UNIVERSITI PUTRA MALAYSIA

DUAL SOLUTIONS FOR UNSTEADY BOUNDARY LAYER FLOW OF NANOFLUIDS OVER STRETCHING/SHRINKING SURFACES AND STABILITY ANALYSIS

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FS 2019 19

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By

NOR FADHILAH BINTI DZULKIFLI

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfillment of the Requirements for the Degree of Doctor of Philosophy

April 2019

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfillment of the requirement for the degree of Doctor of Philosophy.

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April 2019

Chairman: Norfifah binti Bachok @ Lati, PhD
Faculty: Science

There are five unsteady boundary layer flow problems being considered which involved the regular flow, rotating boundary layer flow, the stagnation-point flow over a linear and an exponential stretching/shrinking surface which the flows pass through a flat or a cylindrical surfaces. Besides, the effects such as constant mass flux, velocity slip, Soret and Dufour are also taken into account. The mathematical models for boundary layer problems by considering different nanoparticles namely Copper, Alumina and Titania are dispersed into the water. The nanofluid model by Tiwari and Das are used to study the effect of nanoparticle volume fraction towards the flow and heat transfer behaviors at the surface. The governing equations in the form of partial differential equations are transformed to the ordinary differential equation using the similarity variables and is solved by using bvp4c function in Matlab software to gain the numerical results which focused on obtaining the dual solutions so that the stability analysis can be performed.

The results have shown the dual solutions existed for unsteady accelerating and decelerating flow with the presence of mass suction effect within a certain range of stretching and shrinking surfaces. Increasing the rotation effect, nanoparticle volume fraction, considering different nanoparticles and exponential stretching/shrinking surface is proven can enlarge the range of solutions. Considering Copper-water has resulted in increasing the skin friction coefficient and heat transfer rate at the surface. Performing the stability analysis has found that the first solution is stable solution meanwhile the second solution is unstable solution.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai
memenuhi keperluan untuk ijazah Doktor Falsafah

PENYELESAIAN DUAL BAGI LAPISAN SEMPADAN TIDAK MANTAP BENDALIR NANO TERHADAP PERMUKAAN YANG MEREGANG/MENGECUT DAN ANALISIS KESTABILAN

Oleh

NOR FADHILAH BINTI DZULKIFLI

April 2019

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Fakulti: Sains

Terdapat lima masalah aliran lapisan sempadan tak mantap yang dipertimbang iaitu melibatkan aliran sempadan biasa, aliran putaran dan aliran titik genangan terhadap permukaan meregang/mengecut secara linear dan eksponen di atas permukaan rata dan silinder. Selain daripada itu, kesan-keasan seperti kesan fluks jisim, halaju gelinciran Soret dan Dufour juga diambil kira. Model matematik bagi masalah lapisan sempadan dengan mempertimbangkan zarah nano yang berbeza iaitu Kuprum, Alumina dan Titania yang diserakkan ke dalam air. Model bendalir nano oleh Tiwari dan Das telah digunakan untuk mengkaji kesan pecahan isipadu zarah nano terhadap tingkah laku aliran dan pemindahan haba di permukaan. Persamaan menakluk dalam bentuk persamaan pembezaan separa telah dijelmakan kepada persamaan permbezaan biasa menggunakan penjelmaan keserupaan yang kemudiannya telah diselesaikan menggunakan fungsi bvp4c di dalam perisian Matlab untuk mendapatkan penyelesaian berangka yang menumpukan kepada penyelesaian dual supaya analisis kestabilan dapat dibuat.

Keputusan kajian menunjukan bahawa penyelesaian dual wujud untuk aliran tak mantap yang meningkat dan merosot dengan kehadiran kesan sedutan jisim untuk permukaan yang meregang dan mengecut dalam julat tertentu. Peningkatan kesan putaran, pecahan isipadu zarah nano, mempetimbangkan zarah nano yang berbeza dan permukaan yang merengang/mengecut secara ekponen terbukti meluaskan julat penyelesaian. Pertimbangan Kuprum-air didapati telah meningkatkan pekali geseran dan kadar pemindahan haba dipermukaan. Analisis kestabilan yang telah dibuat telah menunjukkan bahawa penyelesaian pertama adalah penyelesaian yang stabil manakala penyelesaian kedua adalah penyelesaian tidak stabil.

ACKNOWLEDGEMENTS

First of all, I would like to say Alhamdulillah and I am so thankful to Allah S.W.T for giving me a chance to pursue my study in Degree of PhD in Universiti Putra Malaysia, Selangor. I am so grateful and blessed to have Assoc. Prof. Dr. Norfifah Bachok as my supervisor and the chairman of the committee for her time and attention along this journey. I would also to show my gratitude to my co-supervisors Prof. Dr. Norihan Md. Arifin, Dr. Nor Azizah M. Yacob and Dr. Haliza Rosali for the guidances and supervisions from the day I started my study until now. Not forgotten Prof. Ioan Pop from Babes-Bolyai University, Romania for his support and opinions.

Apart of that, I would like to express my appreciation and thought to Universiti Teknologi MARA (UiTM) in Shah Alam and Pahang campus and also the Ministry of Higher Education for the scholarship for my study for this three years. School of Graduate Studies (SGS) as well as Faculty of Science, UPM for an excellent work in doing their work to manage the post graduate students in UPM generally.

