



**UNIVERSITI PUTRA MALAYSIA**

**A LIGHTWEIGHT AND SECURE ALGORITHM OF ELLIPTIC CURVE  
CRYPTOGRAPHY SCALAR MULTIPLICATION USING Q-NAF METHOD  
IN LOPEZ-DAHAB COORDINATE**

**WALEED KHALID AMIN ABDULRAHEEM**

**FSKTM 2019 1**



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By

**WALEED KHALID AMIN ABDULRAHEEM**

**Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia,  
in Fulfilment of the Requirements for the Degree of Doctor of Philosophy**

**April 2019**

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**DEDICATION**

*To my beloved.*



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Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Doctor of Philosophy

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**April 2019**

**Chairman : Sharifah Bte Md Yasin, PhD**  
**Faculty : Computer Science and Information Technology**

Elliptic curve cryptography (ECC) is gaining increasing popularity and acceptance within the research community. This is because it uses shorter keys to achieve security level equivalent to other public-key cryptosystems. Over the years, special attention has been given to improving the scalar recoding algorithm, since it is the most computationally intensive operation of ECC.

The general research objective of this thesis is to improve the efficiency of the scalar multiplication algorithm of ECC in Lopez Dahab coordinate for elliptic curve over binary field. This is targeted at constrained-resource devices for the internet of things (IoT) such as field programmable gate array (FPGA), radio frequency identifications (RFID) and smart cards.

In literature, window  $w$ -NAF method is considered one of the best and most widely used methods for scalar recoding. However, this method does not stand against the recent side channel attack (SCA). The first objective of this thesis is to introduce a new scalar recoding algorithm to achieve better efficiency in terms of security and performance for constrained-resource devices. A new Q-NAF scalar recoding algorithm is proposed to improve the scalar recoding efficiency criteria. Specifically, these criteria includes, hamming weight HW (numbers of non-zeroes), security, and its performance in terms of execution time and memory consumption. To conform to the application requirements of the IoT, the new algorithm improves  $w$ -NAF, where  $w=4$ . The proposed scalar recoding converts the binary scalar into  $\{-1, 0, 1, 3, 5\}$ -NAF using Q-NAF scalar recoding lookup table or a Q-NAF scalar recoding mathematical formula. Markov chain is used to calculate the HW of the lookup table. Q-NAF reduces the HW of the scalar by 81% on average for  $n$ -bit scalar rather than the 80%

HW for w-NAF. By coding the two algorithms, the proposed algorithm improves the execution time and memory consumption with a percentage of about 58% and 93% respectively. Theoretically, Q-NAF scalar recoding is proven to be secure against SCA in terms of timing and simple power attacks.

Since the scalar recoding contains the digit 5 in the representation digits, using quintupling point  $5P$  will increase the efficiency of the scalar multiplication. However,  $5P$  over Lopez-Dahab coordinate in the binary curve has not been considered in literature despite its potential to increase the scalar multiplication performance. A new valid quintupling point  $5P$  arithmetic formula is thus proposed to improve the cost of elliptic curve scalar multiplication method on binary curve over Lopez-Dahab coordinate. The proposed point is formulated as  $(2(2P) + P)$  using Al-Daoud for doubling and mix addition. The cost of the proposed point is  $17M+12S$ .

By combining the proposed scalar arithmetic and the new point arithmetic  $5P$ , a new scalar multiplication algorithm was developed. This scalar multiplication algorithm is named Q-NAF scalar multiplication for binary curve over Lopez-Dahab coordinates. The proposed scalar multiplication is more efficient in term of performance than w-NAF scalar multiplication. This is because Q-NAF method reduces the HW without the need to use the digit 7, which it is highly cost during point arithmetic. So, the proposed cryptosystem is more efficient than w-NAF while scalar recoding cost of points to recode and during the scalar multiplication.

Finally, a new look-up table is proposed to optimize the formula  $\{0, 1, 3\}$ -NAF lookup table. The new lookup table reduces the size of the  $\{0, 1, 3\}$ -NAF lookup table from  $15 \times 6$  into  $4 \times 5$ . This is achieved by scanning two digits to produce one digit instead of three digits, which significantly reduces the time and memory with percentage of about 60% and 75% respectively.

