UNIVERSITI PUTRA MALAYSIA

STRESS INTENSITY FACTOR FOR CRACKS PROBLEMS IN AN ELASTIC HALF PLANE USING SINGULAR INTEGRAL EQUATIONS

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By

NAWARA RAJAB FATHULLAH ELFAKHAKHRE

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirements for the Degree of Doctor of Philosophy

October 2018

DEDICATIONS

To my beloved family and friends

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Doctor of Philosophy

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October 2018

Chair: Associate Professor Nik Mohd Asri Bin Nik Long, PhD
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Single and multiple cracks in two dimensional half plane isotropic elastic solid are considered. The cracks are subjected to uniaxial tension $\sigma_x^\infty = p$ with free traction on the boundary. These problems are formulated into a system of singular integral equations (SIEs) with the distribution dislocation functions as unknown by using the modified complex potential. In solving the obtained SIEs, the cracks configurations are mapped into a straight line on a real axis by using the curved length coordinate method. By applying the appropriate quadrature formulas with the appropriate collocation points the SIEs are reduced to the system of algebraic linear equations with M unknown coefficients. These M unknowns coefficients are solved using the Gauss-Jordan elimination method. The obtained unknown coefficients will later be used in evaluating the stress intensity factor. The stress intensity factor at the tips of single and multiple cracks are obtained for various crack configurations and positions. Numerical results showed that the stress intensity factor influenced by the distance between the cracks, the crack configuration, and the distance between the cracks and the boundary of the half plane. For the test problems, our results are in good agreements with the existence results.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

FAKTOR KEAMATAN REGANGAN UNTUK MASALAH RETAKAN DI DALAM SATAH SEPARUH KENYAL MENGGUNAKAN PERSAMAAN KAMIRAN SINGULAR

Oleh

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Retakan tunggal dan berganda di dalam pepejal kenyal isotropic setengah satah dua dimensi dipertimbangkan. Retakan tertakluk kepada regangan ekapaksi $\sigma_x^\infty = p$ dengan terikan bebas pada sempadan. Masalah ini dirumuskan ke dalam sistem persamaan kamiran singular (PKS) dengan fungsi alihan serakan sebagai anu menggunakan keupayaan kompleks terubah. Dalam menyelesaikan PKS, bentukan retakan dipetakan ke atas garis lurus di atas satah nyata menggunakan kaedah koordinat panjang terlengkung. Dengan menggunakan rumus kuadratur dengan titik kolokasi yang sesuai, PKS diturunkan kepada sistem persamaan linear aljabar dengan M pekali anu. M pekali anu ini diselesaikan menggunakan kaedah penghapusan Gauss-Jordan. Pekali anu yang diperolehi akan digunakan kemudiannya untuk menilai faktor kemaatan regangan. Faktor kemaatan regangan pada hujung retakan tunggal dan berganda boleh diperolehi untuk pelbagai bentukan dan kedudukan retakan. Keputusan berangka menunjukan bahawa faktor keamatan regangan dipengaruhi oleh jarak antara retakan, bentukan retakan, dan jarak antara retakan dan sempadan setengah satah. Bagi masalah percubaan, keputusan berangka kami sangat menepati keputusan sedia ada.

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This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfilment of the requirement for the degree of Doctor of Philosophy.The members of the Supervisory Committee were as follows:

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LIST OF ABBREVIATIONS

LEFM	Linear Elastic Fracture Mechanics
SIF	Stress Intensity Factor
SIEs	Singular Integral Equations
OCP	Original Complex Potential
MCP	Modified Complex Potentials
K	Stress intensity factor
K_{I}	Mode I Stress intensity factor
K_{II}	Mode II Stress intensity factor
K_{III}	Mode III Stress intensity factor
σ_x	Stress in the x-direction

CHAPTER 1

INTRODUCTION

1.1 Overview

Fracture mechanics is a branch of solid mechanics that deals with the study of the propagation of cracks in materials. Methods of analytical solid mechanics are used to calculate the driving force on a crack and those of experimental solid mechanics to characterize the material's resistance to fracture.

