UNIVERSITI PUTRA MALAYSIA

ELASTOHYDRODYNAMIC ANALYSIS OF ROLLING LINE CONTACT USING BOUNDARY ELEMENT METHOD

MAHMOUD HASSAN ONSA

ITMA 2001 7
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By

MAHMOUD HASSAN ONSA

Thesis Submitted in the Fulfilment of the Requirement for the Degree of Doctor of Philosophy in the Institute of Advanced Technology Universiti Putra Malaysia

April 2001
To the soul of my Father.

To my Mother, my Wife, A’Ayah and Iman.
Abstract of thesis presented to the senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Doctor of Philosophy.

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MAHMOUD HASSAN ONSA

April 2001

Chairman: Assoc. Prof. Dr. ShahNor Basri, P. Eng.

Faculty: Institute of Advanced Technology

This study aims at incorporating the use of the boundary element method (BEM) as an efficient and fast numerical method for the solution of the problem of the elastohydrodynamic (EHL) of hard rolling line contact. EHL of hard rolling is the dominant mode of lubrication in many critical, highly stressed machine elements such as gears, cams and followers, and bearings. The study of the stress concentration and deformation is important to predict the performance and the life expectancy against failures. These failures are manifested in wear, fatigue and scuffing. This fundamental study is based on isothermal, steady state, and smooth line contact EHL. The rolling of two cylindrical rollers was approximated by a roller and a plane.

The hard rolling EHL relates to counter-formal contact elements made of high elastic modulus materials such as metals. The problem is to seek a solution, which reconciles the hydrodynamic equation represented by the Reynolds equation, and the elasticity equation while at the same time allowing for the variation of the lubricant properties with pressure. The resultant regime is highly non-linear.
A hybrid solution is utilised to solve the elasticity problem using the BEM, and to solve the Reynolds equation for the pressure using the finite difference method (FDM) in a fully coupled solution. The BEM fundamentally consists of the transformation of the partial differential equations, which describe the behaviour of the variables inside and on the boundary of the domain into integral equation relating to the boundary values, and the numerical solution of these equations. The boundary integral equation is formulated for the elasticity and solved using the BEM. The hydrodynamic equation is solved using FDM. The coupled solution is solved using Newton-Raphson iterative technique. The converged solution gives the pressure distribution and the lubricant film thickness.

The overall result of executing the hybrid BEM-FDM program gives a full agreement when compared to the program using FDM while resulting in reduction in the CPU time. The results also agree with other published numerical works. These verify the use of the developed method. To fully utilize the advancement of the developed program, an extension of the models needs to include a non-Newtonian behaviour of lubricant and the thermal effects.
Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Doktor Falsafah.

ANALISIS HIDRODINAMIK-KENYAL BAGI SENTUHAN GARIS PENGGELEKKAN MENGGUNAKAN KAEDAH UNSUR SEMPADAN

Oleh

MAHMOUD HASSAN ONSA

April 2001

Pengerusi: Prof. Madya Dr. ShahNor Basri, P. Eng.

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Objektif ini adalah untuk menggabungkan penggunaan kaedah unsur sempadan (BEM) sebagai satu kaedah berangka yang berkesan dan cepat bagi penyelesaian masalah pelinciran hidrodinamik-kenyal (EHL), bagi sentuhan garisan penggelekan keras. EHL bagi penggelekan keras adalah ragam yang paling mustahak bagi pelinciran dalam banyak unsur mesin yang kritikal serta bertegasan tinggi seperti gear, sesondol dan pengikut, dan galas. Kajian bagi tumpuan tegasan dan ubah bentuk adalah penting bagi meramalkan prestasi dan jangkaan hayat menentang kegagalan-kegagalan. Kegagalan-kegagalan ini dibuktikan dalam haus, lesu dan penghauslakuran. Kajian asas ini berasaskan EHL sesuhu, keadaan mantap, dan sentuhan garis halus. Penggelekan bagi dua buah penggelek silinder telah dianggarkan oleh sebuah penggelek dan satu satah.

EHL penggelekan keras berhubung dengan unsur sentuhan melawanbentuk yang terdiri daripada bahan modulus kenyal tinggi seperti logam. Masalahnya ialah untuk mencari satu penyelesaian, yang menyesuaikan persamaan hidrodinamik
yang diwakili oleh persamaan Reynolds, dan persamaan kenyal, sementara pada waktu yang sama membenarkan perubahan sifat-sifat bahan pelincir dengan tekanan. Regim yang terhasil adalah tak-lelurus.

