular Liquids*, 237:180–184.

Nadeem, S., Zaheer, S., and Fang, T. (2011). Effects of thermal radiation on the boundary layer flow of a Jeffrey fluid over an exponentially stretching surface. *Numerical Algorithms*, 57(2):187–205.

Narayana, P. S. and Babu, D. H. (2016). Numerical study of MHD heat and mass transfer of a Jeffrey fluid over a stretching sheet with chemical reaction and thermal radiation. *Journal of the Taiwan Institute of Chemical Engineers*, 59:18–25.

Nayak, M. K., Dash, G. C., and Singh, L. P. (2016). Heat and mass transfer effects on MHD viscoelastic fluid over a stretching sheet through porous medium in presence of chemical reaction. *Propulsion and Power Research*, 5(1):70–80.

Parsa, A. and Sayehvand, H.-O. (2016). Analytical solution of MHD flow and heat transfer over a permeable nonlinearly stretching sheet in a porous medium filled by a nanofluid. *Computational Methods for Differential Equations*, 4(1):70–98.

Qasim, M. (2013). Heat and mass transfer in a Jeffrey fluid over a stretching sheet with heat source/sink. *Alexandria Engineering Journal*, 52(4):571–575.

Qawasmeh, B. R., Alrbai, M., and Al-Dahidi, S. (2019). Forced convection heat transfer of Casson fluid in non-Darcy porous media. *Advances in Mechanical Engineering*, 11(1):1–10.

Rapp, B. E. (2016). *Microfluidics: Modeling, Mechanics and Mathematics.* William Andrew.

Rashidi, M., Ganesh, N. V., Hakeem, A. A., and Ganga, B. (2014). Buoyancy effect on MHD flow of nanofluid over a stretching sheet in the presence of thermal radiation. *Journal of Molecular Liquids*, 198:234–238.

Raza, J. (2019). Thermal radiation and slip effects on magnetohydrodynamic (MHD) stagnation point flow of Casson fluid over a convective stretching sheet. *Propulsion and Power Research*, 8(2):138–146.

Reiner, M. (1964). The deborah number. *Physics today*, 17(1):62.

Saidulu, N. and Lakshmi, A. V. (2016). Slip effects on MHD flow of casson fluid over an exponentially stretching sheet in presence of thermal radiation, heat source/sink and chemical reaction. *European journal of advances in engineering and technology*, 3(1):47–55.

Sarkar, J., Ghosh, P., and Adil, A. (2015). A review on hybrid nanofluids: recent research, development and applications. *Renewable and Sustainable Energy Reviews*, 43:164–177.

Sarpkaya, T. (1961). Flow of non-Newtonian fluids in a magnetic field. *AIChE Journal*, 7(2):324–328.

Sheikholeslami, M. and Ganji, D. D. (2015). *Hydrothermal analysis in engineering using control volume finite element method.* Academic Press.

Stölting, M. (1980). Fluid Flow and Heat Transfer. *Chemie Ingenieur Technik*, 52(3):284–284.

Streeter, V. L. (1961). *Handbook of fluid dynamics.* McGraw-Hill Book Company.

Turkyilmazoglu, M. (2011). Multiple solutions of heat and mass transfer of MHD slip flow for the viscoelastic fluid over a stretching sheet. *International Journal of Thermal Sciences*, 50(11):2264–2276.

Turkyilmazoglu, M. (2012). Exact analytical solutions for heat and mass transfer of MHD slip flow in nanofluids. *Chemical Engineering Science*, 84:182–187.

Turkyilmazoglu, M. and Pop, I. (2013). Exact analytical solutions for the flow and heat transfer near the stagnation point on a stretching/shrinking sheet in a Jeffrey fluid. *International Journal of Heat and Mass Transfer*, 57(1):82–88.

Waini, I., Ishak, A., and Pop, I. (2019). Hybrid nanofluid flow and heat transfer over a nonlinear permeable stretching/shrinking surface. *International Journal of Numerical Methods for Heat & Fluid Flow*, 29(9):3110–3127.

Zin, N. A. M., Khan, I., and Shafie, S. (2018). Exact and numerical solutions for unsteady heat and mass transfer problem of Jeffrey fluid with MHD and Newtonian heating effects. *Neural Computing and Applications*, 30(11):3491–3507.