THERMAL CONVECTION IN A BINARY FLUID SATURATED AN ANISOTROPIC POROUS MEDIUM WITH NONLINEAR TEMPERATURE PROFILE

NUR ZARIFAH BINTI ABDUL HAMID

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By

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Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfillment of the Requirements for the Degree of Master of Science

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Chair: Nor Fadzillah binti Mohd Mokhtar, PhD
Faculty: Science

In this thesis, the thermal instability in a binary fluid layer saturated an anisotropic porous medium in the presence of nonlinear temperature profile is formulated mathematically based on Darcy's model and Boussinesq approximation. The linear stability analysis is applied to the system and the resulting eigenvalue obtained are solved numerically using single-term Galerkin method with respect to lower rigid and upper free conducting boundary and lower rigid conducting and upper free insulating boundary.

Several effects are considered in this study, which include the effect of nonlinear temperature profile, anisotropic parameter, binary fluid parameter, internal heat generation, magnetic field and feedback control in order to test the stability of the system. The onset of convection is the most advance in the presence of cooling from above temperature profile while the system is the most stable in the presence of cubic 1 temperature profile. The results obtained are drawn graphically and showed similarities with the result obtained by previous research.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Master Sains

PEROLAKAN TERMA DALAM BENDALIR BINARI TEPU MEDIUM BERLIANG ANISOTROPIK DENGAN PROFIL SUHU TAK LINEAR

Oleh

NUR ZARIFAH BINTI ABDUL HAMID

April 2019

Pengerusi: Nor Fadzillah binti Mohd Mokhtar, PhD
Fakulti: Sains

Dalam tesis ini, ketidakstabilan haba dalam lapisan bendalir binari tepu medium berliang anisotropik dengan kehadiran profil suhu tak linear diformulakan secara matematik berdasarkan model Darcy dan penghampiran Boussinesq. Analisis kestabilan linear digunakan pada sistem dan nilai eigen yang terhasil diselesaikan secara berangka dengan menggunakan kaedah Galerkin dengan mengambil kira sempadan bawah tegar dan sempadan atas bebas berkonduksi, dan sempadan bawah tegar berkonduksi dan sempadan atas tegar berpenebat.

Beberapa kesan dipertimbangkan dalam kajian ini, termasuk kesan profil suhu tak linear, parameter anisotropik, parameter bendalir binari, penjanaan haba dalaman, medan magnet dan kawalan untuk menguji kestabilan sistem. Permulaan perolakan adalah paling awal dengan kehadiran profil suhu penyejukan dari atas manakala sistem adalah paling stabil dengan kehadiran profil suhu kubik 1. Keputusan yang diperolehi dipaparkan secara grafik dan ia menunjukkan persamaan dengan hasil kajian terdahulu.

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This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfilment of the requirement for the degree of Master of Science. The members of the Supervisory Committee were as follows:

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a	wave number
d	depth
D	Differential operator
$f(z)$	non-dimensional temperature profile
g	gravitational force
H	Chandrasekhar number
H_b	uniform magnetic field
K	control gain parameter
$\vec{K}$	inverse of anisotropic permeability tensor
K_x	horizontal permeability
K_z	vertical permeability
Le	Lewis number
p	pressure
Pm	magnetic Prandtl number
Pr	Prandtl number
q	uniformly distributed volumetric internal heat generation
q_x	horizontal wave number in x direction
Q	internal heat generation
r_y	horizontal wave number in y direction
Ra	thermal Rayleigh number
Ra_c	critical thermal Rayleigh number
Ra_s	solute Rayleigh number
S	solute concentration
t	time
T	temperature
T_0	reference temperature
$\vec{u}$	velocity vector
x, y, z	Cartesian coordinates

Greek symbols

α	coefficient of thermal expansion
β	coefficient of solute expansion
γ	ratio of heat capacity
γ_m	magnetic viscosity
σ	growth rate
ϕ	porosity
μ	dynamic viscosity
μ_m	magnetic permeability
κ_s	solute diffusivity
κ_T	anisotropic thermal diffusivity tensor
κ_{T_x}	horizontal thermal diffusivity
κ_{T_z}	vertical thermal diffusivity
ρ	density

ρ_0	reference density
ξ	mechanical anisotropy parameter
η	thermal anisotropy parameter
∇^2	Laplacian operator
ε	thermal depth

Subscripts

b	basic state

Superscripts

$'$	perturbed state

CHAPTER 1

INTRODUCTION

1.1 Heat Transfer

The studies of heat transfer have received much attention of researchers due to its many important in engineering fields. Heat can be transfer through three ways, which include conduction, convection and radiation. Conduction is the process of heat transfer, which takes place in solid, when there is a difference in temperature from one region to another region. Convection is the transfer of heat in fluids such as liquid or gas when there is a movement of molecules within the fluids, which can either take place through advection, diffusion or both. Conduction and convection required a medium, while radiation is the transfer of heat without required any medium. For an example, the transfer of heat from sunlight to the earth through empty space. Advection refers to the movement of substances by bulk motion, while diffusion refers to the movement of a molecule from a region of high concentration to the region of low concentration.

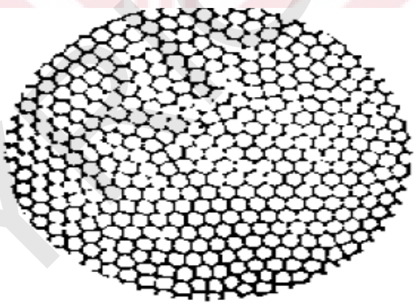

Figure 1.1: Benard cell.

