

On degenerations and invariants of low-dimensional complex nilpotent Leibniz algebra

ABSTRACT

Given two algebras λ and μ , if μ lies in the Zariski closure of the orbit λ , we say that μ is a degeneration of λ . We denote this by $\lambda \rightarrow \mu$. Degenerations (or contractions) were widely applied to a range of physical and mathematical point of view. The most well-known example oriented to the application on degenerations is limiting process from quantum mechanics to classical mechanics under $\hbar \rightarrow 0$ that corresponds to the contraction of the Heisenberg algebras to the abelian ones of the same dimension. Research on degenerations of Lie, Leibniz and other classes of algebras are very active. Throughout the paper we are dealing with mathematical background with abstract algebraic structures. The present paper is devoted to the degenerations of low-dimensional nilpotent Leibniz algebras over the field of complex numbers. Particularly, we focus on the classification of three-dimensional nilpotent Leibniz algebras. List of invariance arguments are provided and its dimensions are calculated in order to find the possible degenerations between each pair of algebras. We show that for each possible degenerations, there exists construction of parameterized basis on parameter t . We proof the non-degeneration case for mentioned classes of algebras by providing some reasons to reject the degenerations. As a result, we give complete list of degenerations and non-degenerations of low-dimensional complex nilpotent Leibniz algebras. In future research, from this result we can find its rigidity and irreducible components.

Keyword: Degenerations; Invariants; Leibniz algebras