UNIVERSITI PUTRA MALAYSIA

FREE AND MIXED CONVECTION BOUNDARY LAYER FLOW, HEAT AND MASS TRANSFER IN NANOFLUID USING BUONGIORNO MODEL

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By

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Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfillment of the Requirements for the Degree of Doctor of Philosophy

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October 2018

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The Buongiorno model is used in the study which takes into account the effects of Brownian motion and thermophoresis on free and mixed convections boundary layer problem. The governing partial differential equations are transformed into a nonlinear ordinary differential equations using similarity transformations. These ordinary differential equations are then solved numerically using shooting method with the help of Maple software and bvp4c codes in Matlab software.

Numerical results for the skin friction coefficient, local Nusselt number and local Sherwood number as well as velocity, temperature and nanoparticle concentration profiles are presented graphically. The governing parameters in this study are Brownian motion parameter Nb, thermophoresis parameter Nt, suction parameter S, mixed convection parameter λ, stretching or shrinking parameter ε, velocity ratio parameter ϖ, velocity slip parameter σ, Biot number Bi, nonlinear parameter n, curvature parameter γ, Soret number Sr and Dufour number Du. It is observed that the skin friction coefficient and local Nusselt and Sherwood numbers both represent the heat and mass transfer rate are significantly controlled by these parameters. Brownian motion and thermophoresis parameters are able to enhance the heat transfer rate when both have small values. An increment of the heat transfer rate increases the cooling process, while the decrement of heat transfer rate enhanced the heating process at the surface.

Dual solutions are found exists for a certain range of suction, stretching or shrinking, mixed convection and moving parameters. It is noticed that suction and partial slip widens the range in which the dual solutions exist. Furthermore, the first solu-

tion is found stable meanwhile the second solution is unstable and it is obtained by performing a stability analysis.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

OLAKAN BEBAS DAN CAMPURAN ALIRAN LAPISAN SEMPADAN, PEMINDAHAN HABA DAN JISIM DALAM NANOBENDALIR MENGGUNAKAN MODEL BUONGIORNO

Oleh

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Model Buongiorno telah digunakan dalam kajian ini yang mengambil kira kesan gerakan Brownan dan termoforesis terhadap masalah lapisan sempadan bagi olakan bebas dan campuran. Persamaan pembezaan separa menakluk telah dijelmakan kepada persamaan pembezaan biasa tak linear menggunakan penjelmaan keserupaan. Persamaan pembezaan biasa ini telah diselesaikan secara berangka menggunakan kaedah meluru dengan bantuan perisian Maple dan kod bvp4c dalam perisian Matlab.

Keputusan berangka untuk pekali geseran kulit, nombor Nusselt setempat dan nombor Sherwood setempat dan juga profil halaju, profil suhu dan profil kepekatan nanozarah telah ditunjukkan dalam bentuk graf. Parameter menakluk dalam kajian ini adalah parameter gerakan Brownan Nb, parameter termoforesis Nt, parameter sedutan S, parameter olakan campuran λ, parameter helaian meregang atau mengecut ε, parameter nisbah halaju ϖ, parameter halaju gelinciran σ, nombor Biot Bi, parameter tak linear n, parameter kelengkungan γ, nombor Soret Sr dan nombor Dufour Du. Didapati bahawa pekali geseran kulit dan nombor Nusselt setempat dan nombor Sherwood setempat yang kedua-duanya mewakili kadar pemindahan haba dan kadar pemindahan jisim telah dikawal dengan ketara oleh parameter ini. Parameter gerakan Brownan dan termoforesis dapat meningkatkan kadar pemindahan haba apabila kedua-duanya nilai kecil. Peningkatan kadar pemindahan haba akan meningkatkan proses penyejukan, sementara pengurangan kadar pemindahan haba akan mempercepatkan proses pemanasan di permukaan.

Penyelesaian dual wujud untuk sebahagian julat bagi parameter sedutan, meregang atau mengecut, olakan campuran dan nisbah halaju. Diperhatikan bahawa sedutan dan gelinciran separa menambah julat bagi penyeselaian dual wujud. Tambahan pula, penyelesaian pertama didapati stabil manakala penyelesaian kedua tidak stabil dan diperhatikan dengan mempersembahkan analisis kestabilan.

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This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfilment of the requirement for the degree of Doctor Philosophy. The members of the Supervisory Committee were as follows:

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LIST OF ABBREVIATIONS

a, c, r	constants
Bi	Biot number
C	fluid concentration
c_p	specific heat at constant pressure
c_s	concentration susceptibility
C_w	wall temperature
C_∞	ambient concentration
D_B	Brownian diffusion coefficient
D_m	mass diffusivity coefficient
D_T	thermophoresis diffusion coefficient
Du	Dufour number
f	dimensionless component of velocity
g	acceleration due to gravity
Gr	Grashof number
h_f	convective heat transfer coefficient
k	thermal conductivity
K_T	thermal diffusion ratio
Le	Lewis number
n	nonlinear parameter
Nb	Brownian motion parameter
Nt	thermophoresis parameter
Nu_x	local Nusselt number
Pr	Prandtl number
q_w	surface heat flux
q_m	surface mass flux
R	radius of cylinder
Re_x	local Reynolds number
S	suction parameter
Sh_x	local Sherwood number
Sr	Soret number
T	fluid temperature
T_f	temperature flowing over plate
T_m	mean fluid temperature
T_w	wall temperature
T_∞	ambient temperature
u	velocity component along x-axis
v	velocity component along y-axis
x	Cartesian coordinate
y	Cartesian coordinate

