UNIVERSITI PUTRA MALAYSIA

GENERATING TOPOLOGIES USING EDGES AND VERTICES IN GRAPHS AND SOME APPLICATIONS

KHALID ABDULKALEK ABDU

GENERATING TOPOLOGIES USING EDGES AND VERTICES IN GRAPHS AND SOME APPLICATIONS

By

KHALID ABDULKALEK ABDU

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfillment of the Requirements for the Degree of Doctor of Philosophy

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DEDICATION

To My Beloved Parents, Brothers & Sisters
To My Beloved wife & My Sons

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfillment of the requirement for the degree of Doctor of Philosophy

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KHALID ABDULKALEK ABDU

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Chairman : Professor Adem Kiliçman, PhD
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The issue of topologizing discrete structures is highlighted by several researches. In that regards, graph theory is one of the major aspects of discrete structures, and the topological graph theory is a crucial branch of it. The investigation of topology on graphs is motivated by the embedding of digital images in a discrete space, interpreted as a graph. Applications in various aspects had been found for topology on graphs, such as in digital geometry, contractions, and strong maps.

In this study, a combination between graph theory and topology has been made. The research adopted a new approach in the investigation of topology on graphs. This is through studying topology on the set of edges of different undirected graphs. It encompasses both simple and non-simple graphs such as multigraph and pseudograph. A subbasis family is introduced to generate a topology on the set of edges of undirected graphs, called the edges topology. Further, properties of this topology are also investigated. In particular, functions between graphs, connectivity, and dense subsets are discussed in this topology. A fundamental step towards studying some properties of undirected graphs by their corresponding topological spaces is displayed.

Additionally, in this research, the new approach is applied to directed graphs by introducing two subbases families to generate two non-similar topologies on the set of edges of any directed graph, called compatible and incompatible edges topologies. Furthermore, the characteristics of these topologies were examined in detail. The relation between directed graphs and their corresponding topologies is presented as well.

In the same vein, the present study generalised the graphic topology defined on the set of vertices of any locally finite simple graph in which every vertex has a finite degree. This is done by presenting a subbasis family to generate a new topology on the set of vertices of simple graphs with vertices of finite/infinite degree, which is called the incidence topology. Accordingly, this study investigated the properties of the incidence topology and made a useful comparison between the two topologies. Moreover, by considering the graphic topology and the incidence topology, this research explored bitopological space on the set of vertices of locally finite simple graphs which was not studied before. Therefore, properties of this bitopological space were discussed in detail. The relation between locally finite graphs and their corresponding bitopological spaces is introduced as well.

Lastly, the edges topology on undirected graphs is used to solve graph problems. This is through identifying all paths between any two distinct vertices, determining all spanning trees (or spanning paths), and finding all Hamilton cycles in simple graphs. In addition, a MATLAB code is written to represent previous applications and allows them to be appropriate for large graphs.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

MENJANA TOPOLOGI DENGAN MENGGUNAKAN TEPI DAN MERCU DALAM GRAF DAN BEBERAPA APLIKASI

Oleh

KHALID ABDULKALEK ABDU

Ogos 2018

Pengerusi : Profesor Adem Kiliçman, PhD
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Isu penjanaan topologi struktur diskret diserlahkan dalam beberapa penyelidikan. Dalam hal ini, teori graf merupakan salah satu aspek utama struktur diskret, and teori graf bertopologi adalah suatu cabang penting. Penyiasatan topologi pada graf didorong oleh pemasukan imej digital ke dalam ruang diskret, yang ditafsirkan sebagai sebuah graf. Aplikasi dalam pelbagai aspek telah dijumpai untuk topologi pada graf, seperti dalam geometri digital, pengecutan, dan pemetaan yang kukuh.

Dalam kajian ini, teori graf telah digabungkan dengan topologi. Kajian ini menggunakan pendekatan baru untuk menyiasat topologi pada graf dengan cara mengkaji set tepi beberapa graf tak berarah yang berlainan. Ia merangkumi kedua-dua graf mudah dan tidak mudah seperti multigraf dan pseudograf. Suatu keluarga sub asas diperkenalkan untuk menjana topologi pada set tepi graf tak berarah, yang dipanggil topologi tepi. Selanjutnya, sifat topologi ini juga disiasat. Khususnya, fungsi antara graf, kesambungan, dan subset padat dibincangkan dalam topologi ini. Langkah asas ke arah mengkaji beberapa sifat graf tak berarah oleh ruang topologi sepadannya dipaparkan.