In the first three contributions, a new Q-NAF scalar multiplication method is proposed for the three scalar levels, such that scalar recoding, point arithmetic and scalar multiplication. Compare to 4-NAF, Q-NAF scalar recoding gives better results in terms of HW, time, memory and security, Q-NAF proposed a new point quintupling which used in the scalar recoding. While 4-NAF used more digits in the first two contributions, Q-NAF scalar multiplication is better than 4-NAF scalar multiplication in terms of precomputed points, security, HW and performance. While for the fourth contribution, a modified lookup table is proposed to improve the original method in terms of time, memory and security.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

**ALGORITMA RINGAN DAN SELAMAT BAGI PENDARABAN SKALAR  
LENGKUNGAN ELLIPTIK KRIPTOGRAFI MENGGUNAKAN KAEDAH  
Q-NAF DALAM KOORDINAT LOPEZ-DAHAB**

Oleh

**WALEED KHALID AMIN ABDULRAHEEM**

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Populariti dan penerimaan kriptografi lengkung eliptik (ECC) dalam komuniti penyelidikan semakin meningkat. Ini kerana ia menggunakan kekunci yang lebih pendek untuk mencapai tahap keselamatan yang sepadan dengan kriptosistem utama awam yang lain. Selama bertahun, perhatian khas diberikan bagi meningkatkan algoritma pengekodan skalar, kerana perkara ini merupakan operasi yang paling intensif dalam komputasi ECC.

Objektif penyelidikan umum tesis ini adalah untuk meningkatkan kecekapan algoritma pendaraban skalar ECC melalui Lopez Dahab bagi menyelaraskan lengkung elips ke atas bidang binari. Ini telah disasarkan pada peranti sumber daya terkurung untuk 'perkara dalam internet' (IoT) seperti jajaran pintu masuk boleh diprogram (FPGA), pengenalan frekuensi radio (RFID) dan kad pintar.

Dalam literatur, kaedah tertingkap  $w$ -NAF dianggap salah satu kaedah terbaik dan paling banyak digunakan untuk pengekalan skalar. Walau bagaimanapun, baru-baru ini kaedah ini tidak menentang serangan saluran sampingan (SCA). Objektif pertama tesis ini adalah untuk memperkenalkan algoritma pengekalan skalar baru untuk mencapai kecekapan yang lebih baik dari segi keselamatan dan prestasi untuk peranti sumber daya terkurung. Algoritma pengekodan skalar Q-NAF telah dicadangkan untuk meningkatkan kriteria kecekapan pengimbasan skalar. Khususnya, kriteria ini termasuk, 'berat hamming' HW (bukan nombor sifar), keselamatan, dan prestasinya dari segi masa pelaksanaan dan penggunaan memori. Untuk mematuhi kehendak aplikasi IoT, algoritma baru meningkatkan  $w$ -NAF, di mana  $w = 4$ . Pengekalan skalar yang dicadangkan bagi menukar skalar binari menjadi  $\{-1, 0, 1, 3, 5\}$ -NAF menggunakan jadual carian pengimbasan skalar Q-NAF atau formula matematik berskalar yang

dikod Q-NAF. Rantaian Markov digunakan untuk mengira HW jadual carian. Q-NAF mengurangkan scalar HW sebanyak 81% secara purata untuk skalar n-bit, bukannya 80% HW untuk w-NAF. Dengan pengekodan kedua algoritma, algoritma yang dicadangkan dapat memperbaiki masa pelaksanaan dan penggunaan memori dengan peratusan masing-masing sekitar 58% dan 93%. Secara teori, pengekaln skalar Q-NAF terbukti selamat terhadap SCA dari segi masa dan serangan mudah kuasa.

Oleh kerana skalar yang dikod mengandungi angka 5 dalam digit perwakilan, dengan menggunakan titik *quintupling* 5P akan meningkatkan kecekapan pendaraban skalar. Walau bagaimanapun, 5P di atas koordinat Lopez-Dahab dalam lengkung binari tidak dipertimbangkan dalam literatur, walaupun berpotensi dalam meningkatkan prestasi pendaraban skalar. Oleh itu, formula aritmetik 5P titik kuantiti yang sah adalah dicadangkan untuk menambah baik kos kaedah pendaraban skalar lengkung elips pada lengkung binari ke atas koordinat Lopez-Dahab. Titik yang dicadangkan formulanya sebagai  $(2(2P) + P)$  dengan menggunakan Al-Daoud untuk menggandakan dan penambahan tambahan. Kos titik yang dicadangkan ialah  $17M + 12S$ .