Greek Symbols

α	thermal diffusivity
η	similarity variables
θ	dimensionless temperature
λ	mixed convection parameter
γ	curvature parameter
ε	stretching or shrinking parameter
μ	dynamic viscosity
ν	kinematic viscosity
ω	smallest eigenvalue
ρ	fluid density
$(\rho c)_p$	effective heat capacity of particle
$(\rho c)_f$	heat capacity of fluid
σ	velocity slip parameter
τ	surface shear stress
ϕ	dimensionless concentration
ϖ	velocity ratio parameter

Subscripts

c	critical value
f	fluid
p	particle
w	condition at the wall
∞	condition at infinity

Superscript

$'$	differentiation with respect to η

CHAPTER 1

INTRODUCTION

1.1 Introduction

Science and engineering devices can be studied either experimentally or analytically. This thesis is focus on analytical approach and numerical approach, which are fast and inexpensive. The results obtained are subject to the accuracy of the assumptions, approximations and idealizations made in the analysis. The study of physical phenomena is provided in precise mathematical model for some physical laws. Thus, the mathematical modeling is used to investigate a wide variety of problems in science and engineering. Sometimes, there is an unrealistic model that obviously give unaccurate and unacceptible results. In that case, the model should be modified and rearrange so that it will be more realistic and gives an accurate results.

One of the field in physical science that deals with both stationary and moving bodies under the influences of forces is called mechanics. Fluid mechanics can be defined as the science that deals with the behaviour of fluids at rest (fluid statics) or fluid in motion (fluid dynamics) and the interactions of fluids with solids or other fluids at the boundaries. Fluid mechanics phenomenon can be observed in natural way, daily activities, an engineering systems and human body. For example in the pumping of blood in the heart, breathing machines, an artificial hearts, refrigerator, fuel pump, lubricants systems, piping systems for water and gas for an individual house, aircraft, wind turbines, power plants and ocean waves.

The conception of heat arises from that particular sensation of warmth and coldness which are immediately experienced on touching a body. This is accomplished by the transfer of energy from the warm medium to the cold one. These phenomenon deals with the determination of the rates of such energy transfers and it is called heat transfer. Heat can be transferred in three different modes, namely conduction, convection and radiation. Conduction can be explained as the transfer of energy from the more energetic particles of a substance to the adjacent less energetic ones as a result of interactions between the particles. Conduction can take place in solids, liquids or gases. In gases and liquids, conduction is due to the collisions and diffusion of the molecules during random motion. In solids, it is due to the combination of vibrations of the molecules in a lattice and the energy transport by free electrons. Convection is the mode of energy transfer between a solid surface and the adjacent liquid or gas that is in motion and it involves the combined effects of conduction and fluid motion. The faster the fluid motion, the greater the convection heat transfer. Convection can be classified into two types which are natural (or free) convection and forced convection. Natural convection happens when the fluid motion is caused by buoyancy forces that are induced by density differences due to the variation of temperature in the fluid. In contrast, forced convection occurs if the fluid is forced to flow over the surface by external sources such as a fan, pump or the wind. However, there has another mechanism which has been called as mixed convection where the phenomenon where both forced and natural (free) convection mechanisms significantly and concurrently contribute to the heat transfer. In this thesis, the natural convection and mixed convection are considered to investigate the behaviour the flow and heat transfer towards these two convections. The last mode of heat transfer is radiation, where it is the energy emitted by matter in the form of electromagnetic waves (or photons) as a result of the changes in the electronic configurations of the atom or molecules.

Mass transfer is another important mechanism in fluid mechanics field. Mass transfer specifically refers to the relative motion of species in a mixture due to concentration gradients. The definition is the movement of a chemical species from a high concentration region towards a lower concentration one relative to other chemical species present in the medium. Heat and mass transfer are analogous to each other and several parallels can be drawn between them. Some applications of mass transfer include the dispersion of contaminants, drying and humidifying, segregation and doping in materials, vaporisation and condensation in a mixture, evaporation (boiling of a pure substance is not mass transfer), combustion and most other chemical processes, cooling towers, sorption at an interface (adsorption) or in a bulk (absorption), and most living-matter processes as respiration (in the lungs and at cell level), nutrition, secretion and sweating.

1.2 Research Background

Fluid dynamics relate to many branches of science and engineering and have considered many aspects in our daily life. In this subsection, some important keywords in the thesis will be introduced.

1.2.1 Boundary Layer Theory

The concept of boundary layer flow was introduced by a German engineer, Ludwig Prandtl in 1904. Boundary layer is a thin layer of fluid near to the solid surface for which the velocity flow changes form zero at the surface to the free stream velocity away from the surface. According to Prandtl's theory, when a real fluid flows past a stationary solid boundary, the flow will be divided into two regions. First, a thin layer adjoining the solid boundary where the viscous force and rotation cannot be neglected, and second, an outer region where the viscous force is very small and can be neglected. The flow behaviour is similar to the upstream flow (Schlichting, 1979). The flow within the boundary layer is useful in many problems, especially in aerodynamics, including, wing stall, the skin friction drag on an object and the heat transfer that occurs in high speed flight. The boundary layer theory is explained and it requires some assumptions in order to express out the boundary layer equations.

