



**UNIVERSITI PUTRA MALAYSIA**

**PROPOSITIONAL SATISFIABILITY METHOD IN ROUGH  
CLASSIFICATION MODELING FOR DATA MINING**

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**PROPOSITIONAL SATISFIABILITY METHOD IN ROUGH  
CLASSIFICATION MODELING FOR DATA MINING**

**By**

**AZURALIZA ABU BAKAR**

**Thesis Submitted in Fulfillment of the Requirements for the degree of  
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*I asked for Strength... and Allah gave me difficulties to make me strong.  
I asked for Wisdom... and Allah gave me problems to solve.  
I asked for Prosperity... and Allah gave me brain and brawn to work.  
I asked for Courage... and Allah gave me danger to overcome.  
I asked for Favours... and Allah gave me opportunities.  
I received nothing I wanted... I received everything I needed.  
My prayer has been answered.*

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in fulfillment of the requirements for the degree of Doctor of Philosophy.

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**Chairman : Md. Nasir Sulaiman, Ph.D.**

**Faculty : Computer Science and Information Technology**

The fundamental problem in data mining is whether the whole information available is always necessary to represent the information system (IS). The goal of data mining is to find rules that model the world sufficiently well. These rules consist of conditions over attributes value pairs called description and classification of decision attribute. However, the set of all decision rules generated from all conditional attributes can be too large and can contain many chaotic rules that are not appropriate for unseen object classification. Therefore the search for the best rules must be performed because it is not possible to determine the quality of all rules generated from the information systems. In rough set approach to data mining, the set of interesting rules are determined using a notion of

reduct. Rules were generated from reducts through binding the condition attribute values of the object class from which the reduct is originated to the corresponding attribute. It is important for the reducts to be minimum in size. The minimal reducts will decrease the size of the conditional attributes used to generate rules. Smaller size of rules are expected to classify new cases more properly because of the larger support in data and in some sense the most stable and frequently appearing reducts gives the best decision rules.

The main work of the thesis is the generation of classification model that contains smaller number of rules, shorter length and good accuracy. The propositional satisfiability method in rough classification model is proposed in this thesis. Two models, **Standard Integer Programming (SIP)** and **Decision Related Integer Programming (DRIP)** to represent the minimal reduct computation problem were proposed. The models involved a theoretical formalism of the discernibility relation of a decision system (DS) into an Integer Programming (IP) model. The proposed models were embedded within the default rules generation framework and a new rough classification method was obtained. An improved branch and bound strategy is proposed to solve the SIP and DRIP models that pruned certain amount of search. The proposed strategy used the conflict analysis procedure to remove the unnecessary attribute assignments and determined the branch level for the search to backtrack in a non-chronological manner.

Five data sets from UCI machine learning repositories and domain theories were experimented. Total number rules generated for the best classification model is recorded

where the 30% of data were used for training and 70% were kept as test data. The classification accuracy, the number of rules and the maximum length of rules obtained from the SIP/DRIP method was compared with other rough set method such as Genetic Algorithm (GA), Johnson, HolteIR, Dynamic and Exhaustive method. Four of the datasets were then chosen for further experiment. The improved search strategy implemented the non-chronological backtracking search that potentially prunes the large portion of search space. The experimental results showed that the proposed SIP/DRIP method is a successful method in rough classification modeling. The outstanding feature of this method is the reduced number of rules in all classification models. SIP/DRIP generated shorter rules among other methods in most dataset. The proposed search strategy indicated that the best performance can be achieved at the lower level or shorter path of the tree search. SIP/DRIP method had also shown promising across other commonly used classifiers such as neural network and statistical method. This model is expected to be able to represent the knowledge of the system efficiently.