Thermal convection is a type of heat transport by the movement of fluid, which takes place due to the changes in density between two regions with different temperature. In 1900, Henri Benard conducted an experiment to investigate the onset of convection in various types of fluid with distinct viscosity. A thin layer of spermaceti is heated from below and observed the appearance of hexagonal cell, which known as Benard cell, formed at the upper layer of the fluid as presented in Figure (1.1). The onset of convection takes place due to buoyancy force is determined.

Later, Rayleigh (1916) found that when heat is applied to the fluid from direction usually from below at small value, heat will diffuse above without causing fluid flow. As the heat flow increased above a critical value of Rayleigh number, the system would change from stable conducting state to the convection state where the bulk motion of fluid due to heat begin as shown in Figure (1.2). The Rayleigh number can be expressed as

$$Ra = \frac{\alpha g \Delta T d^3}{\nu \kappa_{T_z}}$$

where α is the coefficient of thermal expansion, g is the gravity, T is the temperature, d is the depth, $\nu = \frac{\mu}{\rho_0}$ is the kinematic viscosity and κ_{T_z} is the vertical thermal diffusivity.

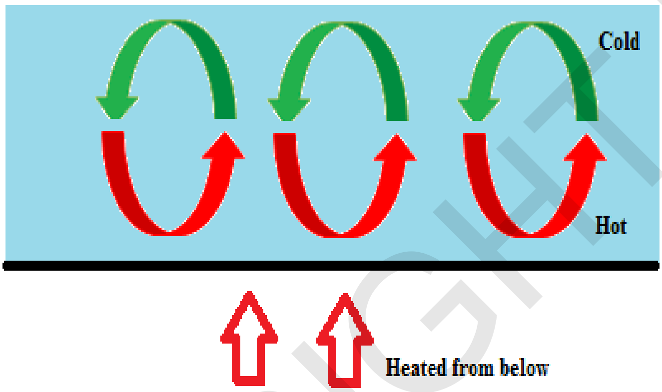

Figure 1.2: Thermal convection in a fluid.

1.2 Binary Fluid

The study of convection in a binary fluid is crucial in understanding the phenomena that happen around us through the field of oceanography, geophysics, astrophysics, chemistry etc. Binary fluid is a type of fluid which formed when two substances are combined such that each substances maintains its own chemical identity. When a fluid contained two components with different diffusivity and their gradient makes opposing contribution to the fluid density, this may lead to a possible source of convection (Bergeon et al., 1998). Example of binary fluids are water ethanol and salts like sodium chloride, magnesium chloride, sodium nitrate etc.

1.3 Anisotropic Porous Medium

Anisotropic porous medium is a type of material containing pores (see Figure 1.3) which has different values of rock properties like density, thermal and electric conductivity and permeability when it is measured with respect to direction. Anisotropic porous can be formed naturally through the process like sedimentation, compaction, frost action and reorientation of solid matrix and can be formed unnaturally through

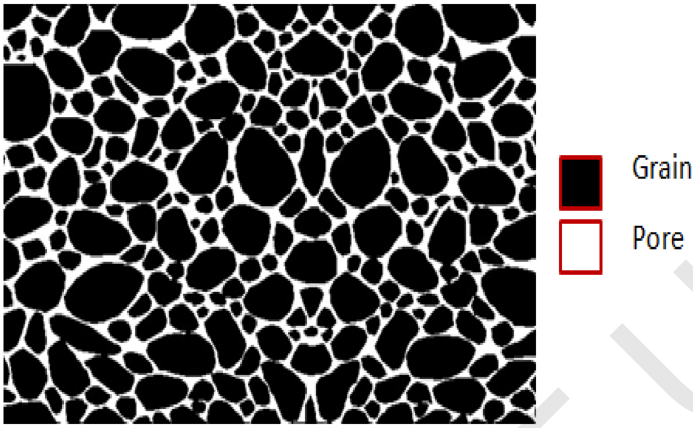

Figure 1.3: Anisotropic porous medium.

the process like pelletizing in chemical industries, fiber materials used in insulating process etc. Examples of material that are considered as an anisotropic porous medium are carbonate rock, wood and composite. The studies of convection in a homogeneous isotropic porous medium have been considered by most of the scientist and researchers. Unfortunately, the assumption made is unphysical since the size of the rock and grain may vary through a reservoir causes the variation in the rock property such as permeability.

Henry Darcy (1856), a French hydraulic engineer, is the first person who mathematically formulated the Eq.(1.3.1) to describe the flow of fluid through the porous medium.

$$Q = KA\frac{h_1 - h_2}{L} \tag{1.3.1}$$

The experiments has been conducted using an inclined column filled with stratified sand. In this experiment, there is no pump been used and the water flows due to gravity force only. The rate of fluid's flow depends directly on the loss of energy and inversely to the length of the flow path. Darcy law has been widely applied in the hydrology field such as in understanding the flow of groundwater through the aquifer.

1.4 Nonlinear Temperature Profile

The study of instability in the presence of nonlinear temperature profile is practically important in the field of science, engineering and technology. The sudden heating or cooling, radiation, throughflow etc are responsible for the formation of nonlinear basic state temperature profile in the system. The effect of nonlinear temperature profile can be used to maintain the stability of the convective system (Nield, 1975;

Rudraiah et al., 1980).

