



UNIVERSITI PUTRA MALAYSIA

**BOOLEAN ROUTING ON HIGH DEGREE
CHORDAL RING NETWORKS**

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By

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**Thesis Submitted in Fulfilment of the Requirement for the Degree of
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Over the past twenty-five years, the telecommunication field has evolved rapidly. Telephone and computer networks, now nearly ubiquitous, provide access to voice, data and video services throughout the world. As networking technologies evolve and proliferate, researchers develop new traffic routing strategies.

The problem of routing in a distributed system has been investigated and issues concerning Boolean routing schemes have been considered. All compact routing techniques minimise time and space complexity. A good routing algorithm optimises the time and space complexity and a routing algorithm that has $O(1)$ time complexity and $O(\log n)$ space complexity for high degree chordal ring has been found.

A Boolean Routing Scheme (BRS) has been applied on ring topology and regular chordal ring of degree three. It was found that the regular chordal ring of degree three



can be represented geometrically. the regular chordal ring of degree three has been categorised into two categories; the first is the regular chordal ring of degree three that satisfies the following formula $n \bmod 4 = 0$ and the second other is $n \bmod 4 \neq 0$, where n is the number of nodes that the graph contains. A BRS that requires $O(\log n)$ bits of storage at each node, $O(1)$ time complexity to compute a shortest path to any destination for the regular chordal ring of degree three and $\theta(\log n)$ bits of storage at each node. $O(1)$ time complexity to compute a shortest path to any destination for the ring topologies has been shown.

The BRS has been applied on chordal ring of degree six. it has been found that the chordal ring of degree six can be represented geometrically and the representation would be in three dimensions (in the space). Very little is known about routing on high degree chordal rings. A BRS that requires $O(\log n)$ bits of storage at each node , and $O(1)$ time complexity to compute a shortest path to any destination for the chordal ring of degree six topologyhas been shown. The chordal ring $G(27;9;3)$ has been considered as a case to apply BRS.

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**PENGHALAAN BOOLEAN KE ATAS RANGKAIAN GELANG PERENTAS
BERDARJAH TINGGI**

Oleh

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Lebih dua puluh lima tahun lalu, bidang telekomunikasi telah berkembang secara meluas. Rangkaian komputer dan telefon kini telah hampir sama pentingnya, menyediakan capaian ke atas perkhidmatan suara, data dan video keseluruh dunia. Seiring dengan perkembangan teknologi rangkaian, para penyelidik membangunkan strategi penghalaan trafik yang baru.

Masalah penghalaan di dalam satu sistem teragih telah dikaji, isu berkenaan skema penghalaan boolean telah diberi perhatian. Semua teknik penghalaan padat mampu untuk meminimumkan masa dan ruang kompleksiti. Satu algoritma penghalaan

yang baik dalam mengoptimalkan masa dan ruang, yang mana ia mempunyai masa $O(1)$ dan ruang $O(\log n)$ untuk gelang perentas berdarjah tinggi telah diperolehi.

Gelang perentas biasa berdarjah tiga merupakan salah satu topologi yang dikaji. Satu perwakilan geometrik untuk tiga gelang perentas biasa ditakrifkan dan satu penghalaan boolean telah diaplikasikan ke atas topologi ini, kekompleksan masa $O(1)$ dan kekompleksan ruang $O(\log n)$ diperolehi.

Skema Penghalaan Boolean (BRS) telah diaplikasikan ke atas topologi gelang dan gelang perentas biasa berdarjah tiga. Kita boleh mewakilkan gelang perentas biasa berdarjah tiga secara geometrik dengan membahagikan perentas berdarjah tiga kepada dua kategori, pertama ialah gelang perentas berdarjah tiga yang memenuhi formula $n \bmod 4 = 0$ dan yang kedua ialah $n \bmod 4 \neq 0$. dimana n merupakan bilangan nod pada graf. Satu BRS yang memerlukan $O(\log n)$ bits storan pada setiap nod ditunjukkan, $O(1)$ kekompleksan masa untuk mengira laluan terpendek ke sebarang destinasi untuk gelang perentas biasa berdarjah tiga dan $\theta(\log n)$ bit storan pada setiap nod, $O(1)$ kekompleksan masa untuk mengira laluan terpendek ke sebarang destinasi ditunjukkan.

