PREDICTOR CORRECTOR BLOCK METHODS FOR SOLVING ORDINARY DIFFERENTIAL EQUATIONS

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DEDICATION

I dedicate this thesis to my beloved family and teachers who always support me in pursuing this degree.

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Master of Science

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In this thesis, the predictor corrector block methods are developed for solving first and higher order initial value problems (IVPs) of ordinary differential equations (ODEs). These methods solve higher order ODEs problem directly without reducing to a system of first order ODEs. The derivation of these proposed block methods are based on the numerical integration method and using an interpolation approach which are similar to the Adams method.

These developed block methods solve higher order ODE problems directly in a single code using variable step size strategy. In order to gain an efficient and reliable numerical approximation, these developed block methods are implemented in the predictor corrector mode using a simple iteration technique. The proposed block methods compute several numerical solutions simultaneously and the number of solutions to be computed depends on the feature of the block methods. The integration coefficients of the developed block methods formulae are stored in the code to avoid tedious and repetitive computation.

Several tested problems of ODEs are taken into account in the numerical experiments. This is to emphasize the main features of the proposed methods by comparing these direct block methods with the existing methods that solve the higher order ODEs by reducing to a system of first order ODEs. The results obtained showed that the developed block methods managed to produce acceptable results in terms of maximum error and computational time for solving higher order ODEs directly.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Master Sains

KAEDAH BLOK PERAMAL PEMBETUL BAGI MENYELESAIKAN PERSAMAAN PEMBEZAAN BIASA

Oleh

NAZREEN WAELEH

November 2011

Pengerusi: Zanariah Abdul Majid, PhD

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Dalam thesis ini, kaedah blok peramal pembetul telah dibangunkan bagi menyelesaikan peringkat pertama dan tinggi masalah nilai awal (MNA) untuk persamaan pembezaan biasa (PPB). Kaedah ini menyelesaikan masalah PPB peringkat lebih tinggi secara langsung tanpa menurunkan kepada satu sistem PPB peringkat pertama. Penerbitan kaedah blok yang dicadangkan ini adalah berdasarkan kaedah pengamiran berangka dan menggunakan pendekatan interpolasi yang serupa seperti kaedah Adams.

Kaedah blok yang dibangunkan ini menyelesaikan masalah PPB peringkat lebih tinggi secara langsung dalam satu kod yang menggunakan strategi panjang langkah berubah. Bagi memperoleh penghampiran berangka yang efisien dan berkesan, kaedah blok yang dibangunkan ini dilaksanakan dengan mod peramal pembetul menggunakan kaedah lelaran mudah. Kaedah blok yang dicadangkan menghitung beberapa penyelesaian berangka secara serentak dan bilangan penyelesaian yang dihitung bergantung kepada ciri kaedah blok. Pekali integrasi bagi formula kaedah

blok yang dibangunkan disimpan di dalam kod bagi mengelakkan pengiraan yang rumit dan berulang.

Beberapa masalah PPB telah diambil kira dalam ujikaji berangka. Ini adalah bagi menekankan ciri-ciri utama kaedah yang dicadangkan dengan membandingkan kaedah blok secara langsung ini dengan kaedah sedia ada yang menyelesaikan PPB peringkat lebih tinggi dengan menurunkan kepada satu sistem PPB peringkat pertama. Hasil penyelesaian yang diperolehi menunjukkan bahawa kaedah blok yang dibangunkan berjaya menghasilkan hasil penyelesaian yang boleh diterima dari segi ralat maksimum dan masa pengiraan bagi menyelesaikan PPB peringkat lebih tinggi secara langsung.