Predicting the fatigue life of cracked components is one of the most important tasks in engineering of fracture mechanics. Fracture mechanics is an important tool in modern materials science which used to improve the performance of mechanical structures. Based on theories of elasticity and plasticity, the stress and strain are applied to the materials in order to predict the mechanical failure of the bodies. Fracture mechanics can be classified into two main categories, Linear Elastic Fracture Mechanics (LEFM), and Elastic Plastic Fracture Mechanics (EPFM).

LEFM work only when the material is an isotropic and linear elastic. The basic assumption of LEFM is that the size of plastic zone is small as compared to the crack size. The crack grow when the stresses near the crack tip exceed the material fracture toughness. The stress field near the crack tip is calculated using the theory of elasticity. In contrast, if large zones of plastic deformation developed before the crack grows then EPFM will be used. Under EPFM, by assuming the material isotropic and elastic-plastic, the strain energy fields or opening displacement near the crack tips can be calculated. The crack will grow when the energy or opening exceeds the critical value.

The fracture mechanics theories are considered as the material contains a crack with infinite stresses at its tips. The fracture mechanics understanding is developed based on linear elasticity from the pioneer work by Inglis (1913), Griffith (1920), and Westergaard (1939). Inglis (1913) studied the unexpected failure of naval ships and constructed the solution of stress for an elliptical hole in a semi-infinite plate subject to remote uniform tension . However, his solution posed mathematical difficulty where the stresses approach infinity at the crack tip and only limited to a perfectly sharp crack. Griffith (1920) extended Inglis's solution to compute the stress concentrations around the elliptical holes (Bowie (1973), Anderson (1991)).

Whereas, Westergaard (1939) assumed the complex stress functions to derive the asymptotic solution for a stationary crack loaded dynamically. His method provides a powerful technique for solving the infinite linear elastic plane containing a crack or array of cracks. Irwin (1957) developed the energy release rate based on Griffith's work into a more useful form for engineering problems. In addition, he used Westgaard's approach to describe the stresses and displacements near the crack tip by a single parameter. This parameter later become known as the stress intensity factor (Anderson (1991)). Therefore, many researchers focused their attention on evaluating the stress intensity factors, and computed data of the stress intensity factors have been mainly used in evaluating the safety of structures. In relation to the stress intensity factors, a set of rules can be obtained for predicting the fatigue life of the cracked structures.

1.2 Background of crack problems

The following definitions are useful for further understanding of the problems under discussion.

1.2.1 Deformation

The movement of points in a solid body relative to each other. In other words, it is the change in shape of objects due to the applied forces (Perez (2017)).

1.2.2 Displacement

The movement of a point in a vector quantity in a body subjected to loading mode. In other word, the displacement of a particle P is a vector u acting the difference between the final and initial position of P. This means it is the distance that P moves during the deformation (Perez (2017)).

1.2.3 Strain

It is a geometric quantity, that depends on the relative movement of two or three points in a body. Also, it can be considered as a measure of deformation of the material based on a reference length (Barber (2002)).

1.2.4 Body and surface forces

Consider a continuous medium, the points of that are referred to rectangular Cartesian system of axes, and let a volume V of arbitrary shape which is bounded by the surface S, and dV an element of volume V. The sum of the external forces that act on the elements of volume dV or mass of the body is called the body forces such as gravity, but that act on the surface of the volume elements dV is the surface forces pressure.

Mathematically a body force acting on a volume element dV can be represented by a vector $\vec{\Phi}dV$ where $\vec{\Phi}$ is some finite vector for any point (x,y,z). It applied to some point of the element dV must be understood in the sense that the resultant force vector $\vec{\Psi}$ acting on any finite volume V. The resultant force may be represented by a triple integral as (Muskhelishvili (1953), Sokolnikoff (1956))

$$\vec{\Psi} = \int\int\int_v \vec{\Phi}dV = \int\int\int_v \vec{\Phi}dxdydz. \tag{1.1}$$

