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GENERALIZED DERIVATIONS AND AUTOMORPHISMS OF SOME CLASSES OF ALGEBRAS

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GENERALIZED DERIVATIONS AND AUTOMORPHISMS OF SOME CLASSES OF ALGEBRAS

By

ABDULKADIR ADAMU

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirements for the Degree of Doctor of Philosophy

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DEDICATIONS

To the memory of my late little daughter KHADIJA

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Doctor of Philosophy

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By

ABDULKADIR ADAMU

February 2019

Chairman: Sharifah Kartini Said Husain, PhD
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This thesis focus on the problem of generalized derivations of some classes of algebras over complex field. It defined the concept of the generalization via some complex parameters, and in particular on some values of the parameters. It uses an algorithm in the computation of the generalized derivations of some algebras of lower dimensional cases. Particularly, the associative algebras and the Leibniz algebras. The results of the computations are presented in a matrix form, and further interpreted. Thus, different subalgebras, subspaces, and two one-parametric sets of linear operators are obtained. Intersection among various subspaces are also found. The classification result of the generalized derivation of algebras are given. Eight different subspaces and their structures are found in the case of associative algebra, which includes, the classical derivation. In the Leibniz algebra case, we had eight subspaces too, with different structures, it includes two one-parametric sets as well. Furthermore, by using the two one-parametric sets, two invariant functions are defined. The functions together with some criteria are used to establish contractions among algebras of lower dimensions. The list of contractions among the Leibniz algebras and the associative dialgebras of dimensions 2 and 3 are given. The work also compute the automorphism group of the Leibniz and the associative dialgebras. The work also described the concept of a generalized automorphism of algebras. The concept is defined with the aid of some sets of automorphisms. It is found out that, there exists an isomorphism between some corresponding sets of generalized derivations and the generalized automorphisms. In the same way, we found an isomorphism among various intersections of the subspaces of the generalized derivations with that of the generalized automorphisms. The inner derivation of algebras is also looked into. As a result, additional invariant characteristic of algebras are found.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

PENERBITAN DAN AUTOMORFISMA TERITLAK BAGI BEBERAPA KELAS ALJABAR

Oleh

ABDULKADIR ADAMU

Februari 2019

Pengerusi: Sharifah Kartini Said Husain, PhD
Fakulti: Institut Penyelidikan Matematik

Tesis ini memfokuskan masalah penerbitan teritlak bagi beberapa kelas aljabar ke atas medan kompleks. Ia mentakrifkan konsep pengitlakkan melalui beberapa parameter kompleks, dan khususnya pada beberapa nilai parameter. Ia menggunakan algoritma dalam pengiraan penerbitan teritlak bagi beberapa aljabar untuk kes berdimensi rendah. Khususnya, aljabar bersekutu dan aljabar Leibniz. Hasil pengiraan dibentangkan dalam bentuk matriks, dan diterjemahkan selanjutnya. Oleh itu, subaljabar, subruang, dan dua set satu parametrik pengoperasi linear yang berbeza diperolehi. Persilangan di antara pelbagai subruang juga dijumpai. Hasil pengkelasan penerbitan teritlak aljabar diberikan. Lapan subruang yang berbeza dan strukturnya didapati dalam kes aljabar bersekutu, termasuk, penerbitan klasik. Dalam kes aljabar Leibniz, kami juga mempunyai lapan subruang dengan struktur yang berbeza, ia termasuk dua set satu parametrik. Seterusnya, dengan menggunakan dua set satu parametrik, dua fungsi tak varian ditakrifkan. Fungsi ini bersama-sama dengan beberapa kriteria digunakan untuk mewujudkan pengecutan di kalangan aljabar berdimensi lebih rendah. Senarai pengecutan di antara aljabar Leibniz dan dwialjabar bersekutu berdimensi 2 dan 3 diberikan. Kerja ini juga mengira kumpulan automorfisma Leibniz dan dwialjabar bersekutu. Kerja ini juga menghuraikan konsep automorfisma teritlak aljabar. Konsep ini ditakrifkan dengan bantuan beberapa set automorfisma. Didapati, terdapat hubungan isomorfisma di antara beberapa set penerbitan teritlak dan automorfisma teritlak. Dengan cara yang sama, kami mendapati isomorfisma di antara pelbagai persilangan subruang dari penerbitan teritlak dengan automorfisma teritlak. Penerbitan dalaman aljabar juga diambil kira. Sebagai hasil, ciri-ciri tambahan tak varian aljabar diperolehi.

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This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfilment of the requirement for the degree of Doctor of Philosophy. The members of the Supervisory Committee were as follows:

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF ABBREVIATIONS

As_p^q	q^{th} isomorphism class of associative algebras in dimension p.
$Dias_p^q$	q^{th} isomorphism class of associative dialgebras in dimension p.
L_n^q	q^{th} isomorphism class of Lie or Leibniz algebras in dimension n.
$\dashv$	Left multiplication of associative dialgebras
$\vdash$	Right multiplication of associative dialgebras
$Der\mathbb{A}$	The set of all derivations of arbitrary algebra $\mathbb{A}$
IC	Isomorphism classes of algebras
Der	Derivation
dim	Dimension of algebra
$LDer(\mathbb{A})$	Left derivation of an arbitrary algebra
$RDer(\mathbb{A})$	Right derivation of an arbitrary algebra
l_a	Left multiplication operator by an element a
r_a	Right multiplication operator by an element a
$\Gamma(L)$	Centroid of Leibniz algebra L
$\diamond$	Product of associative dialgebra
$\cong$	Isomorphism
$Inn(L)$	Inner derivation of L
$Z_L(H)$	Centralizer of H in L
$EndL$	Endomorphism of L

$AutL$	Automorphism of L
$GL(L)$	General Linear Group of L
$End\mathbb{D}$	Endomorphism of $\mathbb{D}$
RHS	Right hand side of equation
LHS	Left hand side of equation
(Trivial)	Null matrix
$C(L)$	Center of L
$CodimL$	Codimension of L

CHAPTER 1

INTRODUCTION

1.1 Background

The concept of Leibniz algebras started with the work of a Russian mathematician, where it is refer to as D-algebras. It was later re-discovered and developed by Loday and Pirashvili (1993). Loday described it as a non-antisymmetric version of Lie algebras, whose bracket satisfies another identity, called Leibniz identity. Loday also described some relationships among different kind of algebras, with the main motivation of searching the periodicity of K-theory. The Loday diagram in Figure 1.1 defines the relationships among these algebras, namely associative, Lie, Leibniz, zinbiel, dendriform and associative dialgebras.

Recently a strong connection is found between these algebras of Loday with other areas of mathematics e.g classical geometry, non-commutative geometry and Physics (Rakhimov and Atan (2012)). Since Leibniz algebras was defined as a non-antisymmetric generalization of the Lie algebras, this fact allows to prove many properties of the Leibniz algebras with the aid of known results in Lie algebras, numerous remarkable results were obtained in the Leibniz algebras as a result, for example analogues of Lie's theorem, Engel's theorem, Cartan criterion, Levi decomposition theorem, etc are extended to the Leibniz algebras. However some results hold differently in Leibniz algebra case, for example concept of representations is defined differently for Lie algebra case. The concept of faithful representations of Leibniz algebra as well as Leibniz algebra cohomology too are also giving differently from Lie case.

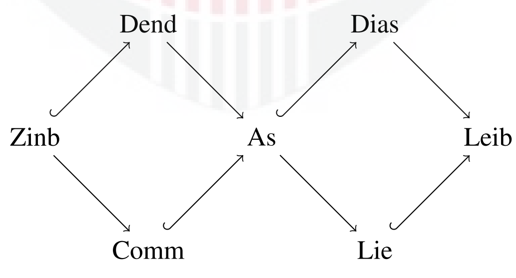

Figure 1.1: Loday diagram

Generalized derivation of finite dimensional algebras form important area of activities in mathematics. It provide useful insight into the nature of the algebras and their invariants. Several important works on the generalized derivation of Lie algebra appeared in Filippov (1998, 1999), Leger and Luks (2000), Hartwig et al. (2006) and Novotnỳ and Hrivnák (2008) which together added new objects to the existing set of invariants of Lie algebra. Another useful application of the generalized derivations of finite-dimensional algebras is the concept of contraction of algebras, for instance the contraction of Lie algebras is an important area of research developed by Segal et al. (1951), and Craig (1990), and also in the work of Nesterenko and Popovych (2006). It was also studied by Burde and Steinhoff (1999),and in the work Hrivnák (2015), and recently in the work of Escobar et al. (2016). Investigation of contraction of Lie algebras was motivated by its numerous applications in different field of physics, and mathematics. For example, in the study of invariants, in representation and in the study of special functions. In a famous work by Burde and Steinhoff (1999) they use contractions as a tool to study Lie algebra varieties. There arose naturally the idea of extending the concept of contraction (sometimes refer to as degenerations) to the fold of Leibniz algebras. Results on contractions of nilpotent Leibniz algebras was presented in the work of Rakhimov (2006), and in Rakhimov and Atan (2012), it was also recently discussed in the work of Kaygorodov et al. (2017). Throughout the works mentioned here, the results of the contraction were achieved with the aid of some contraction invariants, which satisfied some criterion for establishing the contraction among the Leibniz algebras, mostly in low dimensional cases.

In this work we introduce a concept of an $(\alpha, \beta, \gamma)-$derivation of a Leibniz algebras as a form of a generalized derivation. Computations of all the $(\alpha, \beta, \gamma)-$derivations of low-dimensional Leibniz, Lie, associative algebras and the associative dialgebras were given. The focus here is to find some relevant properties of the generalized derivations, with the motive of finding some additional invariants of some algebras, especially Leibniz algebras and associative dialgebras. All subalgebras and subspaces obtained as a result of the derivations are investigated and their properties are described. Invariant functions ξ and ξ' are defined and there values were calculated, the values obtained were used to calculate contractions among two and three dimensional Leibniz and associative dialgebras algebras. The work also introduces the concept of a generalized automorphisms of Leibniz and associative dialgebras. The computations and list of the automorphisms group of the Leibniz and the associative dialgebras are provided, in the same way some important theoretical results on the generalized automorphisms were proven.