Thank you so much to my husband, Anuar for his support, patience, help, consideration and time throughout the journey in completing my study. I also received a lot of motivation and positive thoughts from my family as well as my family in laws to keep the journey going smoothly. Last but not least, to the most supportive and helpful friends from my coursemate Najwa, Ashikin, Alwani, Syamimi, Syuhada, Shahirah, Iera, Mus and also my colleagues in UiTM Pahang for sharing the knowledges and creating the sweet memories along the journey.

I certify that a Thesis Examination Committee has met on 1 April 2019 to conduct the final examination of Nor Fadhilah binti Dzulkifli on her thesis entitled “Dual Solutions for Unsteady Boundary Layer Flow of Nanofluids over Stretching/Shrinking Surfaces and Stability Analysis” in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Master of Science.

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LIST OF ABBREVIATIONS

a,b,c	constants
A	unsteadiness parameter
$\mathrm{Al_2O_3}$	Alumina
c_p	specific heat at a constant pressure
c_s	concentration susceptibility
C	fluid concentration
Cf_x	skin friction coefficient of *x*-direction
Cf_y	skin friction coefficient of *y*-direction
C_p	specific heat at constant pressure
C_w	surface concentration
C_∞	ambient concentration
Cu	Copper
D_f	Dufour number
D_m	mass diffusivity coefficient
f,h	dimensionless components of velocity
k	thermal conductivity
k_T	thermal diffusion ratio
l	characteristic length
L	length of the slip
Nu_x	local Nusselt number
Pr	Prandtl number
q_m	surface mass flux
q_w	surface heat flux
r	radius
Re_x	local Reynolds number
s	constant mass flux
S_r	Soret number
Sh_x	local Sherwood number
Sc	Schmidt number
T	fluid temperature
T_m	mean fluid temperature
T_w	surface temperature
T_∞	ambient temperature
$\mathrm{TiO_2}$	Titania
u	velocity component along *x*-axis
u_s	velocity slip parameter
U_w	velocity of the surface
v	velocity component along *y*-axis
v_w	velocity mass flux parameter
w	velocity component along *z*-axis
w_o	constant mass flux
x,y,z	Cartesian coordinate

Greek Symbol

α	thermal diffusivity
β	expansion/contraction strength constant
η	similarity variables
γ	similarity variables
θ	dimensionless temperature
Ω	rotation parameter
ε	stretching or shrinking parameter
$\bar{\Omega}$	constant angular velocity of the fluid
μ	dynamic viscosity
ν	kinematic viscosity
ϕ	concentration function
ψ	stream function
ρ	fluid density
ρC_p	volumetric heat capacity
σ	velocity slip parameter
τ	dimensionless variable
τ_w	surface shear stress
φ	nanoparticle volume fraction parameter

Subscripts

c	critical value
f	fluid
nf	nanofluid
s	solid nanoparticle
w	condition at the surface
∞	condition at infinity

Superscript

$'$	differentiation with respect to η

CHAPTER 1

INTRODUCTION

1.1 Boundary Layer Flow

Boundary layer theory was introduced by Ludwig Prandtl in 1904 due to the inability to solve the Navier-Stokes equations for calculating the shear force on a surface immersed in a flow. Hence, Prandtls work was considered as the most significant in the field of fluid dynamic since he had pioneered the explanation and description on the concept of boundary layer. As the fluid flows passing through an immersed body, a thin layer, known as boundary layer forms at the adjacent of the surface because of the friction effect, which indicates there is no slip occurs at the surface. In this thin layer, the frictional effect cannot be neglected. On the other hand, in the outside of the boundary layer (inviscid flow), the effect can be ignored (Schlichting, 1979). The formation of the boundary layer can be seen in Figure 1.1 below.

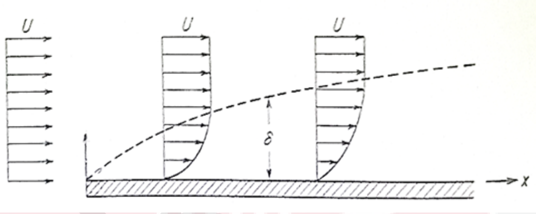

Figure 1.1: Formation of boundary layer on a flat surface

From Figure 1.1, U represents the velocity of undisturbed ambient flow along x-axis. The velocity of the fluid attached to surface in the thin layer is assumed to be zero and the movement of the fluid is delayed due to frictional resistance. Later, the boundary layer starts to extend from the leading edge until a point where the velocity of the fluid in the boundary layer equals to the velocity of fluid in the ambient flow. The boundary layer thickness δ is a distance between the surface and a point with free stream velocity and perpendicular to the surface. Meanwhile the flow in the boundary layer is always to be laminar flow due to the slow flow and low Reynolds number where the viscous forces are dominant as compared to the inertia forces that kept the particles in the flow in line sufficiently.

1.1.1 Steady and Unsteady Boundary Layer Flow

Basically, there are two types of flow which are the steady flow and unsteady flow. Steady flow refers to all of the properties of fluid such as velocity, temperature and density are independent of time. Meanwhile, the unsteady flow indicates that all the fluid properties are time dependent and is important in engineering field, for example in the start-up process and periodic fluid motion as mentioned by Fang and Zhang (2011). Generally, in the study on fluid mechanics, it is assumed that the flow is a steady flow in order to simplify the analysis. However, in reality, the fluid flow and heat transfer are actually unsteady flow due to changes of the velocity ratio and the temperature at the surface (Vajravelu et al. (2013)). Besides that, Fang (2008) mentioned that the characteristic of unsteady flow was different since the parameters in the system were time-dependent which affected the fluid motion and at the same time the behavior of the boundary layer separation.