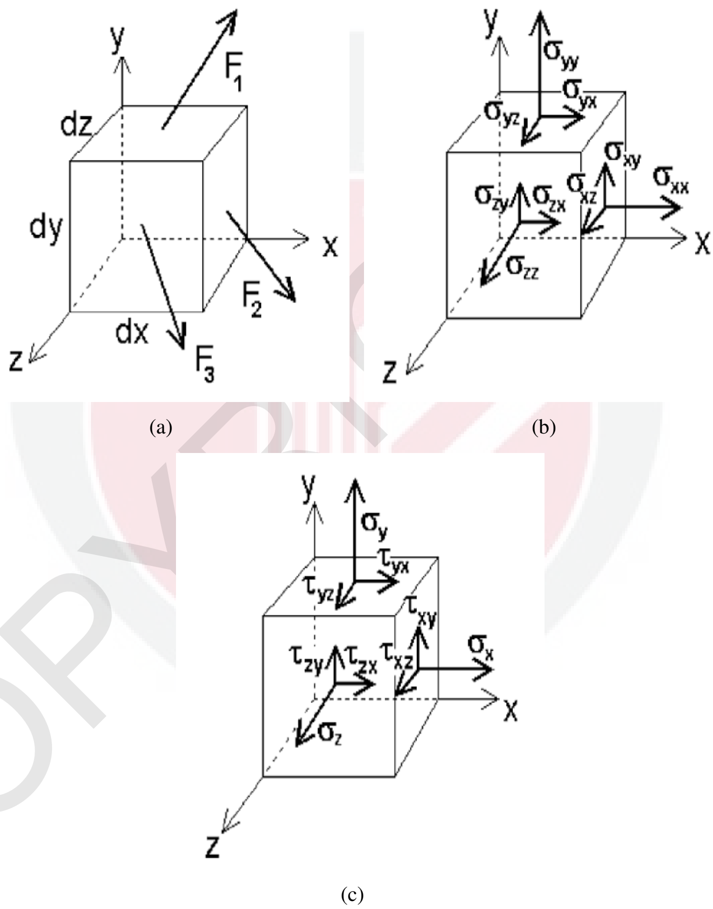

Figure 1.1: Homogeneous stress state. (a) Forces acting on side faces of volume element. (b, c) Stress components given with different notation. [Source: Valberg (2010)]

1.2.5 Stress

Stress is defined as force per unit area across an internal surface in the body. Consider dF is a force that is directed at an angle to surface dA at a particular place inside a material. If the area is made smaller until it approaches to zero, the area will become a point. The stress σ at this point will equal to dF/dA in the same direction of the force. This stress can be analyzed into two components, a normal stress and a shear stress. The stress will become homogeneous when the stress components and force at the back and front face of the element are equal but have opposite signs. The components of stress represented by symbol σ with appropriate suffices. The first and the second suffix denoted to the direction of the outward normal to the surface upon that it acts and the stress component, respectively.

Figure 1.1(b) illustrates the notation for the Cartesian coordinate system x, y, z where the normal stresses have the same suffices (i. e. $\sigma_{xx}, \sigma_{yy}, \sigma_{zz},$), and shear stresses have different suffices (i. e. $\sigma_{xy}, \sigma_{yx}, \sigma_{yz}, \sigma_{zy}, \sigma_{zx}, \sigma_{xz}$). An alternative notation for stress components is given in Figure 1.1(c) where the shear stress is represented by τ. A positive normal stress is a tensile stress while a negative will be a compressive stress. At equilibrium, it is required that the shear stresses be as follow (Barber (2010), Valberg (2010))

$$\tau_{xy} = \tau_{yx}, \tau_{yz} = \tau_{zy}, \tau_{zx} = \tau_{xz}. \tag{1.2}$$

1.2.6 Traction

The traction is defined as the force that the part lying on the positive side of a surface element exerts on the part lying on the negative side. In other words, it is the force acting between the parts of the continuous body adjacent to either side of the surface element dS of the surface S which described by $\vec{F}dS$, where the vector $\vec{F}$ is the traction per unit area or the stress vector (Muskhelishvili (1953)).

