UNIVERSITI PUTRA MALAYSIA

CLASSIFICATION OF TWO-DIMENSIONAL ALGEBRAS USING MATRICES OF STRUCTURE CONSTANTS APPROACH

HOUIDA MOHAMMED HUSSEIN AHMED

FS 2020 19

CLASSIFICATION OF TWO-DIMENSIONAL ALGEBRAS USING MATRICES OF STRUCTURE CONSTANTS APPROACH

By

HOUIDA MOHAMMED HUSSEIN AHMED

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfillment of the Requirements for the Doctor of Philosophy

February 2020

DEDICATIONS

I would like to dedicate this thesis
...to the soul of my beloved father who had dreamed about this day
&
...to my beloved mother, my wonderful husband,
my daughter Lojain and my son Abdulmalek
with my love

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfillment of the requirement for the degree of Doctor of Philosophy

CLASSIFICATION OF TWO-DIMENSIONAL ALGEBRAS USING MATRICES OF STRUCTURE CONSTANTS APPROACH

By

HOUIDA MOHAMMED HUSSEIN AHMED

February 2020

Chairman: Sharifah Kartini Said Husain, PhD
Faculty: Faculty of Science

One of the main issues of modern algebra is the classification problem related to finite-dimensional algebras. So far, there are two approaches to deal with this problem. The first one is the structural approach (basis free and invariant). However, when dealing with general types of algebras, this approach shows that it is ineffective due to the difficulty in dealing with such general algebra types. The second approach that tried to deal with this particular problem is the coordinate-based approach. The above-mentioned approaches somehow are complementary to each other.

Moreover, it is noticed that well known classification theorems of algebras, for example Lie, associative, Jordan and etc., in fixed n-dimensional case cover only small parts of all n-dimensional algebras. In fact, the part out of the consideration is much bigger and it is a dense in Zariski topology subset of the set of all n-dimensional algebras. Therefore, instead of the classification of some classes of algebras one can try to classify all algebras in fixed dimensions. We use a new approach to study the classification problem of finite dimensional algebras and the main advantage of the approach, proposed in Bekbaev (2015) which we apply it here, is the fact that it reduces the classification and some other problems of finite-dimensional algebras to the investigation of a system of equations which can be solved using some computation programs. We give a list of algebras, depending on their matrices of structure constants (MSC). Therefore, any two-dimensional algebra is considered isomorphic to one of the algebras in the provided list.

The organization of the thesis starts with studying the problem of classification over what is known as algebraically closed fields of not two and not three characteristics.

After that we presented the solution for this problem that deals with the state of classification to be over algebraically closed fields of characteristics two and three. In these cases, we provide a list of algebras, using their matrices of structure constants (MSC). Then the automorphism groups and derivation algebras for the all listed canonical algebras are described. We also present complete lists of isomorphism classes of all two-dimensional left (right) unital algebras, (not necessarily commutative) Jordan algebras, power associative algebras and we specify commutative Jordan, associative, Lie, Leibniz and Zinbiel algebras by providing the lists of canonical representatives of their structure constant's matrices. All subalgebras, idempotents, ideals and the left quasiunits of two-dimensional algebras are described and tabulated. We also present all possible evolution algebra structures on two-dimensional vector space over any algebraically closed field. The description of group automorphisms and derivation evolution algebras is given.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

PENGKELASAN ALJABAR MATRA DUA MENGGUNAKAN PENDEKATAN MATRIKS PEMALAR STRUKTUR

Oleh

HOUIDA MOHAMMED HUSSEIN AHMED

Februari 2020

Pengerusi: Sharifah Kartini Said Husain, PhD
Fakulti: Fakulti Sains

Salah satu isu utama aljabar moden ialah masalah pengkelasan aljabar berkait dengan dimensi terhingga. Setakat ini, terdapat dua pendekatan untuk menyelesaikan masalah tersebut. Pendekatan pertama adalah pendekatan struktur (asas bebas, tak variasi). Walau bagaimana pun, apabila dipertimbangkan jenis aljabar yang umum, pendekatan ini menunjukkan bahawa ia kurang berkesan kerana kesukaran dalam berurusan jenis aljabar yang umum sedemikian. Pendekatan kedua untuk penyelesaian masalah ini adalah pendekatan berasaskan koordinat. Pendekatan yang dinyatakan di atas adalah secara dasarnya berkait antara satu sama lain di setiap masa pendekatan saling melengkapkan antara satu sama lain.