BRS telah diaplikasikan ke atas gelang perentas berdarjah enam. Kajian ini menunjukkan bahawa gelang perentas berdarjah enam juga boleh diwakili secara geometrik dan perwakilan ini di dalam tiga dimensi (dalam ruang). Sangat sedikit diketahui berkenaan gelang perentas berdarjah tinggi. Kita dapat membuktikan bahawa BRS memerlukan $O(\log n)$ bits storan pada setiap nod ditunjukkan. $O(1)$ kekompleksan

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LIST OF ABBREVIATIONS

BRS	Boolean Routing Scheme
CR6	Chordal Ring degree 6
E	Set of Edge
FIFO	First In First Out
G	Graph
gcd	greatest common divisor
ILS	Interval Labelling Scheme
IRS	Interval Routing Scheme
LAN	Local Area Network
NP	Nondeterministic in Polynomial time
PRS	Prefix Routing Scheme
UUCP	Unix to Unix CoPy
V	Set of Vertex
VLSI	Very Large-Scale Integration
WAN	Wide Area Network



CHAPTER I

INTRODUCTION

Routing is a key component of every communications network. In the last twenty-five years the telecommunication field has evolved rapidly. Telephone and computer networks, now nearly ubiquitous, provide access to voice, data and video services throughout the world. As networking technologies evolve and proliferate, researchers develop new traffic routing strategies.

Distributed Computing

Starting at the beginning of the 1960s distributed computing has become increasingly important (Flocchini and Flaminia, 1996). Microprocessors have become cheaper and more powerful, and the production of high-speed connection networks has facilitated this need. Local Area Networks (LAN) were the first network produced. A LAN is a network of separate computers that can shares resources such as printers, disks, application programs, etc. In a LAN, the cost of the computers has decreased, communications are fast, and the system is flexible, it is possible to connect new machines. Computation that was once only centralised (the system was using a single CPU) can now be distributed among different processors.



The introduction of Wide Area Networks (WAN) reflects the increasing need for fast communications between distant locations. A WAN is a collection of computers with their own autonomy with no shared memory, devices or applications. The connections are used to exchange information (e.g., e-mail), and to access remote files or databases. The computers are usually located far from each other (e.g., in distant building or cities), and the connections are point-to-point. Along these lines, distributed systems started to become more powerful.

The first example of WAN is the ARPANET that was developed in United State of America by the Department of Defence. Later different networks have been set up (e.g., the Unix-to-Unix-CoPy (UUCP for short), network that connects UNIX machines) and all of them are now interconnected and work as a unique virtual networks via a uniform address (Tanenbaum, 1992).

There are some major differences between WANs and LANs: in both settings communication is a crucial point, although the needs are different. In WANs, transmission errors due to atmospheric problems cannot be ignored since messages can be lost or corrupted, or can even arrive in different order from the one they have been sent with (Gavoille, 2000b). Moreover conversion algorithms are important since systems might support different kinds of software. Also systems have to be protected from external attacks, security protocols are required. In LANs crucial problems include synchronisation among processors, resources allocation (since resources may be shared), mutual exclusion, etc.

A distributed system is a set of entities that are autonomous (i.e., have storage and processing ability), and are able to communicate by exchanging messages. These entities can cooperate to solve a single problem. The power of a distributed system



comes from the possibility of splitting the work among different entities, jobs can be a parallelized.

We model a distributed system with a graph $G (V, E)$, where the entities are the nodes (in the set V), and the communication links are the edges (in the set E). The graph may or may not be oriented, and we assume that is always connected.

The system is called dynamic if it structurally changes over time (i.e., nodes/edges are added/deleted), otherwise it is static. The system is symmetric if every entity executes the same algorithm. The system is synchronous (i.e., local and global clocks are synchronised), if the same step of an algorithm is executed at the same time, otherwise it is asynchronous. Every operation is locally executed and the result can be communicated by message passing. In the synchronous system, extra information can also be deduced from the timing of the clock. For example, an entity waits some clock ticks and if no message arrives it can deduce that something is wrong.

We assume that the channels are FIFO, and that in the absence of the failures the messages will eventually arrive (within a finite amount of time). Each entity communicates with a subset of other entities called neighbours, and distinguish among them (local orientation).