1.1.1 Basic concepts

In this subsection, we introduce some basic concepts and notations which are to be refer to, throughout this thesis. Let V be a vector space over a field $\mathbb{K}$ with a binary operation:

$$\sigma : V \times V \to V.$$

If the binary operation is bilinear, then V is said to be an algebra over the field $\mathbb{K}$ i.e an algebra is a vector space V with a bilinear binary operation which satisfies the conditions:

$$\sigma(\alpha_1 x + \alpha_2 y), z) = \alpha_1 \sigma(x,z) + \alpha_2 \sigma(y,z),$$
$$\sigma(z, \alpha_1 x + \alpha_2 y) = \alpha_1 \sigma(z,x) + \alpha_2 \sigma(z,y), \forall x,y,z \in V \text{ and } \alpha_1, \alpha_2 \in \mathbb{K}.$$

Let us introduce some important well-known class of algebras which are relevant in this thesis, for purpose of clear presentation.

Definition 1.1 [Loday and Pirashvilli, 1993] An associative algebra A is a vector space over a field $\mathbb{K}$ equipped with bilinear map $\sigma : A \times A :\longrightarrow A$ satisfying the associative law:

$$\sigma(\sigma(x,y),z) = \sigma(x, \sigma(y,z)) \text{ for all } x,y,z \in A. \tag{1.1}$$

Sometimes notations $x \cdot y$ or xy will be used for $\sigma(x,y)$. Among simple examples of an associative algebras include a set of linear transformations on a vector space over a fixed field, the square $n \times n$ matrices with entries from a fixed field forms an associative algebra over field of reals, the complex numbers form a 2-dimensional associative algebras over the real numbers.
Another algebra of great importance here is the Lie algebra L, we use the notation $[\cdot,\cdot]$ as our function "σ". It is defined as a vector space over the field $\mathbb{K}$, the function

$$[\cdot,\cdot] : L \times L \to L$$

where the following conditions hold;

$$[x,x] = 0 \tag{1.2}$$

and

$$[x,[y,z]] + [z,[x,y]] + [y,[x,z]] = 0, \forall x,y,z \in L. \tag{1.3}$$

The condition $[x,[y,z]] + [z,[x,y]] + [y,[x,z]] = 0$ is refer to as the Jacobi identity, and the condition $[x,x] = 0$ is called the skew-symmetry. In fact, it follows

$$0 = [x+y, x+y] = [x,x] + [x,y] + [y,x] + [y,y] = [x,y] + [y,x],$$

thus,

$$[x,y] = -[y,x].$$

Several examples of Lie algebras are available in literature. For example, every vector space with the bracket $[x,y] = 0, \forall x,y \in L$ is an abelian Lie algebra. Also consider $(\mathbb{R}^3, [\cdot,\cdot])$, where the bracket are defined by the cross-product of the vectors in $\mathbb{R}^3$, it make an $\mathbb{R}^3$ a non abelian Lie algebra. Another example of a Lie algebra is an $n \times n$ square matrices $M_n(K)$ over an arbitrary field $\mathbb{K}$ where the multiplication $[A,B] = AB - BA$ holds it is denoted as $GL(n)$. The endomorphisms of V i.e $EndV$ over $\mathbb{K}$, with the bracket $[f,g] = f \circ g - g \circ f$ is also a Lie algebra, denoted as $GL(v,k)$ or simply $GL(V)$ if the field is known, it is called a general linear group.

Definition 1.2 [Loday, 2001] A Leibniz algebra L is a vector space over a field $\mathbb{K}$, together with a bilinear map

$$[\cdot,\cdot] : L \times L \to L.$$

If it satisfies the following condition;

$$[x,[y,z]] = [[x,y],z] - [y,[x,z]] \ \forall x,y,z \in L. \tag{1.4}$$

This is actually a left Leibniz algebra, a right Leibniz algebra is defined similarly with the identity written in the following way

$$[[x,y],z]] = [x,[y,z]] - [[x,z],y] \ \forall x,y,z \in L.$$

Note here that if the condition ;

$$[x,x] = 0.$$

holds for all $x,y,z \in L,$ then the Leibniz algebra becomes a Lie algebra, since the Leibniz identity in that case equals the Jacobi identity. Therefore all Lie algebras are Leibniz algebras, in other words Leibniz algebras are the generalization of Lie algebras. Throughout this thesis all algebras are defined over a complex field $\mathbb{C}$. We now consider some examples of a Leibniz algebra.

The following statements of definitions, theorems and propositions on Leibniz algebras can be found in Loday and Pirashvili (1993), Gorbatsevich (2013) and in the work of Lin and Zhang (2010).

Any Lie algebra is an example of Leibniz algebra. Consider an algebra L of dimension two with multiplication as follows $[x,x] = 0, [x,y] = 0, [y,x] = x, [y,y] = x$. Then L is Leibniz algebra. Another example is an associative algebra A over $\mathbb{K}$ equipped with a linear operator $T : A \to A$ with the condition $T^2 = T$, where the multiplication $[\cdot,\cdot] : A \times A \to A$ is defined as $[x,y] : (Tx)y - y(Tx), \ \forall x,y \in A.$

Since, $[x,y] = Tx \cdot y - y \cdot Tx \ \in A$. It is clear that A forms a Liebniz algebra, if we consider the Leibniz identity as follows:

$$[x,[y,z]] = Tx[y,z] - [y,z]Tx.$$

Also,

$$[x,y],z] = T[x,y]z - zT[x,y].$$

Similarly,

$$[y,[x,z] = Ty[x,z] - [x,z]Ty.$$

Comparing the left and the right sides of the identity proves that A forms a Leibniz algebra, however A is a Lie algebra, if and only if $T = id$.

Definition 1.3 [Novotny and Hrivnak, 2008] Let $\{e_k\}$ be a set of a basis vectors for the underlined vector space of an arbitrary algebra. The numbers $\gamma_{ij}^k \in K$ defined as $e_i \cdot e_j = \sum_k \gamma_{ij}^k e_k$ are called the structure constants with respect to the basis $\{e_k\}$.

Let L and $L^{'}$ be two Leibniz algebras and a linear map; $f : L \to L^{'}$ is a homomorphism if it preserves multiplication i.e ; $f[x,y] = [f(x) * f(y)]$, for all $x, y \in L$. The kernel of homomorphism is an ideal in L, if the $ker f = 0$, then the homomorphism becomes an isomorphism, hence L and $L^{'}$ are said to be isomorphic, written $L \cong L^{'}$. All standard morphisms of Lie algebras hold in the Leibniz algebras as well.
Let $GL(V)$ denote the group of all linear maps on an arbitrary vector space V, for the Leibniz algebra $(L, [\cdot,\cdot])$ we define the $GL(L)$ as $GL(V)$. Therefore we can write $EndL = \{f : L \to L | \text{f } is\ linear\}$. So, if $f : L \to L^{'}$ is an isomorphism of Leibniz algebras L and $L^{'}$, then the map

$$g : EndL \to EndL^{'} \ \forall d \in EndL$$

defined as

$$g(d) = fdf^{-1} \tag{1.5}$$

is an isomorphism of Leibniz algebras $EndL$ and $EndL^{'}$. An isomorphism of the Leibniz algebra L to itself ;$f : L \to L$ is called an automorphism of the Leibniz algebra $(L, [\cdot,\cdot])$. The set of all the automorphism forms a multiplicative group $AutL$ which coincides with $GL(L)$ i.e

$$AutL = \{f \in GL(L) | f([x,y]) = [f(x), f(y)] \ \forall x, y \in L\}. \tag{1.6}$$

The isomorphism relation between two Leibniz algebras L and $L^{'}$ is an equivalence relation, therefore the set of all algebras can be decomposed into an isomorphism classes i.e the coset of the equivalence classes of L written as $[L]$. Now $L^{'} \in [L]$ is true if and only if $L \cong L^{'}$. A map $d \in EndL$ is a derivation of a Leibniz algebra if the following holds ;

$$d([x,y]) = [d(x), y] + [x, d(y)], \ \forall x, y \in L. \tag{1.7}$$

The set of all derivations of L is given as;

$$DerL = \{d \in EndL | d([x,y]) = [d(x), y] + [x, d(y)], \ \forall x, y \in L\}. \tag{1.8}$$

Remark 1.1 Obviously, $DerL$ is a Lie algebra. Let L be a Leibniz algebra, a left multiplication operator $l_x : L \to L$ defined as $l_x(y) = [x,y]$ and a right multiplication operator $r_x : L \to L$ defined as $r_x(y) = [y,x]$ for all $x, y \in L$.

Proposition 1.1 [Gorbatsevich, 2013] For every right Leibniz algebra a right multiplication operation is a derivation. Obviously, re-writting the identity, we have:

$$r_z([x,y]) = [r_z(x),y] + [x,[r_z,y]$$

take $r_z = d$, we get

$$d([x,y]) = [d(x),y] + [x,d(y)].$$

Similarly a left multiplication operation is a derivation in a left Leibniz algebra.

Let N be a non empty set and E a subset of $[L]$, a map $h : E \to N$ is an invariant characteristic of E. Consider E to be the set of all isomorphism classes of L and set $N = \mathbb{N}$, for any $[L] \in E$, then we define $h[L] = dimL = n$. Obviously h is well defined, and has values for the isomorphic algebras L and $L^{'}$, it is called a numerical invariant. Similarly another example of a numerical invariant for a Leibniz algebra L can be written as $h_{der}[L] = dim_{derL}$. Therefore, for any $L, L^{'} \in [A]$, we have

$$g(derL) = derL^{'}. \tag{1.9}$$

Therefore, $dimderL = dimderL^{'}$, hence the map h_{der} is well defined. Let $(L,[\cdot,\cdot])$ be a Leibniz algebra, we call $ad_z(x) = [x,z]$, an inner derivation of $L, \forall x,z \in L$, It is denoted as $Inn(L)$, the inner derivation of L is a subspace of $Der(L)$. Let L be a Leibniz algebra and I is a subspace of L , i.e $I \subseteq L$, then I is a subalgebra if $[I,I] \subseteq I$. Let L be a Leibniz algebra and I is a subspace of L , i.e $I \subseteq L$, then I is a left ideal if $[L,I] \subseteq I$. Similarly I is a right ideal if $[L,I] \subseteq I$. Hence I is an ideal of L if it is both left and right ideal. The sum and intersection of two ideals of a Leibniz algebra L is an ideal. Let L be a Leibniz algebra. For any $x,y \in L$ we define a left center of L as $Z^l = \{x \in L | [x,y] = 0, \ \forall \ y \ \in L\}$. Similarly a right center is giving as $Z^r = \{x \in L | [y,x] = 0 \forall \ y \in L\}$. So the center of L is defined as

$$Z(L) = \{x \in L | [x,y] = [y,x] = 0 \ \forall y \ \in L\}. \tag{1.10}$$

The center is clearly the intersection of the left and the right center i.e $Z(L) = Z^l \cap Z^r$.