- All of the viscous effects of the flowfield are confined to the boundary layer, adjacent to the wall. Outside of the boundary layer, viscous effects are not important, so that flow can be determined by inviscid solutions such as potential flow or Euler equations.
- The viscous layer is thin compared to the length of the wall. If L is a characteristic length of the the wall, then $\delta/L \ll 1$. Also, $x = O(L)$ and $y = O(\delta)$. This assumption is obviously not valid near the leading edge of the wall; other methods (such as stagnation flow) are used to determine the upstream boundary condition.
- The boundary conditions of the boundary layer region are the no-slip condition at the wall, and the free-stream condition at infinity; $u(x,0) = 0$, $v(x,0) = 0$, $u(x,\infty) = U$ and $v(x,\infty) = 0$, where u and v are velocity component in x- and y-directions.
- In the boundary layer, $u = O(U)$.

The boundary layer equations in partial differential equation form is discussed in detail in Chapter 3. The boundary layer is divided into two which are velocity boundary layer and thermal boundary layer.

1.2.2 Moving Plate

Moving plate is described as a plate that moves downstream or upstream from the origin in a uniform free stream. Any flow disturbance created by the roll or moving is neglected. The boundary layer behavior here appears to be different from what would be expected if the sheet is considered as a moving flat plate of finite length on which the boundary layer would grow in a direction opposite to the direction of motion of the plate, or away from the leading edge of the plate. The study of moving plate is important in engineering area such as polymer industry, glass fiber drawing, crystal growing or plastic extrusion.

1.2.3 Stretching and Shrinking Surface

Stretching surface is a surface which being stretched in its own plane or it occurs when the velocity at the boundary is stretched from a fixed point. Some applications in engineering and industrial processes are aerodynamic extrusion of plastic and rubber sheets, hot rolling, wire drawing and glass-fiber production. Shrinking surface is a surface that has a shrunk surface when the velocity of the boundary is moving towards a fixed point. It is applied to the shrinking film for packing of bulk products, effects of capillary in small pores and hydraulic behaviors of clay for agriculture purposes. Recently, there are number of studies that considering stretching or shrinking due to a flat plate and cylinder. A flow past a cylinder will acquire vorticity in a thin boundary layer adjacent to a cylinder. Boundary layer separation can occur behind the cylinder and cause the lower pressure to drag the flow to the downstream of the cylinder.

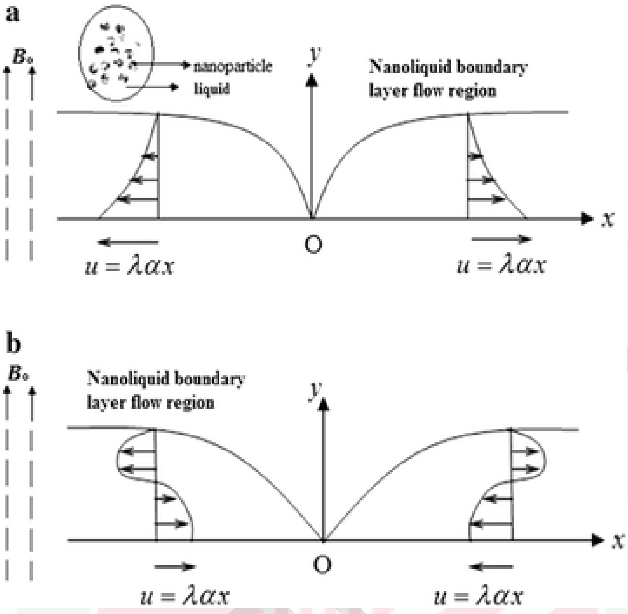

Figure 1.1: Physical model of stretching or shrinking in flat plate

1.2.4 Stagnation-Point

A stagnation point is a point in a flow field where the local velocity of the fluid is zero. Stagnation points exist at the surface of objects in the flow field, where the fluid is brought to rest by the object. Fluid does not accumulate at the stagnation point, it flows away one way or the other. Close to the stagnation point, it flows very slowly and the closer you get, the slower it flows. Streamlines can terminate at a stagnation point. Many attention has been given to the study of stagnation-point flows because of their importance in many engineering disciplines for example cooling of electronic devices by fans, cooling of nuclear reactors, and many hydrodynamics processes. Stuart (1959) among the earlier work who investigated the stagnation-point flow.

1.2.5 Permeable Surface

Permeable surface is described by defining the terms of suction and injection. Suction is the movement of fluid out of the surface or plate, while injection is happens when the fluid move in the surface. So, permeable surface can be defined as a surface