The behaviour of an entity depends on external events such as the arrival of messages or clock ticks. The system is defined by a finite set of states. When an event occurs, an entity executes an action that consists of some local processing, the transmission of messages, and the changing of its state. The behaviour of the entity is defined by a complete and unambiguous set of rules.

A distributed algorithm is defined by the behaviour of all the distinct entities, and has to be correct (i.e., should solve the problem it has been defined for), efficient (i.e.,

should use minimal resources), and flexible (i.e., should recover from failures). The execution of an algorithm starts when one or more entities (spontaneously) receive a (wake-up) message. The other entities are then woken and the real computation starts. The global computation ends when each entity knows the result of the computation.

Communication is a crucial point in a distributed system: why would two entities be connected if they never communicate? At a high level, a global execution of an algorithm is achieved when partial results are exchanged among entities, so communication and synchronisation mechanisms have to be defined. This can be done using message passing, the need for the study of good communication techniques is implied. Routing is a technique that permits each node to decide the path a message has to follow to reach its destination. If the destination node is a neighbour, the message is forwarded through that link; otherwise an outgoing edge has to be chosen. The aim here is to find good routing algorithms, i.e., algorithms that find shortest paths, and minimise both the space needed to memorise routing information (space complexity) and the time to access it.

Related to communication is the cooperation problem (Kloks, 1994). In a distributed network, nodes should cooperate in order to optimise their use of the resources and to speed up computations. A crucial point is how many messages the nodes should exchange in order to reach a mutually agreeable solution to a problem. A further concern is how big these messages should be. In certain regular topologies with particular labelling, some information can be implicitly obtained. However, in anonymous networks (i.e., networks in which nodes do not know the labels of others) this cannot be done. In these networks, defining correct and efficient algorithms is not usually trivial, and sometimes impossible. Many algorithms have been presented to

solve problems on anonymous networks (Krizanc and Flaminia, 1995). A limiting factor in these systems is that symmetry cannot be broken, i.e., there cannot be a leader (a nodes that differs from the others). This is a result of the fact that node labels cannot be used to create a distinction among the nodes (Mans, 1997).

We consider as a distributed system any kind of network whose computers cooperate: starting from WANs, LANs, and multiprocessors with their own processors, going down to a lower level to system where different processors cooperate. Despite the physical hethereogenity of these systems, at the logical level they can be treated in the same way (Flaminia, 1995). Big systems can be more easily extended (implying a better cost/speed performance) and failures can be more easily isolated and recovered. Moreover, a distributed system can be used to increase the reliability of the algorithms since the same computation can be simultaneously executed on different machines.

Motivation of Distributed System

The Distributed system is popular for two reasons: technological change and user needs. Growth microelectronics, VLSI, etc., have changed the price- performance ratio to favour multiple low-performance processors rather than single high-performance processors. The price of communication devices has also dropped dramatically during the past few years. All of these have encouraged the user to change to the distributed system. From the user's perspective, the distributed system provides more sophisticated facilities for the users such as facilities which are faster, more extensible, more reliable and so on. A single high-performance processor cannot fulfil these demands; only a distributed system can support them.

Routing Algorithms

Solving a routing problem involve defining a way in which entities that are physically separated can communicate. As was previously stated, in a WAN the connections are point-to-point. The assumption is that the graph representing the network is connected. If two entities that want to communicate are directly connected, the routing is straightforward, since the message is sent on the edge that connects the two. If the two entities are not directly connected, a path between them has to be found. Routing techniques differ, depending on what kind of information has to be stored in the nodes and in the messages. We assume that labels are associated to nodes and edges; a destination address coincides with a node label. A good routing algorithm follows these rules:

- It is correct (gives an admissible solution);
- It is fair (serves each node equally);
- It optimises space (to memorise node/edges labels and routing function) and time (to access the information);
- It searches for shortest paths (in weighted networks), or minimum hops paths (in unweighted networks);
- It minimises traffic congestion (giving alternative paths);
- It is robust (it allows topological changes);
- It Avoids deadlock.