1.5 Magnetic Field

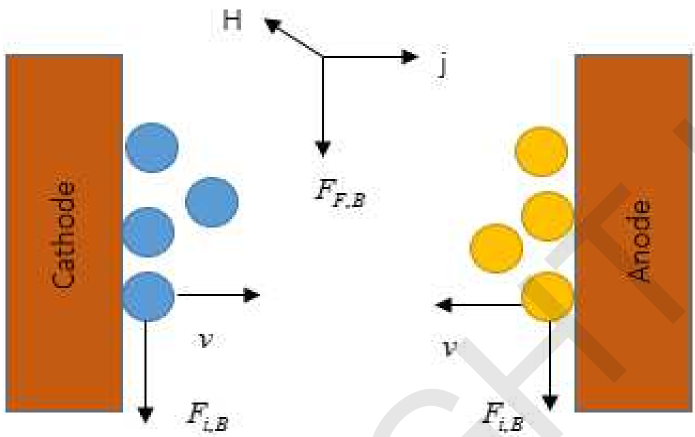

Figure 1.4: Illustration of magnetic field in fluid.

Magnetic force, $F_{F,B}$ which act on an electrically conducting fluid such as salt water and liquid metals with a magnetic flux are perpendicular to magnetic field, H (see Figure 1.4). The positive ion (orange) attract to anode and gain electron while the negative ion (blue) attract to cathode and lose electron by redox reactions. This individual ion will moving with a velocity $v \cdot F_{i,B}$, where $F_{i,B}$ is the magnetic force which acting on the individual ion. The forces, which generated by magnetic fields, can control of fluid motion (Weston et al., 2010). In 1861, the macroscopic formulation for electromagnetic field known as Maxwell's equation been introduced by James Clerk Maxwell to replace the microscopic version named as Lorentz force. Maxwell's equation are widely been used in solving the real field problem which involve electricity and magnetism.

1.6 Internal Heat Generation

The study of convection in the system with internal heat generation is crucial in understanding the convection in Earth's mantle (Tritton and Zaragga, 1967). Internal heat generation, which utilized to a part of the medium, can either function as a heat source or heat sink throughout the system. The presence of internal heat generation will cause the system to experience the nonlinear temperature profile. Therefore, thermal convection can occur even when the temperature at the upper layer is higher than the temperature at the bottom layer as long as the adverse slope of temperature profile occur somewhere within the system (Gasser and Kazimi, 1976).

1.7 Feedback Control

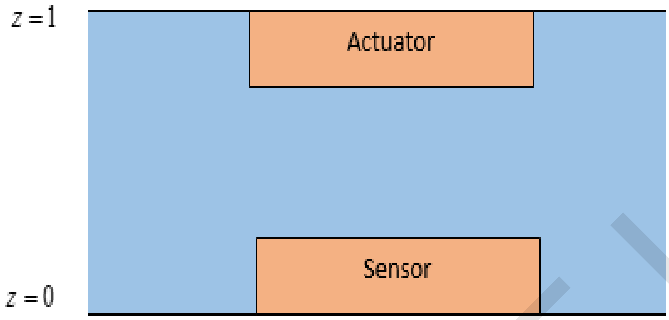

Figure 1.5: Feedback control in fluid.

Feedback control, which are used to control heat in fluids, can give many benefits in the real field especially in simulating phenomena such as in modeling and animating clouds (Dobashi et al., 2008). Feedback control is a system that consist of sensor and actuator (see Figure 1.5). The role of sensor is to detect any changes of the free surface temperature from its conductive state and send the signal to the actuator. The function of an actuator is to adjust the heated surface temperature by dropping slightly the surface temperature below the rising of fluid's temperature.

1.8 Application of Convection

The numberless applications of convection in the real field is the main reason that had captured the interest of researchers to explore on the convection problem in a binary fluid saturated an anisotropic porous medium.

1.8.1 Solar Pond

The research done on the convection problem can help to understand the transfer of heat in the solar pond, which is useful for various thermal applications including green house heating, desalination of salt water and production of electricity. Solar pond which consist of a binary fluid named as salt has been develop to collect and store solar thermal energy. The solar pond can be divided into three zones, which include the upper convective zone, middle non-convective zone and lower convective zone. The heat loss through evaporation can be preventing by inhibit the movement of warmer fluid from the bottom surface to the upper surface of the pond. This can be done by increasing the salinity with depth by continuously adding the concentrated brine and fresh water at the bottom and upper layer of the pond respectively (Velmurugan and Srithar, 2008). The solar pond can stored the thermal energy up to

100°C (Tahat et al., 2000).

1.8.2 Seismic Wave and Earth's Interior

The study of convection in an anisotropic porous medium is crucial in understanding the natural phenomena like earthquakes, which may lead to disaster like Tsunami, ground shacking and landslides. Thermal convection that takes place in the interior of Earth can create a tectonic force leading the rocks of the lithosphere to generate elastic strain. The rock will be cracked if the strained get too large. The sudden release of kinetic energy due to the brittle fracture of the rock is known as an earthquake.

1.8.3 Carbon Sequestration

Nowadays fossil fuels has become the main energy sources in the electricity and transportation sectors. The combustion of fossil fuels produces carbon dioxide, which can give a serious impacts to our health and environment. One way to reduce the accumulation of carbon dioxide in the atmosphere is by storing the carbon into underground brine-filled aquifers (Juanes et al., 2006). However, the convection that happens in the aquifer may cause the carbon dioxide to leak from the upper surface of the aquifer. Therefore, it is crucial to understand the effect of temperature and anisotropic porous medium on the flow of carbon dioxide in the underground water (Abbaszadeh and Shariatipour, 2018; Hill and Morad, 2014).