Remark 1.2 The left center of a left and a right Leibniz algebra is a two sided ideal, However the right center of a left and also of a right Leibniz algebra is a subalgebra.

Suppose L is a Leibniz algebra and $L^{'}$ is a subspace of L. A left centralizer is defined as $Z^l_L(L^{'}) = \{x \in L | [L^{'},x] = 0$. Similarly a right centralizer $Z^r_L(L^{'}) = \{x \in L | [x,L^{'}] = 0$. Hence a centralizer of L is the intersection of the left and the right ;

$$Z_L(L^{'}) = \{x \in L | [L^{'},x] = [x,L^{'}] = 0\}. \tag{1.11}$$

Suppose L is a Leibniz algebra. For any $x,y \in L$, we define a left centroid of L as $\Gamma^l_L = \{d \in EndL | d([x,y]) = [d(x),y], \forall x,y \in L\}$. Similarly a right centroid is given as $\Gamma_L^r = \{d \in EndL | d([x,y]) = [x,d(y)], \forall x,y \in L.\}$ Hence a centroid of L is

$$\Gamma_L = \{d \in EndL | d([x,y]) = [d(x),y] = [x,d(y)], \forall x,y \in L\}. \qquad (1.12)$$

A representation of G on V is a group homomorphism $\rho : G \to GL(V)$ which induces an action of G on V by the relation $(g \cdot f)(x) := f(g^{-1}(x))$ on $K[V]$ where $K[V]$ is a space of polynomial function on V. For all $x \in V, g \in G$ and $f \in K[V]$. Note that the function $f \in K[V]$ is called an invariant if $f(x) = f(gx)$, for all $x \in V$ and $g \in G$.
Consider the following derived series of a Leibniz algebra
$D^0(L) \supset D^1(L) \supset D^2(L) \supset ....$ written as:

$$D^0(L) = L,\ D^1(L) = [L,L],\ D^{n+1}(L) = [D^nL, D^nL],\ \forall n \in \mathbb{N}^*.$$

Whenever the derived series comes to $\{0\}$ after some finite steps, then the Leibniz algebra L is called solvable. Another the series $L^1 \supset L^2 \supset ...$ defined as

$$L^1 = L,\ L^{n+1} = [L^n, L], \forall n \in \mathbb{N}$$

is called a descending central series. Whenever the descending series comes to $\{0\}$ after some finite steps, then the Leibniz algebra L is called nilpotent. An ascending central series $C^0(L) \subset C^1(L) \subset ....$ is defined by

$$C^0(L) = 0, C^{n+1}(L)/C^n(L) = C(L/C^n(L))\ \forall n \in \mathbb{N}^*.$$

Remark 1.3 From the definitions of the series, some important numerical invariants are observed and defined as follows:

$$d_n(L) = dimD^n(L);\ l_n(L) = dimL^{n+1};\ c_n(L) = dimC^{n+1}(L).$$

Theorem 1.1 [Gorbatsevich, 2013] A Leibniz algebra L is nilpotent if and only if the right multiplicative operator $L_x(y) = [y,x]$ is nilpotent for any $x,y \in L$.

A representation of Leibniz algebra L is $k-$module M equipped with two actions of L;

$$[\cdot,\cdot] : L \times M \to M$$

and

$$[\cdot,\cdot] : M \times L \to M$$

which satisfies the following conditions;

$$[m,[x,y]] = [[m,x],y] - [[m,y],x],$$

together with

$$[x,[m,y]] = [[x,m],y] - [[x,y],m]$$

and

$$[x,[y,m] = [[x,y],m] - [[x,m],y]$$

for any $m \in M$ and $x, y \in L$.

A Leibniz algebra representation is called symmetric if for all $x, y \in L$ and $m \in M$, the relation

$$[m,x] + [x,m] = 0$$

is true, and anti-symmetric if

$$[x,m] = 0,$$

and a trivial representation is when

$$[x,m] = 0 = [m,x].$$

A Leibniz algebra cohomology: Let L be Leibniz algebra and M be a representation of L. Let

$$C^n(L,M) := Hom_{\mathbb{K}}(L^{\bigotimes n}, M),\ n > 0$$

and

$$d^n : C^n(L,M) \to C^{n+1}(L,M)$$

be a $\mathbb{K}-$ homomorphism defined by;

$$(d^n f)(x_1,...,x_{n+1}) := [x_1, f(x_2,...,x_{n+1})] + \sum_{i=2}^{n+1}(-1)^i[f(x_1,...,\hat{x_i},...,x_{n+1}),x_i]$$

$$+ \sum_{1\leq i<j\leq n}(-1)^{j+1} f(x_1,...,x_{i-1},[x_i,x_j],x_{i+1},...,\hat{x_j},...,x_n).$$

Then, $(CL^*(L,M),d)$ is a cochain complex, whose cohomology is called the cohomology of the Leibniz algebra L with coefficient in the representation of M:

$$HL^*(L,M) := H^*(C^*(L,M),d).$$

Consider the case of $n = 0$, the Leibniz cohomology becomes a submodule of a left invariant of the representation, defined as ;

$$HL^0(L,M) = \{m \in M | [x,m] = 0 \forall\, x \in L\}.$$

When $n = 1$, a 1-cocycle is a k-module homomorphism $\delta : L \to M$ which satisfies $\delta([x,y]) = [\delta(x),y] + [x,\delta(y)]$. which is actually a derivation from L to M. The k-module of derivation is denoted as $der(L,M)$. It is called a coboundary if it is of the form, $ad_m = [x,m]$ for some $m \in M$, therefore the 1-cocycle is given as;

$$HL^1(L,M) = der(L'M)/ad_m$$

where ad_m is an inner derivation. The following statements and propositions can be found in Novotnỳ and Hrivnák (2008).
The set of complex $n \times n$ invertible matrices forms a group with respect to multiplication, it is called a general linear group denoted as $GL(n,\mathbb{C})$, its subsets $SL(n,\mathbb{C})$, are the matrices with +1 determinants which also form a group in the same way, usually

referred to as special linear groups. The vector spaces $gl(n,\mathbb{C})$ and $sl(n,\mathbb{C})$ are Lie algebras. The Lie algebra structure is given with respect to the Lie brackets defined as ; $[X,Y] = XY - YX$. It is also clear that $GL(n,\mathbb{C})$ and $SL(n,\mathbb{C})$ are complex Lie groups.
Let G be a subgroup of a group $GL(n,\mathbb{C})$, if there exists a set of polynomials $\mathbb{Q} \subset \mathbb{C}[u_{11}, u_{12}, ..., u_{nn}]$ such that

$$G = \{(v_{ij}) \in GL(n,\mathbb{C}) | p(v_{11}, v_{12}, ..., v_{nn}) = 0, \forall p \in P\}$$

then G is called an algebraic group. Let $\mathbb{A}$ be an arbitrary algebra, giving as $\mathbb{A} = (V, f)$. A subgroup $GL(\mathbb{A})$ is an algebraic group if it is represented by an algebraic group in $GL(n,\mathbb{C})$ with respect to the basis $\{e_1, e_2, ..., e_n\}$ of the underlined vector space V. Let $\mathbb{A} = (V, f)$ be an arbitrary algebra over a complex field $\mathbb{C}$. Then the automorphism group Aut$\mathbb{A}$ is an algebraic group in $GL(\mathbb{A})$. The property of an exponential map is important in the study of Lie groups. Consider the map

$$exp : gl(n,\mathbb{C}) \to GL(n,\mathbb{C})$$

given as:

$$e^X = \sum_{n=0}^{\infty} 1/n!X^n = I + 1/2X^2 + \cdots,$$

for any $X \in gl(n,\mathbb{C})$. Obviously the map is well defined and surjective too. Since the group G is linear, it follows that our set:

$$g = \{Y \in gl(n,\mathbb{C} | exp(\lambda Y) \in G, \forall \lambda \in \mathbb{R}\}$$

is a Lie algebra over $\mathbb{R}$.

Remark 1.4 Recall that a linear group G in $GL(n,\mathbb{C})$ is complex if and only if, its Lie algebra g is a subset of $gl(n,\mathbb{C})$, and if it also satisfies the Lie bracket over the field $\mathbb{C}$. Additionally for any two linear groups $G, G^{'}$ from the Lie group, the intersection of $g, g^{'}$ is the Lie algebra of the two linear groups. Moreover the subgroups $G, G^{'}$ can be regarded as an algebraic groups, hence the algebraic groups of the Lie groups here, are complex linear groups.

We now bring the definition of contraction (or continuous contraction) of Leibniz algebras as follows:

Definition 1.4 [Rakhimov and Atan, 2012] Let $L_1 = (V, [\cdot,\cdot]_1)$ be a complex Leibniz algebra and $f_t : (0,1] \to GL(V)$ a continuous mapping where $f_t(\varepsilon) \in GL(V)$ for $0 < \varepsilon \leq 1$. If the limit $[x,y]_2 = \lim_{\varepsilon \to 0+} f_t(\varepsilon)^{-1}[f_t(\varepsilon)x, f_t(\varepsilon)y]$ exists $\forall x, y \in V$, then algebra $L_2 = (V, [\cdot,\cdot]_2)$ is called a one-parametric continuous contraction(or in short contraction)of $L_1 = (V, [\cdot,\cdot]_1)$ written as $L_1 \mapsto L_2$.

The following propositions can be found in Novotnỳ and Hrivnák (2008),Nesterenko and Popovych (2006) and Burde (2005) describe the criteria for computing the contraction of algebras.