Lebih-lebih lagi, diketahui bahawa teori pengkelasan aljabar yang terkenal, contohnya Lie, kalis sekutuan, Jordan dan sebagainya, dalam kes matra-n yang ditetapkan hanya meliputi sebahagian kecil daripada kesemua aljabar bermatra-n. Malah, sebahagian daripada pertimbangan lebih besar dan ia adalah padat dalam subset topologi Zariski dari set semua aljabar bermatra-n. Oleh itu, daripada pengkelasan beberapa kelas aljabar seseorang boleh mencuba untuk mengkelaskan semua aljabar dalam matra yang ditetapkan. Kami menggunapakai pendekatan baharu untuk mengkaji masalah pengkelasan aljabar bagi matra terhingga dan kelebihan utama pendekatan ini,yang telah dicadangkan oleh Bekbaev (2015) yang kami praktikkan di sini, adalah sebenarnya ia mengurangkan pengkelasan dan beberapa masalah lain bagi aljabar bermatra terhingga kepada penyiasatan sistem persamaan yang boleh diselesaikan dengan menggunakan program pengkomputeran. Kami memberi suatu senarai aljabar, bergantung kepada matriks pemalar strukturnya (MSC). Oleh itu, sebarang aljabar bermatra dua yang dipertimbangkan berisomorfik kepada salah satu aljabar dalam senarai yang disediakan.

Penyusunan tesis bermula dengan mengkaji masalah pengkelasan ke atas medan tertutup aljabar bagi cirian bukan dua dan tiga. Selepas itu kami telah membentangkan untuk masalah berkaitan pengkelasan ke atas medan tertutup aljabar bagi cirian dua dan tiga. Dalam setiap kes ini, kami menyenaraikan aljabar melalui matriks pemalar strukturnya. Kemudian kumpulan automorfisma dan pembezaan aljabar untuk semua aljabar berkanun yang tersenarai dinyatakan. Kami juga membentangkan senarai lengkap kelas isomorfisma dari semua matra dua aljabar unital kiri (kanan), (tidak semestinya kalis tukar tertib) aljabar Jordan dan kami menentukan aljabar Jordan kalis tukar tertib, kalis sekutuan, Lie, Leibniz dan Zinbiel dengan menyediakan senarai perwakilan berkanun bagi struktur matriks pemalarnya. Semua subaljabar, idempoten, unggulan dan kuasiunit kiri aljabar matra dua diterangkan dan ditabulasi. Kami juga membentangkan semua struktur aljabar evolusi yang mungkin pada ruang vektor matra dua ke atas sebarang medan tertutup aljabar. Penerangan mengenai kumpulan automorfisma dan pembezaan aljabar diberikan.

ACKNOWLEDGEMENTS

First and foremost, all praise is for *Allah Subhanahu Wa Taala* for giving me the strength, guidance and patience to complete this thesis. I thank Allah for His immense grace and blessing every stage of my entire life. May blessing and peace be upon Prophet Muhammad *Sallalahu Alaihi Wasallam*, who was sent for mercy to the world.

First and foremost, I would like to express my sincere gratitude to all my supervisors especially to Associate Professor Dr. Ural Bekbaev and Prof. Dr. Isamiddin Rakhimov for the continuous support of my study and research, for their patience, motivation, enthusiasm, and immense knowledge. Their guidance helped me in all the time of research and writing of this thesis. Their encouragement and help made me feel confident to overcome every difficulty I encountered in all the stages of this research. What I really learned from them, however, is his attitude to work and life - always aiming for excellence. I also thank Associate Professor Dr. Sharifah Kartini Said Husain and Dr. Witriany Basri for supervision.

I am very grateful to my family, my father, , my mother, , my brothers, and my sisters for their unflagging love and support throughout my life. I have no suitable words that can fully describe my everlasting love to them except, I love you all.

Words fail me to express my appreciation to my wonderful husband whose dedication, love and persistent confidence in me, has taken the load off my shoulder. Special thank goes to my kids, and, you are my joy and my guiding lights. Thanks for giving me your valuable time through this long process. I promise I will never let you alone anymore.

Finally, I would like to thank everybody who was important to the successful realization of this thesis, as well as I express my apology that I could not mention you all personally.