The acceleration of the flow can be defined as the rate of change of the velocity with time, in which for the steady flow, the acceleration equals to zero since the velocity is independent on time. As for unsteady flow, the acceleration has its own value and cannot be zero. The positive acceleration is defined as accelerating flow and the negative acceleration represents decelerating flow. The accelerating flow tends to make the boundary layer thickness grows slowly since the flow suppresses the thin layer and delays the boundary layer separation. Meantime, the boundary layer thickness grows faster for the decelerating flow and speeds up the boundary layer separation off the surface. Nevertheless, the unsteady boundary layer problem has an important role in the industrial and engineering applications.

1.1.2 Stagnation-Point Flow

Stagnation-point flow is a flow that explains the behavior of the fluid motion near the stagnation region. This flow occurs when the flow impinges on the solid surface and the fluid velocity at the stagnation-point equals to zero. The idea of stagnation-point flow is applied in various engineering and manufacturing field such as cooling of electronic devices by fans, cooling of nuclear reactors and hydrodynamics processes. The stagnation-point flow develops where the streamline is perpendicular to the surface and the Navier-Stokes equations characterized the flow near the stagnation point (Sin and Chio (2012)) as can be seen in figure below

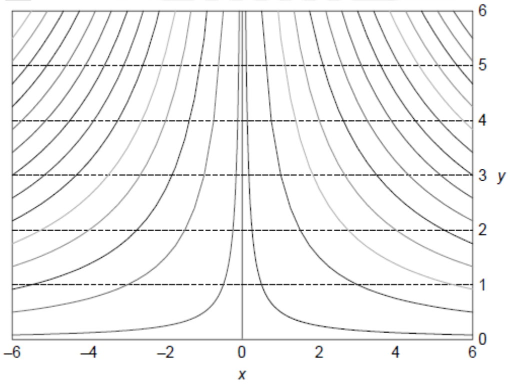

Figure 1.2: Formation of stagnation-point flow on a flat plane (Graebel (2007))

1.2 Heat Transfer

The heat-transport property due to the molecular motion is significantly depending on the rate of heat transfer. Heat transfer is the kinematic process and by increasing motion, molecules gain heat or energy where the thermal energy flows from the higher temperature substance to the lower temperature substance due to the thermal non-equilibrium or temperature difference. In the real world situation, the applications of heat transfer can be found in various areas such as manufacturing, industrial and environmental processes which involves the energy utilization, thermal processing and thermal control. The movement of heat from one substance to another substance can be explained in three modes or methods which are conduction, convection and radiation as illustrated in Figure 1.3.

Conduction is the heat transfer within materials where the kinematic energy moves from one molecule to other molecule that is adjacent to it. Convection is the heat transfer between fluid flow and wall where the heat is being extracted from the flow if the temperature of the surface is lower than the temperature of the ambient flow. Conversely, when the temperature of the ambient flow is greater than the surface, the heat will be transfered from the surface to the flow and the heat transfer process will continue until the equilibrium temperature is obtained.

Convection of heat transfer can also be divided into three types which are mixed convection, forced convection and free convection. The radiation transmits the heat transfer through an empty space and the energy which is transferred by radiation mode is known as radiant heat. The heat travels in the form of emit radiation, touch another particle and transfers the radiant heat as the kinematic energy to that particle. Some examples of heat transfer via radiation is the way Earth receives the energy or heat from the sun, when microwaves is applied in heating or cooking in the oven.

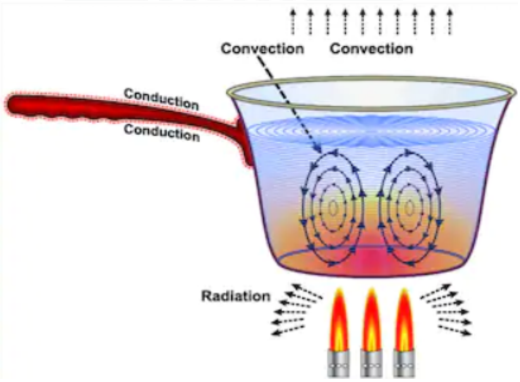

Figure 1.3: Physical model of different heat transfer modes

1.3 Stretching and Shrinking Surfaces

There are different conditions of the surface that may exist either it is quiescent, stretching or shrinking surface. A quiescent surface denotes that the surface is not moving and standing still at its place. A stretching surface is a surface that stretch and the velocity on the boundary is moving away from a fixed point. The applications of stretching surface can be found in the manufacturing, engineering and industrial process generally. The polymer extrusion process is one of the examples where the stretching surface is applied in fluid mechanics. Its mechanical characteristic was found to improve when the stretching conveys the indirectional orientation to the extruder.