1.2.7 Safety factor

It is a parameter utilized for designing structural components to assure structural integrity. It is can be described as follow

$$S_F = \frac{Strength}{Stress} > 1, \tag{1.3}$$

where the strength is denoted to a material's property, such as yield strength σ_{ys}, and the stress σ is the variable to be applied to structure. The role of S_F in this simple relationship is to control the design stress so that $\sigma < \sigma_{ys}$ in designing applications, while it is represent to have a prolong design life for assuring structural integrity. Usually, the safety factor is in the order of two, however its magnitude depends on the designer's experience or on a design code (Perez (2017)).

1.3 Stress analysis of cracks

There are two approaches which are equivalent each to other in certain circumstances that can be used to crack analysis: the energy criterion and the stress intensity approach. The energy approach states that if the energy available for crack growth is enough to overcome the resistance of the material will cause the crack extension. The material resistance may include the surface energy, plastic work, or other types of energy dissipation associated with a propagating crack. The first researcher who proposed the energy criterion for fracture was Griffith (1920). Irwin (1956) is primarily responsible for developing the present version of this approach. Assume that $\mathfrak{G}$ and $\mathfrak{G}_c$ are represented the rate of change in potential energy with the crack area for a linear elastic material and the critical energy release rate, respectively. A measure of crack toughness is at the moment of fracture $\mathfrak{G} = \mathfrak{G}_c$. For a crack in an infinite plate subject to a remote tensile stress, $\mathfrak{G}$ can be described by

$$\mathfrak{G} = \frac{\pi\sigma^2 a}{E}, \tag{1.4}$$

where a is the half length of the crack, E is Young's modulus, and σ is the remotely applied stress. Since $\mathfrak{G} = \mathfrak{G}_c$ then Equation (1.4) can be rewritten to describe the critical combinations of stress and crack size for failure as

$$\mathfrak{G}_c = \frac{\pi\sigma_f^2 a_c}{E}. \tag{1.5}$$

The applied stress can be viewed as the driving force for plastic deformation, since the yield strength is a measure of the material's resistance to deformation. One of the fundamental assumptions of fracture mechanics is that the crack toughness ($\mathfrak{G}_c$ in this case) does not depend of the size and geometry of the cracked body; a crack toughness measurement on a laboratory specimen should be applicable to a structure. As long as this assumption is valid, all configuration effects are taken into account by the driving force $\mathfrak{G}$.

For an isotropic linear elastic material, the stress distribution, σ_{ij} near the crack tip in polar coordinate system as shown in Figure 1.2 with origin at the crack tips is given by (Anderson (1991))

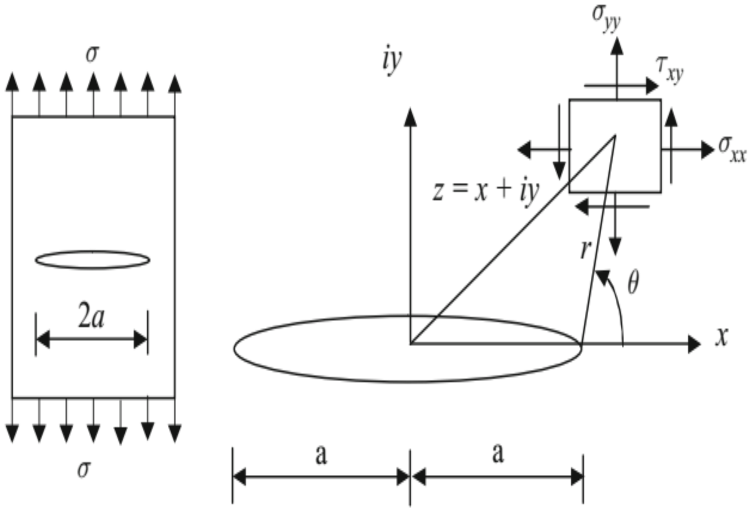

Figure 1.2: Stress field in the vicinity of crack tip for mode *I* crack using complex coordinates. [Source: Kumar and Barai (2011)]

$$\sigma_{ij} = (\frac{k}{\sqrt{r}})f_{ij}(\theta) + \sum_{m=0}^{\infty} A_m r^{\frac{m}{2}} g_{ij}^{(m)}(\theta), \tag{1.6}$$

where σ_{ij} = stress tensor, r and θ are defined as in Figure 1.2 , k = constant, f_{ij} = dimensionless function of θ in the leading term, A_m is the amplitude and $g_{ij}^{(m)}$ is a dimensionless function of θ for the higher-order terms.