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LIST OF ABBREVIATIONS

BDF	:	Backward Differentiation Formula
BVPs	:	Boundary Value Problems
IVPs	:	Initial Value Problems
LMM	:	Linear Multistep Method
ODE	:	Ordinary Differential Equation
PDE	:	Partial Differential Equation
2P1FI	:	2-Point 1 Block Fully Implicit Method by Majid (2004)
2PHODE	:	2-Point Multistep Block Method for Solving Higher Order ODEs Directly
3P1FI	:	3-Point 1 Block Fully Implicit Method by Majid (2004)
3PHODE	:	3-Point Multistep Block Method for Solving Higher Order ODEs Directly
4P1FI	:	4-Point 1 Block Fully Implicit Method by Majid (2004)
4PHODE	:	4-Point Multistep Block Method for Solving Higher Order ODEs Directly

CHAPTER 1

INTRODUCTION

1.1 Introduction

Many natural processes or real-world problems can be translated into the language of Mathematics (Order and Chaos, 2000). This formulation will explain the behaviour of the phenomenon in detail. The result is that an appropriate strategy and technique can be applied in order to solve any perceived problem.

The mathematical formulation of physical phenomena in science and engineering often leads to differential equation. Differential equation can be categorized into ordinary differential equation (ODE) and partial differential equation (PDE). An ODE is a differential equation involving ordinary derivatives of one or more dependent variables with respect to a single independent variables. Meanwhile, a PDE is a differential equation involving partial derivatives of one or more dependent variables with respect to more than one independent variable. The order of the differential equation can be referred to the highest derivative that appears in the differential equation and the power of the highest derivative in the equation is called the degree.

ODE plays a key role in modelling real-world problems such as those seen in physics, engineering, chemistry, biology, astronomy and other fields. In many applications where real-world problems are transformed into mathematical equations, they usually take the form of nonlinear differential equation that cannot be solved using analytic techniques (Chapra and Canale, 2003). There is therefore a need for different methods in treating such problems efficiently and obtaining the desired solutions.

Numerical approximation techniques are powerful tools to meet this need and the role of numerical methods in engineering problems solving has increased dramatically in recent years. The classes of the numerical methods can be divided into a single step method and a multistep method. Generally, a single step method utilises information only at one previous point x_i and its derivative in order to predict a value of the dependent variable y_{i+1} at a future point x_{i+1} whereas a multistep method refers to a set of previous points and its derivative values for computing the next solution. Furthermore, a multistep method needs starting values to start the computation of the solution and these proposed block methods are based on a multistep method.

1.2 Objective of the Thesis

The aim of this research is to develop the predictor corrector block methods for solving ODEs directly. This goal can be achieved by:

i. Deriving the formulae of the 2-point, 3-point, and 4-point multistep block methods and implementing variable step size strategy.
ii. Comparing the block method in terms of total number of steps, maximum error, total functions evaluation and computational time with the existing method for solving higher order ODEs.

iii. Developing a code for each block method for solving first and higher order ODEs.

1.3 Scope and Limitation

This study focuses on developing the block methods that consist of 2-point, 3-point and 4-point in the computing block to solve first order up to sixth order ODE problems. The stability region of 2-point, 3-point and 4-point block methods is presented in Chapter 3, Chapter 4 and Chapter 5 respectively and the results show that the stability region decrease as the number of points in the block increased. Therefore, this might affect the accuracy of the developed method if we increase the number of points in the block. In this thesis, we will only focus of deriving the block method consists of 2-point up to 4-point. Furthermore, these methods are developed for solving IVPs type and particularly non-stiff problems. Lagrange polynomial is used in the derivation of the method with unevenly spaced data.

1.4 Outline of the Thesis

This thesis is organized in six chapters. The first chapter introduces briefly about the importance of numerical method as one of the numerical approaches for solving ODEs problem. Apart from that, the intentions for developing these new methods have also been clarified in order to convey the intended message clearly.

Some of the theorems and definitions related to the numerical solution have been presented as elementary concept in Chapter 2. Apart from that, previous works related to this area have also been discussed.

In Chapter 3, the derivation of 2-point multistep block method and the algorithm are presented for solving ODEs. The stability analysis of the developed method is discussed in order to make the method to be practical. Eight numerical problems are tested for illustrating the effectiveness of the proposed method by analysing the numerical results obtained.