Dengan menggabungkan aritmetik skalar yang dicadangkan dan aritmetik titik 5P baru, algoritma pendaraban skalar baru telah dibangunkan. Algoritma pendaraban skalar ini dinamakan multiplikasi skalar Q-NAF untuk lengkung binari berbanding koordinat Lopez-Dahab. Pendaraban skalar yang dicadangkan lebih berkesan dari segi prestasi berbanding pendaraban skalar w-NAF. Ini kerana kaedah Q-NAF mengurangkan HW tanpa perlu menggunakan angka 7, yang sangat tinggi semasa aritmetik titik. Oleh itu, kriptosistem yang dicadangkan adalah lebih cekap daripada w-NAF melalui kos pengiraan skalar untuk dikod serta semasa pendaraban skalar.

Akhir sekali, jadual paparan baru dicadangkan untuk mengoptimumkan jadual carian  $\{0, 1, 3\}$ -NAF. Jadual carian baru mengurangkan saiz jadual carian  $\{0, 1, 3\}$ -NAF dari  $15 \times 6$  ke  $4 \times 5$ . Ini dapat dilaksanakan dengan mengimbas dua digit untuk menghasilkan satu digit dan bukannya tiga digit, secara signifikan dapat mengurangkan masa dan memori dengan peratusan masing-masing sekitar 60% dan 75%.

Dalam tiga sumbangan pertama, kaedah pendaraban skalar Q-NAF baru dicadangkan untuk tiga tahap skalar, seperti pengekaln skalar, aritmetik titik dan pendaraban skalar. Perbandingan antara 4-NAF dan Q-NAF, pengimbasan skalar Q-NAF memberi hasil yang lebih baik dari segi HW, masa, memori dan keselamatan, Q-NAF mencadangkan titik mata baru yang digunakan untuk pengimbasan skalar. Sementara 4-NAF menggunakan lebih banyak digit dalam dua sumbangan pertama, pendaraban skalar Q-NAF pula adalah lebih baik daripada pendaraban skalar antara 4-NAF dari segi titik, keselamatan, HW dan prestasi. Walaubagaimanapun, untuk sumbangan keempat, jadual carian yang telah diubahsuai pula dicadangkan untuk memperbaiki kaedah asal dari segi masa, memori dan keselamatan.



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Thank You All.

This thesis was submitted to the senate of the Universiti Putra Malaysia and has been accepted as fulfilment for the degree of Doctor of Philosophy. The members of the Supervisory Committee were as follows:

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## LIST OF ABBREVIATIONS

AES	Advance encryption standard
DBNS	Double Base Number System
DES	Data Encryption Standard
DLP	Discrete logarithm problem
DSA	Digital signature algorithm
EC	Elliptic curve
ECC	Elliptic curve cryptography
ECDH	Elliptic Curve Diffie-Hellman
ECDLP	Elliptic curve discrete logarithm problem
ECDSA	Elliptic Curve Digital Signature Algorithm
ECE	Elliptic Curve ElGamal
FPGA	Field programmable gate array
GF	Galois field
HW	Hamming weight
IFP	Integer factoring problem
IoT	Internet of Things
L2R	Left-to-right
LD	López-Dahab
LSB	Least significant bit
MSB	Most significant bit
NAF	Non-adjacent form
NIST	National institute of standards and technology

PDA	Personal digital assistants
PKC	public key cryptography
Q-NAF	Quintupling Non-adjacent form
R2L	Right-to-left
RFID	Radio frequency identifications
SCA	side channel attack
WSN	Wireless sensor network
<i>w</i> -NAF	Width Non-Adjacent Form
<i>wmb</i> NAF	Window Multi-base Non-Adjacent Form

# CHAPTER 1

## INTRODUCTION

### 1.1 Introduction

Technological advances made in the development of smart devices and e-communication has affected almost all facets of life. By 2020, more than 30 billion devices are expected to be connected to the internet in what is called Internet of Things (IoT). This unprecedented number of devices, along with their expected fast increase, puts two principal concerns, i.e. security and power consumption on the spot light. Constrained-resource devices are a major component of this recent technology, where memory and power are limited, while secure communication channel is crucial. These constrained-resource devices access and connection flexibility that recent technology makes occur with various threats and introduces the need for computational and performance-demanding security mechanisms (Suárez-Albela, Fraga-Lamas, & Fernández-Caramés, 2019).

Constrained-resource devices like RFID and sensors are applied in diverse areas to sense, transmit and store sensitive information. The most appropriate technique to ensure privacy and confidentiality of the sensitive information is cryptography. Usually, digital signature and encryption are used to secure documents during storage and transmission. Due to the device limited computational strength, capability of the device resources such as time, memory, energy and security by encryption schemes must be minimized. This way, constrained-resource devices can also participate in secure communication channel (Aditia, 2019).