Proposition 1.2 Any two of two three- dimensional complex Lie algebras L_1 and L_2 are isomorphic if and only if $\xi L_1 = \xi L_2$ or $\xi^{'} L_1 = \xi^{'} L_2$.

Proposition 1.3 Let L_2 be a proper contraction of L_1 where L_1 and L_2 are complex Lie algebras, then $dimDerL_1 < dimDerL_2$.

Proposition 1.4 Let L_2 be a proper contraction of L_1 where L_1 and L_2 are complex Lie algebras, then it holds:
1. $\xi L_1 \leq \xi L_2$.
2. $\xi L_1(1) < \xi L_2(1)$.

Here we introduce in this work the concept an algebra with two product operations which are refer to as dialgebras. Basic definitions and properties of dialgebras are giving in the works of Loday (2001), Lin and Zhang (2010) and also in Rakhimov and Fiidow (2015). We now focus on an associative dialgebra.

Definition 1.5 [Loday, 2001] **Associative dialgebras**: An associative dialgebra is a vector space $\mathbb{D}$, over a field $\mathbb{K}$, equipped with two associative multiplication $\vdash$ and $\dashv$ called right and left multiplications respectively, together with a bilinear maps;

$$\vdash : \mathbb{D} \times \mathbb{D} \to \mathbb{D},$$
$$\dashv : \mathbb{D} \times \mathbb{D} \to \mathbb{D}.$$

If it satisfies the following five conditions;

$$\begin{aligned}
&(x \dashv y) \dashv z = x \dashv (y \vdash z),\\
&(x \dashv y) \dashv z = x \dashv (y \dashv z),\\
&(x \vdash y) \dashv z = x \vdash (y \dashv z),\\
&(x \dashv y) \vdash z = x \vdash (y \vdash z),\\
&(x \vdash y) \vdash z = x \vdash (y \vdash z), \forall x, y, z \in \mathbb{D}.,
\end{aligned}$$

However, we can have the following equivalent formulation, i.e three di-associativity conditions instead of the five conditions above,

$$\begin{aligned}
&x \dashv (y \dashv z) = x \dashv (y \vdash z),\\
&(x \vdash y) \dashv z = x \vdash (y \dashv z),\\
&(x \vdash y) \vdash z = (x \dashv y) \vdash z, \forall x, y, z \in \mathbb{D}.
\end{aligned}$$

We denote by *Dias* the category of dialgebras.

Remark 1.5 A bar unit in a dialgebra $\mathbb{D}$ is the element $e \in \mathbb{D}$, such that

$$x \vdash e = e \dashv x, \forall x \in \mathbb{D}.$$

The subset of a bar units of associative dialgebra $\mathbb{D}$ is called its halo.

Let A be an associative algebra over the field $\mathbb{K}$, then the formulas $x \dashv y = xy = x \vdash y$, defines a dialgebra structure on A. If the unit of an associative algebra $e = 1$, then $e = 1$ is the unit of the dialgebra $\mathbb{D}$ and the set $\{1\}$ is the halo of $\mathbb{D}$.

Let (A, d) be a differential associative algebra over the field $\mathbb{K}$, and define the left and the right product on A by the relations: $x \dashv y := xdy$ and $x \vdash y := dxy$, together with $d^2 = 0$ defines an associative dialgebra structure on A.
Suppose $\mathbb{K}[x, y]$ is a polynomial algebra over $\mathbb{K}(Char = 0)$ with two indeterminates x and y. Let defines the right product $\vdash$ and the left product $\dashv$ on $\mathbb{K}[x, y]$ as follows:

$$f(x,y) \vdash g(x,y) = f(x,y)g(y,y)$$

and

$$f(x,y) \dashv g(x,y) = f(x,x)g(x,y).$$

Obviously $(\mathbb{K}[x,y], \vdash, \dashv)$ is an associative dialgebra.

Remark 1.6 It is clear here that $\mathbb{K}[x, y]$ can be considered as a Leibniz algebra, having the Leibniz bracket as

$$[f(x,y) \cdot g(x,y)] = f(x,y)(g(y,y) - g(x,x)).$$

In fact any associative dialgebra can be naturally defined as a Leibniz algebra.

Suppose that I is the subspace of the associative dialgebra $\mathbb{D}$. If for any $x, y \in \mathbb{D}$, we have $x \vdash y \in I$ and $x \dashv y \in I$, then I is called a subdialgebra of $\mathbb{D}$. Suppose that I is the subspace of the associative dialgebra $\mathbb{D}$. If for any $x \in \mathbb{D}$, and $y \in I$, we have $x \vdash y \in I$ and $x \dashv y \in I$, and $y \vdash x \in I$ and $y \dashv x \in I$, then I is called an ideal of $\mathbb{D}$. Let $(\mathbb{D}', \vdash, \dashv), (\mathbb{D}'', \vdash, \dashv)$ be two associative dialgebras over a field $\mathbb{K}$, then a homomorphism from $\mathbb{D}^{'}$ to $\mathbb{D}^{''}$ is a $\mathbb{K}$ - linear map such that

$$f(x \vdash y) = f(x) \vdash f(y)$$

and

$$f(x \dashv y) = f(x) \dashv f(y)$$

for all $x, y \in \mathbb{D}$.

Remark 1.7 If $\mathbb{D}^{'} = \mathbb{D}^{''}$ then the homomorphism becomes an endomorphism.

Let $(\mathbb{D}, \vdash, \dashv)$ be an associative dialgebra over a field $\mathbb{K}$, and $f \in End\mathbb{D}$ if f satisfies the condition:

$$f(x \vdash y) = f(x) \vdash f(y)$$

and

$$f(x \dashv y) = f(x) \dashv f(y)$$

for all $x, y \in \mathbb{D}$ and f is bijective, then f is called an automorphism of the associative dialgebra $\mathbb{D}$. The set of all automorphisms of $\mathbb{D}$ is denoted as $Aut\mathbb{D}$.

A linear transformation $d : \mathbb{D} \to \mathbb{D}$ which satisfies the following conditions:

$$d(x \vdash y) = d(x) \vdash y + x \vdash d(y)$$

and

$$d(x \dashv y) = d(x) \dashv y + x \dashv d(y)$$

for all $x, y \in \mathbb{D}$, is called a derivation of the associative dialgebra. The set of all derivations of $\mathbb{D}$ is a subspace of $End_{\mathbb{K}}\mathbb{D}$.

Remark 1.8 The subspace above equipped with the bracket $[d_1, d_2] = d_1 \circ d_2 - d_2 \circ d_1$, is a Lie algebra and is denoted as $Der\mathbb{D}$.

Theorem 1.2 [Rikhsiboev et al, 2014] Any two-dimensional complex associative dialgebra is included in the following isomorphism classes:

$$\begin{aligned}
&Dias_2^1 : e_1 \dashv e_1 = e_1,\, e_2 \dashv e_1 = e_2,\, e_1 \vdash e_1 = e_1; \\
&Dias_2^2 : e_1 \dashv e_1 = e_1,\, e_1 \vdash e_2 = e_2,\, e_1 \vdash e_1 = e_1; \\
&Dias_2^3 : e_1 \dashv e_1 = \alpha e_2,\, e_1 \vdash e_1 = e_2,\, \alpha \in \mathbb{C}; \\
&Dias_2^4 : e_1 \dashv e_1 = e_1,\, e_2 \dashv e_1 = e_2,\, e_1 \vdash e_1 = e_1,\, e_1 \vdash e_2 = e_2;
\end{aligned}$$

where $\{e_1, e_2\}$ is a basis.

Theorem 1.3 [Rakhimov et al, 2015] Any three-dimensional complex associative dialgebra is included in the following:

$$
\begin{aligned}
Dias_3^1 &: e_1 \dashv e_2 = e_1, e_2 \dashv e_2 = e_2, e_3 \dashv e_3 = e_3, e_2 \vdash e_2 = e_2, e_3 \vdash e_3 = e_3;\\
Dias_3^2 &: e_1 \dashv e_2 = e_1, e_2 \dashv e_2 = e_2, e_3 \dashv e_3 = e_3, e_2 \vdash e_2 = e_2, e_3 \vdash e_3 = e_3,\\
&\quad e_2 \dashv e_1 = e_1;\\
Dias_3^3 &: e_1 \dashv e_2 = e_1, e_2 \dashv e_2 = e_2, e_3 \dashv e_3 = e_3, e_2 \vdash e_2 = e_2, e_3 \vdash e_1 = e_1;\\
Dias_3^4 &: e_1 \dashv e_3 = e_2, e_2 \dashv e_3 = e_2, e_3 \dashv e_3 = e_3, e_3 \vdash e_3 = e_3;\\
Dias_3^5 &: e_1 \dashv e_3 = e_2, e_2 \dashv e_3 = e_2, e_3 \dashv e_3 = e_3, e_3 \vdash e_3 = e_3, e_3 \vdash e_1 = e_1 - e_2;\\
Dias_3^6 &: e_1 \dashv e_3 = e_2, e_2 \dashv e_3 = e_2, e_3 \dashv e_3 = e_3, e_3 \vdash e_3 = e_3, e_3 \vdash e_1 = e_1,\\
&\quad e_3 \vdash e_2 = e_2;\\
Dias_3^7 &: e_1 \dashv e_3 = e_2, e_2 \dashv e_3 = e_2, e_3 \dashv e_3 = e_3, e_3 \vdash e_3 = e_3, e_3 \vdash e_1 = e_2,\\
&\quad e_3 \vdash e_2 = e_2;\\
Dias_3^8 &: e_1 \dashv e_3 = e_2, e_2 \dashv e_3 = e_2, e_3 \dashv e_3 = e_3, e_3 \vdash e_3 = e_3, e_1 \vdash e_3 = e_2,\\
&\quad e_3 \vdash e_2 = e_2, e_3 \vdash e_1 = e_1 - e_2;\\
Dias_3^9 &: e_3 \dashv e_1 = e_2, e_3 \dashv e_2 = e_2, e_3 \dashv e_3 = e_3, e_3 \vdash e_3 = e_3, e_3 \vdash e_1 = e_1,\\
&\quad e_3 \vdash e_2 = e_2;\\
Dias_3^{10} &: e_3 \dashv e_1 = e_1, e_2 \dashv e_3 = e_2, e_3 \dashv e_3 = e_3, e_3 \vdash e_3 = e_3, e_3 \vdash e_1 = e_1;\\
Dias_3^{11} &: e_3 \dashv e_1 = e_1, e_2 \dashv e_3 = e_2, e_3 \dashv e_3 = e_3, e_3 \vdash e_3 = e_3, e_3 \vdash e_1 = e_1,\\
&\quad e_3 \vdash e_2 = e_2;\\
Dias_3^{12} &: e_1 \dashv e_3 = e_1, e_2 \dashv e_3 = e_2, e_3 \dashv e_1 = e_1, e_3 \vdash e_3 = e_3, e_1 \vdash e_3 = e_1,\\
&\quad e_3 \vdash e_1 = e_1, e_3 \dashv e_3 = e_3;\\
Dias_3^{13} &: e_1 \dashv e_3 = e_1, e_2 \dashv e_3 = e_2, e_3 \dashv e_1 = e_1, e_3 \vdash e_3 = e_3, e_1 \vdash e_3 = e_1,\\
&\quad e_3 \dashv e_1 = e_1, e_3 \vdash e_2 = e_2, e_3 \dashv e_3 = e_3;\\
Dias_3^{14} &: e_1 \dashv e_3 = e_1, e_2 \dashv e_3 = e_2, e_3 \dashv e_1 = e_1, e_3 \dashv e_3 = e_3, e_1 \vdash e_3 = e_1 + e_2,\\
&\quad e_3 \vdash e_1 = e_1, e_3 \vdash e_2 = e_2, e_3 \vdash e_3 = e_3;\\
Dias_3^{15} &: e_1 \dashv e_1 = e_2, e_3 \dashv e_3 = e_3, e_3 \vdash e_3 = e_3;\\
Dias_3^{16} &: e_1 \dashv e_3 = e_2, e_3 \dashv e_1 = ke_2, e_1 \vdash e_1 = me_2, e_3 \vdash e_1 = pe_2,\\
&\quad e_3 \vdash e_3 = qe_2, e_1 \vdash e_3 = ne_2 \, where \, k, m, p, q \in \mathbb{C};\\
Dias_3^{17} &: e_1 \dashv e_1 = e_2, e_1 \dashv e_2 = e_3, e_2 \dashv e_1 = e_3, e_1 \vdash e_1 = e_2 + e_3, e_1 \vdash e_2 = e_3,\\
&\quad e_2 \vdash e_1 = e_3;\\
Dias_3^{18} &: e_3 \dashv e_3 = e_3, e_3 \vdash e_1 = e_2, e_3 \vdash e_2 = e_2, e_3 \vdash e_3 = e_3;\\
Dias_3^{19} &: e_1 \dashv e_3 = e_1, e_2 \dashv e_3 = e_2, e_3 \dashv e_1 = e_2, e_3 \vdash e_3 = e_3, e_3 \vdash e_2 = e_2,\\
&\quad e_3 \vdash e_3 = e_3;\\
Dias_3^{20} &: e_3 \dashv e_1 = e_2, e_2 \dashv e_3 = e_2, e_1 \dashv e_3 = e_1 - e_2, e_3 \dashv e_3 = e_3,\\
&\quad e_3 \vdash e_1 = e_2, e_3 \vdash e_2 = e_2, e_3 \vdash e_3 = e_3;
\end{aligned}
$$

$Dias_3^{21}: e_1 \dashv e_3 = e_1, e_2 \dashv e_3 = e_2, e_3 \dashv e_3 = e_3, e_1 \vdash e_3 = e_2, e_2 \vdash e_3 = e_2,$
$e_3 \vdash e_3 = e_3;$

$Dias_3^{22}: e_1 \dashv e_3 = e_1, e_1 \vdash e_3 = e_1, e_3 \vdash e_2 = e_2, e_3 \dashv e_3 = e_3, e_3 \vdash e_3 = e_3;$

$Dias_3^{23}: = e_1 \dashv e_3 = e_1, e_2 \vdash e_3 = e_2, e_1 \vdash e_3 = e_1, e_3 \dashv e_3 = e_3, e_2 \dashv e_2 = e_2$
$e_3 \vdash e_3 = e_3;$

$Dias_3^{24}: e_3 \dashv e_1 = e_1, e_1 \vdash e_3 = e_1, e_3 \vdash e_1 = e_1, e_3 \dashv e_3 = e_3, e_3 \vdash e_2 = e_2,$
$e_3 \vdash e_3 = e_3;$

$Dias_3^{25}: e_1 \dashv e_1 = e_1, e_1 \vdash e_1 = e_1, e_1 \vdash e_2 = e_2, e_3 \dashv e_3 = e_3, e_3 \vdash e_3 = e_3;$

$Dias_3^{26}: e_1 \dashv e_1 = e_1, e_1 \dashv e_2 = e_1, e_1 \dashv e_3 = e_1, e_2 \dashv e_1 = e_2, e_2 \dashv e_2 = e_2,$
$e_2 \dashv e_3 = e_2, e_3 \dashv e_1 = e_3, e_3 \dashv e_2 = e_3, e_3 \dashv e_3 = e_3, e_1 \vdash e_1 = e_1,$
$e_1 \vdash e_2 = e_2, e_1 \vdash e_3 = e_3, e_2 \vdash e_1 = e_1, e_2 \vdash e_2 = e_2, e_2 \vdash e_3 = e_3$
$e_3 \vdash e_1 = e_1, e_3 \vdash e_2 = e_2, e_3 \vdash e_3 = e_3;$

The following tables described the generalized derivations of associative dialgebras [Fiidow et al, 2016] and the "trivial" refers to a null matrix.

Table 1.1: Description of generalized derivations of two dimensional associative dialgebras

IC	(α,β,γ)	$Der_{(\alpha,\beta,\gamma)}\mathbb{D}$	Dim	(α,β,γ)	$Der_{(\alpha,\beta,\gamma)}\mathbb{D}$	Dim
$\mathbb{D}_2^1$	(1,1,1)	$\begin{pmatrix}0&0\\0&d_{22}\end{pmatrix}$	1	(1,1,0)	$\begin{pmatrix}d_{11}&0\\0&d_{22}\end{pmatrix}$	2
	(1,0,1)	$\begin{pmatrix}d_{11}&0\\0&d_{11}\end{pmatrix}$	1	(1,0,0)	$(trivial)$	0
	(0,1,1)	$(trivial)$	0	(0,0,1)	$\begin{pmatrix}0&0\\d_{21}&d_{22}\end{pmatrix}$	2
	(0,1,0)	$(trivial)$	2	$(0,1,\delta)$	(trivial)	3
$\mathbb{D}_2^2$	(1,1,1)	$\begin{pmatrix}0&0\\0&d_{22}\end{pmatrix}$	1	(1,1,0)	$\begin{pmatrix}d_{11}&0\\0&d_{11}\end{pmatrix}$	1
	(1,0,1)	$\begin{pmatrix}d_{11}&0\\0&d_{22}\end{pmatrix}$	2	(1,0,0)	$(trivial)$	0
	(0,1,1)	$(trivial)$	0	(0,0,1)	$(trivial)$	0
	(0,1,0)	$\begin{pmatrix}0&0\\d_{21}&d_{22}\end{pmatrix}$	2	$(0,1,\delta)$	(trivial)	4
$\mathbb{D}_2^3$	(1,1,1)	$\begin{pmatrix}d_{11}&0\\d_{21}&2d_{11}\end{pmatrix}$	2	(1,1,0)	$\begin{pmatrix}d_{11}&0\\d_{21}&d_{11}\end{pmatrix}$	2
	(1,0,1)	$\begin{pmatrix}d_{11}&0\\d_{21}&d_{11}\end{pmatrix}$	0	(1,0,0)	$\begin{pmatrix}d_{11}&0\\d_{21}&0\end{pmatrix}$	2
	(0,1,1)	$\begin{pmatrix}0&0\\d_{21}&d_{22}\end{pmatrix}$	2	(0,0,1)	$\begin{pmatrix}0&0\\d_{21}&d_{22}\end{pmatrix}$	2
	(0,1,0)	$\begin{pmatrix}0&0\\d_{21}&d_{22}\end{pmatrix}$	2	$(0,1,\delta)$	$\begin{pmatrix}0&0\\d_{21}&d_{22}\end{pmatrix}$	2
$\mathbb{D}_2^4$	(1,1,1)	$\begin{pmatrix}0&0\\d_{21}&d_{22}\end{pmatrix}$	2	(1,1,0)	$\begin{pmatrix}d_{11}&0\\0&d_{11}\end{pmatrix}$	1
	(1,0,1)	$\begin{pmatrix}d_{11}&0\\0&d_{11}\end{pmatrix}$	1	(1,0,0)	(trivial)	0
	(0,1,1)	(trivial)	0	(0,0,1)	(trivial)	0
	(0,1,0)	(trivial)	0	$(0,1,\delta)$	(trivial)	0

Table 1.2: Description of generalized derivations of three dimensional associative dialgebras