1.9 Motivation

Most of the studies considered the convection problem in the presence of linear temperature profile and not much attention given to the nonlinear temperature profile problem. Besides, only a few researchers considered the studies in an anisotropic porous medium and most of the researchers focused their studies of convection in an isotropic porous medium where in real field, almost all types of porous medium have an anisotropic properties. Due to this motivation, we decided to study the convection problem in a binary fluid saturated an anisotropic porous medium in the presence of nonlinear temperature profile. The linear stability analysis and single-term Galerkin method are used to solve this problem.

1.10 Problem Statements

The studies of convection in an anisotropic porous medium can give great significance in many disciplines of oil field. Getting an ignorance on the anisotropy property of the earth can result in the failure on evaluating and developing the reservoir.

For the present study, the problem relating to convection in a binary fluid saturated an anisotropic porous medium is studied. The problem statements of this thesis are:

1. What are the differences in the values of critical Rayleigh number when considering linear and nonlinear temperature profile for different types of boundary conditions?

2. What are the effect on critical Rayleigh number when magnetic field is applied on the system?

3. What will happen to the onset of convection when there is a combined effects of magnetic field and internal heating on the system?

4. How the convection can be control in the presence of internal heating and feedback control?

1.11 Objectives

The objectives of this thesis are to analyze the mathematical model for the convection problem in a binary fluid saturated an anisotropic porous medium for the following cases:

- effect of nonlinear temperature profile.

- effect of magnetic field and nonlinear temperature profile.

- combined effects of magnetic field and internal heating in the presence of nonlinear temperature profile.

- combined effects of internal heating and feedback control in the presence of nonlinear temperature profile.

In this study, the linear stability analysis on the above formulated problem are applied and the system of homogeneous ordinary differential equations are solved numerically using single-term Galerkin method.

1.12 Scope

The scope of this study is limited to stationary thermal convection in a binary fluid layer saturated an anisotropic porous medium. This study is mainly focus on the effect of non linear temperature profile on the onset of stationary thermal convection in a binary fluid layer saturated an anisotropic porous medium, where several other effects are included in their respective problem.

1.13 Problem Formulation

An infinite horizontal incompressible binary fluid layer saturated an anisotropic porous medium of depth d, which is heated from below with gravity force, $\vec{g} = (0,0,-g)$ acting vertically downward on it is considered as shown in Figure (1.6). The uniformly adverse slope of the fluid's temperature and concentration, which $\Delta T = T_l - T_u$ and $\Delta S = S_l - S_u$, where $T_l > T_u$ and $S_l > S_u$ are maintained between the plane. The system is subjected to nonlinear temperature profile. The governing equation for the flow of binary fluid through the anisotropic porous medium is characterized based on the well known Darcy law. We assumed that the mechanical and thermal properties of the porous medium to be anisotropy in the vertical direction and isotropy in the horizontal direction.

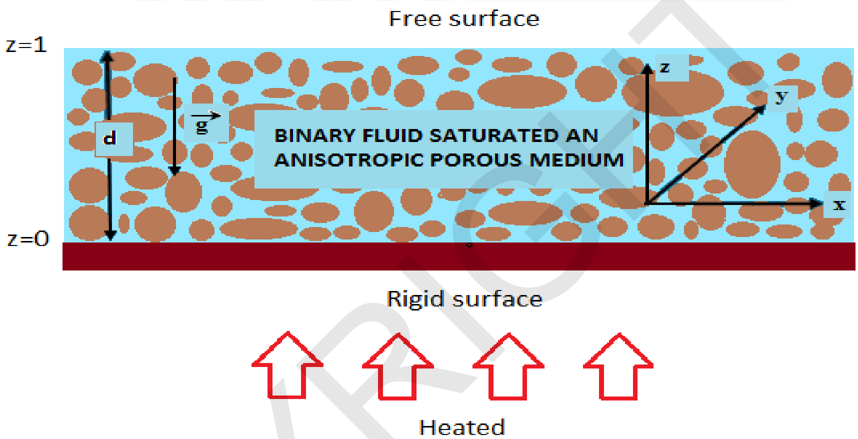

Figure 1.6: Physical model of a binary fluid layer saturated an anisotropic porous medium.

In order to study the convection problem, we need to understand the conservation of mass, conservation of momentum and conservation of energy in the fluid. Let us consider an infinitesimal control volume from the system, the conservation of mass can be understood as follow, the mass flow in the control volume is equal to the mass flow out of the control volume. It describes that the mass can neither be destroyed nor created. In the fluid mechanics, the conservation of mass can be mathematically express as

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0. \tag{1.13.1}$$

Following the Boussinesq approximation, the density variation does not effect the flow of fluid except the buoyancy force, thus we can neglect $\frac{\partial \rho}{\partial t}$ term and by considering a constant density $\rho = \rho_{constant}$, thus equation (1.13.1) reduce to continuity equation of the form

$$\nabla \cdot \vec{u} = 0, \tag{1.13.2}$$

where $\vec{u} = (u, v, w)$ is the velocity vector.

Based on the conservation of momentum theory, the Navier stokes equation takes the form (Malashetty and Swamy, 2010)

$$\frac{\rho_0}{\phi}\frac{\partial \vec{u}}{\partial t} + \nabla p + \mu \vec{K} \cdot \vec{u} - \rho \vec{g} = 0, \tag{1.13.3}$$

where $\vec{K} = K_x^{-1}(\hat{i}\hat{i} + \hat{j}\hat{j}) + K_z^{-1}(\hat{k}\hat{k})$ is the inverse of anisotropic permeability tensor.