Lightweight cryptography scheme is a branch of cryptography supported by the National Institute of Standards and Technology (NIST). Lightweight cryptography consumes minimal execution time, memory consumption, energy and bandwidth. Devices such as embedded frameworks, RFID, wireless sensor network (WSN) and field programmable gate array (FPGA) are on the lower end of the constrained-resource devices range (Shah & Engineer, 2019).

To secure digital information on constrained-resource devices from attackers, different cryptosystem algorithms are implemented at the core of the cryptographic protocol to encrypt and decrypt the data. Two main subsets of the method are symmetric and asymmetric cryptography (Baccarini & Hayajneh, 2019).

#### 1.1.1 Cryptography

Cryptography is mainly concerned with establishing the secrecy between the communication entities. The term cryptography expresses the art of writing or solving

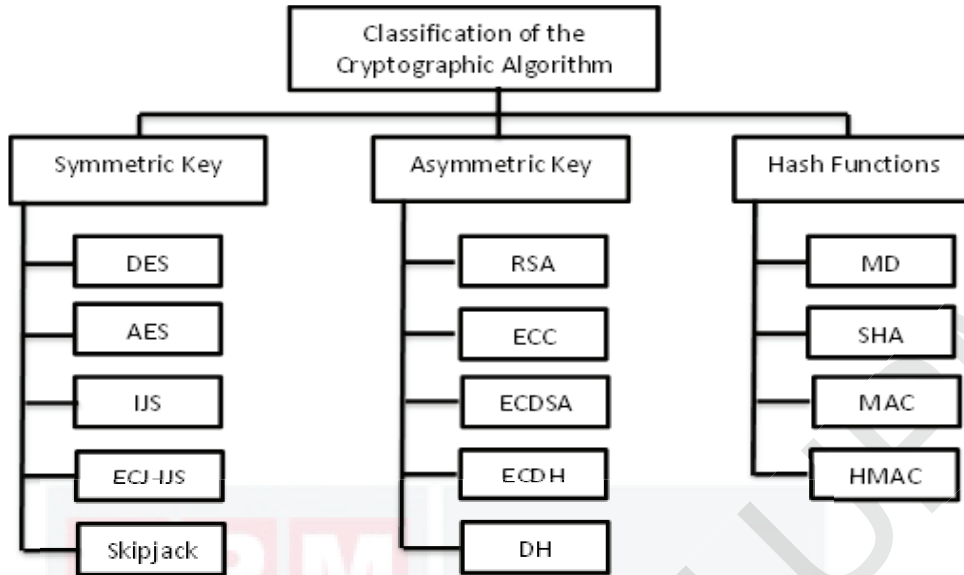
the codes in this world either everything is private, or everything is public to all for free access. Cryptography rises from the old Greeks; it means encoding message to be shared with other people. This encoded message is transferred through the public communication way so that no eavesdropper can understand the plain message even he carries that cipher message (Ajith, Balaji Ganesh Kumar, Latha, Samiappan, & Muthu, 2018).

Recently, cryptosystem protocols are being designed to improve the strength of security algorithms with information transmission. Different branches of knowledge are involved in these cases. These include using number theory, mathematics, information theory, statistics and computational complexity. For the purpose of data transmission and data storage, these areas of knowledge applied to cryptography aims to provide four cryptographic services which are confidentiality, integrity, authentication and non-repudiation. Confidentially service ensures information remains secret, and integrity service ensure that the information has not been changed, authentication service ensures the origin of the message is correctly identified, whereas non-repudiation service ensures that both sender and receiver will not deny the transmission commitment (Bhat & Kapoor, 2019).

### **1.1.2 Public-Key Cryptosystem**

Generally, cryptology has two major types, i.e., private key and public key cryptography. The term private key or symmetric key cryptography refers to any system that utilizes one key for both encryption and decryption of the plain text for communicating parties involved. On the other hand, the public key cryptography refers to any system that utilizes a pair of keys, one is used for data encryption (the public key), while the other is used for data decryption (the private key). Although private key cryptography (symmetric) is achieving highly efficient, it has drawbacks with key management, non-repudiation service and efficient secure key distribution through a communication channel (Hodgson, 2019).