Meanwhile, the shrinking surface occurs when the surface is shrunk after it is stretched away and the velocity on the boundary layer is approaching the fixed point as illustrated in Figure 1.4. The idea of abilities of surface shrinkage is applied in various fields such as in shrink wrap packaging using a shrink film, polymer processing, glass sheet production and textiles industries. The shrink wrap is one of the most common applications in manufacturing where the film is stretched with the help of heat to orient the particle from the initial shape and then, the film shrinks back to the initial dimension after it is cooled. There are various types of surface that are considered in the boundary layer flow problems such as flat plate, cylinder and cone. In addition, the velocity ratio of the surface can also be considered in different trends which are linear, exponential and non-linear stretching/shrinking surface.

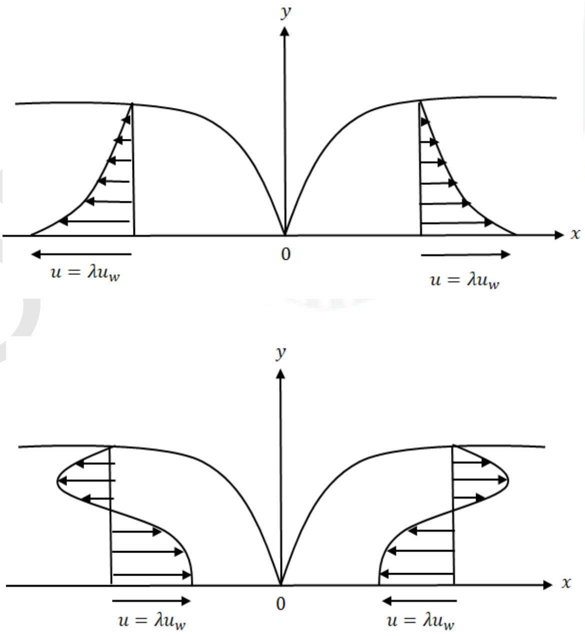

Figure 1.4: Physical model of stretching ($\lambda > 0$) and shrinking ($\lambda < 0$) surfaces

1.4 Permeable and Impermeable Surfaces

Permeable surface is defined by the presence of suction and injection effects at the surface while the surface is said to be impermeable with the absent of both effects as depicted in Figure 1.5. The presence of both effects shows the occurrence of fluid movement through the bounding surfaces, for example in mass transfer cooling which affects the fluid flow and heat transfer rate at the surface. In addition, the effects also reduce the drag at the surface which tends to delay the boundary layer separation of the laminar flow. It consequently enlarges the range of the solution that can be obtained. Due to their effects on the boundary layer control which influence the heat transfer rate at the surface that enhance the heating or cooling process, the consideration of both effects is found to significantly reduce the cost. This has developed into interest in various physical applications such as film cooling, engineering, chemical process, aerodynamics and space sciences.

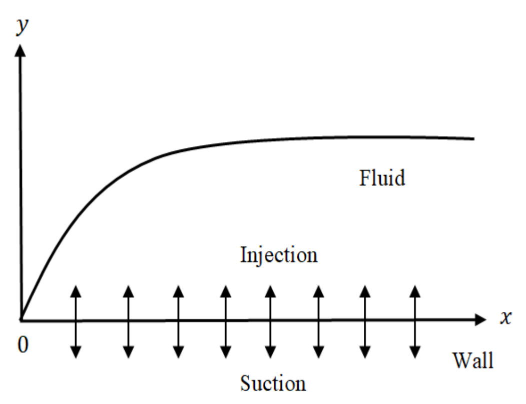

Figure 1.5: Physical model of permeable surface

1.5 Nanofluid

Nanofluid is a fluid that can be produced by combining the nanoparticles dan base fluid. This term was first introduced by Choi (1995). Many researchers have issued several definition of nanofluid, for example, Wong and Leon (2010) have pointed out that nanofluids are nanoparticles that are suspended in the manner of the principal dimensions is lower than 100 nm. The same idea about the size of nanoparticles has been discussed by Keblinski et al. (2005). They mentioned that nanofluids are composite materials in the form of solid liquid that have 1-100 nm sized nanoparticles which is suspended in liquid.

There are two types of nanofluids which are metallic nanofluids and nonmetallic nanofluids. The difference between them is in the form of material that has been used in dispersion nanoparticles process. Metallic nanofluid comes from metals for example Alumina, Copper and Nickel where nonmetallic nanofluid is made from nonmetals such as metal oxides. There are several methods in producing the nanofluid such as direct evaporation technique, chemical reduction, submerged arc nanoparticle synthesis system, laser ablation, microwave irradiation, polyp process and phase-transfer method.

Water, mineral oil and ethylene glycol are the three types of fluid that commonly used as heat transfer fluid. However, these kinds of fluid are identified to have low thermal conductivity of heat transfer Li et al. (2009). Therefore, by proposing nanofluid as an alternative fluid, the performance of heat transfer fluids is expected to be optimized Manca et al. (2010). Some high quality features of this fluid are ultrafast heat transfer ability, increasing thermal conductivity and have better stability than colloids. Nanofluid also has several traits in reduction reaction which are the reduction of erosion and clogging in micro channels, in pumping power and friction coefficient. Heat transfer fluids have an important function in many industrial activities involving chemical, microelectronics, cooling and heating processes.

Recently, nanofluids have gained popularity in various fields that involve processes which related to heat and thermal such as in engineering, automotive, electronic, biomedical applications, for instance in shell and tube exchanges by Afshoon and Fakhar (2014). Therefore, any deficiency of these heat transfer fluids may cause an obstruction to the effectiveness and compactness of heat converter. Due to weak performance of the base fluids, the industry will face some disadvantages in terms of production and costs due to their lower thermal conductivity (Goharshadi et al. (2013)). Therefore, according to Anuar and Bachok (2016) the nanofluid is one of the way and option that will help to enhance thermal conductivity and also the heat transfer at the surface effectively compared to the base fluid.