The conservation of energy state that the rate of change of energy in the control volume is equal to the summation of net heat flux into the element with the rate of work done on the control volume by body and surface forces. The energy equation in the conservation form (Malashetty and Swamy, 2010)

$$\gamma \frac{\partial T}{\partial t} + (\vec{u} \cdot \nabla) T = \kappa_T (\nabla^2 T), \tag{1.13.4}$$

where $\kappa_T = \kappa_{T_x}(\hat{i}\hat{i} + \hat{j}\hat{j}) + \kappa_{T_z}(\hat{k}\hat{k})$ is the anisotropic thermal diffusivity tensor.

The concentration equation take the form (Malashetty and Swamy, 2010)

$$\phi \frac{\partial S}{\partial t} + (\vec{u} \cdot \nabla) S = \kappa_s (\nabla^2 S). \tag{1.13.5}$$

The density of fluid is directly proportional to temperature and concentration of the fluid which given by (Malashetty and Swamy, 2010)

$$\rho = \rho_0 [1 - \alpha (T - T_0) + \beta (S - S_0)]. \tag{1.13.6}$$

Consider two type of boundary conditions for the system as follows:

(a) lower rigid conducting and upper free conducting

$$\begin{aligned} &w = 0, \ \ \frac{\partial w}{\partial z} = 0, \ \ T = T_l, \ \ S = S_l, \ \ \text{at} \ \ z = 0, \\ &w = 0, \ \ \frac{\partial^2 w}{\partial z^2} = 0, \ \ T = T_u, \ \ S = S_u, \ \ \frac{\partial S}{\partial z} = 0, \ \ \text{at} \ \ z = 1. \end{aligned} \tag{1.13.7}$$

(b) lower rigid conducting and upper free insulating

$$\begin{aligned} &w = 0, \ \ \frac{\partial w}{\partial z} = 0, \ \ T = T_l, \ \ S = S_l, \ \ \text{at} \ \ z = 0, \\ &w = 0, \ \ \frac{\partial^2 w}{\partial z^2} = 0, \ \ T = T_u, \ \ \frac{\partial T}{\partial z} = 0, \ \ S = S_u, \ \ \frac{\partial S}{\partial z} = 0, \ \ \text{at} \ \ z = 1. \end{aligned} \tag{1.13.8}$$

1.13.1 Basic State

We assumed the basic state of the fluid to be quiescent. In order to study the effect of nonlinear temperature profile on the onset of thermal convection, we consider the basic state for the temperature in the forms (Nield, 1975)

$$\vec{u}_b = (0,0,0), \;\; p = p_b(z), \;\; \rho = \rho_b(z),$$
$$T = T_b(z), \;\; \frac{-d}{\Delta T}\frac{dT_b}{dz} = f(z), \;\; S = S_b(z), \tag{1.13.9}$$

where $f(z)$ refer to non-dimensional temperature profile which hold the following condition

$$\int_0^1 f(z)dz = 1. \tag{1.13.10}$$

Substitute Eq.(1.13.9) into Eqs.(1.13.2)-(1.13.6) to get

$$\frac{dp_b}{dz} = -\rho_b g, \;\; \frac{d^2T_b}{dz^2} = 0, \;\; \frac{d^2S_b}{dz^2} = 0,$$
$$\rho_b = \rho_0[1-\alpha(T_b - T_0) + \beta(S_b - S_0)], \tag{1.13.11}$$

where subscript b indicate the basic state. The conduction state solution for the fluid takes the form

$$T_b = \frac{-\Delta T}{d}z + T_l, \;\; S_b = \frac{-\Delta S}{d}z + S_l. \tag{1.13.12}$$

1.13.2 Perturbed State

The motionless state of the fluid subjected to an infinitesimal perturbation is given by

$$\vec{u} = \vec{u}_b + \vec{u}', \;\; T = T_b + T', \;\; S = S_b + S', \;\; p = p_b + p', \;\; \rho = \rho_b + \rho', \tag{1.13.13}$$

which the primes refer to the infinitesimal perturbation quantities. Substitute Eq.(1.13.13) together with the basic state solution into Eqs.(1.13.2)-(1.13.6) to obtain (see APPENDIX A for details)

$$\nabla \cdot \vec{u}' = 0, \tag{1.13.14}$$

$$\frac{\rho_0}{\phi}\frac{\partial \vec{u}'}{\partial t} + \nabla p' + \mu \vec{K} \cdot \vec{u}' + \rho_0(\alpha T' - \beta S')\vec{g} = 0, \tag{1.13.15}$$

$$\gamma\frac{\partial T'}{\partial t} + (\vec{u}' \cdot \nabla)T' - w'\frac{\Delta T}{d}f(z) = \kappa_{T_x}(\nabla_h^2 T') + \kappa_{T_z}\frac{\partial^2 T'}{\partial z^2}, \tag{1.13.16}$$

$$\phi\frac{\partial S'}{\partial t} + (\vec{u}' \cdot \nabla)S' + w'\frac{dS_b}{dz} = \kappa_s(\nabla^2 S'), \tag{1.13.17}$$

$$\rho' = \rho_0[-\alpha T' + \beta S']. \tag{1.13.18}$$

Boundary conditions (1.13.7) and (1.13.8) under perturbation are given as follows:

(a) lower rigid conducting and upper free conducting

$$
\begin{aligned}
w' = 0, \ \frac{\partial w'}{\partial z} = 0, \ T' = 0, \ S' = 0, \ \text{at} \ z = 0, \\
w' = 0, \ \frac{\partial^2 w'}{\partial z^2} = 0, \ T' = 0, \ \frac{\partial S'}{\partial z} = 0, \ \text{at} \ z = 1.
\end{aligned} \tag{1.13.19}
$$

(b) lower rigid conducting and upper free insulating

$$
\begin{aligned}
w' = 0, \ \frac{\partial w'}{\partial z} = 0, \ T' = 0, \ S' = 0, \ \text{at} \ z = 0, \\
w' = 0, \ \frac{\partial^2 w'}{\partial z^2} = 0, \ \frac{\partial T'}{\partial z} = 0, \ \frac{\partial S'}{\partial z} = 0, \ \text{at} \ z = 1.
\end{aligned} \tag{1.13.20}
$$

We nondimensionalized Eqs.(1.13.14)-(1.13.18) using the following transformations

$$
\begin{aligned}
(x,y,z) = (x^* d, y^* d, z^* d), t = \frac{\gamma d^2 t^*}{\kappa_{T_z}}, p' = \frac{\mu \kappa_{T_z}}{K_z} p^*, T' = (\Delta T) T^*, \\
(u', v', w') = \left(\frac{\kappa_{T_z} u^*}{d}, \frac{\kappa_{T_z} v^*}{d}, \frac{\kappa_{T_z} w^*}{d} \right), S' = (\Delta S) S^*,
\end{aligned} \tag{1.13.21}
$$

and eliminate the pressure term from the resulting Eq.(1.13.15) by applying the curl twice on Eq.(1.13.15). After dropped $(*)$ from the resulting Eqs.(1.13.14)-(1.13.18), we obtain (see APPENDIX A for details)

$$
\left[\frac{Da}{\gamma \phi Pr} \frac{\partial}{\partial t} \nabla^2 + \nabla_h^2 + \frac{1}{\xi} \frac{\partial^2}{\partial z^2} \right] w - Ra \nabla_h^2 T + Ra_s \nabla_h^2 S = 0, \tag{1.13.22}
$$

$$
\left[\frac{\partial}{\partial t} - \eta \nabla_h^2 - \frac{\partial^2}{\partial z^2} + \vec{u} \cdot \nabla \right] T - f(z) w = 0, \tag{1.13.23}
$$

$$
\left[\frac{\phi}{\gamma} \frac{\partial}{\partial t} - \frac{1}{Le} \nabla^2 + \vec{u} \cdot \nabla \right] S - w = 0, \tag{1.13.24}
$$

where $Da = \frac{K_z}{d^2}$ is the Darcy number, $Pr = \frac{\mu}{\rho_0 \kappa_{T_z}}$ is the Prandtl number, $Ra = \frac{\rho_0 \alpha g \Delta T d K_z}{\mu \kappa_{T_z}}$ is the thermal Rayleigh number, $Ra_s = \frac{\rho_0 \beta g \Delta S d K_z}{\mu \kappa_{T_z}}$ is the solute Rayleigh number, $Le = \frac{\kappa_{T_z}}{\kappa_s}$ is the Lewis number, $\xi = \frac{K_x}{K_z}$ is the mechanical anisotropy parameter, $\eta = \frac{\kappa_{T_x}}{\kappa_{T_z}}$ is the thermal anisotropy parameter, $\nabla_h^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ and $\nabla^2 = \nabla_h^2 + \frac{\partial^2}{\partial z^2}$.

1.13.3 Linear Stability Analysis

The linear stability analysis is applied in order to eliminate all the nonlinear term from Eqs.(1.13.22)-(1.13.24). We considered the vertical velocity, temperature and concentration to be periodic waves and hence we find the solutions in the form of normal mode expansion as

$$(w,T,S) = (W(z),\Theta(z),\Phi(z))exp[i(q_x x + r_y y) + \sigma t], \tag{1.13.25}$$

where q_x and r_y are horizontal wave number in x and y direction respectively and σ is the growth rate parameter, which generally a complex quantity. Substituting Eq.(1.13.25) into the linearized version of Eqs.(1.13.22)-(1.13.24), (see APPENDIX A for details) we get

$$\left[\frac{\sigma Da}{\gamma\phi Pr}(D^2-a^2)+\left(\frac{D^2}{\xi}-a^2\right)\right]W + a^2 Ra\Theta - a^2 Ra_s\Phi = 0, \tag{1.13.26}$$

$$\left[\sigma-(D^2-\eta a^2)\right]\Theta - f(z)W = 0, \tag{1.13.27}$$

$$\left[\frac{\phi\sigma}{\gamma}-\frac{1}{Le}(D^2-a^2)\right]\Phi - W = 0, \tag{1.13.28}$$

where $D = d/dz$ and $a^2 = q_x^2 + r_y^2$. Since our objective is to study the onset of stationary thermal convection in the system, we substitute $\sigma = 0$ into Eqs.(1.13.26)-(1.13.28) and get

$$\left(\frac{D^2}{\xi}-a^2\right)W + a^2 Ra\Theta - a^2 Ra_s\Phi = 0, \tag{1.13.29}$$

$$(D^2-\eta a^2)\Theta + f(z)W = 0, \tag{1.13.30}$$

$$\frac{1}{Le}(D^2-a^2)\Phi + W = 0. \tag{1.13.31}$$

Boundary conditions (1.13.19) and (1.13.20) under normal expansion are given as follows:

(a) lower rigid conducting and upper free conducting

$$\begin{aligned} W = DW = \Theta = \Phi = 0 \ \text{ at } \ z = 0, \\ W = D^2W = \Theta = D\Phi = 0 \ \text{ at } \ z = 1. \end{aligned} \tag{1.13.32}$$

(b) lower rigid conducting and upper free insulating

$$\begin{aligned} W = DW = \Theta = \Phi = 0 \ \text{ at } \ z = 0, \\ W = D^2W = D\Theta = D\Phi = 0 \ \text{ at } \ z = 1. \end{aligned} \tag{1.13.33}$$

1.13.4 Method of Solution

The single-term Galerkin method is used in order to find the eigenvalues of the Eqs.(1.13.29)-(1.13.31). The basis functions of the variables takes the forms

$$W = \sum_{n=1}^{N} A_n W_n, \quad \Theta = \sum_{n=1}^{N} B_n \Theta_n, \quad \Phi = \sum_{n=1}^{N} C_n \Phi_n, \tag{1.13.34}$$

where A_n, B_n and C_n are constants. W_n, Θ_n and Φ_n are the trial functions that satisfied the boundary condition (1.13.32) and (1.13.33) respectively, where

i) for boundary conditions (1.13.32):

$$\begin{aligned} W_n &= \left(z^4 - \frac{5}{2}z^3 + \frac{3}{2}z^2\right)T_{n-1}, \\ \Theta_n &= \left(z - z^2\right)T_{n-1}, \\ \Phi_n &= \left(2z - z^2\right)T_{n-1}. \end{aligned} \tag{1.13.35}$$

ii) for boundary conditions (1.13.33):

$$\begin{aligned} W_n &= \left(z^4 - \frac{5}{2}z^3 + \frac{3}{2}z^2\right)T_{n-1}, \\ \Theta_n &= \left(z^2 - 2z\right)T_{n-1}, \\ \Phi_n &= \left(2z - z^2\right)T_{n-1}, \end{aligned} \tag{1.13.36}$$

where T_{n-1} is the Chebyshev polynomial of the first kind. For the single-term Galerkin method, we only consider $n = 1$. Therefore, we replaced $T_0 = 1$ into Eq.(1.13.35)-(1.13.36) and substitute the resulting equations into Eq.(1.13.34) to get

i) for boundary condition (1.13.32):

$$W = A_1\left(z^4 - \frac{5}{2}z^3 + \frac{3}{2}z^2\right), \quad \Theta = B_1\left(z - z^2\right), \quad \Phi = C_1\left(2z - z^2\right). \tag{1.13.37}$$

ii) for boundary condition (1.13.33):

$$W = A_1\left(z^4 - \frac{5}{2}z^3 + \frac{3}{2}z^2\right), \quad \Theta = B_1\left(z^2 - 2z\right), \quad \Phi = C_1\left(2z - z^2\right). \tag{1.13.38}$$

We multiplied Eq.(1.13.29) by W_1, Eq.(1.13.30) by Θ_1 and Eq.(1.13.31) by Φ_1. Then, the resulting equation are integrate by parts from $z = 0$ to $z = 1$, we have

$$W_1\left[\left(\frac{D^2}{\xi} - a^2\right)W_1 + a^2 Ra\Theta_1 - a^2 Ra_s \Phi_1\right] = 0,$$

$$\frac{1}{\xi} < W_1 D^2 W_1 > -a^2 < W_1^2 > + a^2 Ra < W_1 \Theta_1 > -a^2 Ra_s < W_1 \Phi_1 > = 0,$$
$$-\frac{1}{\xi} < (DW_1)^2 > -a^2 < W_1^2 > + a^2 Ra < W_1 \Theta_1 > -a^2 Ra_s < W_1 \Phi_1 > = 0,$$

$$\begin{aligned}
\Theta_1[(D^2 - \eta a^2)\Theta_1 + f(z)W_1] &= 0, \\
< \Theta_1 D^2 \Theta_1 > -\eta a^2 < \Theta_1^2 > + < f(z)\Theta_1 W_1 > &= 0, \\
- < (D\Theta_1)^2 > -\eta a^2 < \Theta_1^2 > + < f(z)\Theta_1 W_1 > &= 0, \\
< (D\Theta_1)^2 > +\eta a^2 < \Theta_1^2 > - < f(z)\Theta_1 W_1 > &= 0,
\end{aligned}$$

$$\begin{aligned}
\Phi_1[\frac{1}{Le}(D^2 - a^2)\Phi_1 + W_1] &= 0, \\
\frac{1}{Le} < \Phi_1 D^2 \Phi_1 > -\frac{a^2}{Le} < \Phi_1^2 > + < \Phi_1 W_1 > &= 0, \\
-\frac{1}{Le} < (D\Phi_1)^2 > -\frac{a^2}{Le} < \Phi_1^2 > + < \Phi_1 W_1 > &= 0, \\
\frac{1}{Le} < (D\Phi_1)^2 > +\frac{a^2}{Le} < \Phi_1^2 > - < \Phi_1 W_1 > &= 0,
\end{aligned}$$

where $< ... >$ represent the integration from $z = 0$ to $z = 1$.