Examples of symmetric cryptosystem Data Encryption Standard (DES), Advance Encryption Standard (AES), Carlisle Adams and Stafford Tavares (CAST), Blowfish, Two fish, International Data Encryption Algorithm (IDEA), and secure and fast encryption routine (SAFER) algorithms. Other cryptographic algorithms such as Rivest, Shamir and Adleman (RSA), digital signature algorithm (DSA), Elgamal, and elliptic curve cryptography (ECC) are examples of asymmetric cryptosystem algorithm. Besides the aforementioned categories, there is hash function, which uses a mathematical transformation to iterate a compression function on the input message, such as message digest (MD) and SHA family. These categories of cryptography algorithms are as illustrated in Figure 1.1 (Dixit, Gupta, & Trivedi, 2018) .



**Figure 1.1 : Classification of the cryptographic algorithm**

Public key cryptography simplifies the mechanism of key distribution, and ensures information can be made confidential by encrypted with public key (Baek et al., 2019). If using asymmetric cryptosystem offers a better key distribution solution, however, performance is slower than symmetric cryptosystem (Baccarini & Hayajneh, 2019). The security of modern asymmetric cryptosystem depends on the hardness of mathematical problems, such as integer factoring problem (IFP), discrete logarithm problem (DLP), elliptic curve discrete logarithm problem (ECDLP) and others (Liu, Choo, & Grossschadl, 2018).

## 1.2 Problem Statement

Working on the security of constrained-resource devices is essential. Cryptography is one of the significant ways to ensure privacy and to protect information from unauthorized personnel. In ECC applications and protocols, the scalar multiplication algorithm is considered as a major operation (Thangarasu & Selvakumar, 2018). The elliptic curve scalar multiplication computational process is the most time and resource consuming operation.

Implementation of ECC is a challenge for constrained-resource devices such as mobile technology devices, personal digital assistants (PDA's), embedded systems, sensors and smart cards (Bafandehkar, Yasin, Mahmud, & Hanapi, 2013). These devices have constraints in memory, CPU, energy consumptions and battery, while the performance like timing and speed are highly required with privacy (security) usage.

The width non-adjacent form (*w*-NAF) method was proposed by (Okeya & Takagi, 2003a). The main objective of this method is to provide fast scalar multiplication using



small memory, and to be secure against side channel attack (SCA). Recently,  $w$ -NAF method has been still widely used to reduce the computational cost in ECC and it is considered as one of the best methods known so far (Dou, Weng, Ma, & Wei, 2017). However,  $w$ -NAF, double-and-add algorithm and width-4 NAF methods are prone to and not more secure against SCA, (Abdulrahman & Reyhani-Masoleh, 2015) & (Järvinen & Balasch, 2017). The algorithm representation of the key in  $w$ -NAF is vulnerable to several SCA attacks, such as cache-timing attacks. This is due to its non-constant time execution which targeted previously using cache-timing techniques (Tuveri, Hassan, Garcia, & Brumley, 2018), this prove that the algorithm of  $w$ -NAF needs to be improved in term of security. The suitable width for  $w$ -NAF to be used in constrained-resource devices is  $w = 4$  for its number operation required (M. F. De Oliveira & Henriques, 2015), since 4-NAF requires only 3P, 5P and 7P precomputed points. Related literature on  $w$ -NAF attempted to reduce its HW such as multi-base  $mb$ -NAF method (Longa & Gebotys, 2010) and *frac*  $w$ -NAF method (Méloni & Hasan, 2016). Reducing the HW will increase the performance of scalar multiplication (Musa & Xu, 2017). However, the high precomputed point also is costly in terms of time, memory, CPU and battery power for constrained-resource devices (Bafandehkar et al., 2013). Selecting less precomputing points such as 3P and 5P only in the scalar recoding with the same HW of 4-NAF scalar recoding will increase the efficiency of scalar multiplication, without losing its security characters.