Abstrak tesis dikemukakan kepada Senat Universiti Putra Malaysia  
sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

**KAEDAH KEPUASAN USULAN DALAM PEMODELAN PENGELASAN  
KASAR UNTUK PERLOMBONGAN DATA**

**Oleh**

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Masalah utama dalam melombongi data ialah sama ada keseluruhan maklumat yang ada sentiasa perlu untuk mewakili satu sistem maklumat. Matlamat melombongi data ialah mencari petua yang memodelkan alam dengan baik. Petua-petua ini mengandungi syarat-syarat ke atas pasangan nilai atribut yang dipanggil deskripsi dan pengelasan atribut kataputus. Walau bagaimanapun, set semua petua yang dijanakan dari semua atribut bersyarat boleh menjadi terlalu besar dan boleh mengandungi banyak petua yang berserabut yang tidak dipaerlukan untuk pengelasan objek baru. Oleh itu pencarian petua yang terbaik mesti dilaksanakan kerana adalah tidak mungkin untuk menentukan kualiti semua petua yang dijana daripada sistem maklumat. Dalam pendekatan set kasar ke atas perlombongan data, set petua yang menarik ditentukan menggunakan notasi reduksi. Petua dijana daripada reduksi dengan mengikat nilai atribut syarat kepada satu kelas

objek daripada reduksi yang diperolehi, keatas atribut berkaitan. Adalah penting untuk reduksi bersaiz minimum. Reduksi minima mengurangkan saiz atribut syarat yang digunakan untuk menjana petua. Saiz petua yang lebih kecil dijangka dapat mengelaskan kes-kes baru dengan lebih baik kerana sokongan yang lebih besar ke atas data.

Kerja utama tesis ini ialah penjanaan model pengelasan yang mengandungi bilangan petua yang sedikit, petua yang lebih pendek dan ketepatan yang baik. Kaedah kepuasan usulan dalam pemodelan pengelasan kasar dicadangkan di dalam tesis ini. Dua model, Standard Integer Programming (SIP) dan Decision Related Integer Programming (DRIP) untuk mengira reduksi minimal dibincangkan. Model-model tersebut melibatkan satu formalisma teoritikal keatas hubungan ketaksamaan bagi satu sistem kataputus kepada satu model pengaturcaraan integer. Model yang dicadangkan kemudiannya di bina di dalam rangka kerja penjanaan petua lalai menghasilkam satu kaedah pengelasan kasar yang baru. Satu strategi cabang dan sempadan dicadangkan untuk menyelesaikan model-model SIP/DRIP untuk mengurangkan sebilangan jumlah carian. Strategi yang dicadangkan menggunakan tatacara analisis knoflik untuk memangkas umpukan atribut yang tidak perlu dan menentukan paras cabang untuk carian jejak kebelakang dengan keadaan tidak bertertib.

Lima set data dari *UCI machine learning repositories and domain theories* diuji. Jumlah petua yang dijana untuk model pengelasan terbaik direkod iaitu 30% daripada data digunakan untuk latihan dan 70% lagi disimpan sebagai data ujian. Ketepatan pengelasan, bilangan petua dan panjang maksimum petua yang diperolehi daripada kaedah SIP/DRIP dibandingkan dengan kaedah set kasar yang lain seperti kaedah



algoritma genetik, Johnson, Holte1R, Dynamic dan Exhaustive. Empat daripada set data kemudian dipilih untuk eksperimen seterusnya. Set data latihan kemudiannya dieksperimen ke atas strategi carian yang dicadangkan. Strategi pembaikan carian tersebut melaksanakan carian jejak kebelakang tak tertib yang berpotensi mengurangkan sebahagian besar ruang carian sambil mengekalkan carian lengkap. Hasil eksperimen menunjukkan kaedah SIP/DRIP merupakan kaedah yang berjaya dalam memodelkan pengelasan kasar. Satu fitur yang terbaik ialah bilangan petua yang minimum dalam semua model pengelasannya. Kaedah SIP/DRIP juga menjana petua yang terpendek dalam kebanyakan set data. Strategi carian yang dicadangkan menunjukkan pencapaian yang terbaik boleh diperolehi pada paras carian pohon yang lebih bawah atau laluan yang lebih pendek. Kaedah SIP/DRIP juga menunjukkan hasil yang setanding dan lebih baik berbanding dengan kaedah pengelasan yang biasa digunakan iaitu rangkaian neural dan kaedah statistik. Model ini dijangka dapat mewakili pengetahuan sistem secara cepak.