We obtained the system of homogeneous algebraic equations in the forms

$$\begin{aligned}
A_1 C_{11} + B_1 D_{11} + C_1 E_{11} &= 0, \\
A_1 F_{11} + B_1 G_{11} &= 0, \\
A_1 H_{11} + C_1 I_{11} &= 0,
\end{aligned} \tag{1.13.39}$$

where the coefficient $C_{11} - I_{11}$ required the inner products of basis functions

$$\begin{aligned}
C_{11} &= -\frac{1}{\xi} < (DW_1)^2 > -a^2 < W_1^2 >, \\
D_{11} &= a^2 Ra < W_1 \cdot \Theta_1 >, \\
E_{11} &= -a^2 Ra_s < W_1 \cdot \Phi_1 >, \\
F_{11} &= - < f(z) \cdot \Theta_1 \cdot W_1 >, \\
G_{11} &= < (D\Theta_1)^2 > +\eta a^2 < \Theta_1^2 >, \\
H_{11} &= - < \Phi_1 \cdot W_1 >, \\
I_{11} &= \frac{1}{Le} < (D\Phi_1)^2 > +\frac{a^2}{Le} < \Phi_1^2 > .
\end{aligned} \tag{1.13.40}$$

Equation (1.13.39) can be written in the matrix form as

$$MX = 0, \tag{1.13.41}$$

where

$$M = \begin{bmatrix} C_{11} & D_{11} & E_{11} \\ F_{11} & G_{11} & 0 \\ H_{11} & 0 & I_{11} \end{bmatrix}, \ X = \begin{bmatrix} A_1 \\ B_1 \\ C_1 \end{bmatrix}.$$

Based on the invertible matrix theorem, Equation (1.13.41) has a nontrivial solution if and only if matrix M is not invertible, which means $|M| = 0$.

In this thesis, we further the study of convection in a binary fluid saturated an anisotropic porous medium done by (Malashetty and Swamy, 2010) by considering the effects of nonlinear temperature profile, magnetic field, internal heat generation and feedback control.

A linear and five nonlinear basic state temperature profile models that satisfied the condition stated in Eq.(1.13.10) are included as shown in Table 1.1 (Siddheshwar and Pranesh, 1998; Idris et al., 2009).

Table 1.1: Six models of basic state temperature profiles

Model	Basic state temperature profile	Critical Rayleigh number	$f(z)$
1	Linear	(Ra_{c1})	$f1 = 1$
2	Inverted parabola	(Ra_{c2})	$f2 = 2(1-z)$
3	Cubic 1	(Ra_{c3})	$f3 = 3(z-1)^2$
4	Cubic 2	(Ra_{c4})	$f4 = 0.6 + 1.2(z-1)^2$
5	Heating from below	(Ra_{c5})	$f5 = \varepsilon^{-1}$ for $0 \leq z < \varepsilon$
			$f5 = 0$ for $\varepsilon < z \leq 1$
6	Cooling from above	(Ra_{c6})	$f6 = 0$ for $0 \leq z < 1-\varepsilon$
			$f6 = \varepsilon^{-1}$ for $1-\varepsilon < z \leq 1$

By taking the parameters at $Ra_s = 10$, $Le = 5$, $\xi = 0.5$, $\eta = 0.3$, $H = 5$, $Q = 3$ and $K = 5$, we obtain the critical value of thermal depth, ε_c corresponding to the critical Rayleigh number, Ra_c as shown in Table 1.2.

Table 1.2: Critical value of thermal depth, ε_c. (a) lower rigid conducting and upper free conducting and (b) lower rigid conducting and upper free insulating.

Model	Boundary profiles	ε_c (a)	ε_c (b)
5	heating from below	0.77	0.87
6	cooling from above	0.66	0.55

1.14 Outline of Thesis

The thesis consist of seven chapters. Chapter 1 explained a brief introduction on the research area which include heat transfer, binary fluid, anisotropic porous medium, nonlinear temperature profile, magnetic field, internal heat generation, feedback control, applications of convection, motivation, problem statements and objectives of this studies, problem formulation and method of solution.

Chapter 2 discussed the literature review on the study of thermal convection, anisotropic porous medium, nonlinear temperature profile, magnetic field, internal heat generation and feedback control.

Chapter 3 investigated the onset of thermal convection in a binary fluid saturated an anisotropic porous medium in the presence of nonlinear temperature profile using linear stability analysis. We discussed the numerical solution obtained for this problem with respect to lower rigid conducting and upper free conducting plate and lower rigid conducting and upper free insulating plate. The effects of nonlinear temperature profile are discussed.

Chapter 4 studied the effect of nonlinear temperature profile on the onset of thermal convection in the binary fluid saturated an anisotropic porous medium in the presence of magnetic field using linear stability analysis. We performed the single-term Galerkin method on this problem by considering the lower rigid conducting and upper free conducting plate and lower rigid conducting and upper free insulating plate. We explained the effect of magnetic field on the system.

Chapter 5 investigated the effect of magnetic field and internal heat generation on the onset of thermal convection in the binary fluid saturated an anisotropic porous medium in the presence of nonlinear temperature profile using linear stability analysis. We solved the problem numerically using single-term Galerkin method by considering the lower rigid conducting and upper free conducting plate and lower rigid conducting and upper free insulating plate. We studied the influence of magnetic field and internal heat generation on the system.

Chapter 6 discussed the effect of nonlinear temperature profile on the onset of thermal convection in a binary fluid saturated an anisotropic porous medium in the presence of internal heat generation and feedback control using linear stability analysis. We discussed the effect of internal heat generation and feedback control by solving the problem using single-term Galerkin method with respect to lower rigid conducting and upper free insulating plate.

Chapter 7 gives the overall conclusion and the future research suggestion.

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