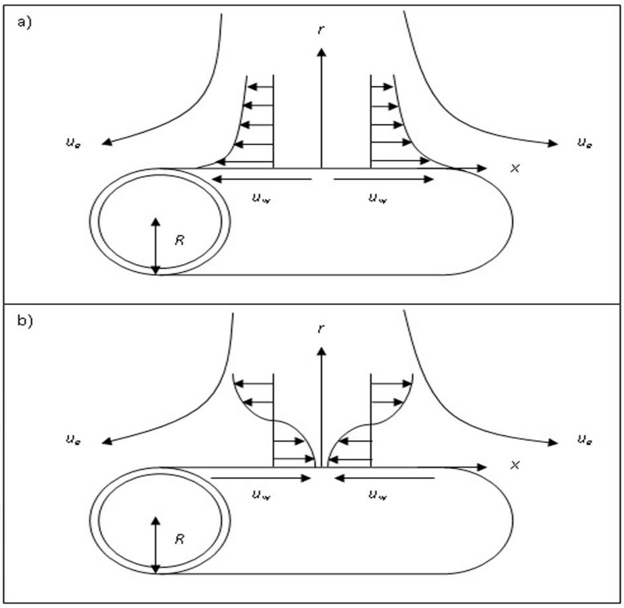

Figure 1.2: Physical model of stretching or shrinking in cylinder

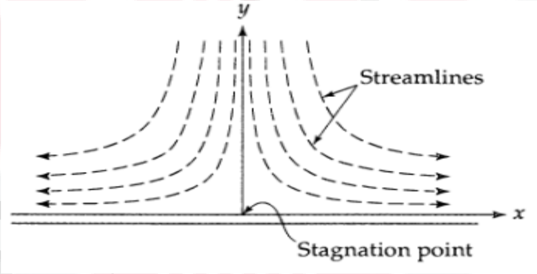

Figure 1.3: Physical model of stagnation-point

that allows the fluid to move in and out. An interest has been shown in the application of suction or injection through the surface, the former to maintain laminar flow and to prevent or postpone boundary layer separation to reduce drag, the latter to provide surface cooling on the wings of high-speed aircraft or on turbine blades. It is also well known that suction or injection of fluid through the surface, as in mass transfer cooling can significantly modify the flow field and affect the rate of heat transfer convection (Pop and Watanabe, 1992).

1.2.6 Partial Slip

Partial slip occurs when the fluid and the plate cannot stick together due to slippery surface of the plate. Some of the reseachers investigated the boundary layer flow

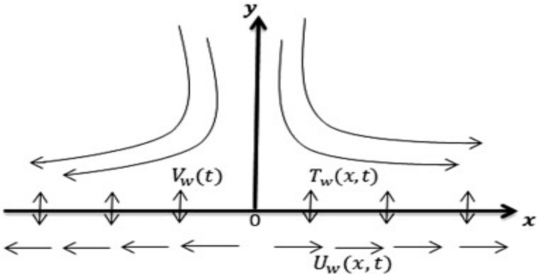

Figure 1.4: Physical model of permeable surface

with no slip condition. Sometimes, in certain cases, the no slip condition can be change to partial slip condition, which is given by

$$u(x,y) = L\frac{\partial u}{\partial y},$$

where u is velocity of the fluid, L is the length of slip.

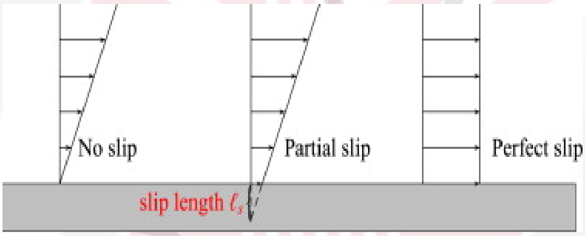

Figure 1.5: Physical model of partial slip

1.2.7 Linear and Nonlinear

In this research, the fluid velocity u for similarity solutions comes in the form of linear and nonlinear. A simple equation for linear case is given by $ax+b=0$, where a and b are constant and $a \neq 0$. A linear equation gives straight line when graphed. It has a constant slope value and the degree of a linear equation is always 1. A nonlinear equation is written as $ax^n+b=0$, where n is number of degree and a and b are constant. This equation look like a curve when plotted in a graph and has a variable slope value. The degree of a nonlinear equation is at least 2 or other higher integer values. As the number of degree increases, the curvature of the graph increases. In this study, it is important to examine the linear and nonlinear case on the behaviour of the existence of dual solutions.

1.2.8 Nanofluid

Nanofluid technology has emerged as a new enhanced heat transfer technique in recent years. It is formed by adding nanoparticles and a base fluid for which can greatly enhance the thermal conductivity and convective heat transfer. The size of nanoparticles is very small within the range 1 to 100 nanometer. Nanofluid can be classified as a new class of fluid mixtures engineered by suspending nanometer-sized particles in conventional base fluids. The applications of nanofluids in engineering area such as microelectronics, coolant, fuel cells, pharmaceutical processes, domestic refrigerator and chiller. With the recent improvements in nanotechnology, there are two successful modeling of heat transfer convection in nanofluids. The first modeling was pioneered by Buongiorno (2006) and second, Tiwari and Das (2007). Since then, a large number of studies about nanofluid have been widely investigated toward various aspects. These two nanofluid are described as follows

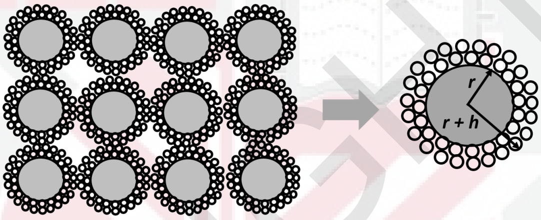

Figure 1.6: Physical model of nanofluid

1.2.8.1 Buongiorno Model

This model is a two phase model where the slip velocity between base fluid and nanoparticles are not equal to zero. Buongiorno (2006) takes into account seven slip mechanisms which produce a relative velocity between the base fluid and nanoparticles, including, inertia, thermophoresis, diffusiophoresis, Brownian motion, fluid drainage, gravity and Magnus effect. However, only two out of these seven slip mechanisms are totally important in nanofluids, which are Brownian motion and thermophoresis. These two mechanisms play an important role in heat transfer convection and can be defined as follows

- Brownian motion: is a random movement of particles suspended in a nanofluid. Brownian motion has ability to enhance the thermal conductivity. It only exists when the size of nanoparticles is small enough.