IC	(α,β,γ)	$Der_{(\alpha,\beta,\gamma)}\mathbb{D}$	Dim	(α,β,γ)	$Der_{(\alpha,\beta,\gamma)}\mathbb{D}$	Dim
$\mathbb{D}_3^1$	(1,1,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	1	(1,1,0)	$\begin{pmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & 0 \\ 0 & 0 & d_{33} \end{pmatrix}$	3
	(1,0,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & 0 \\ 0 & 0 & d_{33} \end{pmatrix}$	3	(1,0,0)	$(trivial)$	0
	(0,1,1)	$(trivial)$	0	(0,0,1)	$\begin{pmatrix} d_{11} & d_{12} & d_{13} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	3
	(0,1,0)	$(trivial)$	2	$(0,1,\delta)$	$\begin{pmatrix} t_6 & 0 & 0 \\ 0 & l_6 & 0 \\ 0 & 0 & l_{7\int} \end{pmatrix}$	3
$\mathbb{D}_3^2$	(1,1,1)	$\begin{pmatrix} d_{11} & d_{12} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	2	(1,1,0)	$\begin{pmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & 0 \\ 0 & 0 & d_{33} \end{pmatrix}$	3
	(1,0,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & 0 \\ 0 & 0 & d_{33} \end{pmatrix}$	3	(1,0,0)	$(trivial)$	0
	(0,1,1)	$(trivial)$	0	(0,0,1)	$(trivial)$	0
	(0,1,0)	$(trivial)$	2	$(0,1,\delta)$	$\begin{pmatrix} t_6 & 0 & 0 \\ 0 & l_6 & 0 \\ 0 & 0 & l_{7\int} \end{pmatrix}$	4
$\mathbb{D}_3^3$	(1,1,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	1	(1,1,0)	$\begin{pmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & 0 \\ 0 & 0 & d_{33} \end{pmatrix}$	3
	(1,0,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & 0 \\ 0 & 0 & d_{11} \end{pmatrix}$	2	(1,0,0)	$(trivial)$	0
	(0,1,1)	$(trivial)$	0	(0,0,1)	$(trivial)$	0
	(0,1,0)	$(trivial)$	0	$(0,1,\delta)$	$\begin{pmatrix} t_6 & 0 & 0 \\ 0 & t_6 & 0 \\ 0 & 0 & l_7 \end{pmatrix}$	2
$\mathbb{D}_3^4$	(1,1,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & m_2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	2	(1,1,0)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & m_2 & 0 \\ 0 & 0 & d_{33} \end{pmatrix}$	3
	(1,0,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & d_{22} & 0 \\ 0 & 0 & d_{22} \end{pmatrix}$	3	(1,0,0)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & 0 & 0 \\ d_{31} & 0 & 0 \end{pmatrix}$	3
	(0,1,1)	$\begin{pmatrix} d_{11} & d_{12} & d_{13} \\ -d_{11} & -d_{12} & -d_{13} \\ 0 & 0 & 0 \end{pmatrix}$	3	(0,0,1)	$\begin{pmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ 0 & 0 & 0 \end{pmatrix}$	6
	(0,1,0)	$\begin{pmatrix} d_{11} & d_{12} & d_{13} \\ -d_{11} & -d_{12} & -d_{23} \\ 0 & 0 & 0 \end{pmatrix}$	3	$(0,1,\delta)$	$\begin{pmatrix} t_3 & d_{12} & d_{13} \\ d_{21} & t_4 & -d_{13} \\ 0 & 0 & t_7 \end{pmatrix}$	4

Table 1.2

IC	(α,β,γ)	$Der_{(\alpha,\beta,\gamma)}\mathbb{D}$	Dim	(α,β,γ)	$Der_{(\alpha,\beta,\gamma)}\mathbb{D}$	Dim
$\mathbb{D}_3^5$	(1,1,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & m_2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	2	(1,1,0)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & m_2 & 0 \\ 0 & 0 & d_{22} \end{pmatrix}$	2
	(1,0,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & m_2 & 0 \\ 0 & 0 & m_2 \end{pmatrix}$	3	(1,0,0)	$(trivial)$	0
	(0,1,1)	$(trivial)$	0	(0,0,1)	$\begin{pmatrix} 0 & 0 & 0 \\ d_{21} & d_{22} & d_{23} \\ 0 & 0 & 0 \end{pmatrix}$	3
	(0,1,0)	$(trivial)$	2	$(0,1,\delta)$	$\begin{pmatrix} t_6 & 0 & 0 \\ 0 & l_6 & 0 \\ 0 & 0 & l_7 \end{pmatrix}$	3
$\mathbb{D}_3^6$	(1,1,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & m_2 & d_{23} \\ 0 & 0 & 0 \end{pmatrix}$	3	(1,1,0)	$\begin{pmatrix} d_{11} & 0 & 0 \\ 0 & d_{11} & 0 \\ 0 & 0 & d_{11} \end{pmatrix}$	1
	(1,0,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & d_{22} & 0 \\ 0 & 0 & d_{22} \end{pmatrix}$	3	(1,0,0)	$(trivial)$	0
	(0,1,1)	$(trivial)$	0	(0,0,1)	$(trivial)$	0
	(0,1,0)	$\begin{pmatrix} d_{11} & d_{12} & d_{13} \\ -d_{11} & -d_{12} & -d_{13} \\ 0 & 0 & 0 \end{pmatrix}$	2	$(0,1,\delta)$	$\begin{pmatrix} t_9 & 0 & 0 \\ 0 & t_9 & 0 \\ 0 & 0 & t_9 \end{pmatrix}$	1
$\mathbb{D}_3^7$	(1,1,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & m_2 & d_{23} \\ 0 & 0 & 0 \end{pmatrix}$	3	(1,1,0)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & m_2 & 0 \\ 0 & 0 & m_2 \end{pmatrix}$	2
	(1,0,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & m_2 & 0 \\ 0 & 0 & m_2 \end{pmatrix}$	2	(1,0,0)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & 0 & 0 \\ d_{31} & 0 & 0 \end{pmatrix}$	0
	(0,1,1)	$\begin{pmatrix} d_{11} & d_{12} & d_{13} \\ -d_{11} & -d_{12} & -d_{13} \\ 0 & 0 & 0 \end{pmatrix}$	3	(0,0,1)	$\begin{pmatrix} d_{11} & d_{12} & d_{13} \\ -d_{11} & -d_{12} & -d_{13} \\ 0 & 0 & 0 \end{pmatrix}$	0
	(0,1,0)	$\begin{pmatrix} d_{11} & d_{12} & d_{13} \\ -d_{11} & -d_{12} & -d_{13} \\ 0 & 0 & 0 \end{pmatrix}$	3	$(0,1,\delta)$	$\begin{pmatrix} t_3 & d_{12} & d_{13} \\ d_{21} & t_4 & -d_{13} \\ 0 & 0 & t_7 \end{pmatrix}$	2
$\mathbb{D}_3^8$	(1,1,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & m_2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	2	(1,1,0)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & m_2 & d_{23} \\ 0 & 0 & d_{11} \end{pmatrix}$	3
	(1,0,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & m_2 & 0 \\ 0 & 0 & m_2 \end{pmatrix}$	2	(1,0,0)	$(trivial)$	0
	(0,1,1)	$(trivial)$	0	(0,0,1)	$\begin{pmatrix} 0 & 0 & 0 \\ d_{21} & d_{22} & d_{23} \\ 0 & 0 & 0 \end{pmatrix}$	3
	(0,1,0)	$\begin{pmatrix} d_{11} & d_{12} & d_{13} \\ -d_{11} & -d_{12} & -d_{23} \\ 0 & 0 & 0 \end{pmatrix}$	3	$(0,1,\delta)$	$\begin{pmatrix} t_7 & 0 & 0 \\ 0 & t_7 & 0 \\ 0 & 0 & t_7 \end{pmatrix}$	1

Table 1.2

IC	(α,β,γ)	$Der_{(\alpha,\beta,\gamma)}\mathbb{D}$	Dim	(α,β,γ)	$Der_{(\alpha,\beta,\gamma)}\mathbb{D}$	Dim
$\mathbb{D}_3^9$	(1,1,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & m_2 & d_{23} \\ 0 & 0 & 0 \end{pmatrix}$	3	(1,1,0)	$\begin{pmatrix} d_{11} & 0 & 0 \\ 0 & d_{11} & 0 \\ 0 & 0 & d_{11} \end{pmatrix}$	1
	(1,0,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & m_2 & d_{23} \\ d_{31} & d_{31} & d_{33} \end{pmatrix}$	3	(1,0,0)	$(trivial)$	0
	(0,1,1)	$(trivial)$	0	(0,0,1)	$(trivial)$	3
	(0,1,0)	$\begin{pmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ 0 & 0 & 0 \end{pmatrix}$	6	$(0,1,\delta)$	$\begin{pmatrix} t_9 & 0 & 0 \\ 0 & t_9 & 0 \\ 0 & 0 & t_9 \end{pmatrix}$	3
$\mathbb{D}_3^{10}$	(1,1,1)	$\begin{pmatrix} d_{11} & 0 & d_{13} \\ 0 & d_{22} & 0 \\ 0 & 0 & 0 \end{pmatrix}$	3	(1,1,0)	$\begin{pmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & 0 \\ 0 & 0 & d_{11} \end{pmatrix}$	2
	(1,0,1)	$\begin{pmatrix} d_{11} & 0 & d_{13} \\ 0 & d_{22} & 0 \\ 0 & 0 & d_{22} \end{pmatrix}$	3	(1,0,0)	$(trivial)$	0
	(0,1,1)	$(trivial)$	0	(0,0,1)	$(trivial)$	0
	(0,1,0)	$\begin{pmatrix} d_{11} & d_{12} & d_{13} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	3	$(0,1,\delta)$	$\begin{pmatrix} t_9 & 0 & 0 \\ 0 & t_9 & 0 \\ 0 & 0 & t_9 \end{pmatrix}$	1
$\mathbb{D}_3^{11}$	(1,1,1)	$\begin{pmatrix} d_{11} & 0 & d_{13} \\ 0 & d_{22} & d_{23} \\ 0 & 0 & 0 \end{pmatrix}$	4	(1,1,0)	$\begin{pmatrix} d_{11} & 0 & 0 \\ 0 & d_{11} & 0 \\ 0 & 0 & d_{11} \end{pmatrix}$	1
	(1,0,1)	$\begin{pmatrix} d_{11} & 0 & d_{13} \\ 0 & d_{22} & 0 \\ 0 & 0 & d_{22} \end{pmatrix}$	3	(1,0,0)	$(trivial)$	0
	(0,1,1)	$(trivial)$	0	(0,0,1)	$(trivial)$	0
	(0,1,0)	$\begin{pmatrix} d_{11} & d_{12} & d_{13} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	3	$(0,1,\delta)$	$\begin{pmatrix} t_9 & 0 & 0 \\ 0 & t_9 & 0 \\ 0 & 0 & t_9 \end{pmatrix}$	2
$\mathbb{D}_3^{12}$	(1,1,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & 0 \\ 0 & 0 & 0 \end{pmatrix}$	2	(1,1,0)	$\begin{pmatrix} d_{11} & 0 & d_{13} \\ 0 & d_{22} & 0 \\ 0 & 0 & d_{11} \end{pmatrix}$	3
	(1,0,1)	$\begin{pmatrix} d_{11} & 0 & d_{13} \\ 0 & d_{11} & 0 \\ 0 & 0 & d_{11} \end{pmatrix}$	3	(1,0,0)	$(trivial)$	3
	(0,1,1)	$(trivial)$	3	(0,0,1)	$\begin{pmatrix} 0 & 0 & 0 \\ d_{21} & d_{22} & d_{23} \\ 0 & 0 & 0 \end{pmatrix}$	3
	(0,1,0)	$(trivial)$	3	$(0,1,\delta)$	$\begin{pmatrix} t_9 & 0 & -\delta d_{13} \\ 0 & t_9 & 0 \\ 0 & 0 & t_9 \end{pmatrix}$	2