Selain itu, dalam kajian ini, pendekatan baru digunakan terhadap graf berarah dengan memperkenalkan dua keluarga sub asas untuk menjana dua topologi yang tidak serupa pada set tepi setiap graf berarah, yang disebut sebagai topolgi tepi serasi dan tidak serasi. Tambahan pula, ciri-ciri topologi ini diperiksa secara terperinci. Hubungan antara graf berarah dan topologi mereka yang berkaitan juga ditunjukkan.

Pada masa yang sama, kajian ini merumuskan topologi grafik yang ditakrifkan pada set bucu mana-mana graf mudah terhingga setempat, di mana setiap bucu mempunyai darjah terhingga. Ini dilakukan dengan membentangkan suatu keluarga sub asas untuk menjana topologi baru pada set bucu graf mudah dengan bucu darjah terhingga/tak terhingga, yang dipanggil topologi kejadian. Sewajarnya, kajian ini menyiasat sifat topologi dan membuat perbandingan yang berguna antara dua topologi tersebut. Lagipun, dengan mempertimbangkan topologi grafik dan topologi kejadian, kajian ini meneroka ruang bitopologi pada set bucu graf terhingga mudah setempat yang tidak pernah dikaji sebelum ini. Kemudian, sifat ruang bitopologi ini dibincangkan secara terperinci. Hubungan antara graf terhingga setempat dan ruang bitopologi mereka yang berpadanan juga diperkenalkan.

Akhir sekali, topologi tepi pada graf tak berarah digunakan untuk menyelesaikan masalah graf dengan mengenal pasti semua laluan di antara mana-mana dua bucu yang berbeza, menentukan semua pepohon rentang (atau lintasan rentang), dan mencari semua kitaran Hamilton dalam graf mudah. Di samping itu, kod MATLAB ditulis untuk mewakili aplikasi terdahulu dan membolehkan mereka menjadi sesuai untuk graf yang besar.

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This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfillment of the requirement for the degree of Doctor of Philosophy. The members of the Supervisory Committee were as follows:

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

Figure **Page**

LIST OF ABBREVIATIONS

$G = (V(G), E(G), \varphi_G)$	Undirected graph
$V(G)$	Set of vertices of G
$E(G)$	Set of edges of G
φ_G	Incidence function of G
$\lvert V(G) \rvert$	Order of a graph G
$d(v)$	Degree of a vertex v
K_n	Complete graph of order n
$K_{m,n}$	Complete bipartite graph
$G_1 \cong G_2$	Isomorphic graphs
C_n	Cycle with n vertices
P_n	Path graph with n vertices
W_n	Wheel
$D = (V(D), E(D), \varphi_D)$	Directed graph
$(X, \mathcal{T})$	Topological space
$\bar{A}, cl(A)$	Closure of a set A
$A^\circ, int(A)$	Interior of a set A
$\mathcal{B}$	Basis for a topology
S	Subbasis for a topology
U_x	Minimal open set containing x
$\mathcal{T}\vert_Y$	Subspace topology on Y
$(X, \mathcal{T}_1, \mathcal{T}_2)$	Bitopological space

$\mathcal{T}_G$	The graphic topology of G
A_v	Set of all vertices adjacent with a vertex v
I_v	Set of all edges incident with a vertex v
$\mathcal{T}_{EG}$	The edges topology of G
I_e	Set of vertices incident with an edge e
C	Cut set
$\mathcal{T}_{CE}(D)$	Compatible edges topology of D
$\mathcal{T}_{IE}(D)$	Incompatible edges topology of D
$A_C(e)$	Set of all edges adjacent with e which has the same direction of e
$A_I(e)$	Set of all edges adjacent with e which has different direction to e
$\mathcal{T}_{IG}$	The incidence topology of G
S_n	Star graph with n vertices

CHAPTER 1

INTRODUCTION

1.1 General Introduction

Graph theory is considered one of the most valuable structures in discrete mathematics. Its origin can be found in games and puzzles, including Hamilton's icosian game and Euler's Konigsburg bridge problem. The field has exploded and established itself as a prominent mathematical tool in many subjects, ranging from geography and electrical engineering to architecture and sociology, and from chemistry and operational research to linguistics and genetics. Although graph theory is one of the combinatorics branches, it is considered as a cross-disciplinary tool between math, operations research, electrical engineering and computer sciences. An indication of the growing maturity of graph theory is the strong increase in the links between graph theory and other branches of mathematics. A prominent example is the analysis situs, known today as topology.