- Thermophoresis: it occurs due to kinetic energy in which the molecule with high temperature at higher energy produces greater momentum compared to the molecules at low temperature. This causes the movement of particles in opposite directions with the temperature gradient.

1.2.8.2 Tiwari and Das Model

This model is a phase model which considered the viscosity models proposed by Brinkman (1952) and Maxwell-Garnet thermal conductivity. Since this model is a phase model, base fluid and nanoparticles are said to be in thermal equilibrium, flowing at the constant velocity and no-slip condition between them. This model takes into account the effect of nanoparticles volume fraction. An increment in the nanoparticle volume fraction rate increases the effective thermal conductivity of nanofluid. According to Jang and Choi (2007), only a small amount of the solid volume fraction is needed to make sure its effectiveness. Futhermore, increase the thermal conductivity leads to enhance the performance of the heat transfer occurs on the wall.

1.2.9 Dimensionless Parameters

There are some dimensionless parameters that that have an important role in the behaviour of fluids. Nondimensional scaling provides a method for developing dimensionless groups that can provide physical insight into the importance of various terms in the system of governing equations. Furthermore, dimensionless number is able to solve a problem more easily.

1.2.9.1 Prandtl Number

Prandtl number, Pr can be referred as a dimensionless parameter used in the calculation of heat transfer between a moving fluid and a solid body. The main use of the Prandtl number in heat transfer problems is to control the relative thickness of the momentum and thermal boundary layers. It is noted that the heat diffuses very quickly compared to the velocity (momentum) when the Pr is small. This means that the thickness of the thermal boundary layer is much bigger than the momentum boundary layer for liquid metals. This Prandtl number was proposed by Ludwig Prandtl in 1904. The equation of the Prandtl number can be expressed as follows

$$\mathrm{Pr} = \frac{\text{viscous diffusion rate}}{\text{thermal diffusion rate}} = \frac{\mu C\rho}{k} = \frac{\nu}{\alpha},$$

where μ is the dynamic viscosity, C_p is the specific heat, ρ is the fluid density, k is the thermal conductivity, ν is the kinematic viscosity and α is thermal diffusivity. The Prandtl number is often used in heat transfer and free and forced convection calculations. Prandtl number gives the information about the type of fluid. Besides, it provides the information about the thickness of thermal and hydrodynamic boundary layer.

1.2.9.2 Reynolds Number

Reynolds number was introduced by Sir George Stokes in 1851, but Osborne Reynolds was popularized the usage of this number in 1883. Reynolds number is a dimensionless number used in fluid mechanics to indicate whether fluid flow past a body or in a duct is steady or turbulent. This is evaluated as the ratio of inertial forces to that of viscous forces. It is given by the following relation

$$Re = \frac{\text{inertial force}}{\text{viscous force}} = \frac{\rho u L}{\mu} = \frac{uL}{\nu},$$

where ρ is density of the fluid, u is velocity of the fluid, L is characteristic linear dimension, μ is dynamic viscosity of the fluid and ν is kinematic viscosity of the fluid. On the other hand, when the Reynolds number is less than about 2000, flow in a pipe is generally laminar, where as, at values greater than 2000, flow is usually turbulent.

1.2.9.3 Grashof Number

Grashof number was named after Franz Grashof in 1921. Grashof number is a nondimensional number used both in fluid mechanics and heat transfer. This number is frequently used in cases where natural convection is involved. Essentially, it is use where buoyancy force is predominant. Grashof number can be defined as the ratio of buoyancy force to viscous force, and it is written as

$$Gr = \frac{\text{buoyancy force}}{\text{viscous force}} = \frac{g\beta(T_w - T_\infty)L^3}{\nu^2},$$

where g, β, T_w, T_∞, L and ν are defined as acceleration due to gravity, coefficient of thermal expansion , surface temperature, bulk temperature, vertical length and kinematic viscosity, respectively. Grashof number is very similar to the Reynold number. Only difference is that Reynolds number is used for forced convection cases where Grashof number is used for natural convection phenomenon. When Gr $>>$ 1, the viscous force is negligible compared to the buoyancy and inertial forces.

1.2.9.4 Lewis Number

Lewis number is the ratio of thermal diffusivity and mass diffusivity. It is used to characterize fluid flows where there is simultaneous heat and mass transfer. The Lewis number is therefore a measure of the relative thermal and concentration boundary layer thicknesses. The Lewis number can also be expressed in terms of the Prandtl number and the Schmidt number, Sc. Here, the term is written as

$$Le = \frac{\text{thermal diffusion rate}}{\text{mass diffusion rate}} = \frac{Sc}{\mathrm{Pr}} = \frac{\alpha}{D},$$

where α is thermal diffusivity and D is mass diffusivity. The Lewis number physically relates the relative thickness of the thermal layer and concentration boundary layer. Besides, it indicates that thermal boundary layer and mass transfer by diffusion are comparable, and temperature and concentration boundary layers almost coincide with each other.

1.2.9.5 Biot Number

A French physicist Jean-Baptiste Biot was introduced the Biot number which is a dimensionless quantity used in heat transfer calculations. It gives a simple index of the ratio of the heat transfer resistances inside of and at the surface of a solid. The Biot number is defined as

$$Bi = \frac{\text{conductive resistance in solid}}{\text{convective resistance in thermal boundary layer}} = \frac{hL}{k_{\text{solid}}},$$

where h is heat transfer coefficient, L is characteristic length and k is thermal conductivity of the solid. Biot number determines uniformity of temperature in solid. The heat flow experiences two resistances. First, within the solid and second, at the surface of the solid. $Bi \ll 1$ If the thermal resistance of the fluid or solid interface exceeds that thermal resistance offered by the interior of the solid and when $Bi >> 1$, the interior resistance to heat flow will exceed that of the fluid or solid boundary.