In solving the problem of boundary layer flow and heat which involves the nanofluid in the system, there are few nanofluid approaches have been developed and the most frequent models are two phase model where the nanofluid is considered as two-component mixture known as Boungiorno model, Boungiorno (2006). The second approach is a single phase model namely Tiwari and Das model which was discovered by Tiwari and Das (2007). Both models have their own focus and purpose in determining the flow behavior and heat transfer rate at the surface when nanofluid is taken into account. The physical model of nanofluid is illustrated in Figure 1.6.

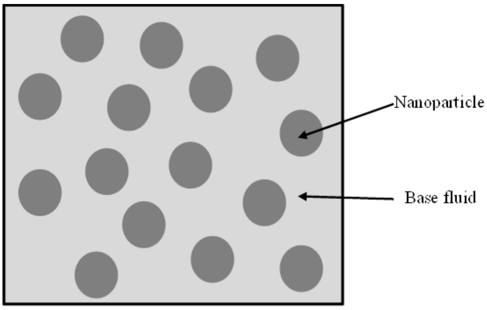

Figure 1.6: Cross section physical model of nanofluid

1.5.1 Boungiorno Model

Due to the emergence of nanofluid in the system of fluid dynamic, it has ability to enhance the thermal conductivity as well as the heat transfer process because of the nanoparticle motion. Hence Boungiorno (2006) has tested seven slip mechanisms which relates between the nanoparticle and the base fluid. These mechanisms are inertia, Brownian diffusion, thermophoresis, diffusiopherasis, Magnus effect, fluid drainage and gravity. It can be concluded that out of these seven slip mechanisms, only Brownion diffusion and thermophoresis have significant in nanofluid. Brownion diffusion is defined as a choatic and random movement of particle in nanofluid due to the collisions between the nanoparticle and the molecule of the base fluid. Meanwhile, the themophoresis represents the diffusion of particle due to the temperature gradient.

1.5.2 Tiwari and Das Model

This single phase model is proposed by Tiwari and Das (2007) to investigate the behavior of the nanofluids inside two-sided lid driven square cavity. As compared to the Boungiorno model, Tiwari and Das developed a model to analyze the effects of nanoparticle volume fraction in the base fluid towards the flow behavior. This model considered the range of the nanoparticle volume fraction was between 0% to 20% and it was found that the presence of nanoparticle increased the heat transfer capacity of the water as the base fluid. Besides, by increasing the volume of Copper in the water, the heat transfer rate was found proportionally increased.

Tiwari and Das model applied the Brinkman effective viscosity model and the Maxwel-Grannett model for spherical-particle to measure the effective thermal conductivity of the fluid. Meanwhile, the effective density and the heat capacitance of the nanofluid are taken from Xuan and Li (2003). In addition, the water was chosen as the base fluid and the nanoparticle used to be dispersed in the base fluid was Copper where the thermo physical properties of both substance can be seen in Table

1.1

Table 1.1: Thermophysical properties of fluid phase and solid phase

Physical Properties	Fluid phase (water)	Solid Phase (Copper)
Spesific heat $C_p(J/kgK)$	4179	385
Density $\rho(kg/m^3)$	997.1	8954
Thermal conductivity $k(W/mK)$	0.6	400

1.6 Velocity Slip

Classical Navier-Stokes equation has an assumption of no-slip condition at the surface where this assumption should be replaced since the slip between fluid and a surface may occur in certain cases as shown in Figure 1.7. Maxwell in 1897 described the relationship between slip and no-slip condition as the length scales in the fluid flowed approaching the continuum limit which was defined as the velocity difference between the wall and the fluid to the strain rate at the wall which can be represented as follows

$$u_s = L_s \frac{\partial u}{\partial y},$$

where u_s is the velocity slip, L_s is the length of the slip and $(\partial u/\partial y)_{wall}$ denotes the strain rate at the wall. The amount of the slip depends on the roughness of the surface where the flow passed through and also the rate of interaction between surface and the fluid

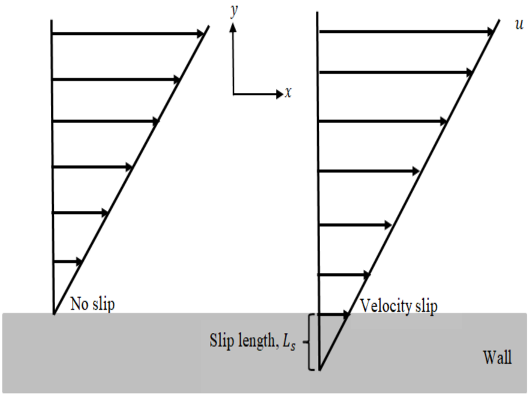

Figure 1.7: Physical model of velocity slip

1.7 Stability Analysis

Stability analysis is a mean to analyze the stability of solutions whether the solution is stable or unstable. The stability of the solution is determined by investigating the affect of small disturbance on the laminar flows whether the disturbance shows the growth or decay trend. The decay of disturbance with time indicates that the flow is stable while the growth of disturbance represents an unstable flow and there is a possibility the occurrence of transition from laminar flow to turbulent flow. The implementation of stability analysis is essential in solving the boundary layer problems in order to choose a physically realizable solution in real life application.