The higher-order terms depend on geometry, while the solution for any given configuration contains a leading term which is proportional to $1/\sqrt{r}$. If $r \to 0$ the leading term approaches infinity, however the other terms remain finite or approach zero. Therefore, stress near the crack tip varies with $1/\sqrt{r}$, regardless of the configuration of the cracked body. Equation (1.6) characterizes a stress singularity, since stress is asymptotic to $r = 0$.

There are three types of loading which a crack can experience, as shown in Figure 1.3. A crack body can be described by one of these modes, or a combinations of more than one. These basic fracture modes are called Mode I, Mode II, and Mode III.

Mode I is a normal or tensile mode where the crack surfaces move directly apart. Mode II is slide or shearing mode where the crack surfaces slide over one another in a di-

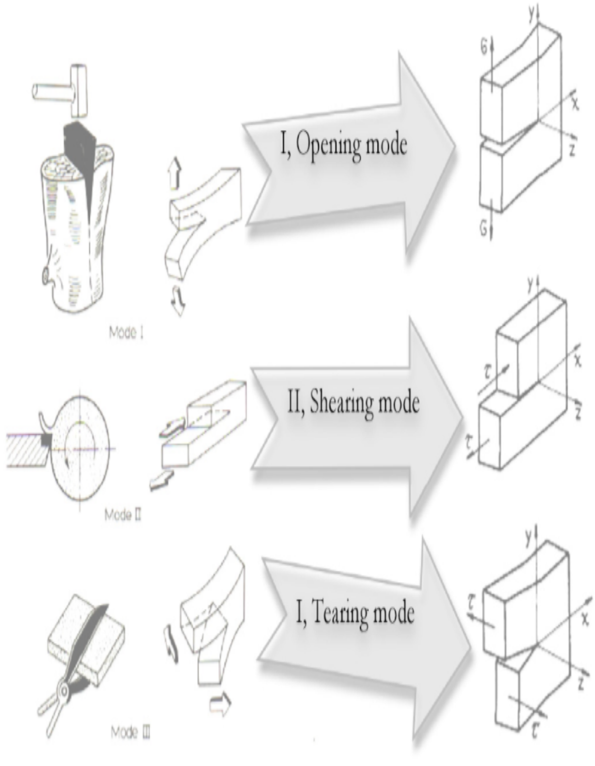

Figure 1.3: Three modes of fracture mechanics. [Source: Prawoto (2011)]

rection perpendicular to the leading edge of the crack. While the tearing mode (i. e. mode III) acting where the crack surfaces move relative to one another and parallel to the leading edge of the crack.

The loading produces for any of these modes is the $1/\sqrt{r}$ singularity at the crack tip, however the proportionality constants k and f_{ij} depend on the mode where k can be replaced by the stress intensity factor K as

$$K = k\sqrt{2\pi}. \tag{1.7}$$

1.4 Stress intensity factor

The stress intensity factor K is an important quantity in mechanics of solids and plays an essential role to study the strength of the material. It can be defined as a measure of the singular stress field near the tip of the crack. Several methods have been developed for determining stress intensity factors such as analytical, numerical, and experimental approaches. Consider $K_{\mathrm{I}}, K_{\mathrm{II}}$, and K_{III} that represent the stress intensity factors corresponding to Modes I, II, and III, respectively, then for an isotropic linear elastic material the stress fields ahead of a crack tip can be expressed as

$$\lim_{r\to 0} \sigma_{ij}^{(\mathrm{I,\ II,\ III})} = \frac{K_{\mathrm{I,\ II,\ III}}}{\sqrt{2\pi r}} f_{ij}^{(\mathrm{I,\ II,\ III})}(\theta). \tag{1.8}$$

Thus the three factors can be defined by

$$\begin{aligned} K_{\mathrm{I}} &= \lim_{r\to 0} \sqrt{2\pi r}\sigma_{yy}(r,0), \\ K_{\mathrm{II}} &= \lim_{r\to 0} \sqrt{2\pi r}\sigma_{xy}(r,0), \\ K_{\mathrm{III}} &= \lim_{r\to 0} \sqrt{2\pi r}\sigma_{yz}(r,0). \end{aligned} \tag{1.9}$$

1.5 Basic equations of plane elasticity and Airy stress function

The basic equations of elasticity contain equilibrium equations of stresses and strain displacement relations, and Hooke's law regarding stresses and strain. Assume a system of stress components are applied on a body together with the body force stresses F_x, F_y, and F_z.