Within the field of mathematics, topology is of great value. It began as a milestone development in geometry during the middle of the nineteenth century and then, become a significant force in modern mathematics. Topological structures are important modification for extraction and processing knowledge, used in analyzing data without the notion of distance. Topological concepts like denseness, connectedness, and compactness are basic knowledge for mathematicians.

A large number of publications have considered the problem of topologizing discrete structures. Two reasons essentially caused these efforts: First, as a robust tool, topology leads to several valuable concepts such as homotopy, continuity, and connectivity. The other reason is that a discrete topology is needed at each time spatial relations are modeled on a computer. As close as possible this discrete topology resembles an ordinary topology in the sense of implicitly containing the "intrinsic" spatial information as much as possible.

The development in computer science, especially in computer graphic and image analysis makes the topologies on graphs much more essential. Studying topological properties of digital images is known as digital topology. Any digital image can be “embedded” in a discrete space (interpreted as a graph); the vertices are representing the pixels (grey level intensity and geometric points) of the image and the edges define connectedness and nearness. This background has paved the way to study topology on graphs.

A number of authors studied topology on the set of vertices of directed graphs and simple undirected graphs while some others studied it on the union of vertices and edges (Amiri et al., 2013; Baby Girija and Pilakkat, 2013; Shokry and Aly, 2013; Marijuan, 2010; Bretto, 2007; Vella, 2005; Nogly and Schladt, 1996; Neumann-Lara and Wilson, 1995; Préa, 1992; Lieberman, 1972; Bhargava and Ahlborn, 1968; Evans et al., 1967). So far, however, there has been little discussion about topology on the edge set of a given graph.

Plenty of important applications had been found for the topology on graphs. Lieberman (1972) defined topologies on the set of vertices of directed graphs and studied the relationships between graph theoretical concepts and standard topological properties. He also presented applications involving contractions and strong maps. Bretto (2007) studied compatible topologies on graphs, described particular properties of these topological spaces and developed some applications to digital geometry. James and Klette (2008) introduced the topology of incidence pseudographs in a comprehensive overview. They indicated that this topology has numerous applications, such as in digital picture analysis of two or three dimensions. Shokry and Aly (2013) introduced a new method of taking neighborhood to generate a topology on a graph. The new way of taking neighborhood builds on the distance between two vertices. They linked the new concepts on the graph to the technique of nerve repair as an application in the field of peripheral nervous system.

It is expected that topology on graphs could provide more flexible solutions for the fundamental problems of graphs, such as the enumeration of all paths, finding the Hamilton cycles and spanning trees.

The issue of enumerating paths in a graph was described as NP-hard, non-deterministic problem by computation (Borgwardt and Kriegel, 2005; Sedgewick, 2003). Many applications such as networks signaling paths, stationary, topological features, reliability and fault tolerance are desired to be extracted or investigated for design or development (Yang et al., 2015).

The Hamilton cycle (HC) name belong to the Irish mathematician William Rowan Hamilton in 1856 when he represented the icosahedral group by generators and relations (Gould, 2003). The Hamilton cycle issue is closely related to a series of problems, applications, and puzzles such as traveling salesman problem, choice of travel routes, time scheduling, Icosian game and network topology. Therefore, the resolution of the HC problem is a significant issue in graph theory and computational methods in mathematics and computer science as well. A Hamiltonian path in an undirected graph is a path that visits each vertex only once, and, a Hamiltonian cycle is a closed path of once-visited vertices. Hence, a Hamilton graph is the graph that consists of a Hamilton cycle. For a general and some particular graphs, a decision whether a graph is Hamiltonian is NP-complete. Although this problem was extensively studied and investigated using different algorithms, appropriate solutions has not been reached yet (Nishiyama et al., 2018; Alhalabi et al., 2016; Ibarra, 2009).

So many systems that have flow/connection between elements can be represented by undirected graphs with flows/connections between nodes. These systems such as computer network, computer-aided design, circuit analysis, telecommunication systems, image segmentation, particular chemical isomers and cluster analysis, have problems that can be solved by determining (enumerating or counting) the spanning trees in the graph. This problem is NP-hard, so that the determination can be based on different criteria, approximation or optimization depending on the problem considered and the chosen bounds and constraints (Jothi et al., 2018; Nikolopoulos et al., 2014; Galbiati et al., 1997; Mai and Evans, 1984). When the problem is to minimize the cost in the system, the problem concerns with searching and finding the minimal spanning trees in a weighted graph within a cost/power consumption function. However different algorithms have been presented and developed to change the problem of spanning tree to a solvable problem using the heuristic, backtrack, intelligent and hybrid algorithms (Consoli et al., 2015), all solutions were limited and cannot be generalized to undirected graphs. Furthermore, there is so far no evaluation showing the exact solution, especially for large size graphs.