1.2.9.6 Soret Number

Soret number or thermal-diffusion was first observed and reported by Carl Ludwig in 1856 and further understood by Charles Soret in 1879. Soret number is a phenomenon observed in mixtures of mobile particles where the different particle types exhibit different responses to the force of a temperature gradient. When heat and mass transfer occur simultaneously in a moving plate, the relations between the fluxes and the driving potentials are of a more intricate nature. Hence, mass flux can be created by temperature gradients. It can defined as

$$Sr = \frac{\text{thermodiffusion coefficient}}{\text{diffusion coefficient}} = \frac{D_T}{D} = \frac{D_m K_T (T_w - T_\infty)}{T_m \nu (C_w - C_\infty)},$$

where D_m is coefficient of mass diffusivity, K_T is thermal diffusion ratio, T_w is wall temperature, T_∞ is bulk temperature, T_m is mean fluid temperature, ν is kinematic viscosity, C_w is wall concentration and C_∞ is bulk concentration.

1.2.9.7 Dufour Number

Dufour number was first observed by L. Dufour in 1873. Dufour number or diffusion-thermo is the phenomenon where the energy flux caused by a composition gradient. This happen when heat and mass transfer occurs simultaneously between the fluxes, the driving potential is of more intricate nature, as energy flux can be generated not only by temperature gradients but by composition gradients as well. Dufour number is equal to the increase in enthalpy of a unit mass during isothermal mass transfer divided by the enthalpy of a unit mass of mixture.

$$Du = \frac{D_m K_T (C_w - C_\infty)}{c_s c_p \nu (T_w - T_\infty)},$$

where D_m is coefficient of mass diffusivity, K_T is thermal diffusion ratio, C_w is wall concentration, C_∞ is bulk concentration, c_s is concentration susceptibility, c_p is specific heat at constant pressure, ν is kinematic viscosity, T_w is wall temperature and T_∞ is bulk temperature.

1.2.9.8 Brownian Motion

The random movement of microscopic particles suspended in a liquid or gas, caused by collisions between these particles and the molecules of the liquid or gas. This movement is named after a Scottish botanist, Robert Brown (1773-1858) namely Brownian motion. He investigated the movement of pollen suspended in water. It provided strong evidence in support of the kinetic theory of molecules. The equation is mentioned as

$$Nb = \frac{(\rho c)_p D_B (T_w - T_\infty)}{(\rho c)_f \nu (C_w - C_\infty)},$$

where $(\rho c)_p$ is heat capacity of the nanofluid, $(\rho c)_f$ is heat capacity of the fluid, D_B is Brownian diffusion coefficient, T_w is wall temperature, T_∞ is bulk temperature, C_w is wall concentration, C_∞ is bulk concentration and ν is kinematic viscosity. Brownian motion plays an important role in the heat transfer.

1.2.9.9 Thermophoresis

Thermophoresis is the phenomenon where the particle motion in a temperature gradient, from a hotter to a colder region. The thermophoresis number can be written as

$$Nt = \frac{(\rho c)_p D_T (T_w - T_\infty)}{(\rho c)_f T_\infty \nu (C_w - C_\infty)},$$

where $(\rho c)_p$ is heat capacity of the nanofluid, $(\rho c)_f$ is heat capacity of the fluid, D_T is thermophoresis diffusion coefficient, T_w is wall temperature, T_∞ is bulk temperature, C_w is wall concentration, C_∞ is bulk concentration and ν is kinematic viscosity. Samilar to Brownian motion, the thermophoresis plays an important role in the heat transfer as well as the thicknesses of the thermal and concentration boundary layer.

1.2.9.10 Skin Friction Coefficient

Skin friction coefficient is a dimensionless skin shear stress which is nondimensionalized by the dynamic pressure of a free stream. It can be defined the shearing stress exerted by the wind at the earth's surface, and the square of the surface wind speed. The friction may occurs between a fluid and the surface of a solid moving through it or between a moving fluid and its enclosing surface. The skin friction coefficient can be expressed as

$$C_f = \frac{\tau_w}{\rho {U_\infty}^2},$$

where τ_w is local wall shear stress, ρ is fluid density and U_∞ is free stream velocity. The local wall shear stress τ_w is defined as

$$\tau_w = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0},$$

with μ is kinematic viscosity.

1.2.9.11 Nusselt Number

Nusselt number is a dimensionless parameter that can solve the thermal convection and it is describe as the ratio of convective to conductive heat transfer across (normal to) the boundary. The Nusselt number is

$$Nu = \frac{\text{convective heat transfer}}{\text{conductive heat transfer}} = \frac{hL}{k_{\text{fluid}}},$$

where h is the convective heat transfer coefficient of the flow, L is the characteristic length, k is the thermal conductivity of the fluid. Moreover, when Nu is smaller, the conduction is more significant, while Nu is greater, the convection is more prominent. Besides, the heat is transferred accross the boundary layer by pure conduction when $Nu = 1$.