Chapter 4 is devoted for elaborating the derivation of the 3-point multistep block method. The stability regions of the devised method are also determined. Ten IVPs are tested by using this proposed method and the outcomes are interpreted.

The derivation of the 4-point multistep block method based on the same strategies as in Chapter 3 and 4 is presented in Chapter 5. The stability regions regarding to this method are also determined and discussed. The obtained numerical results which are carried out using the identical set of test problems such as in Chapter 4, are presented, and the significant outputs of this method are discussed.

Finally, this thesis is concluded in Chapter 6 by making some suggestions which could be taken for future work.

BIBLIOGRAPHY

Abdel-Halim Hassan, I.H. (2004). Differential transformation technique for solving higher-order initial value problems. *Applied Mathematics and Computation.* 154: 299-311.

Awoyemi, D.O. (2001). A new sixth-order algorithm for general second order ordinary differential equations. *International Journal of Computer Mathematics.* 77: 117-124.

Awoyemi, D.O. (2003). A p-stable linear multistep method for solving general third order ordinary differential equations. *International Journal of Computer Mathematics.* 80(8): 985-991.

Awoyemi, D.O. (2005). Algorithmic collocation approach for direct solution of fourth-order initial-value problems of ordinary differential equations. *International Journal of Computer Mathematics.* 82(3): 321-329.

Awoyemi, D.O. and Idowu, O.M. (2005). A class of hybrid collocation methods for third-order ordinary differential equations. *International Journal of Computer Mathematics.* 82(10):1287-1293.

Brugnano, L. and Trigiante, D. (1998). *Solving Differential Problems by Multistep Initial and Boundary Value Methods.* Amsterdam B. V..

Burden, R.L. and Faires, J.D. (2005). *Numerical Analysis.* Eighth Edition. Thomson Brooks-Cole.

Cash, J.R. (1983). Block runge-kutta methods for the numerical integration of initial value problems in ordinary differential equations part I. The nonstiff case. *Mathematics of Computation.* 40(161): 175-191.

Celia, M.A. and Gray, W.G. (1992). *Numerical Methods for Differential Equations.* Prentice-Hall, Inc.

Chapra, S.C. and Canale, R.P. (2003). *Numerical Methods for Engineers with Software and Programming Applications.* Fourth Edition. The McGraw-Hill Companies, Inc.

Chu, M.T. and Hamilton, H. (1987). Parallel solution of ODE's by multi-block methods. *SIAM Journal on Scientific and Statistical Computing.* 8(3): 342-353.

Cong, N.H. (1998). RKN-type parallel block PC methods with lagrange-type predictors. *Computers and Mathematics with Applications.* 35(9): 45-57.

Dahaghin, M.SH. and Eskandari, Z. (2010). Stability analysis of special general linear multi-step method for special third-order differential equations.

International Journal of Computational and Applied Mathematics. 5(3): 383-392.

Fatunla, S.O. (1991). Block methods for second order ODES. *International Journal of Computer Mathematics.* 41: 55-63.

Hairer, E. Norsett, S.P. and Wanner, G. (1987). *Solving Ordinary Differential Equations I: Nonstiff Problems.* Springer-Verlag Berlin Heldelberg.

Henrici, P. (1962). *Discrete Variable Methods in Ordinary Differential Equations.* New York: John Wiley & Sons, Inc.

Ibrahim. Z.B., Suleiman, M. and Othman, K.I. (2008). Fixed coefficients block backward differentiation formulas for the numerical solution of stiff ordinary differential equations. *European Journal of Scientific Research.* 21(3): 508-520.

Jain, M.K., Iyengar, S.R.K. and Saldanha, J.S.V. (1977). Numerical solution of a fourth-order ordinary differential equation. *Journal of Engineering Mathematics.* 11(4): 373-380.

Jator, S.N. (2008). Numerical integrators for fourth order initial and boundary value problems. *International Journal of Pure and Applied Mathematics.* 47(4): 563-576.