This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfilment of the requirement for the degree of Doctor of Philosophy. The members of the Supervisory Committee were as follows:

Sharifah Kartini Said Husain, PhD
Assosiate Professor
Faculty of Science
Universiti Putra Malaysia
(Chairperson)

Isamiddin Rakhimov, PhD
Professor
Institute for Mathematical Research
Universiti Putra Malaysia
(Member)

Ural Bekbaev, PhD
Associate Professor
Faculty of Engineering
International Islamic University Malaysia
(Member)

Witriany Basri, PhD
Senior Lecturer
Faculty of Science
Universiti Putra Malaysia
(Member)

ZALILAH MOHD SHARIFF, PhD
Professor and Dean
School of Graduate Studies
Universiti Putra Malaysia

Date: 16 July 2020

Declaration by graduate student

I hereby confirm that:

- this thesis is my original work;
- quotations, illustrations and citations have been duly referenced;
- this thesis has not been submitted previously or concurrently for any other degree at any other institutions;
- intellectual property from the thesis and copyright of thesis are fully-owned by Universiti Putra Malaysia, as according to the Universiti Putra Malaysia (Research) Rules 2012;
- written permission must be obtained from supervisor and the office of Deputy Vice-Chancellor (Research and Innovation) before thesis is published (in the form of written, printed or in electronic form) including books, journals, modules, proceedings, popular writings, seminar papers, manuscripts, posters, reports, lecture notes, learning modules or any other materials as stated in the Universiti Putra Malaysia (Research) Rules 2012;
- there is no plagiarism or data falsification/fabrication in the thesis, and scholarly integrity is upheld as according to the Universiti Putra Malaysia (Graduate Studies) Rules 2003 (Revision 2012-2013) and the Universiti Putra Malaysia (Research) Rules 2012. The thesis has undergone plagiarism detection software.

Signature:________________________ Date:________________________

Name and Matric No: Houida Mohammed Hussein Ahmed, GS45099

Declaration by Members of Supervisory Committee

This is to confirm that:

- the research conducted and the writing of this thesis was under our supervision;
- supervision responsibilities as stated in the Universiti Putra Malaysia (Graduate Studies) Rules 2003 (Revision 2012-2013) are adhered to.

Signature: ______________________________
Name of
Chairman of
Supervisory
Committee: Assosiate Professor Dr. Sharifah Kartini Said Husain

Signature: ______________________________
Name of
Member of
Supervisory
Committee: Professor Dr. Isamiddin Rakhimov

Signature: ______________________________
Name of
Member of
Supervisory
Committee: Assosiate Professor Dr. Ural Bekbaev

Signature: ______________________________
Name of
Member of
Supervisory
Committee: Dr. Witriany Basri

TABLE OF CONTENTS

LIST OF TABLES

CHAPTER 1

INTRODUCTION

1.1 Introduction

The classification problem of finite dimensional algebras is one of the important problems of modern algebra. So far there are two approaches to deal with this problem. The first one is the structural approach (basis free and invariant). Two examples for this approach can be taken from Cartan and Wedderburn. That is, Cartan classified finite dimensional simple and semi-simple Lie algebras, and Wedderburn deals with simple and semi-simple associative algebras. However, when dealing with general types of algebras, this approach shows that it is ineffective due to the difficulty in dealing with such general algebra types. The second approach that tried to deal with this particular problem is the coordinate-based approach (Díaz et al. (2003); Encinas et al. (2004); Goze and Remm (2011); Popov (2014); Bekbaev (2015) for the latest results). The above-mentioned approaches are in essence related to each other as every approach completes the other one. In the thesis, we study the classification problem of finite-dimensional algebras using a new approach. We apply this approach to solve the classification problem of 2-dimensional algebras. The same result is obtained by Goze and Remm (2011), but there are differences between the results obtained in this study and the results obtained by them. The authors state the existence without ensuring the uniqueness. Furthermore, the approach that deals with the problem in this study is totally different from the approach used by Goze and Remm (2011).

The study of the automorphism groups of algebra has a long history, and the previous literature shows that it is really difficult to determine the full automorphism group of an algebra. In the thesis, we use a general rule to define the automorphism groups of m-dimensional algebras using our approach.

It is known that many important algebras in use are so called PI-algebras, that is, algebras satisfying a certain set of polynomial identities. Therefore, the classification of such algebras, up to isomorphism, is of a great interest. We give classification results for some important classes of two-dimensional PI-algebras, for instance, Jordan algebras,power-associative algebras, left (right) unital algebras, associative algebras, Lie algebras, Leibniz algebras and Zinbiel algebras.