1.2 Problem Statement

The edges play an essential role in the structure of graphs since they give the links between vertices and in some applications in which the edges stand for distance, affinity, construction costs, or capacities relying upon the applications that were considered. However, far too little attention has been paid to topologies on the set of edges of a given graph. Such topology on edges could be more appropriate for the problems that are well described by edges because the topology is directly defined on the active variables of the problem. This debate was a motivation for studying a new approach in the investigation of topology on graphs by linking the set of edges of directed and undirected graphs with topology. In addition, the proposed approach is supposed to present appropriate topology to solve some particular graph problems.

Another issue is related to assuming a graph with vertices of finite degree. The graphic topology presented by Amiri et al. (2013) is associated with the set of vertices of locally finite simple graphs only. Hence this topology is not appropriate to be associated with simple graphs that have vertices of infinite degree. Thus, a generalisation of the topology on the set of vertices of any simple graph is desired. This can be achieved through presenting a new topology and comparing with the previous. Furthermore, the new topology and the graphic topology could cover bitopological space on the set of vertices of locally finite simple graphs, which was not studied before.

1.3 Research Objectives

The present study covers the following objectives:

1. To associate a topology with the set of edges of every locally finite undirected graph and investigate the properties of this topology.
2. To associate topologies with the set of edges of every directed graph and examine the attributes of these topologies. Further, introduce a relation between directed graphs and their corresponding topologies.
3. To associate a topology with the set of vertices of simple graphs with vertices of (finite or infinite) degree and study bitopological space on the set of vertices of locally finite simple graphs.

4. To solve the graph problems (identifying all paths, finding the Hamilton cycles, and determining spanning trees) as applications in graphs through using the topology on the set of edges of locally finite undirected graphs. Additionally, represent the applications by MATLAB program.

1.4 Outline of the Thesis

This thesis is organized into seven chapters.

The first chapter contains an introduction on the topic and problem statement. Also, the objectives of the work are listed.

In chapter 2, we give basic notions of graph theory and topology. In addition, we provide a brief review of the previous works that had been done by other researchers.

Chapter 3 deals with the topology on undirected graphs. We associate a topology with the set of edges of every locally finite undirected graph. Properties of this topology are investigated, and a relation between undirected graphs and their corresponding topologies is presented.

In chapter 4, two topologies are associated with edge set of any given directed graph. Two subbases are introduced to generate these two topologies. Then, their characteristics are figured out. Notably, the connectivity and dense subsets in these topologies are studied.

In chapter 5, we generalised the graphic topology presented by Amiri et al. (2013) on the set of vertices of locally finite simple graphs by presenting a new topology on the set of vertices of any simple graph. Properties of the new topology are investigated and a comparison between the two topologies is displayed. Furthermore, we studied bitopological space on the set of vertices of locally finite simple graphs, which was not studied before, by considering these two topologies (the new topology and the graphic topology). Then, properties of locally finite graphs are studied by their corresponding bitopological spaces.

Chapter 6 includes some applications of topology on the edge set of undirected graphs. This topology is used to solve graph problems. Steps of identifying all paths between any two distinct vertices, determining all spanning trees (or spanning paths), and finding all Hamilton cycles in simple graphs are presented. In addition, the previous applications are represented by a MATLAB program to be more convenient for large graphs.

In chapter 7, our results are summarized and concluded by highlighting unsolved and open problems.

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LIST OF PUBLICATIONS

The following papers were derived from this study.

A. Kiliçman and K. Abdulkalek, Topological spaces associated with the edges set of graphs, submitted to *Electronic Journal of Graph Theory and Applications (EJGTA)*.

K. Abdulkalek and A. Kiliçman (2018). Bitopological spaces on undirected graphs, *Journal of Mathematics and Computer Science*, 18: 232-241.

K. Abdulkalek and A. Kiliçman (2018). Topologies on the edges set of directed graphs, *International Journal of mathematical Analysis*, 12(2): 71-84.

A. Kiliçman and K. Abdulkalek (2018), Topological spaces associated with simple graphs, *Journal of Mathematical Analysis,* 9(4): 44-52.

K. Abdulkalek, A. Kilicman and S. M. Saleh Ahmed, Framework and topology of shortest route problem in a maze undirected graph, submitted to *Periodica Polytechnica Transportation Engineering.*

K. Abdulkalek, A. Kilicman and S. M. Saleh Ahmed, SPGPIM: Solving Particular Graph Problems in MATLAB, submitted to *SoftwareX*.

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