## On some specific patterns of $\boldsymbol{\tau}$-Adic non-adjacent form expansion over ring $\mathbf{Z}$ ( $\tau$ )


#### Abstract

Let $\tau=(-1)^{1-a}+\sqrt{ }-7 / 2$ for $\mathrm{a} \in\{0,1\}$ is Frobenius map from the set $\mathrm{E}_{\mathrm{a}}\left(\mathrm{F}_{2} m\right)$ to it self for a point ( $\mathrm{x}, \mathrm{y}$ ) on Koblitz curves $\mathrm{E}_{\mathrm{a}}$. Let P and Q be two points on this curves. $\tau$-adic Non-Adjacent Form (TNAF) of $\alpha$ an element of the ring $Z(\tau)=\{\alpha=c+d \tau \mid c, d \in Z\}$ is an expansion where the digits are generated by successively dividing $\alpha$ by $\tau$, allowing remainders of $-1,0$ or 1 . The implementation of TNAF as the multiplier of scalar multiplication $n P=Q$ is one of the technique in elliptical curve cryptography. In this study, we find the formulas for TNAF that have specific patterns $\left[0, \mathrm{c}_{1}, \ldots, \mathrm{c}_{1-1}\right],\left[-1, \mathrm{c}_{1}, \ldots, \mathrm{c}_{1-1}\right],\left[1, \mathrm{c}_{1}, \ldots, \mathrm{c}_{1-1}\right]$ and $\left[0,0,0, \mathrm{c}_{3}, \mathrm{c}_{4}, \ldots\right.$, $\left.c_{1-1}\right]$.