The stability analysis was first performed by Merkin (1986) in his work since the dual solutions were obtained for a certain range of parameter for mixed convection in porous medium of steady boundary layer flow. The dual solutions were divided into two types namely upper solution and lower solution. Stability of the solutions can be identified by considering the unsteady boundary layer problem where the variables are time-dependent.

1.8 Dimensionless Numbers

Dimensionless numbers represent a property of a physical system that has no scale of physical units which are suitable to be used in any system of units. They reduce the number of variables in the system and consequently reduce the amount of data required to correlate the physical phenomena in the system. Using dimensionless numbers has given some advantages such as the problems can be solved more easily, the comparison between different systems can be made, the behavior of the system can be observed, various significant relationships between dimensionless number which describes their influences towards the system can be recognized.

1.8.1 Prandtl Number

Prandtl number was introduced by Ludwig Prandtl in 1904 where it is a dimensionless number which defined the ratio between momentum diffusivity to thermal diffusivity. The Prandtl number depends only on the fluid and the state of the fluid. Generally, the Prandtl number can be represented as follows:

$$Pr = \frac{\text{momentum diffusivity}}{\text{thermal diffusivity}} = \frac{\nu}{\alpha} = \frac{\mu\rho}{k/\rho c_p} = \frac{c_p\mu}{k},$$

where ν is the kinematic viscosity, α is the thermal diffusivity, μ is the dynamic viscosity, k is the thermal conductivity, ρ is the density and c_p is the specific heat. The Prandtl number is applied in the problem which involves the heat transfer, free and forced convection to control the relative thickness of momentum and thermal boundary layer. The value of Prandtl number depends on the types and properties of the fluid as listed below:

- Gases: Pr ranges are from 0.7 to 1.0
- Water: Pr ranges are from 1 to 10
- Liquid metals: Pr ranges are from 0.001 to 0.03
- Oils: Pr ranges are from 50 to 2000

Based on the list above, there are two ranges of Prandtl number which are $Pr < 1$ and $Pr > 1$. The small Prandtl number ($Pr < 1$) indicates that the thermal diffusivity dominates whilst when the Prandtl number is large ($Pr > 1$), it shows that the momentum diffusivity dominates the behavior.

1.8.2 Reynolds Number

The Reynolds number discovered by Osborne Reynolds experimentally in 1883 is a dimensionless number which defines the type and the behavior of the fluid flow system based on velocity, density, dynamic viscosity and characteristics of the fluid . Technically, the Reynold number is the ratio between the inertia forces and the viscous forces as follows:

$$Re = \frac{\text{inertia force}}{\text{viscous force}} = \frac{u/\rho}{\mu/L} = \frac{\rho u L}{\mu} = \frac{uL}{\nu},$$

where ρ is the density, u is the velocity, μ represents dynamic viscosity, L is the characteristic length and $\nu = \mu/\rho$ is the kinematic viscosity. The flow with low Reynolds number is laminar flow whilst the turbulent flow has high Reynolds number.

1.8.3 Schmidt Number

Schmidt number is a dimensionless number which is named after Ernst Heinrich Wilhelm Schmidt that relates the transport of momentum and mass of a fluid. It is defined as the ratio between kinematic viscosity and mass diffusivity. This number is applied to illustrate the diffusivity which the momentum and mass diffusion convection processes occur simultaneously. The Schmidt number is given as follows:

$$Sc = \frac{\text{kinematic viscosity}}{\text{mass diffusivity}} = \frac{\nu}{D_m},$$

where ν is the kinematic viscosity and D_m is the mass diffusivity.Besides, Schmidt number also expresses the interrelation between the velocity and concentration of a fluid which occurs simultaneously in the boundary layer as the momentum and mass transfers happen at the surface.

1.8.4 Soret Number

In 1856, the Soret effect or thermal-diffusion effect was discovered by a German scientist named C. Ludwig and further study on this effect was expanded by a Swiss scientist, C. Soret in 1991. This effect refers to a situation where mass flux occurs due to the temperature gradient and consequently affecting the characteristic of mass transfer rate at the surface. The Soret effect can be defined as follows:

$$S_r = \frac{D_m k_T (T_w - T_\infty)}{T_m \nu (C_w - C_\infty)},$$

where D_m is mass diffusivity coefficient, T_∞ is ambient temperature,T_w is surface temperature, T_m is mean fluid temperature, k_T is thermal diffusion ratio, C_w is surface concentration, C_∞ is ambient concentration and ν is kinematic viscosity.

1.8.5 Dufour Number

Dufour effect or diffusion thermo effect represents the occurrence of heat flux due to the concentration gradient. Thus, the existence of Dufour effect in the boundary layer influences the behavior of heat transfer rate at the surface. This effect is defined as

$$D_f = \frac{D_m k_T (C_w - C_\infty)}{c_s c_p \nu (T_w - T_\infty)},$$

where D_m is mass diffusivity coefficient, T_∞ is ambient temperature,T_w is surface temperature, T_m is mean fluid temperature, k_T is thermal diffusion ratio, C_w is surface concentration, C_∞ is ambient concentration, c_p is the specific heat at a constant pressure and c_s is concentration susceptibility.