The description of all subalgebras, idempotents, left (right) ideals and quasi-units of 2-dimensional algebras is important. We consider only nontrivial 2-dimensional algebras and by a subalgebra (left (right) ideal) we mean nontrivial, that is one dimensional, subalgebra (respectively, left (right) ideal). It is shown that every such an algebra may have only one, two, three or infinitely many subalgebras and may have zero, one, two or infinitely many ideals. We provide a classification for each class of algebras with n different subalgebras (left (right) ideals), where $n = 1, 2, 3, \infty$ (respectively, $n = 0, 1, 2, \infty$) and subsequently, give a classification of 2-dimensional simple algebras.

Recently a new type of genetic algebras, denominated evolution algebras, has emerged to enlighten the study of non-Mendelian Genetics, which is the basic language of the molecular Biology. In particular, evolution algebras can be applied to the inheritance of organelle genes. A classification of 2-dimensional complex evolution algebras is considered in Cali and Josephy (1985). This result is clarified and generalized in Casado (2016) for such algebras over any field where every element has quadratic and cubic roots. In this work, we give classification of 2-dimensional evolution algebras over any field where each element has quadratic root and apply it to investigate their automorphisms, derivatives and describe the isomorphism classes of 2-dimensional left (right) evolution unital algebras.

1.2 Motivation

One of the main issues of modern algebra is the classification problem related to finite-dimensional algebras. The problem is to try and determine the isomorphism classes of the algebraic structure. There are some approaches to the solution of the problem. In some researches, lists of some classes of algebras are introduced but some algebras are missed in the list or some of them are isomorphic. On the other side, it is so difficult to study some identities of algebras, for example, the unite, the idempotent, ideal,... etc. This motivates us to find a general approach to solve this problem such that this approach can cover all 2-dimensional algebras in the same time it can describe any identity related to that classified algebras and can apply for high dimension.

1.3 Problem Statement

It is noticed that well known classification theorems of algebras, for example Lie, associative, Jordan and etc, in fixed n-dimensional case cover only small parts of all n-dimensional algebras. In fact, the part out of the consideration is much bigger and it is a dense in Zariski topology subset of the set of all n-dimensional algebras. Therefore, instead of the classification of some classes of algebras one can try to classify all algebras in fixed dimensions. We use a new approach to study the classification problem of finite dimensional algebras and the main advantage of the approach, proposed in Bekbaev (2015) which we apply it here, is the fact that it reduces the classification and some other problems of finite-dimensional algebras to the investigation of a system of equations which can be solved using some computation programs as Wolfram Mathematica.

Furthermore in our research, the set of all n-dimensional algebra structures Alg_n on a vector space V over a field $\mathbb{F}$ is simply the vector space of all $\mathbb{F}$-bilinear maps from $V \times V$ to V and G a linear algebraic group over an algebraically closed field $\mathbb{F}$ which is a subgroup of $GL(n, \mathbb{F})$, whose elements are precisely the solutions of a set of polynomial equations in the matrix coordinates. For a fixed basis $\mathbf{e} = \{e_1, e_2, ..., e_n\}$, one can represent the bilinear map $\cdot$ by a matrix $A \in Mat(n \times n^2; \mathbb{F})$ such that

$$\mathbf{u} \cdot \mathbf{v} = \mathbf{e}A(u \otimes v)$$

for any $\mathbf{u} = \mathbf{e}u, \mathbf{v} = \mathbf{e}v$, where u and v are column coordinate vectors of $\mathbf{u}$ and $\mathbf{v}$, respectively. We construct a G-invariant open dense Zariski subset

$$V_0 = \{A : \ f(A) \neq 0 \text{ where } f \text{ is a polynomial in } n \text{ variables over } \mathbb{F}\}.$$

We provide a list of algebras in the 2-dimensional case, using their matrices of structure constants (MSC). The list shows that all the 2-dimensional algebras are isomorphic to one algebra provided in the list. Using the main approach also we study and describe some identities especially thats ones which have not been given yet as: unital algebras, subalgebras, the idempotent and the ideals for all 2-dimensional algebras.

1.4 Objectives of the research

The main objectives of this research are:

1. To classify 2-dimensional algebras over algebraically closed field.
2. To establish groups of automorphisms and derivation algebras of 2-dimensional algebras.
3. To find the classification of 2-dimensional left (right) unital algebras.
4. To give the list of isomorphism classes of 2-dimensional Jordan and power associative algebras together with their properties.
5. To determine subalgebras, idempotent and ideals of 2-dimensional algebras.
6. To classify 2-dimensional evolution algebras.