1.2.9.12 Sherwood Number

Lastly, Sherwood number is one of the dimensionless parameter that related to mass transfer. Sherwood number represents the dimensionless concentration gradient at the solid surface and it can be expressed as the ratio of convective to diffusive mass transport.

$$Sh = \frac{\text{convective mass transfer}}{\text{diffusive mass transfer}} = \frac{h_D L}{D_{\text{fluid}}},$$

where h_D is the mass transfer coefficient of the flow, L is the characteristic length, D is the mass diffusivity of the fluid. Sherwood number is able to find the diffusion boundary layer thickness and hence the concentration gradient.

1.3 Problem Statement

The problems regarding the boundary layer flow due to a stretching or shrinking surface in nanofluid have been given an attention by many authors. For the present study, the term rotating boundary layer flow over three different problems which are linear, exponential and nonlinear with suction effects at the surface are studied. Some of the issues about the rotating flow are:

1. How does the mathematical model for free convection (and mixed convection) past a stretching or shrinking sheet (and moving plate) are formulated?
2. What happens to the nature of skin friction coefficient, local Nusselt number and local Sherwood number when considering moving plate and stretching or shrinking sheet?
3. What are the values of suction on boundary layer flow over a stretching or shrinking sheet for cases of cylinder and nonlinear to get dual solutions?
4. What are the distinction in the values of partial slip for the dual solutions exist when deal with free and mixed convection?
5. How does the impact of Brownian motion and thermophoresis in nanofluid to the flow, heat and mass transfer characteristics?
6. How does the thermal convective boundary condition, Soret and Dufour effects would impact the behaviour of the flow, heat and mass transfer rate?

1.4 Objective and Scopes

The main objectives of the study are:

1. To extend the problem of
   (a) Free convection boundary layer stagnation-point flow over a stretching or shrinking sheet to a cylinderical case.
   (b) Free convection boundary layer flow over a nonlinearly stretching or shrinking sheet to suction, partial slip, Soret and Dufour effects.
   (c) Free convection boundary layer flow over a stretching or shrinking sheet to a cylinderical case, suction, Soret and Dufour effects.
   (d) Mixed convection boundary layer flow over a moving plate to the case of partial slip and thermal convective boundary condition.

    (e) Mixed convection boundary layer flow over a moving plate to the case of Soret and Dufour effects.

2. To construct the mathematical formulation, design an algorithm and interpret numerically the free and mixed convection boundary layer flow, heat and mass transfer of nanofluid toward the problems of 1(a), 1(b), 1(c), 1(d) and 1(e) utilizing the shooting technique in Maple programming.

3. To perform stability analysis for dual solutions exist in problem 1(a) by finding the smallest unknown eigenvalues.

The scope is limited to the two-dimensional free and mixed convection boundary layer flow, steady (no change of velocity, temperature or pressure at a point with time), laminar (smooth layer of fluid) and incompressible (density is constant) in nanofluid for which the nanofluid model is proposed by Buongiorno (2006). This model contemplates the impact of the Brownian motion and thermophoresis.

1.5 Significant of the Study

Free convection heat transfer can be observed in our daily life. It is also extensively used in the areas of engineering. One of the applications can be found in telecommunication where the air flow and temperature distribution are analyzed in a sealed telecommunications module that is cooled only by free convection. The aluminum enclosure contains several heat-generating components. No fans or other active devices are used to provide component cooling. All heat transfer is caused by buoyancy-driven flow within the enclosure and by conduction to the outer casing. The module is simulated in a still-air environment. This means that free convection of the surrounding air is the primary mechanism for removing heat dissipated by the components. Due to the external air is not simulated, it is simulate this with a film coefficient (convection) boundary condition applied to the external surfaces of the enclosure. Sometimes, free convection alone is not enough to dissipate all the necessary heat generated (Ahmad et al., 2016). It has been solved by considering combined of free and force convection, called mixed convection. Some purpose of mixed convection are are nuclear reactor technology and some aspects of electronic cooling (Chaurasia et al., 2016).

The rapid development of nanofluid technology aimed to enhance heat transfer. Throughout the transfer of heat energy, it require heat to be added, removed or moved from one process to another process and this situation provide the fluid heating or cooling. New technological developments are increasing thermal loads and requiring faster cooling. Nanofluid is one of the medium to taking part in the enhancing the heat transfer. The motivations of using nanofluid can be explained by adding of nanoparticle with higher thermal conductivity into a conventional fluid with low thermal conductivity where the mixture is able to enhance the thermal conductivity. Due to small size of nanoparticle, it has better dispersion behaviour, less clogging and abrasion as well as it has the larger surface area. The using of nanofluid may create a saving in energy, reduce process time, raise thermal rating and lengthen the working life of equipment. Cooling is one of the most important technical challenges facing numerous industries such as automobiles, biomedical and electronics.

The most important part in the automobile cooling system of an vehicle engine is radiator (Dhale et al., 2015). When the coolant flows through the radiator tubes, heat is dissipated along the tube walls and fins due to the flow of air through conduction and convection. Some of the conventional fluids used in the radiator are water, coolants, engine oil or ethylene glycol. However, it give less adequacy. To acquire more viability, the nanoparticles are added in the fluid. Radiator system plays a vital role in preventing the vehicle engine from overheating due to friction. Conventionally, a car radiator pumps water as the heat transfer medium through the chambers within the engine block to absorb the heat and spread it away from other important parts. A radiator is designed with louvered fins, so that the heat transfer at the surface area can be created and thus, interrupt the growth of a boundary layer formed along the surface (Sidik et al., 2015).