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## **LIST OF ABBREVIATIONS**

AUS	Australian Credit Card
BCO	Breast Cancer of Ontology Institute
CLEV	Cleveland Heart Disease
CNF	Conjunctive Normal Form
DRIP	Decision Related Integer Programming
DS	Decision System
GA	Genetic Algorithms
GERM	German Credit
IP	Integer Programming
IS	Information System
LYM	Lymphography
MLP	Multi Layer Perceptron
MR	Multiple Regression
NN	Neural Network
SAT	Satisfiability
SIP	Standard Integer Programming

# CHAPTER 1

## INTRODUCTION

### 1.1 Background

As the amount of information in the world is steady increasing, there is a growing demand for tools for analyzing the information with the aim of finding patterns in terms of implicit dependencies in data. Realizing that much of the collected data will not be handled or even seen by human beings, systems that are able to generate summaries from large amount of information will be important currently. Although several statistical techniques for data analysis were developed long ago, advanced techniques for intelligent data analysis are not yet mature. As a result, there is a growing gap between data generating and data understanding. On a high level, different formalisms for machine learning that uses the notion of knowledge are expected to be able to extract interesting and useful patterns from large collection of data. Several different paradigms have been developed over the years.

The concept of *knowledge discovery* has recently been brought to attention of the business community. One main reason for this is the need of the general recognition that perform the nontrivial extraction of implicit, previously unknown, and potentially useful information from data. The term *data mining* is used to denominate the process of automatic extraction of information in knowledge discovery process. In this thesis, the extracted knowledge is represented as a set of propositional rules that that can be

formally defined as a relationship between a set of attribute values and a decision. Rules are the easiest pattern for human interpretation and understandings. A set of propositional rules forms a model, which explains how different values for the attribute lead to different decisions. Two main tasks for which the model is useful are *prediction* and *classification*. Basically prediction involves using some variables or fields in the database to predict unknown or future values of other variables of interest. Classification is the process of finding the common properties among different objects and classifying the objects into classes.

Classification is probably the most well known data mining problem. It has been widely studied by researchers in the artificial intelligence (AI) field. The database community focuses on searching efficient classification algorithms. Their work involves either developing new and efficient classification algorithms (Agrawal et al., 1992;1993) or further advancing the existing AI techniques for example extracting rules in “if ... then ...” form that can be applied to large database. In the real world applications, the number of attributes of a dataset could be very large. It is common for a class label of an object depends only on the values of a few attributes. In such cases, presenting the original data with all the attributes into a classifier may confuse the classifier to generate unnecessary complex classification rules. The knowledge extraction in the classification context is the process of selecting the most important attributes from the information systems or a dataset.

There are few advantages if a small set of attributes can be determined before the actual objects are fed into a classifier. First, the time required to extract rules is reduced

because a smaller input data set compared to the original one. For example, if neural networks are used for classification, less number of attributes means less number of nodes at the input layer and less training time required. For decision tree based classifiers, less number of attributes means less computation required for classification criteria and less comparison is made. Secondly, as the attributes that do not contribute to the classification are removed, the rules generated by the classifiers are expected more concise than the original dataset used (Lu et al., 1995).

With a number of the classification algorithms available, a natural task is developed appropriate performance metrics that can be used to compare the goodness of these algorithms. Classification accuracy is the most important performance metric of a classification method. It is defined as the ratio between the number of correctly classified objects and the total number of objects in the test set. However when two different methods are applied to the same classification problem, they most likely will extract different set of rules. Certain criteria need to be introduced to compare the goodness of the extracted rules. An ideal rule set has a minimum number of rules and each rule is as short as possible. In practice, it is quite often that a rule set contains fewer rules but they usually have more conditions. Therefore, shorter rules should rather be generated although they will not be perfect on the known cases there are good chances of good classification quality when classifying new cases.