These stress components acting in the x-direction, y-direction, and z-direction are described, respectively as (Helena (2017))

$$\begin{aligned} \frac{d\sigma_x}{dx} + \frac{d\tau_{yx}}{dy} + \frac{d\tau_{zx}}{dz} + F_x &= 0, \\ \frac{d\sigma_y}{dy} + \frac{d\tau_{xy}}{dx} + \frac{d\tau_{zy}}{dz} + F_y &= 0, \\ \frac{d\sigma_z}{dz} + \frac{d\tau_{xz}}{dx} + \frac{d\tau_{yz}}{dy} + F_z &= 0. \end{aligned} \tag{1.10}$$

These equations are called the general stress equations in equilibrium. In two dimensional problem where the body forces are absent the equilibrium equation will be rewritten as

$$\begin{aligned} \frac{d\sigma_x}{dx} + \frac{d\tau_{xy}}{dy} &= 0, \\ \frac{d\sigma_y}{dy} + \frac{d\tau_{xy}}{dx} &= 0. \end{aligned} \tag{1.11}$$

In this case the stress components can be expressed by means of one single auxiliary function χ in the following manner (Muskhelishvili (1953))

$$\sigma_x = \frac{d^2\chi}{dy^2}; \quad \tau_{xy} = -\frac{d^2\chi}{dxdy}; \quad \sigma_y = \frac{d^2\chi}{dx^2}. \tag{1.12}$$

This functions χ is known as Airy stress function. Also, this function can be satisfied the compatibility condition which expressed in terms of stresses as (Sun and Jin (2012))

$$\nabla^2(\sigma_x + \sigma_y) = 0, \tag{1.13}$$

when

$$\nabla^4(\chi) = \nabla^2\nabla^2(\chi) = 0, \tag{1.14}$$

where the Laplace operator ∇^2, and the biharmonic operator ∇^4 are defined as

$$\begin{aligned} \nabla^2 &= \frac{d^2}{dx^2} + \frac{d^2}{dy^2}, \\ \nabla^4 = \nabla^2\nabla^2 &= \frac{d^4}{dx^4} + 2\frac{d^4}{dx^2}dy^2 + \frac{d^4}{dy^4}. \end{aligned} \tag{1.15}$$

Any function χ satisfying Equation (1.14) is defined a biharmonic function. If the function g is harmonic i. e. $\nabla^2 g = 0$ then g will be biharmonic but the converse is not true. Thus if the Airy stress function is known then the stresses can be obtained by Equation (1.12). Also, the strains and the displacements can be obtained by the following equation

$$e_x = \frac{du}{dx}; \quad e_y = \frac{dv}{dy}; \quad e_{xy} = \frac{1}{2}\left(\frac{du}{dy} + \frac{dv}{dx}\right), \tag{1.16}$$

where e_x, e_y, and e_{xy} are tensorial strain components, and u and v are displacements.

The stress-strain relations are given by

$$\begin{aligned} \sigma_x &= \lambda^*\left(e_x + e_y\right) + 2\mu e_x, \\ \sigma_y &= \lambda^*\left(e_x + e_y\right) + 2\mu e_y, \\ \sigma_{xy} &= 2\mu e_{xy}, \end{aligned} \tag{1.17}$$

or inversely

$$
\begin{aligned}
e_x &= \frac{1}{2\mu}\left[\sigma_x - \frac{\lambda^*}{2(\lambda^*+\mu)}(\sigma_x+\sigma_y)\right],\\
e_y &= \frac{1}{2\mu}\left[\sigma_y - \frac{\lambda^*}{2(\lambda^*+\mu)}(\sigma_x+\sigma_y)\right],\\
e_{xy} &= \frac{1}{2\mu}\sigma_{xy},
\end{aligned}
\tag{1.18}
$$

where μ is the shear modulus and

$$\lambda^* = \frac{3-\kappa}{\kappa-1}\mu, \tag{1.19}$$

while

$$\kappa = \begin{cases} 3-4\nu & \text{for plane strain} \\ \dfrac{3-\nu}{1+\nu} & \text{for plane stress} \end{cases} \tag{1.20}$$

and ν denotes the Poisson's ratio.