Table 1.2

IC	(α,β,γ)	$Der_{(\alpha,\beta,\gamma)}\mathbb{D}$	Dim	(α,β,γ)	$Der_{(\alpha,\beta,\gamma)}\mathbb{D}$	Dim
$\mathbb{D}_3^{13}$	(1,1,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & d_{23} \\ 0 & 0 & 0 \end{pmatrix}$	3	(1,1,0)	$\begin{pmatrix} d_{11} & 0 & 0 \\ 0 & d_{11} & 0 \\ 0 & 0 & d_{11} \end{pmatrix}$	1
	(1,0,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ 0 & d_{11} & 0 \\ 0 & 0 & d_{11} \end{pmatrix}$	1	(1,0,0)	$(trivial)$	0
	(0,1,1)	$(trivial)$	0	(0,0,1)	$(trivial)$	3
	(0,1,0)	$(trivial)$	2	$(0,1,\delta)$	$\begin{pmatrix} t_7 & 0 & 0 \\ 0 & t_7 & 0 \\ 0 & 0 & t_7 \end{pmatrix}$	1
$\mathbb{D}_3^{14}$	(1,1,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ 0 & d_{11} & d_{23} \\ 0 & 0 & 0 \end{pmatrix}$	2	(1,1,0)	$\begin{pmatrix} d_{11} & 0 & 0 \\ 0 & d_{11} & d_{13} \\ 0 & 0 & d_{11} \end{pmatrix}$	2
	(1,0,1)	$\begin{pmatrix} d_{11} & 0 & d_{13} \\ 0 & d_{11} & 0 \\ 0 & 0 & d_{11} \end{pmatrix}$	2	(1,0,0)	$(trivial)$	0
	(0,1,1)	$(trivial)$	0	(0,0,1)	$(trivial)$	0
	(0,1,0)	$(trivial)$	2	$(0,1,\delta)$	$\begin{pmatrix} t_9 & 0 & 0 \\ 0 & t_9 & 0 \\ 0 & 0 & l_9 \end{pmatrix}$	4
$\mathbb{D}_3^{15}$	(1,1,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & 2d_{11}0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	2	(1,1,0)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & d_{11} & 0 \\ 0 & 0 & d_{33} \end{pmatrix}$	3
	(1,0,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & d_{11} & 0 \\ 0 & 0 & d_{33} \end{pmatrix}$	3	(1,0,0)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & 0 & 0 \\ d_{31} & 0 & 0 \end{pmatrix}$	3
	(0,1,1)	$\begin{pmatrix} 0 & 0 & 0 \\ d_{21} & d_{22} & d_{22} \\ 0 & 0 & 0 \end{pmatrix}$	2	(0,0,1)	$\begin{pmatrix} 0 & 0 & 0 \\ d_{21} & d_{22} & d_{23} \\ 0 & 0 & 0 \end{pmatrix}$	3
	(0,1,0)	$\begin{pmatrix} 0 & 0 & 0 \\ d_{21} & d_{22} & d_{23} \\ 0 & 0 & 0 \end{pmatrix}$	3	$(0,1,\delta)$	$\begin{pmatrix} 0 & 0 & 0 \\ d_{21} & d_{22} & d_{23} \\ 0 & 0 & 0 \end{pmatrix}$	3
$\mathbb{D}_3^{16}$	(1,1,1)	$\begin{pmatrix} t & 0 & t_1 \\ d_{21} & t_2 & d_{23} \\ d_{31} & 0 & d_{33} \end{pmatrix}$	4	(1,1,0)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & d_{11} & d_{23} \\ d_{31} & 0 & d_{33} \end{pmatrix}$	5
	(1,0,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & d_{22} & 0 \\ 0 & 0 & d_{22} \end{pmatrix}$	3	(1,0,0)	$\begin{pmatrix} d_{11} & 0 & d_{13} \\ d_{21} & d_{22} & d_{23} \\ 0 & 0 & d_{22} \end{pmatrix}$	5
	(0,1,1)	$\begin{pmatrix} d_{13}r & 0 & d_{13} \\ d_{21} & d_{22} & d_{23} \\ -d_{13}s & 0 & -d_{13}s \end{pmatrix}$	4	(0,0,1)	$\begin{pmatrix} 0 & 0 & 0 \\ d_{21} & d_{22} & d_{23} \\ 0 & 0 & 0 \end{pmatrix}$	3
	(0,1,0)	$\begin{pmatrix} 0 & 0 & 0 \\ d_{21} & d_{22} & d_{23} \\ 0d_{31} & d_{32} & d_{33} \end{pmatrix}$	6	$(0,1,\delta)$	$\begin{pmatrix} t_{11} & 0 & -\delta k d_{13} \\ d_{21} & d_{22} & d_{23} \\ t_{12} & 0 & -\delta t_{14} \end{pmatrix}$	4

Table 1.2

IC	(α,β,γ)	$Der_{(\alpha,\beta,\gamma)}\mathbb{D}$	Dim	(α,β,γ)	$Der_{(\alpha,\beta,\gamma)}\mathbb{D}$	Dim
$\mathbb{D}_3^{17}$	(1,1,1)	$\begin{pmatrix} 0 & 0 & 0 \\ d_{21} & 0 & 0 \\ d_{31} & 2d_{21} & 0 \end{pmatrix}$	2	(1,1,0)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & d_{11} & 0 \\ d_{31} & d_{21} & d_{11} \end{pmatrix}$	3
	(1,0,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & d_{11} & 0 \\ d_{31} & d_{21} & d_{11} \end{pmatrix}$	3	(1,0,0)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & 0 & 0 \\ d_{31} & 0 & 0 \end{pmatrix}$	3
	(0,1,1)	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ d_{31} & d_{32} & d_{33} \end{pmatrix}$	3	(0,0,1)	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ d_{31} & d_{32} & d_{33} \end{pmatrix}$	3
	(0,1,0)	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ d_{31} & d_{32} & d_{33} \end{pmatrix}$	3	(0,1,δ)	$\begin{pmatrix} t_9 & 0 & 0 \\ \delta d_{21} & t_9 & 0 \\ d_{31} & d_{32} & d_{33} \end{pmatrix}$	5
$\mathbb{D}_3^{18}$	(1,1,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & m_2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	2	(1,1,0)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & m_2 & 0 \\ 0 & 0 & d_{22} \end{pmatrix}$	3
	(1,0,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & m_2 & 0 \\ 0 & 0 & d_{33} \end{pmatrix}$	3	(1,0,0)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & 0 & 0 \\ d_{31} & 0 & 0 \end{pmatrix}$	0
	(0,1,1)	$\begin{pmatrix} d_{11} & d_{12} & d_{13} \\ -d_{11} & -d_{12} & -d_{13} \\ 0 & 0 & 0 \end{pmatrix}$	3	(0,0,1)	$\begin{pmatrix} d_{11} & d_{12} & d_{13} \\ -d_{11} & -d_{12} & -d_{13} \\ 0 & 0 & 0 \end{pmatrix}$	3
	(0,1,0)	$\begin{pmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ 0 & 0 & 0 \end{pmatrix}$	6	(0,1,δ)	$\begin{pmatrix} t_{12} & d_{12} & d_{13} \\ -d_{12} & d_{22} & -d_{13} \\ 0 & 0 & l_{7\int} \end{pmatrix}$	4
$\mathbb{D}_3^{19}$	(1,1,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & m_2 & d_{23} \\ 0 & 0 & 0 \end{pmatrix}$	3	(1,1,0)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & d_{22} & 0 \\ 0 & 0 & d_{22} \end{pmatrix}$	3
	(1,0,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ 0 & d_{11} & 0 \\ 0 & 0 & d_{11} \end{pmatrix}$	1	(1,0,0)	$(trivial)$	0
	(0,1,1)	$(trivial)$	0	(0,0,1)	$\begin{pmatrix} d_{11} & d_{12} & d_{13} \\ -d_{11} & -d_{12} & -d_{13} \\ 0 & 0 & d_{11} \end{pmatrix}$	3
	(0,1,0)	$(trivial)$	0	(0,1,δ)	$\begin{pmatrix} t_7 & 0 & 0 \\ 0 & t_7 & 0 \\ 0 & 0 & t_7 \end{pmatrix}$	1
$\mathbb{D}_3^{20}$	(1,1,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & m_2 & d_{23} \\ 0 & 0 & 0 \end{pmatrix}$	3	(1,1,0)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & m_2 & 0 \\ 0 & 0 & {}_2 \end{pmatrix}$	2
	(1,0,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & m_2 & d_{23} \\ 0 & 0 & d_{11} \end{pmatrix}$	3	(1,0,0)	$(trivial)$	3
	(0,1,1)	$(trivial)$	3	(0,0,1)	$\begin{pmatrix} d_{11} & d_{12} & d_{13} \\ -d_{11} & -d_{12} & -d_{13} \\ 0 & 0 & 0 \end{pmatrix}$	3
	(0,1,0)	$\begin{pmatrix} 0 & 0 & 0 \\ d_{21} & d_{22} & d_{23} \\ 0 & 0 & 0 \end{pmatrix}$	3	(0,1,δ)	$(trivial)$	0