The most essential part in the car cooling arrangement of a vehicle motor is radiator (Dhale et al., 2015). The coolant courses through radiator tubes, warm is disseminated through the tube dividers and balances because of stream of air through conduction and convection. The customary liquids utilized for warmth move in radiator are water, coolants, motor oil or ethylene glycol. However, this is not effect enough. To acquire more viability, nanoparticles are included the liquid. Radiator framework assumes an essential job in keeping the vehicle motor from overheating because of rubbing. Customarily, an auto radiator directs water as the warmth exchange medium through the chambers inside the motor square to retain the warmth and spread it far from other essential parts. A radiator is structured with louvered balances so extra warmth exchange at the surface zone can be made and interfere with the development of a limit layer framed along the surface (Sidik et al., 2015).

One of the biomedical applications is photodynamic cancer therapy which is based on the destruction of the cancer cells by laser generated atomic oxygen, which is cytotoxic (Salata, 2004). A greater quantity of a special dye that is used to generate the atomic oxygen is taken in by the cancer cells when compared with a healthy tissue. Hence, only the cancer cells are destroyed then exposed to a laser radiation. Unfortunately, the remaining dye molecules migrate to the skin and the eyes and make the patient very sensitive to the daylight exposure. This effect can last for up to six weeks. To avoid this side effect, the hydrophobic version of the dye molecule was enclosed inside a porous nanoparticle. The dye stayed trapped inside the Ormosil nanoparticle and did not spread to the other parts of the body. At the same time, its oxygen generating ability has not been affected and the pore size of about 1 nanometer freely allowed for the oxygen to diffuse out.

Due to higher density of chips, design of electronic components with more compact makes heat dissipation more difficult. Advanced electronic devices face thermal management challenges from the high level of heat generation and the reduction of available surface area for heat removal. So, the reliable thermal management system is vital for the smooth operation of the advanced electronic devices. In general, there are two approaches to improve the heat removal for electronic equipment. One is to find an optimum geometry of cooling devices and second, is to increase the heat transfer capacity. Nanofluids with higher thermal conductivities are predicated convective heat transfer coefficients compared to those of base fluids. Recent researches illustrated that nanofluids could increase the heat transfer coefficient by increasing the thermal conductivity of a coolant. Jang and Choi (2006) introduced a cooler, combined microchannel heat sink with nanofluids. Higher cooling performance was obtained when compared to the device using pure water as working medium. Nanofluids reduced both the thermal resistance and the temperature difference between the heated microchannel wall and the coolant. A combined microchannel heat sink with nanofluids had the potential as the next generation cooling devices for removing ultra-high heat flux .

1.6 Outline of Thesis

This thesis is divided into ten chapters. Chapter 1 starts with an introduction, the research backgrounds for which the basic explanations of important keywords is presented, followed by the problem statements, objectives and scope, the significance of the study and lastly the thesis outline. The literature reviews of the previous studies related to the problem identified in the thesis are discussed in Chapter 2.

The steps in obtaining the mathematical formulation is presented in Chapter 3 where the governing partial differential equations (PDEs) are transformed into the ordinary differential equations (ODEs) by using similarity transformation and taking into consideration an appropriate initial and boundary conditions.

Chapter 4 presented the mathematical formulation for free convection stagnation-point boundary layer flow over a stretching or shrinking cylinder. Followed by Chapter 5 which enlighten the mathematical formulations for free convection boundary layer flow over a nonlinearly stretching or shrinking sheet and Chapter 6 considers the mathematical formulations for free convection boundary layer flow over a permeable stretching or shrinking cylinder.

Chapter 7 elucidate the mathematical formulation for mixed convection over a moving plate in the presence of partial slip and thermal convective boundary condition. Then, Chapter 8 is extension of Chapter 7 where the Soret and Dufour effects are taken into consideration to continue the investigations. In Chapter 9, the stability analysis is carried out to identify that the first solution is stable, meanwhile the second solution is unstable.

Next, an overall conclusions and some recommendations for future research are presented in Chapter 10.

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LIST OF PUBLICATIONS

The following are the list of publications that arise from this study.

Journal articles:

Bakar, N. A. A., Bachok, N. and Arifin, N. M. Nanofluid Flow using Buongiorno Model over a Stretching Sheet and Thermophysical Properties of Nanoliquids. In *Indian Journal of Science and Technology*, 9(31): 1-9 (2016).

Bakar, N. A. A., Bachok, N. and Arifin, N. M. Moving Plate in a Nanofluid using Buongiorno Model and Thermophysical Properties of Nanoliquids. In *JP Journal of Heat and Mass Transfer*, 14(1): 119-129 (2017).

Bakar, N. A. A., Bachok, N. and Arifin, N. M. Rotating Flow over a Shrinking Sheet in Nanofluid using Buongiorno Model and Thermophysical Properties of Nanoliquids. In *Journal of Nanofluids*, 6(6): 1-12 (2017).

Bakar, N. A. A., Bachok, N., Arifin, N. M. and Pop, I. Stability Analysis on the Flow and Heat Transfer of Nanofluid past a Stretching or Shrinking Cylinder with Suction Effect. In *Results in Physics*, 9: 1335-1344 (2018).

Proceedings:

Bakar, N. A. A., Bachok, N. and Arifin, N. M. Boundary Layer Flow and Heat Transfer on a Moving Plate in a Copper-water Nanofluid. In *AIP Conference Proceedings*, 1739, 020021 (2016). (International Conference on Mathematical Science and Statistics (ICMSS), Kuala Lumpur, Malaysia).

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