The scalar multiplication performance depends on the performance of the elliptic curve point operations. Point multiplication is important and dominates the execution time of the elliptic curve (Rashidi, 2017). Basically in the operation, there is  $Q = kP = P + P + P + \dots + P$  ( $k$  times), where  $k$  the secret (private) key is a positive integer,  $P$  and  $Q$  are two points on a curve. Binary curve according to NIST requires a smaller key size to be secure than the prime curve (NIST, 2013). Over binary curve, Lopez-Dahab (LD) coordinate gives the best performance for elliptic curve over the binary field (S. Yasin & Muda, 2015) (Rashidi, 2017) and it is the most studied coordinate system for binary elliptic curves (T. Oliveira, López, Aranha, & Rodríguez-Henríquez, 2014). According to (Musa & Xu, 2017), the point quintupling 5P using LD is not available in the literature. Since scalar multiplication can be improved by using efficient point operations (S. Yasin & Muda, 2015), introducing point quintupling 5P will be efficient for scalar multiplication

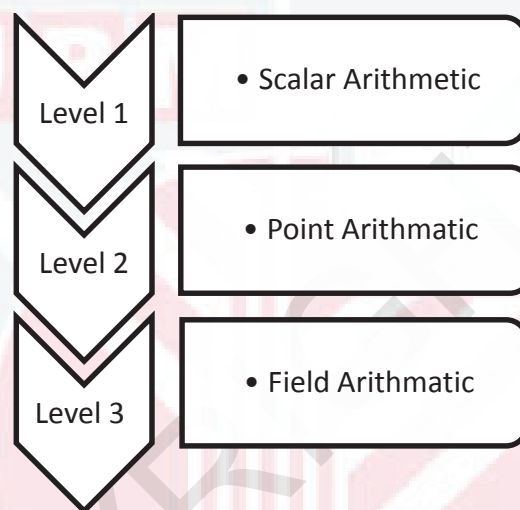
Introducing a new scalar recoding and point arithmetic should be accomplished with scalar multiplication algorithm. Different scalar multiplication methods have been proposed, however, double-and-add method has remained the most straightforward form to compute scalar multiplication (Mostafa, 2018). Double-and-add method entails doubling all scalar digits while adding the nonzero digits only (Allan, Brumley, & Falkner, 2016).

$\{0, 1, 3\}$ -NAF scalar recoding method is introduced by (S. M. Yasin, 2011). The method uses lookup table of size 15 rows and 6 columns. During the recoding process, the proposed lookup table scans three consecutive digits to produce one. The original lookup table contains two special cases during recoding execution, which require more

time than other, non-constant time execution of these two special cases according to other cases make the lookup table vulnerable to SCA. Improving a lookup table which reduces the scanning digits from three into two and contains no special cases will be more efficient and secure.

### 1.3 Research Objectives

In general, working on ECC scalar multiplication contains three levels of computation (Rezai & Keshavarzi, 2011) and (Bafandehkar, Yasin, & Mahmud, 2016) as shown in Figure 1.2. For an efficient scalar multiplication algorithm, it is necessary to accomplish the three levels of arithmetic operation.



**Figure 1.2 : Computational levels in the scalar multiplication**

The principal objective of this thesis is to propose a scalar multiplication algorithm with a simple form and less cost for elliptic curve over binary field. A number of other objectives must be accomplished. These objectives are discussed in what follows:

#### **Propose New Scalar Recoding Algorithm**

Hamming weight (HW) is the number of nonzero digit in the binary number. The efficiency of the scalar multiplication  $kP$  depends on the number of hamming weight in the scalar  $k$ . The proposed Q-NAF scalar recoding algorithm converts the binary number of  $k$  into  $\{-1, 0, 1, 3, 5\}$  digits since it is the only digits use in 3 bits lookup table. The Q-NAF scalar recoding has the non-adjacent form (NAF) property. Q-NAF scalar recoding algorithm can be represented either in form of lookup table or mathematical formula. In order to cater for the application requirement of resource constrained devices The new algorithm makes a trade-off between the performances of the recoding algorithm with respect to hamming weight, security, time, and memory consumption, so as to be suitable for constrained-resource devices. Binary field is

chosen since its secured key size is smaller than prime key size, which is more suitable for constrained-resource devices.

### **Propose a New Point Quintupling (5P) for General Binary Curve Using Lopez-Dahab Coordinate**

To achieve an efficient elliptic curve scalar multiplication performance, precomputed points help to realize a faster computation (T. Oliveira et al., 2014) which takes away the need to repeat the addition process every time. Mixed addition in Affine and López-Dahab is used for point formula since the mixed addition computation cost is better than the traditional addition in Lopez-Dahab coordinates (S. Yasin & Muda, 2015). While using digit 5 in the scalar recoding, new point arithmetic namely Quintupling (5P) is proposed for the general binary curve for implementation in scalar multiplication. To save the quintupling (5P) cost, it is computed as  $(2(2P) + P)$  where two doubling and one mixed addition is required. The quintupling cost is measured by calculating the number of field operation in the computation.