1.8.6 Skin Friction Coefficient

Skin friction coefficient is a dimensionless number which describes the frictional force between the fluid and a surface in the boundary layer and physically personalizes the ratio between local surface shear stress to free stream dynamic pressure. The friction between the fluid and the surface occurs as the fluid particles passes through the surface which causes the drag exerted on the surface due to the viscous resistance to the flow and becomes a force that retards the forward motion. The skin friction coefficient can be denoted as

$$C_f = \frac{\tau_w}{\rho U^2},$$

where τ_w is local surface shear stress, ρ is density of the fluid and U is velocity of the flow and τ_w is defined as

$$\tau_w = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0},$$

where μ represents dynamic viscosity.

1.8.7 Nusselt Number

Nusselt number is a dimensionless number which is named after a German engineer, Wilhelm Nusselt. It shows the measurement of convective heat transfer occurs at the surface. The heat transfer takes place when there is temperature difference between the surface and the fluid. The Nusselt number is denoted as

$$N_u = \frac{l q_w}{k_f (T_w - T_\infty)},$$

where q_w is heat flux at the surface, l is a characteristic geometrical length, k_f is thermal conductivity of fluid, T_w is temperature at the surface and T_∞ is ambient temperature. q_w is defined as

$$q_w = -k_f \left(\frac{\partial T}{\partial y} \right)_{y=0}.$$

1.8.8 Sherwood Number

Sherwood number, which is named after Thomas Kilgore Sherwood, represents the di- mensionless number that measures the mass convection at the surface and is used to determine the effectiveness of mass transfer at the surface. The mass transfer in the boundary layer occurs due to the concentration difference between the surface and the fluid. The Sherwood number can be expressed as

$$S_h = \frac{l q_m}{D_m (C_w - C_\infty)},$$

where l is a characteristic geometrical length, D_m is mass diffusivity coefficient, C_w is concentration at the surface, C_∞ is ambient concentration and q_m is mass flux at the surface which is defined as

$$q_m = -D_m \left(\frac{\partial C}{\partial y} \right)_{y=0}.$$

1.9 Problem Statement

The study on the unsteady boundary layer flow has gained an interest of many researchers since this kind of boundary layer is more similar to the real life situation. The nanofluid is considered in all problems since the heat transfer enhancement has become an issue in real life applications due to the low thermal conductivity of the base fluid. In addition, since the problems are simulated using mathematical model and have been solved numerically, it is necessary to obtain all of the solutions as long as the solution exists and determines its stability. Hence, the issues that arise in the problems are:

1. What are the parameters that contribute to the existence of the dual solutions and expand the range of solutions?
2. What are the effects of the nanoparticle volume fraction in the base fluid towards the skin friction coefficient and the heat transfer rate at the surface?
3. What are the effects of considering different type of nanoparticle on the skin friction coefficient and heat transfer rate at the surface?
4. What are the stability of the first solution and second solution?

1.10 Objective and Scopes of The Study

The objectives of the thesis are to study the fluid flow and heat transfer behavior at the surface for unsteady boundary layer flow immersed in nanofluid using Tiwari and Das (2007) model by

1. formulating and deriving the mathematical model,
2. solving the mathematical model numerically using bvp4c function in Matlab software,
3. formulating and deriving the mathematical model for stability analysis purpose,
4. solving mathematical model numerically using bvp4c function in Matlab software in order to determine the stability of the solutions,

for the following problems:

1. Unsteady boundary-layer flow and heat transfer of a nanofluid over a permeable stretching/shrinking sheet to Soret, Dufour and velocity slip effects with stability analysis.
2. Unsteady shrinking sheet with mass transfer in a rotating fluid to stretching/shrinking surface in nanofluid with stability analysis.

3. Unsteady boundary layer and heat transfer analysis over a stretching/shrinking cylinder to nanofluid with stability analysis.
4. The boundary layers of an unsteady stagnation-point flow in a nanofluid to stretching/shrinking surface with velocity slip effect and stability analysis.
5. Unsteady stagnation-point flow in a nanofluid to a permeable exponential stretching/shrinking surface with velocity slip effect and stability analysis.

Tiwari and Das model investigates the influence of the nanoparticle volume fraction in the base fluid on the fluid flow characteristics and the heat transfer rate at the surface. As proposed by the model, the water is considered as the base fluid whilst the Copper Cu, Alumina Al_2O_3 and Titania TiO_2 aare the chosen nanoparticles where each of the substance has different physical properties.

1.11 Significance of The Study

The study on unsteady boundary layer flow has an important role in real life applications such as the start-up process and periodic fluid motion in engineering. The unsteady boundary layer flow is different from steady boundary layer flow since the unsteady flow has an additional contribution to the acceleration/deceleration which changes the velocity with respect to time at a fixed point. The addition of the properties has significantly affected the behavior of the flow and consequently influences the fluid motion and boundary layer separation. Thus, the consideration of unsteady boundary layer flow is an added-value in boundary layer flow study to find a suitable solution which contributes to optimizing the sources usage, cost reduction and time management.