1.5 Thesis Organization

The thesis comprises of ten chapters, we briefly mention the layout of this thesis as follows.

Chapter 1 contains the motivation to study the problem and the statement of the problem. Then the objectives of the research are listed.

Chapter 2 covers a review of relevant literature.

Chapter 3 contains some definitions and relevant background information related to the research conducted, and motivation of the method applied.

In Chapter 4, firstly, we introduce a classification approach of n-dimensional algebras over algebraically closed fields then we use this approach to study the problem of classification over what is known as algebraically closed fields of not 2 and not 3 characteristics. After that, we present the solution for this problem that deals with the state of classification to be over algebraically closed fields of characteristics 2 and 3. In these cases, we provides a list of algebras, using their matrices of structure constants (MSC). Then we specify classes of well-known algebras as left (right) unital algebras, associative algebras, Lie algebras, Leibniz algebras and Zinbiel algebras by providing the lists of canonical representatives of their structure constant's matrices.

In Chapter 5, the automorphism groups and derivation algebras for the all listed canonical algebras are described.

In Chapter 6, as an application of the previous work a complete list of 2-dimensional (not necessarily commutative) Jordan algebras is given and the automorphism groups and derivation algebras for the all listed Jordan algebras are described then all unital Jordan algebras are presented.

Chapter 7 contains results on the class of power-associative algebras. The automorphism groups and derivation algebras for the all listed power-associative algebras are described then all unital power-associative algebras are given.

In Chapter 8, all subalgebras, idempotents, ideals and the left quasiunits of 2-dimensional algebras are described and tabulated.

In Chapter 9, we introduce a new technique to classify finite dimensional evolution algebras then we present all possible evolution algebra structures on 2-dimensional vector space over any algebraically closed field. The description of group automorphisms and derivation algebras is given. Finally, all unital evolution algebras are described.

Chapter 10, contains the conclusions on the results of the thesis and suggest a few problems for future work in these areas.

REFERENCES

Abdulkareem, A. O., Rakhimov, I. S., and Husain, S. K. S. (2015). Isomorphism classes and invariants of low-dimensional filiform Leibniz algebras. *Linear and Multilinear Algebra*, 63(11):2254–2274.

Albert, A. A. (1946). On Jordan algebras of linear transformations. *Trans. Amer. Math. Soc*, 59(31):524–555.

Albert, A. A. (1948). On power-associativity of rings. *Summa Brasiliensis Math*, 2(12):21–33.

Albeverio, S., Omirov, B. A., and Rakhimov, I. S. (2006). Classification of 4-dimensional nilpotent complex Leibniz algebras. *Extracta Mathematicae*, 21(3):197–210.

Alev, J. and Chamarie, M. (1992). Derivations et automorphismes de quelques algebras quantiques. *Communications in Algebra*, 20(6):1787–1802.

Alev, J. and Dumas, F. (1996). Rigidité des plongements des quotients primitifs minimaux de $U_g(sl(2))$ dans lálgebre quantique de Weyl-Hayashi. *Nagoya Mathematical Journal*, 143:119–146.

Andruskiewitsch, N. and Dumas, F. (2003). On the automorphisms of $U_q^+(g)$. *preprint, arXiv:0301066*.

Arezina, P., Caldwell, S., Davis, J., Frederick, R., Julius, G., and Ringel, M. (1996). Three-dimensional associative unital algebras. *Journal of PGSS*, 15:227–237.

Bavula, V. and Jordan, D. (2001). Isomorphism problems and groups of automorphisms for generalized weyl algebras. *Transactions of the American Mathematical Society*, 353(2):769–794.

Bekbaev, U. (2015). On classification of finite dimensional algebras. *preprint, arXiv:1504.01194*.

Bekbaev, U. (2017). Complete classifications of two-dimensional general, commutative, commutative Jordan, division and evolution real algebras. *preprint, arXiv:1705.01237*.

Cali, A. L. and Josephy, M. (1985). Two-dimensional real division algebras. *Rev. Un. Mat. Argentina*, 32(1):58–63.

Camacho, L., Gómez, J., Omirov, B., and Turdibaev, R. (2010). Some properties of evolution algebras. *arXiv preprint arXiv:1004.1987*.