1.6 Analytic function and Cauchy-Riemann equations

The complex variable ζ and its conjugate $\bar{\zeta}$ in a Cartesian coordinate system (x,y) are defined as

$$\zeta = x+iy; \quad \bar{\zeta} = x-iy, \quad \text{where } i=\sqrt{-1}. \tag{1.21}$$

While in polar coordinates (r,θ) are expressed as

$$\zeta = r(\cos\theta + i\sin\theta); \quad \bar{\zeta} = r(\cos\theta - i\sin\theta), \quad \text{where } i=\sqrt{-1}. \tag{1.22}$$

Now, consider the complex function $f(\zeta)$ then the derivative of $f(\zeta)$ respect to ζ is

$$\frac{df(\zeta)}{d\zeta} = \lim_{\Delta\zeta\to 0}\frac{f(\zeta+\Delta\zeta)-f(\zeta)}{\Delta\zeta}. \tag{1.23}$$

If $f(\zeta)$ has a derivative at point ζ_0 and also at each point in some neighborhood of ζ_0, then $f(\zeta)$ is said to be analytic at ζ_0. The complex function can be described into this form

$$f(\zeta) = u(x,y)+iv(x,y), \tag{1.24}$$

where u and v are real functions. If $f(\zeta)$ is analytic, we have

$$\frac{d}{dx}f(\zeta) = f'(\zeta)\frac{d\zeta}{dx} = f'(\zeta), \tag{1.25}$$

and

$$\frac{d}{dy}f(\zeta) = f'(\zeta)\frac{d\zeta}{dy} = if'(\zeta), \tag{1.26}$$

where a prime stands for differentiation with respect to ζ. Therefore,

$$\frac{d}{dx}f(\zeta) = -i\frac{d}{dy}f(\zeta), \tag{1.27}$$

or

$$\frac{du}{dx} + i\frac{dv}{dx} = \frac{dv}{dy} - i\frac{du}{dy}. \tag{1.28}$$

By this equation, the Cauchy-Riemann equations will be obtained as follow

$$\frac{du}{dx} = \frac{dv}{dy}, \quad \frac{du}{dy} = -\frac{dv}{dx}. \tag{1.29}$$

These equations can be shown to be sufficient for $f(\zeta)$ to be analytic. By the Cauchy-Riemann equations it is easy to derive the following

$$\nabla^2(u) = \nabla^2(v) = 0, \tag{1.30}$$

this means the real and imaginary parts of an analytic function are harmonic.

1.7 Integrals equations

A single crack problem and the multiple cracks problem either for an infinite or half plane elasticity can be solvable by the boundary integral equations (BIE). In generally, these integral equations may be expressed as

$$\int_L A(\omega, \omega_0)g(\omega)d\omega = p(\omega_0), \quad (\text{or } p(\omega_0) + c, \ \omega_0 \in L), \tag{1.31}$$

where L is the crack configuration, and $A(\omega, \omega_0)$ is kernel specified by the choice of the unknown function $g(\omega)$ and the known function $p(\omega_0)$ (Chen (1994), Chen et al. (2003)). Since the displacements are discontinuous along the line L, we have two possibilities to choose the unknown function either be the displacement jump or the dislocation distribution. Then, we can choose the traction or the resultant force function along the crack as the right hand term.

Table 1.1 lists the possibilities to classification of the integral equations which depend on the choice of the functions $g(\omega)$ and $p(\omega_0)$. The kernel is weakly singular (WS) if the unknown function $g(\omega)$ is chosen as dislocation distribution function and the right hand term $p(\omega_0)$ is the resultant force. This is named as weakly singular integral equation because the kernel is a logarithmic function and the integration is in weaker singularity.