Satu penyelesaian hibrid telah digunakan bagi menyelesaikan masalah kekenyalan menggunakan BEM; dan menyelesaikan persamaan Reynolds bagi tekanan menggunakan kaedah perbezaan terhingga (FDM) dalam penyelesaian terganding sepenuhnya. BEM secara dasarnya mengandungi penjelasan bagi persamaan kebezaan separa yang menerangkan kelakuan bagi pembolehubah yang tidak diketahui di dalam dan di atas sempadan domain kepada persamaan kamiran berkaitan dengan nilai-nilai sempadan, dan penyelesaian berangka bagi persamaan ini. Persamaan kamiran sempadan ini telah dirumuskan bagi kekenyalan dan diselesaikan menggunakan BEM. Persamaan hidrodinamik telah diselesaikan menggunakan FDM. Penyelesaian terganding telah diselesaikan menggunakan teknik lelaran Newton-Raphson. Penyelesaian tertumpu memberikan tekanan dan ketebalan filem pelincir.

Keputusan-keputusan keseluruhan dalam melaksanakan aturcara hibrid BEM-FDM memberikan persetujuan sepenuhnya jika dibandingkan dengan aturcara menggunakan FDM, sementara menghasilkan pengurangan dalam masa CPU. Keputusan-keputusan ini juga menawarkan persetujuan yang baik dengan kerja-kerja berangka yang lain yang telah diterbitkan. Ini mengesahkan penggunaan kaedah yang telah dibangunkan. Untuk memanfaatkan sepenuhnya kerja penyelidikan ini, aturcara yang telah dibangunkan memerlukan tambahan dalam model dengan memasukkan kelakuan tak-Newtonian bagi pelincir dan kesan haba.
ACKNOWLEDGEMENTS

Praise be to ALLAH the ALMIGHTY in giving me the help, guidance and patience to complete this thesis.

I would like to express my sincere thanks to the chairman of the supervisory committee, Assoc. Prof. Dr. Ir. ShahNor Basri for his continuous help, support, friendship, hospitality and encouragement throughout this work. I am truly grateful to Dr. Mohd. Sapuan Salit, who is a member of the supervisory committee and served as the chairman during Dr. ShahNor’s sabbatical leave. I thank him for the brotherly handling of all matters concerning my research. I am also indebted to Dr. Megat Mohamad Hamdan Megat Ahmad for his technical advice and comments. I thank them all for their patience in reading, revising and commenting my thesis, and I thank their constructive advices and guidance.

I am most grateful to the University of Khartoum (Sudan) and the European Commission for the financial support conceded to my study. Thanks to Prof. David T. Gethin of University of Wales, Swansea for the guidelines given during his short visits to Malaysia. In addition, thanks are offered to Prof. H. P. Evans of University of Wales Cardiff, and Prof. Arto Lehtovaara of Tampere University of Technology, Finland for their valuable suggestions and comments.

I would like to thank the staff of Institute of Advanced Technology and the staff of Department of Aerospace Engineering, UPM for availing their computing facilities to conduct this work, and for not hesitating to give any needed resource.
I wish to extend my sincere gratitude to Dr. Mohamed Musadaq El Awad, who was a lecturer in the Department of Mechanical and Manufacturing Engineering, UPM for his invaluable suggestions in the early stages of the work. Great appreciation is also extended to all Sudanese, Libyans, Malaysians and other nationalities colleagues and good friends and to all those who contributed directly or indirectly to the success of this study, may ALLAH bless them all.

Last, but not least, I wish to express my gratefulness to my wife who sacrificed a lot to give me a good environment to finish this work, and my thanks are to my daughters, mother, brothers and sisters for their moral support and encouragement.
I certify that an Examination Committee met on 11th April 2001 to conduct the final examination of Mahmoud Hassan Onsa on his Doctor of Philosophy thesis entitled “Elastohydrodynamic Analysis of Rolling Line Contact Using Boundary Element Method” in accordance with Universiti Pertanian Malaysia (Higher Degree) Act 1980 and Universiti Pertanian Malaysia (Higher Degree) Regulations 1981. The Committee recommends that the candidate be awarded the relevant degree. Members of the examination committee are as follows:

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I hereby declare that the thesis is based on my original work except for quotations and citations, which have been duly acknowledged. I also declare that it has not been previously or concurrently submitted for any other degree at UPM or other institutions.

MAHMOUD HASSAN ONSA

Date: 17 April 2001
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NOMENCLATURE

\( a_i \) Weighting factor used in \( \frac{dP}{dX} \) formula at node \( i \)

\( b \) Hertzian half width

BEM Boundary element method

\( C_j \) Weighting factor used to integrate \( P \)

CPU Central processing unit

\( E \) Modulus of elasticity

\( E' \) Reduced modulus of elasticity

EHL Elastohydrodynamic lubrication

FDM Finite differance method

FEM Finite element method

\( G \) Dimensionless material property

\( G(X, Q) \) Weighting factor

\( G_{ij} \) G matrix

\( h \) Film thickness

\( h' \) Reynolds equation term = \( \rho h^2 / 12 \eta \)