Camacho, L., Gómez, J., Omirov, B., and Turdibaev, R. (2013). The derivations of some evolution algebras. *Linear and Multilinear Algebra*, 61(3):309–322.

Casado, Y. (2016). *Evolution algebras*. PhD thesis, Universidad de Málaga.

Casado, Y. C., Molina, M. S., and Velasco, M. V. (2016). Evolution algebras of arbitrary dimension and their decompositions. *Linear Algebra and its Applications*, 495:122–162.

Casado, Y. C., Molina, M. S., and Velasco, M. V. (2017). Classification of three-dimensional evolution algebras. *Linear Algebra and its Applications*, 524:68–108.

Casas, J., Ladra, M., Omirov, B., and Rozikov, U. (2014). On evolution algebras. *Algebra Colloquium*, 21:331 – 342.

Casas, J. M., Insua, M. A., Ladra, M., and Ladra, S. (2012). An algorithm for the classification of 3-dimensional complex Leibniz algebras. *Linear Algebra and its Applications*, 436(9):3747–3756.

Ceken, S., Palmieri, J. H., Wang, Y.-H., and Zhang, J. (2015). The discriminant controls automorphism groups of noncommutative algebras. *Advances in Mathematics*, 269:551–584.

de Graaf, W. A. (2005). Classification of solvable Lie algebras. *Experimental Mathematics*, 14(1):15–25.

Díaz, R. D., Masqué, J. M., and Domínguez, A. P. (2003). Classifying quadratic maps from plane to plane. *Linear Algebra and its Applications*, 364:1–12.

Dixmier, J. (1960). Sur les representations unitaires des groupes de Lie nilpotents. vi. *Canadian Journal of Mathematics*, 12:324–352.

Dzhumadil'Daev, A. and Tulenbaev, K. (2005). Nilpotency of Zinbiel algebras. *Journal of Dynamical and Control Systems*, 11(2):195–213.

Elduque, A., Laliena, J., and Sacristán, S. (2004). Maximal subalgebras of associative superalgebras. *Journal of Algebra*, 275(1):40–58.

Elduque, A., Laliena, J., and Sacristán, S. (2008). Maximal subalgebras of Jordan superalgebras. *Journal of Pure and Applied Algebra*, 212(11):2461 – 2478.

Encinas, L. H., del Rey, A. M., and Masqué, J. M. (2004). Non-degenerate bilinear alternating maps $f : V \times v \to v$, dim (v)= 3, over an algebraically closed field. *Linear Algebra and its Applications*, 387:69–82.

Etherington, I. M. H. (1939). Genetic algebras. *Proc. Roy. Soc. Edinburgh*, 59:242–258.

Etherington, I. M. H. (1941). Non-associative algebra and the symbolism of genetics. *Proc. Roy. Soc. Edinburgh, Section B: Biological Sciences*, 61(1):24–42.

Fialowski, A., Penkava, M., and Phillipson, M. (2011). Deformations of complex 3-dimensional associative algebras. *Journal of Generalized Lie Theory and Applications*, 5:1–22.

Gabriel, P. (1975). Finite representation type is open. In *Representations of algebras*, pages 132–155. Springer Berlin Heidelberg.

Gainov, A. T. (1970). Power-associative algebras over a finite-characteristic field. *Algebra and Logic*, 9(1):5–19.

Goodearl, K. R. and Yakimov, M. T. (2015). Unipotent and nakayama automorphisms of quantum nilpotent algebras. *Commutative Algebra and Noncommutative Algebraic Geometry*, 2:181–212.

Goze, M. and Remm, E. (2011). 2-dimensional algebras. *African Journal of Mathematical Physics*, 10:81–91.

Jordan, P., von Neumann, J., and Wigner, E. (1934). On an algebraic generalization of the quantum mechanical formalism. *Annals of Mathematics*, 35(1):29–64.

Kaygorodov, I. and Volkov, Y. (2019). The variety of 2-dimensional algebras over an algebraically closed field. *arXiv preprint arXiv:1701.08233v2*, 2(18):1–18.

Koecher, M. (1970). Jordan algebras and differential geometry. *Proc. Internat. Congress I*, pages 279–283.

Koecher, M. (1999). *The Minnesota notes on Jordan algebras and their applications*, volume 1710. Springer Science & Business Media.

Ladra, M. and Rozikov, U. (2013). Evolution algebra of a bisexual population. *Journal of Algebra*, 378:153–172.