Table 1.1: The classification of the integral equations in crack problem.

Type	$g(\omega)$	$p(\omega_0)$	property of $A(\omega,\omega_0)$
WS	Dislocations	Resultant forces	Weakly singular
$S1$	Dislocations	Tractions	Cauchy singular
$S2$	Displacement jump (COD)	Resultant forces	Cauchy singular
HS	Displacement jump (COD)	Tractions	Hypersingular
F1A	Dislocations	–	Fredholm/Regular
F1B	Tractions	Tractions	Fredholm/Regular
F2	Displacement Jump	–	Fredholm/Regular

If $g(\omega)$ is the dislocation distribution function, and $p(\omega_0)$ is traction then $A(\omega,\omega_0)$ is a Cauchy singular kernel (S1). This integral called as S1 because the integral belongs to Cauchy principle value integral. A second kind of Cauchy singular equation (S2) is formulated where $g(\omega)$ is crack opening displacement (COD) and $p(\omega_0)$ is resultant force. Also, this type of integral is Cauchy principle value integral.

For hypersingular (HS) integral equation, we choose $g(\omega)$ as the crack opening displacement (COD) and $p(\omega_0)$ is traction. This type of integration allows the COD function be obtained directly from the solution. Regularization of the suggested singular integral equations gives three types of the Fredholm integral equations (F1A, F1B, and F2) for the relevant problem. By regularize the singular integral equation of type S1, we can obtain the type F1A. This type of integration denotes the regular integral. For the type F1B, the traction applied on the individual crack and the traction applied on the actual problem are chosen as the unknown function and the right hand term respectively. The advantage of this type is that we do not need to utilize a complicated mathematical works to solve the multiple cracks problems. Also, this type is a regular integral. Finally, by using the formulation of the type S2, we can obtain the type F2 of the Fredholm integral equation.

1.8 Research objectives

The main objectives of this research are:

1. To formulate the physical problem of the multiple cracks in an elastic half plane into a system of singular integral equations (SIEs) by using complex potentials.

2. To reduce the obtained SIEs for the above mentioned problems for the unknown coefficients (dislocation distribution functions) to the system of linear equations by quadrature formula and curve length coordinate method.

3. To analysis the behavior of SIF at the crack tips as the cracks far/close to each other or to the boundary.

4. To investigate the interaction between two and three cracks.

1.9 Motivation

The study of crack geometry becomes an increasingly important in engineering design due to the presence of the cracks that affect the stability and safety of component significantly. The safety of the components can be determined from the computed data of stress intensity factor. In addition, the stress intensity factor can be used to predict the fatigue life of cracked components. To this end, accurate and efficient technique are required for determining stress intensity factor for these problems. Thus, the focus of this research is to investigate the interaction between the cracks in an elastic half plane. The singular integral equation is used to formulate this problem and solved numerically.

1.10 Scope of the study

This research will be focused on the modeling of the multiple cracks subjected to uniaxial tension $\sigma_x^\infty = p$ in an isotropic elastic half plane with free traction boundary condition. The problem is formulated into a system of singular integral equations and is solved numerically for the different cracks configurations.

1.11 Structures of the thesis

The thesis contains six chapters which are organized as follows:
In Chapter 1, a brief overview and some basic definitions related to fracture mechanics are presented. Also, the basic equations in plane elasticity are introduced. Chapter 2 focuses on the literature review on the different approaches used in solving the cracks problems and the modified complex potential for elastic half plane. The formulation of the multiple cracks problem in an isotropic half plane using the modified complex potential is presented, also the methodology for solving the crack problem is included.

Chapter 3 covers the details of the formulations for single crack problem into singular integral equation with free traction boundary condition where the distribution dislocation function is taken as unknown. The final solution is obtained with the help of the curve coordinate method in conjugation with Gauss quadrature rules. Several numerical examples are given.

Chapter 4 deals with the interaction between two cracks and to study the effect of the cracks for each to other that including different configurations of the cracks. The interaction between three cracks is discussed in Chapter 5. Finally, Chapter 6 presents a summary of the study and the future recommendations.

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