### **Propose New Scalar Multiplication Algorithm**

Scalar multiplication algorithm is the main computation in the ECC. Thus, after introducing a new scalar recoding arithmetic and a new point arithmetic, there is a need to have a new Q-NAF scalar multiplication algorithm to integrate the scalar recoding and the new point quintupling. The complexity of Q-NAF is measured by counting the number of point operations per scalar during its execution.

### **Improve the lookup table of {0, 1, 3}-NAF method**

The  $\{0, 1, 3\}$  –NAF was introduced as an efficient and lightweight method used to recode the scalar with NAF property. The original method uses lookup table which is of size  $15 \times 6$ , contains special cases and scan three digits to produce one through scalar recoding process. For more efficiency, this study aims to propose a modified lookup table smaller in size than the original, scanning two digits instead of three digits and contains no special case while recoding to be secure against SCA. The proposed lookup table aims to be more efficient in terms of time, memory and security.

## **1.4 Research Contributions**

This research contains four contributions at different ECC computational levels as in Figure 1.1 above. These contributions are highlighted as following:

### **1.4.1 Q-NAF Scalar Recoding Algorithm**

Q-NAF scalar recoding method is proposed as an improvement to the  $w$ -NAF method where  $w = 4$  for specifically caters for constrained-resource devices. The proposed

method recodes the scalar from binary into  $\{-1, 0, 1, 3, 5\}$ -NAF. Q-NAF can be executed using either lookup table or its associated mathematical formula. HW of Q-NAF is  $\frac{2}{11}n$  (where  $n$  is scalar size). Q-NAF is more efficient compared with 4-NAF in terms of time, memory and security.

#### **1.4.2 New Quintupling Formula for Point Arithmetic**

A new 5P point arithmetic formula for Lopez-Dahab coordinate system is proposed over the binary curve. The new quintupling point (5P) is introduced using the formula  $5P = 4P + P = 2(2P) + P$  using Al-Daoud formula (Al-Daoud, Mahmod, Rushdan, & Kiliçman, 2002) for doubling and mix addition with cost  $17M+12S$ .

#### **1.4.3 Q-NAF Scalar Multiplication Algorithm**

The Q-NAF scalar recording and the new quintupling point 5P arithmetic formula are integrated into the design of Q-NAF Scalar Multiplication for Lopez-Dahab coordinate over the binary curve using the double-and-add method.

#### **1.4.4 Improving the Lookup Table of $\{0, 1, 3\}$ -NAF Method**

A new Lookup table is proposed to improve the  $\{0, 1, 3\}$ -NAF lookup table. The new lookup table reduces table size from  $15 \times 6$  into  $6 \times 5$  without special cases and with constant time during execution for all rows. The improved table scans two digits only, which improved execution time and memory consumption.

### **1.5 Research Scope**

This work concentrates on the first two levels of scalar multiplication, the scalar recoding and point arithmetic. It includes proposing, enhancement, and analysis of the scalar multiplication for an elliptic curve using LD in the binary field. For the improvement on scalar arithmetic,  $w$ -NAF is chosen to compare where  $w = 4$ .

### **1.6 Thesis Outline**

This thesis is organised as follows:

Chapter 2 introduces the related literature in general to this work and the associated background information on cryptography and the elliptic curve cryptography. It gives an introduction to each level in the computational scalar multiplication. In Chapter 3, the methodology of this research is proposed by introducing a general scheme which contains four phases related to problem identification, suggested solution, analysis and

performance evaluation and expected results phase. Chapters 4, 5, 6 and 7 discuss the proposed contributions 1, 2, 3 and 4 respectively and their associated results. Diverse techniques used for deriving and substantiating these results such as algorithms, lemmas, mathematical proofs and coding are also detailed in these chapters. Chapter 8 concludes this thesis with an overall summary of these contributions and makes suggestions for future research.

## **1.7 Summary**

This implication is summarized as in Table 1.1 below.