Table 1.2

IC	(α,β,γ)	$Der_{(\alpha,\beta,\gamma)}\mathbb{D}$	Dim	(α,β,γ)	$Der_{(\alpha,\beta,\gamma)}\mathbb{D}$	Dim
$\mathbb{D}_3^{21}$	(1,1,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & m_2 & d_{23} \\ 0 & 0 & 0 \end{pmatrix}$	3	(1,1,0)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & m_2 & d_{23} \\ d_{31} & d_{31} & d_{33} \end{pmatrix}$	5
	(1,0,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ 0 & d_{11} & 0 \\ 0 & 0 & d_{11} \end{pmatrix}$	1	(1,0,0)	$(trivial)$	0
	(0,1,1)	$(trivial)$	0	(0,0,1)	$\begin{pmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ 0 & 0 & 0 \end{pmatrix}$	6
	(0,1,0)	$(trivial)$	2	(0,1,δ)	$\begin{pmatrix} t_7 & 0 & 0 \\ 0 & t_7 & 0 \\ 0 & 0 & t_7 \end{pmatrix}$	1
$\mathbb{D}_3^{22}$	(1,1,1)	$\begin{pmatrix} d_{11} & 0 & d_{13} \\ 0 & d_{22} & 0 \\ 0 & 0 & d_{22} \end{pmatrix}$	3	(1,1,0)	$\begin{pmatrix} d_{11} & 0 & d_{13} \\ 0 & d_{22} & 0 \\ 0 & 0 & d_{22} \end{pmatrix}$	3
	(1,0,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & 0 \\ 0 & 0 & d_{22} \end{pmatrix}$	2	(1,0,0)	$(trivial)$	0
	(0,1,1)	$(trivial)$	0	(0,0,1)	$\begin{pmatrix} d_{11} & d_{12} & d_{13} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	3
	(0,1,0)	$\begin{pmatrix} 0 & 0 & 0 \\ d_{21} & d_{22} & d_{23} \\ 0 & 0 & 0 \end{pmatrix}$	2	(0,1,δ)	$\begin{pmatrix} t_7 & 0 & 0 \\ 0 & -(/\delta)d_{33} & 0 \\ 0 & 0 & t_7 \end{pmatrix}$	1
$\mathbb{D}_3^{23}$	(1,1,1)	$\begin{pmatrix} d_{11} & 0 & d_{13} \\ 0 & d_{22} & d_{23} \\ 0 & 0 & d_{11} \end{pmatrix}$	4	(1,1,0)	$\begin{pmatrix} d_{11} & 0 & d_{13} \\ 0 & d_{22} & 0 \\ 0 & 0 & d_{22} \end{pmatrix}$	3
	(1,0,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ 0 & d_{11} & 0 \\ 0 & 0 & d_{11} \end{pmatrix}$	1	(1,0,0)	$(trivial)$	0
	(0,1,1)	$(trivial)$	0	(0,0,1)	$\begin{pmatrix} d_{11} & d_{12} & d_{13} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	3
	(0,1,0)	$(trivial)$	0	(0,1,δ)	$\begin{pmatrix} t_7 & 0 & 0 \\ 0 & t_7 & 0 \\ 0 & 0 & t_7 \end{pmatrix}$	1
$\mathbb{D}_3^{24}$	(1,1,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{22} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	2	(1,1,0)	$\begin{pmatrix} d_{11} & 0 & 0 \\ 0 & d_{11} & 0 \\ 0 & 0 & d_{11} \end{pmatrix}$	1
	(1,0,1)	$\begin{pmatrix} d_{11} & 0 & d_{13} \\ 0 & d_{22} & 0 \\ 0 & 0 & d_{11} \end{pmatrix}$	3	(1,0,0)	$(trivial)$	3
	(0,1,1)	$(trivial)$	3	(0,0,1)	$(trivial)$	6
	(0,1,0)	$\begin{pmatrix} 0 & 0 & 0 \\ d_{21} & d_{22} & d_{23} \\ 0 & 0 & 0 \end{pmatrix}$	3	(0,1,δ)	$\begin{pmatrix} t_9 & 0 & 0 \\ 0 & t_9 & 0 \\ 0 & 0 & t_9 \end{pmatrix}$	1

Table 1.2

IC	(α,β,γ)	$Der_{(\alpha,\beta,\gamma)}\mathbb{D}$	Dim	(α,β,γ)	$Der_{(\alpha,\beta,\gamma)}\mathbb{D}$	Dim
$\mathbb{D}_3^{25}$	(1,1,1)	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & d_{22} & 0 \\ 0 & 0 & 0 \end{pmatrix}$	1	(1,1,0)	$\begin{pmatrix} d_{11} & 0 & 0 \\ 0 & d_{11} & 0 \\ 0 & 0 & d_{33} \end{pmatrix}$	2
	(1,0,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & 0 \\ 0 & 0 & d_{33} \end{pmatrix}$	3	(1,0,0)	$(trivial)$	0
	(0,1,1)	$(trivial)$	0	(0,0,1)	$(trivial)$	3
	(0,1,0)	$\begin{pmatrix} 0 & 0 & 0 \\ d_{21} & d_{22} & d_{23} \\ 0 & 0 & 0 \end{pmatrix}$	3	$(0,1,\delta)$	$\begin{pmatrix} t_9 & 0 & 0 \\ 0 & -(1/\delta)d_{11} & 0 \\ 0 & 0 & t_7 \end{pmatrix}$	3
$\mathbb{D}_3^{26}$	(1,1,1)	$\begin{pmatrix} t_4 & d_{12} & d_{13} \\ d_{21} & t_5 & d_{23} \\ d_{31} & d_{32} & t_6 \end{pmatrix}$	6	(1,1,0)	$\begin{pmatrix} d_{11} & 0 & 0 \\ 0 & d_{11} & 0 \\ 0 & 0 & d_{11} \end{pmatrix}$	1
	(1,0,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ 0 & d_{11} & 0 \\ 0 & 0 & d_{11} \end{pmatrix}$	1	(1,0,0)	$(trivial)$	0
	(0,1,1)	$(trivial)$	0	(0,0,1)	$(trivial)$	0
	(0,1,0)	$(trivial)$	0	$(0,1,\delta)$	$\begin{pmatrix} t_9 & 0 & 0 \\ 0 & -(1/\delta)d_{11} & 0 \\ 0 & 0 & t_9 \end{pmatrix}$	1

Now we recall that various forms of generalization of derivation exists, of importance for instance, Novotnỳ and Hrivnák (2008) defined it as follows;

Definition 1.6 Let $\mathbb{A} = (V,\cdot)$ be an arbitrary algebra. We call a linear operator $d \in End\mathbb{A}$ an $(\alpha,\beta,\gamma)-$derivation of $\mathbb{A}$ if $\exists\, \alpha,\beta,\gamma \in \mathbb{C} \ni, \forall x,y \in \mathbb{A}$, the following expression holds true, i.e, $\alpha d(x \cdot y) = \beta(dx)\cdot y + \gamma x \cdot y.$ The set of all $(\alpha,\beta,\gamma)-$derivations is denoted as $der_{(\alpha,\beta,\gamma)}\mathbb{A}$. It is also a linear subspace of $End\mathbb{A}$.

The "." in the above expression represent the operation in the arbitrary algebra. Important results emerges from this definition, among which we mention few fundamental theorems in this part, which are found to be relevant throughout this thesis.

Theorem 1.4 [Novotny and Hrivnak, 2008] Let $\mathbb{A} = (V,\cdot)$ be an arbitrary algebra, then for $\alpha,\beta,\gamma \in \mathbb{C},\ \exists\, \delta \in \mathbb{C}, \ni$ the subspace $der_{(\alpha,\beta,\gamma)}\mathbb{A} \subset End\mathbb{A}$, is equal to some of the followings ;
1. $der_{(\delta,0,0)}\mathbb{A}$,
2. $der_{(\delta,1,-1)}\mathbb{A}$,
3. $der_{(\delta,1,0)}\mathbb{A}$,
4. $der_{(\delta,1,1)}\mathbb{A}$.

1.1.2 Motivation

It is well known that a generalized derivation of algebras is a very important area of research, for example Leger and Luks (2000) studied a systematic approach to generalization of Lie and Lie subalgebras. As a result of which some important properties were obtained, for instance the centroids and quasiderivations were described. More importantly they studied the structure of the generalized derivation and thereby characterized the Lie algebras according to some conditions. They also showed in their work the relations between quasi-derivations and cohomology of Lie algebras. Another important work on the generalized derivations of Lie algebras is by Hartwig et al. (2006), in that work crucial results were obtained, which enable them to develop an approach to the ”deformations” of Witt and Virasoro algebras, which are some forms of a Lie algebras, It was also shown among the results ways to construct new deformation of Lie algebras and its central extensions. Hrivnák (2015) described the concept of generalized derivations of Lie algebras using some complex parameters, and presented the results related to its structure and invariants. They defined some invariant functions and used the calculated values of the invariant function as a classification tool. Hence employed the calculated values of the invariant functions to find possible contractions among the Lie algebras. Associative dialgebra structure was presented by Loday (2001). Other properties of the associative dialgebra, like derivations, automorphisms, generalized derivations et cetera were studied in work of Lin and Zhang (2010) and also in Rikhsiboev et al. and Fiidow et al. (2016).

1.2 Objectives of the study

The main objectives of this research are :

- To describe and compute the generalized (α, β, γ)-derivations of low-dimensional associative and Lie algebras.
- To describe and compute the generalized (α, β, γ)-derivations of Leibniz algebras.
- To describe the automorphism group of low-dimensional Leibniz algebras and associative dialgebras.
- To describe the concept of a generalized automorphism of low-dimensional Leibniz algebras and associative dialgebras.
- To find and describe additional invariant functions of Leibniz algebras and associative dialgebras.
- To apply a contraction criteria to find possible contractions among low-dimensional Leibniz algebras and associative dialgebras .

1.3 Outline of the thesis

This work comprises of six chapters, it is given briefly as follows:

- Chapter one is basically concerned about a review of some important results on the associative algebras, Lie algebras, Leibniz algebras and the associative dialgebras. It includes basic definitions, fundamental concepts, and other relevant statements etc.
- In the second chapter. The review of literature and a description and computations of the generalized (α, β, γ)-derivations of low-dimensional associative and Lie algebras were given, including some important results.
- In the third chapter, we introduce, describe and compute the generalized (α, β, γ)-derivations of Leibniz algebras, some important properties are also presented.
- Chapter 4 focusses on the concept of a generalized automorphism of Leibniz algebras and associative dialgebras, and also provide some important results.
- In chapter 5, the invariants and contractions of Leibniz algebras and associative dialgebras are considered. The list of the contractions on lower dimensional cases are given.
- Chapter six is about the conclusion and the summary of the work, it also carries some recommendations for future work.

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