Loday, J.-L. (1993). Une version non commutative des algèbres de Lie : les algèbres de Leibniz. *Les Rencontres Physiciens-Mathématiciens de Strasbourg -RCP25*, 44.

Lyubich, Y. I., Vulis, D., Karpov, A., and Akin, E. (1992). *Mathematical structures in population genetics*. Springer-Verlag.

Malcev, A. (1944). On semi-simple subgroups of Lie groups. *Izv. Akad. Nauk SSSR Ser. Mat.*, 8(4):143–174.

Mazzola, G. (1979). The algebraic and geometric classification of associative algebras of dimension five. *Manuscripta Mathematica*, 27(1):81–101.

Omirov, B. (2002). Classification of two-dimensional Zinbiel algebras. *Uzbek Math. Journal*, 2:55–59.

Patera, J. and Zassenhaus, H. (1990). Solvable Lie algebras of dimension ≤ 4 over perfect fields. *Linear Algebra and its Applications*, 142:1–17.

Peirce, B. (1881). Linear associative algebra. *Amer. J. Math.*, 4:97–221.

Petersson, H. P. (2000). The classification of two-dimensional nonassociative algebras. *Results in Mathematics*, 37:120–154.

Popov, V. L. (2014). Generic algebras: rational parametrization and normal forms. *arXiv preprint arXiv:1411.6570*.

Rakhimov, I., Rikhsiboev, I., and Basri, W. (2009). Complete lists of low dimensional complex associative algebras. *arXiv preprint arXiv:0910.0932*.

Reed, M. (1997). Algebraic structure of genetic inheritance. *Bulletin of the American mathematical society*, 34(2):107–130.

Rikhsiboev, I. M. and Rakhimov, I. S. (2012). Classification of three dimensional complex Leibniz algebras. In *AIP Conference Proceedings*, volume 1450, pages 358–362. AIP.

Ritchie, R. (1959). A generalization of non-commutative Jordan algebras. *Proceedings of the American Mathematical Society*, 10(6):926–930.

Romdhani, M. (1989). Classification of real and complex nilpotent Lie algebras of dimension 7. *Linear and Multilinear Algebra*, 24:167–189.

Schafer, R. (1955). Noncommutative Jordan algebras of characteristic 0. *Proceedings of the American Mathematical Society*, 6(3):472–475.

Schafer, R. (1958). On noncommutative Jordan algebras. *Proceedings of the American Mathematical Society*, 9(1):110–117.

Shestakov, I. and Umirbaev, U. (2004). The tame and the wild automorphisms of polynomial rings in three variables. *Journal of the American Mathematical Society*, 17(1):197–227.

Suárez-Alvarez, M. and Vivas, Q. (2015). Automorphisms and isomorphisms of quantum generalized weyl algebras. *Journal of Algebra*, 424:540–552.

Tian, J. P. (2007). *Evolution algebras and their applications*. Springer.

Vergne, M. (1966). Réductibilité de la variété des algèbres de Lie nilpotentes. *C. R. Acad. Sci., Paris, Sér. A*, 263:4–6.

Vinberg, È. B. (1963). The theory of convex homogeneous cones. *Trans. Moscow Math. Soc.*, 12:340–403.

Wallace, E. (1970). Two-dimensional power-associative algebras. *Mathematics Magazine*, 43(3):158–162.

Wörz-Busekros, A. (1980). *Algebras in genetics*. Springer.

Yakimov, M. (2013). The Launois-Lenagan conjecture. *Journal of Algebra*, 392:1–9.

Yakimov, M. (2014). Rigidity of quantum tori and the Andruskiewitsch-Dumas conjecture. *Selecta Mathematica*, 20(2):421–464.

BIODATA OF STUDENT

Houida Mohammed Hussein Ahmed was born in January 1979 in Taiz city, Yemen. In July 2002, she received her Bachelor's degree in Mathematics / Physics from Taiz University, Yemen. In February 2012, she obtained the Master degree from King Abdulaziz University, Jeddah, KSA. From 2003 to 2007 she worked as a tutor in Department of Mathematics, Faculty of Science in Turba branch, Taiz University. Until now, she is a staff in the same faculty. She has taught Calculation (I,II), Statistics and probabilities, Linear Algebra, Discrete Mathematics, Differentia Equation and Analytical Geometry. In October 2015, she began her Ph.D study. The main results of her research on classification of finite-dimension algebras and their applications using a new approach.