**Table 1.1 : Summary of Research Plan**

Current Technique	Research Problem	Research Objective	Proposed Solution	Analysis Methods	Findings
<p>1. <math>w</math>-NAF technique is still widely used for scalar recoding.</p> <p>2. <math>w = 4</math> is suitable for constrained-resource devices</p>	<p>1. <math>w</math>-NAF insecure against SCA.</p> <p>2. Need to improve in terms of performance.</p>	<p>Objective 1:</p> <p>1. Improve HW, time and memory.</p> <p>2. Secure against SCA.</p>	<p>Contribution 1:</p> <p>Q-NAF method using lookup table or mathematical formula with a special case where <math>w = 4</math> using the digits <math>\{1, 0, 1, 3, 5\}</math>.</p>	<p>1. Markov chain for HW.</p> <p>2. Java code: timing and RAM.</p> <p>3. Mathematical proofs: Efficiency.</p> <p>4. Security analysis: SCA.</p>	<p>1. HW=81%, and reduce time 10-60%.</p> <p>3. Secure against SCA.</p> <p>4. Suitable for constrained-resource devices.</p>
<p>1. Binary curve requires small secure key.</p> <p>2. Lopez-Dahab best coordinate over binary curve.</p> <p>3. Mixed addition minimize cost</p>	<p>1. While using the digit 5 in the scalar recoding, point quintupling 5P need to be implemented using proper curve and coordinate.</p>	<p>Objective 2:</p> <p>Propose a new point quintupling 5P over binary curve using LD coordinate</p>	<p>Contribution 2:</p> <p>Formulate new 5P point quintupling with forms using LD coordinate and mixed addition operation of forms <math>5P=3P+2P</math> and <math>2 \cdot 5P=2(2P)+P</math></p>	<p>1. The two forms are proven as a valid using affine coordinate method equation over the mixed addition and LD coordinate.</p> <p>2. Calculating the cost.</p>	<p>1. The form of <math>5P=3P+2P</math> cost is <math>23M+13S</math> with <math>2D+2A</math></p> <p>2. The form <math>5P=2(2P)+P</math> cost is <math>17M+12S</math> with <math>2D+1A</math></p>
<p>Double-and-add still the most straightforward form to compute scalar multiplication.</p>	<p>1. <math>w</math>-NAF scalar multiplication recodes the digits <math>\{0, \bar{1}, \bar{3}, \bar{5}, \bar{7}\}</math>.</p> <p>2. Q-NAF recodes digits <math>\{-1, 0, 1, 3, 5\}</math>-NAF, a new scalar multiplication algorithm should be implemented.</p>	<p>Objective 3:</p> <p>Propose a new scalar that comprises Q-NAF scalar recoding digits and the proposed 5P</p>	<p>Contribution 3:</p> <p>Q-NAF scalar multiplication algorithm is proposed considers the recoding digits and the new point 5P using double-and-add algorithm since it is the most straightforward method.</p>	<p>1. The scalar multiplication algorithm using the proposed lookup table and the point quintupling and the multiplication algorithm. Evaluate the omit 7P in <math>w</math>-NAF with HW and multiplication cost.</p>	<p>Theoretically, Q-NAF is better than 4-NAF in terms of:</p> <p>1. HW with 1% while 7P is highly cost in <math>w</math>-NAF.</p> <p>2. Time and memory</p> <p>3. Q-NAF multiplication is secure only by adding additional anti-SCA software and hardware</p>
<p>1. <math>\{0, 1, 3\}</math>-NAF method proved as efficient for scalar recoding using lookup table and mathematical formula.</p>	<p>1. The lookup table is of size <math>15 \times 6</math>.</p> <p>2. Not secured against SCA.</p> <p>3. The lookup table scans three digits.</p>	<p>Objective 3:</p> <p>1. To improve the <math>\{0, 1, 3\}</math>-NAF lookup table with smaller size, and the same HW.</p> <p>2. Secure against SCA.</p> <p>3. Scan two digits only.</p>	<p>Contribution 4:</p> <p>1. Modified <math>\{0, 1, 3\}</math>-NAF lookup table of size <math>6 \times 5</math> and its mathematical formula are proposed.</p> <p>3. Scans two digits only.</p> <p>3. Lookup table contain no special case while execution with no special case.</p>	<p>1. Java code used to compare time and memory consumed to recode different scalar size.</p> <p>2. Security analysis for SCA.</p>	<p>1. Execution time reduced by 50-60%, memory consumed reduced by 60-75% with same HW.</p> <p>2. Secure against SCA.</p>



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## LIST OF PUBLICATIONS

Waleed K AbdulRaheem, Sharifah Bte Md Yasin, Nur Izura Binti Udzir and Muhammad Rezal bin Kamel Ariffin, "Improving the Performance of  $\{0,1,3\}$ -NAF Recoding Algorithm for Elliptic Curve Scalar Multiplication" *International Journal of Advanced Computer Science and Applications (IJACSA)*, 10(4), 2019. <http://dx.doi.org/10.14569/IJACSA.2019.0100432>.

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