Jator, S.N. (2010). Solving second order initial value problems by a hybrid multistep method without predictors. *Applied Mathematics and Computation.* 217(8): 4036-4046.

Kayode, S.J. (2008a). An efficient zero-stable numerical method for fourth-order differential equations. *International Journal of Mathematics and Mathematical Sciences.* 2008: 1-10.

Kayode, S.J. (2008b). An order zero-stable method for direct solution of fourth order ordinary differential equations. *American Journal of Applied Sciences.* 5(11): 1461-1466.

Kayode, S.J. and Awoyemi, D.O. (2010). A multiderivative collocation method for 5^{th} order ordinary differential equations. *Journal of Mathematics and Statistics.* 6(1): 60-63.

Lambert, J.D. (1973). *Computational Methods in Ordinary Differential Equaitions.* New York: John Wiley & Sons, Inc.

Logan, J.D. (2006). *Applied Mathematics.* Third Edition. John Wiley and Sons, Inc.

Majid, Z.A. (2004). *Parallel Block Methods for Solving Ordinary Differential Equations.* PhD Thesis, Universiti Putra Malaysia, Malaysia.

Majid, Z.A. and Suleiman, M. (2006). Direct integration implicit variable steps method for solving higher order systems of ordinary differential equations directly. *Sains Malaysiana.* 35(2): 63-68.

Majid, Z.A. and Suleiman, M. *Two point fully implicit block method direct integration variable step method for solving higher order system of ordinary differential equations,* Proceedings of the World Congress on Engineering 2007 Vol II WCE 2007, London, U.K., July 2-4, 2007.

Majid, Z.A., Azmi, N.A. and Suleiman, M. (2009). Solving second order ordinary differenrial equations using two point four step direct implicit block method. *European Journal of Scientific Research.* 31(1): 29-36.

Mehrkanoon, S. (2010). A direct variable step block multistep method for solving third-order ODEs. *Numerical Algorithms*. 57(1): 53-66.

Milne, W.E. (1953). *Numerical Solution of Differential Equations*. John Wiley, New York.

Olabode, B.T. (2009a). An accurate scheme by block method for third order ordinary differential equations. *The Pacific Journal of Science and Technology*. 10(1): 136-142.

Olabode, B.T. (2009b). A six-step scheme for the solution of fourth order ordinary differential equations. *The Pacific Journal of Science and Technology.* 10(1): 143-148.

Omar, Z. (1999). *Developing Parallel Block Methods for Solving Higher Order ODEs Directly.* PhD Thesis, Universiti Putra Malaysia, Malaysia.

Omar, Z. and Suleiman, M. (2005). Solving higher order ordinary differential equations using parallel 2-point explicit block method. *Matematika*. 21(1): 15-23.

Order and Chaos, (2000). *An Introduction to Differential Equations.* W. H. Freeman.

Rosser, J.B. (1967). A runge-kutta for all seasons. *Siam Review.* 9(3): 417-452.

Russell, R.D. and Shampine, L.F. (1972). A collocation method for boundary value problems. *Numerische Mathematik*. 19(1): 1-28.

Sagara, N. and Fukushima, M. (1987). An efficient predictor-corrector method for solving nonlinear equations. *Journal of Computational and Applied Mathematics.* 19(1987): 343-349.

Shampine, L.F. and Watts, H.A. (1969). Block implicit one-step methods. *Mathematics of Computation.* 23: 731-740.

Suleiman, M. (1979). *Generalized Multistep Adams and Backward Differentiation Methods for the Solution of Stiff and Non-Stiff Ordinary Differential Equations.* PhD Thesis, University of Manchester.

Suleiman, M. (1989). Solving nonstiff higher order ODEs directly by the direct integration method. *Applied Mathematics and Computation*. 33(3): 197-219.

Vigo-Aguiar, J. and Ramos, H. (2006). Variable stepsize implementation of multistep methods for $y'' = f(x, y, y')$. *Journal of Computational and Applied Mathematics*. 192(1): 114-13