All these rules are not necessarily considered at the same time. Recently, particular attention has been given to the study of some techniques called Compact Routing Techniques whose principal aim is to optimise the trade-off between the length of the paths and the spaces required for routing. In this thesis some of these techniques that search for shortest paths and try to optimise the space in which these are Interval Routing Schemes, Prefix Routing Schemes and Boolean Routing will be discussed. Boolean Routing will be studied in detail, the time/space required for the routing analysed, and we will also show how this method can apply to a particular topology of interest: a chordal ring. We will also extend and generalise some results in (Narayanan & Opatrny 1999), where the authors present Boolean Routing schemes on chordal ring degree four.

Problem Statement

The standard model of point-to-point communication network is described as finite connected symmetric digraphs $G(V, E)$. In the following the number of vertices of such a graph are denoted by n . The vertices represent the processors or the nodes of the network, and the edges represent bidirectional communication links between the nodes (to each edge correspond two symmetric arcs). A vertex can directly communicate with its neighbours only and messages between nonadjacent vertices are sent along a path which connects them in the network

If $R=(I, H, P)$ is a routing function on a graph G , the path built by R is the following. If the vertex x wants to send message M to the vertex y in G it prepares a header h for the message M by computing $h=I(x, y)$, and attaches it to M and it selects

the output port p by computing $P=p(x, h)$, and sends the message out by this port. Now suppose that when a message M with a header h' arrives at some vertex z . the first task of z consists in computing a new header $h''=H(z, h')$. Then z determines the output port by computing $p'=P(z, h')$. If $p'=e$, then the routing process ends and M is sent to the memory of the corresponding processor. Otherwise, z forwards M on the output port p after having replaced the old header h' attached to M by the new one h'' .

We consider routing function such that every message follows a path that depends only on the source and the destination of the message. The compact routing is related to the following definition:

Definition 1.1 (memory requirement of a router) Let G be any graph and let x be any vertex of G . For every routing function R on G . We denote by $\text{MEM}(G, R, x)$ the minimum memory requirement of R in x , i.e., the minimum number of bits necessary to store the function R .

Objectives

The major goal of this research is to find a Boolean routing scheme on such topologies as regular chordal ring degree three and regular chordal ring degree six.

To find a Boolean routing schemes for the chordal rings it is necessary to study the characteristics of the graph and the way in which the graph can be represented. For this reason the search is to find a way to represent the chordal ring. Before finding the Boolean routing scheme for the chordal ring there is another goal: finding a way to represent chordal rings. The objectives of this thesis are to:

1. apply the BRS on the geometrical representation of the regular chordal ring of degree three.
2. find the BRS that has optimal time and space complexity.
3. compute the time and space complexity for the BRS on regular chordal ring of degree three and six.

Scope of the Research

The scope of the research in this thesis is to find a Boolean Routing Scheme for the chordal ring of high degree topologies. These Boolean functions are supposed to have a minimum time and space complexity. We are interested in distributed computing. As usual, the network is modelled by a graph $G(V,E)$ whose set of vertices V presents, for example, are the routers, processors, PCs, and whose set of edges E represents as the communication links between the entities. It is assumed that the links are bidirectional (that is if an entity x is able to send messages to one of its neighbours y , then y is also able to send messages to x) and the graph is an unweighted graph. Of course, the topic of interest here is connected networks, so all the statements of this thesis assume that the graph are connected that there is a path between any couple of vertices. The intention is to apply the BRS on the regular chordal ring of degree three and six and the graph $G(27;3;9)$ is taken as an example for the regular chordal ring of degree six. An edge of extremities x and y is therefore denoted by (x, y) .

of the research is to find a Boolean function for regular chordal rings.

Organisation of the Thesis

The remainder of this thesis is organised as follows: chapter II provides a historical perspective on compact routing such as its types. In addition, in this chapter we give an example for each type of compact routing and the previous work that has been done in this research area. In chapter III, a Boolean Routing function on ring topology and regular chordal ring of degree three is introduced and a new method to present the ring and chordal ring topology (geometrical representation) is also introduced, we base on this presentation to find the shortest path between any two vertices, and in addition we compute the time and space complexity for this routing function.

In chapter IV, a Boolean routing function on chordal ring of degree six and we introduce a new method to present the chordal ring topology (geometrical representation) is introduced. In the last chapter some final observations are made and some open problems presented.