LIST OF PUBLICATIONS

Ahmed, H., Bekbaev, U., and Rakhimov, I. (2017). Classification of two-dimensional Jordan algebras. AIP 1905, (2017) 030003. In *4th-7th December 2017 The 13th IMT-GT International Conference On Mathematics, Statistics And Their Applications (Icmsa 2017) UUM , Malaysia*, 8 pages.

Ahmed, H., Bekbaev, U., and Rakhimov, I. (2017). Complete classification of two-dimensional algebras, AIP 1830, (2017) 070016. In *15-17 November 2016 ICMS4 Conference, Malaysia*, 11 pages.

Ahmed, H., Bekbaev, U., and Rakhimov, I. (2017) Classification of 2-dimensional evolution algebras, their groups of automorphisms and derivation algebras. In *23-26 January 2017 4th Biennial International Group Theory Conference (4BIGTC), UTM, Malaysia*, 12 pages.

Ahmed, H., Bekbaev, U., and Rakhimov, I. (2018). Classification of two-dimensional Jordan algebras over $\mathbb{R}$. Malaysian Journal of Mathematical Sciences 12, 287-303.

Ahmed, H., Bekbaev, U., and Rakhimov, I. (2018). The automorphism groups and derivation algebras of two-dimensional algebras. Journal of Generalized Lie Theory and Applications, 12, 9 pages.

Ahmed, H., Bekbaev, U., and Rakhimov, I. (2019). On two-dimensional power associative algebras over algebraically closed fields and $\mathbb{R}$. Lobachevskii Journal of Mathematics, 40, 1-13.

Ahmed, H., Bekbaev, U., and Rakhimov, I. (2019). Two-dimensional left (right) unital algebras over algebraically closed fields and R. IOP 1489, (2020) 012002. In *30-31 October 2019 ICMAE2019, IIUM, Malaysia*, 13 pages.

Ahmed, H., Said Husain, Sh.K. and Basri Witriany (2019). 3-algebras generated by binary algebras. In *30-31 October 2019 IFSC2019, UPM, Malaysia*.

Ahmed, H., Bekbaev, U., and Rakhimov, I. (2019). On classification of 2-dimensional evolution algebras and its applications. IOP 1489, (2020) 012001. In *30-31 October 2019 ICMAE2019, IIUM, Malaysia*, 11 pages.

Ahmed, H., Bekbaev, U., and Rakhimov, I. (2020). Subalgebras, idempotents, ideals and quasi-units of two-dimensional algebras. International Journal of Algebra and Computation, 27 pages.

Ahmed, H., Said Husain, Sh.K. and Basri Witriany (2020). On certain 3-algebras generated by binary algebras. In *4-6 February 2020 ICRAAM2020, UPM, Malaysia*.

UNIVERSITI PUTRA MALAYSIA

STATUS CONFIRMATION FOR THESIS / PROJECT REPORT AND COPYRIGHT

ACADEMIC SESSION : Second Semester 2019/2020

TITLE OF THESIS / PROJECT REPORT :

CLASSIFICATION OF TWO-DIMENSIONAL ALGEBRAS USING MATRICES OF STRUCTURE CONSTANTS APPROACH

NAME OF STUDENT: HOUIDA MOHAMMED HUSSEIN AHMED

I acknowledge that the copyright and other intellectual property in the thesis/project report belonged to Universiti Putra Malaysia and I agree to allow this thesis/project report to be placed at the library under the following terms:

1. This thesis/project report is the property of Universiti Putra Malaysia.
2. The library of Universiti Putra Malaysia has the right to make copies for educational purposes only.
3. The library of Universiti Putra Malaysia is allowed to make copies of this thesis for academic exchange.

I declare that this thesis is classified as :

*Please tick (√)

☐	**CONFIDENTIAL**	(Contain confidential information under Official Secret Act 1972).
☐	**RESTRICTED**	(Contains restricted information as specified by the organization/institution where research was done).
☐	**OPEN ACCESS**	I agree that my thesis/project report to be published as hard copy or online open access.

This thesis is submitted for :

☐	**PATENT**	Embargo from_____________ (date) until _____________ (date)

Approved by:

_____________________ (Signature of Student)
New IC No/ Passport No.:

Date :

_____________________ (Signature of Chairman of Supervisory Committee)
Name:

Date :

[Note : If the thesis is CONFIDENTIAL or RESTRICTED, please attach with the letter from the organization/institution with period and reasons for confidentially or restricted.]