\( h_{\text{min}} \) Minimum film thickness

\( H \) Dimensionless film thickness

\( H_{\text{min}} \) Dimensionless minimum film thickness

\( H_c \) Dimensionless central film thickness

\( H_{ij} \) H matrix

\( i, j, k \) Indexes

\( L \) Dimensionless material parameter, \( L = G (2U)^{1/4} \)

\( M \) Dimensionless load parameter, \( M = W (2U)^{-1/2} \)

\( N \) Number of nodes
Dimensionless pressure $= p/p_H$

Pressure $p$

Hertzian pressure $p_H$

Reduced radius $R, R'$

Radius of roller 1 $R_1$

Radius of roller 2 $R_2$

Effective radius in x and y direction $R_x, R_y$

Traction $t$

Traction matrix $T_y$

Average rolling velocity $U$

Dimensionless speed parameter $U$

Displacement, velocity $u$

Roller 1 velocity $U_1$

Roller 2 velocity $U_2$

Displacement matrix $U_y$

Dimensionless load $W$

Load per unit length $w$

Dimensionless coordinate $X$

$x$-coordinate along rolling direction $x$

$x, y, z$ coordinates

Coordinate X at end of calculation zone (exit) $X_{\text{end}}$

Coordinate X at node N $X_N$
Greek Symbols

\( \alpha \)  Roelands pressure-viscosity
\( \varepsilon \)  Infinitesimal distance from a point
\( \delta \)  Elastic deformation
\( \delta(x - a) \)  Dirac delta function
\( \delta_y \)  Dirac delta function
\( \Gamma \)  Boundary
\( \eta \)  Viscosity
\( \eta_0 \)  Viscosity at standard temperature and pressure
\( \eta_1, \eta_2 \)  Transformation coordinate system
\( \rho \)  Density
\( \zeta_0 \)  Point of under consideration
\( \zeta \)  Any point of calculation
\( \varphi \)  Reynolds equation term = \( \frac{d}{dx}[\rho hU] \)

Subscripts

1, 2  Rolling body 1 and 2
\( H, h \)  Hertzian
\( i, j, k \)  Indices
\( min \)  Minimum value
\( N \)  Node number N

Note: other symbols are defined in the text.
CHAPTER 1

INTRODUCTION

1.1 Overview

The main aim of this research is to use an efficient and fast numerical method for the solution of the challenging problem of the elastohydrodynamic of hard rolling line contact. The boundary element method (BEM) is utilised to achieve this goal. The investigations lead to two numerical models; one uses BEM and the other uses the finite difference method (FDM). The novel contribution of this study to the field of hard rolling elastohydrodynamic lubrication (EHL) is to incorporate the BEM to the numerical solution of the problem.

EHL is a form of fluid film lubrication where the elastic deformation of the lubricated bodies becomes significant. The elastic deformation together with the hydrodynamic behaviour of the lubricant film determine the regime. The EHL study describes the separation using a lubricant film between two elastic machine elements loaded against each other in a relative motion. The lubricant film is capable of reducing friction and wear between surfaces by the mechanism of separating the contact surfaces. Nevertheless, the elastic deformation is the prominent characteristic of EHL, as the materials under contact can have high modulus of elasticity that denote the hard EHL, as well as a low modulus value which in turn designates the soft EHL. Since the EHL of hard rolling is the dominant mode of lubrication in many critical, highly stressed machine elements, the study of the stress concentration and deformation is important to predict the performance and the life expectancy of the machine element. In these machine
elements such as gears, cams and followers and bearings, most of the failures are manifested in wear, fatigue and scuffing.

EHL is a sort of fluid film lubrication, which is related to the hydrodynamic lubrication by the common hydrodynamic action. The Hydrodynamic lubrication is generally characterized geometrically by the conformal contact, and its applications in journal and thrust bearings are well developed. The confirmation of theoretical prediction through experiments has led to satisfactory design procedures. The conformity between the bearing components enables a substantial load to be carried at a relatively small lubricant film pressure. EHL however, is characterized by a non-conformal contact, in which the load is carried out by a small lubricant footprint area at relatively high pressure.

The concepts of lubrication come to exist in the prehistoric period. When human invented the wheel and drove it in an axle, they found that it ran easier and faster when lubricated. It is likely that even in prehistory, men were interested in two aspects of friction. Firstly, man is interested in its effect in generating heat and then producing fire, and secondly, its effect in reducing the motion of his first cart. It is known that in the Pheronic Egypt, lubrication was used to facilitate the movement of sledges carrying large statues and building blocks of stone. Cowls Encyclopaedia of Science and Technology mentions that the chariots found in the Egyptian tombs have traces of lubricant, which upon analysis proved to be a mutton or beef tallow. Although lubrication is an old subject, it is still advancing with the advances in the machinery and industrial technologies, and even advances rapidly with the need to conserve energy and reduce wear.