Fractional Riccati equation and its applications to rough Heston model using numerical methods

ABSTRACT

Rough volatility models are recently popularized by the need of a consistent model for the observed empirical volatility in the financial market. In this case, it has been shown that the empirical volatility in the financial market is extremely consistent with the rough volatility. Currently, fractional Riccati equation as a part of computation for the characteristic function of rough Heston model is not known in explicit form and therefore, we must rely on numerical methods to obtain a solution. In this paper, we will be giving a short introduction to option pricing theory (Black–Scholes model, classical Heston model and its characteristic function), an overview of the current advancements on the rough Heston model and numerical methods (fractional Adams–Bashforth–Moulton method and multipoint Padé approximation method) for solving the fractional Riccati equation. In addition, we will investigate on the performance of multipoint Padé approximation method for the small u values in $D^{\alpha}h(u-i/2,x)$ as it plays a huge role in the computation for the option prices. We further confirm that the solution generated by multipoint Padé (3,3) method for the fractional Riccati equation is incredibly consistent with the solution generated by fractional Adams–Bashforth–Moulton method.

Keyword: Fractional Riccati equation; Rough volatility models; Classical Heston model; Rough Heston model; Characteristic function; Fractional Adams–Bashforth–Moulton method; Multipoint Padé approximation method