The emergence of nanofluid in the boundary layer flow problem has opened the opportunities and spaces to improve and enhance heat transfer performance in the fluid dynamics field. This study applied Tiwari and Das model instead of Boungiorno model to study the influence of different types of nanoparticle where each nanoparticle has its own thermal physical properties as compared to Boungiourno model which focuses on the Brownion motion and thermophoresis in the fluid. Therefore, Tiwari and Das model offers an option to choose type of nanoparticles and the base fluid to be considered in solving a boundary layer problems.

The advantages of nanofluids can be dicussed in terms of the importance of the nanofluid presence in the process. Bang (2009) in his work has mentioned that the nanofluid can be an efficient heat removal agent also known as a coolant in the thermal-fluid system. Besides considering nanofluid technology is also important in thermal vehicle management as reported by Sidik et al. (2015) since the vehicles are the necessity in this modern world.The nanoparticle suspensions in fluid changes the transport properties (Devi and Andrews (2011)) and is said can enhance the thermo physical features for example thermal conductivity, thermal diffusivity, stickiness and convective heat transfer compared the process that apply base fluid such as water or oil (Wong and Leon (2010)). Nanofluid existence indicates the thermal conductivity rise up along with the increasing volumetric fragment of nanoparticles. Therefore the development of nanofluid helps to manage the sources efficiently so that the optimize output can be produced throughout the production.

In solving boundary layer problems, there are some methods that can be applied which are experimental and theoretical method. Experimental method studies the problem practically. However, performing the experimental method is actually costly since the materials and instrumentals are needed in the experiment. Besides, they might be expensive and may turn into wastage if the material cannot be used repeatedly especially if the instrumental is broken. Meanwhile, the cost issue can be solved by considering the theoretical method. The material and an expensive machine are not required since they are not used in the method. Theoretical method can be divided into two types. The first type is numerical method and the second type is analytical method. The analytical method gives the exact solution but this method has limited use in practical system. Numerical method can solve a problem when the analytical method cant. In addition, this method has the abilities to solve a problem with large system of equations, different degrees of nonlinear problem, and various physical geometries which occurs commonly in engineering area. Hence, with all the superiority of the numerical method, this method is the best method to be chosen which manages to avoid the financial requirements, possible wastage of materials and the complex boundary layer problems with various geometrical shape and consideration can be solved.

The boundary layer problems are solved using numerical method which multiple solutions can be obtained. Since there are more than a solution exist, the best solution needs to be chosen among the solutions which can be done by performing the stability analysis. The analysis of the solution is important in order to identify the suitable parameter that can give the optimum results

1.12 Outline of Thesis

This thesis is divided into nine chapters including this. Chapter 1 discusses the background of the research which includes the definitions and the explanation of the main terms considered in this thesis. Besides, the objectives, which are the direction of the study as well as the scope which is the limitation of the study are also stated in Chapter 1. Next, the significant of the study describes the advantages of the study towards the mathematical field as well as the real life applications. The literature review is based on the previous studies which related to this study is discussed in Chapter 2.

Meanwhile, Chapter 3 explains the procedures that have been used in order to solve the boundary layer problems until the numerical solutions are obtained and the stability of the solutions is identified. The procedures begin by introducing the elliptic equations for an unsteady boundary layer problem over a stretching/shrinking immersed in the nanofluid using Tiwari and Das model. The elliptic equations will be reduced to the the parabolic boundary layer equation using boundary layer approximation. Then, the similarity transformation is applied to transform the parabolic equations to the ordinary differential equations system. The stability analysis is introduced since the focus of the study is to obtain multiple solutions. The numerical solutions and the stability of the solutions are obtained by solving the system of ordinary differential equation and the linearized eigenvalue problem in the form of ODE using bvp4c function in Matlab software.

Five main problems as highlighted in the objectives of the study which are studied in this thesis as presented in Chapter 4 to 8. These five chapters can be divided into two types of unsteady boundary layer problems in which Chapter 1 to 3 study the stretching/surface whilst Chapter 4 and 5 consider the unsteady boundary layer problem on stretching/shrinking stagnation-point flow. Each of the chapter presents five sections which are introduction, problem formulation, numerical solutions, results, discussion and the last section is the conclusion of that particular chapter.

Chapter 4 discusses the mathematical formulation of unsteady boundary layer, heat and mass transfer over stretching/shrinking sheet in nanofluid with the effects of Soret, Dufour, constant mass flux and slip are considered. Different surface which is cylinder is considered in Chapter 5 for unsteady boundary layer and heat transfer with the presence of constant mass flux parameter. Meanwhile, Chapter 6 presents the three-dimensional unsteady rotating boundary layer flow on stretching/shrinking surface over impermeable surface in the nanofluid.

Chapter 7 and 8 present the unsteady stagnation-point flow over a stretching/shrinking surface and an exponentially stretching/shrinking surface, respectively. The results for every chapter show the parameters that affect the duality of the solution, the influences of the parameters towards the skin friction coefficient and heat transfer rate at the surface and also the boundary conditions are fulfilled asymptotically.

The last chapter, Chapter 9 gives a summary for the whole thesis and extension of the problem so that more problems that involve boundary layer problems can be resolved.

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LIST OF PUBLICATIONS

Journal articles:

Dzulkifli, N. F., Bachok, N., Yacob, N. A. M., Arifin, N. M. and Rosali, H. Unsteady Boundary Layer Rotating Flow and Heat Transfer in a Copper-water Nanofluid over a Stretching Sheet. In *Malaysian Journal of Mathematical Sciences*, 11(S):21-33 (2017).

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