

UNIVERSITI PUTRA MALAYSIA

ELASTOHYDRODYNAMIC ANALYSIS OF ROLLING LINE CONTACT USING BOUNDARY ELEMENT METHOD

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By

MAHMOUD HASSAN ONSA

Thesis Submitted in the Fulfilment of the Requirement for the Degree of Doctor of Philosophy in the Institute of Advanced Technology Universiti Putra Malaysia

April 2001



To the soul of my Father.

To my Mother, my Wife, A'Ayah and Iman.



Abstract of thesis presented to the senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Doctor of Philosophy.

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Chairman: Assoc. Prof. Dr. ShahNor Basri, P. Eng.

Faculty: Institute of Advanced Technology

This study aims at incorporating the use of the boundary element method (BEM) as an efficient and fast numerical method for the solution of the problem of the elastohydrodynamic (EHL) of hard rolling line contact. EHL of hard rolling is the dominant mode of lubrication in many critical, highly stressed machine elements such as gears, cams and followers, and bearings. The study of the stress concentration and deformation is important to predict the performance and the life expectancy against failures. These failures are manifested in wear, fatigue and scuffing. This fundamental study is based on isothermal, steady state, and smooth line contact EHL. The rolling of two cylindrical rollers was approximated by a roller and a plane.

The hard rolling EHL relates to counter-formal contact elements made of high elastic modulus materials such as metals. The problem is to seek a solution, which reconciles the hydrodynamic equation represented by the Reynolds equation, and the elasticity equation while at the same time allowing for the variation of the lubricant properties with pressure. The resultant regime is highly non-linear. A hybrid solution is utilised to solve the elasticity problem using the BEM, and to solve the Reynolds equation for the pressure using the finite difference method (FDM) in a fully coupled solution. The BEM fundamentally consists of the transformation of the partial differential equations, which describe the behaviour of the variables inside and on the boundary of the domain into integral equation relating to the boundary values, and the numerical solution of these equations. The boundary integral equation is formulated for the elasticity and solved using the BEM. The hydrodynamic equation is solved using FDM. The coupled solution is solved using Newton-Raphson iterative technique. The converged solution gives the pressure distibution and the lubricant film thickness.

The overall result of executing the hybrid BEM-FDM program gives a full agreement when compared to the program using FDM while resulting in reduction in the CPU time. The results also agree with other published numerical works. These verify the use of the developed method. To fully utilize the advancement of the developed program, an extension of the models needs to include a non-Newtonian behaviour of lubricant and the thermal effects.



ANALISIS HIDRODINAMIK-KENYAL BAGI SENTUHAN GARIS PENGGELEKKAN MENGGUNAKAN KAEDAH UNSUR SEMPADAN

Oleh

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Objektif ini adalah untuk menggabungkan penggunaan kaedah unsur sempadan (BEM) sebagai satu kaedah berangka yang berkesan dan cepat bagi penyelesaian masalah pelinciran hidrodinamik-kenyal (EHL), bagi sentuhan garisan penggelekan keras. EHL bagi penggelekan keras adalah ragam yang paling mustahak bagi pelinciran dalam banyak unsur mesin yang kritikal serta bertegasan tinggi seperti gear, sesondol dan pengikut, dan galas. Kajian bagi tumpuan tegasan dan ubah bentuk adalah penting bagi meramalkan prestasi dan jangkaan hayat menentang kegagalan-kegagalan. Kegagalan-kegagalan ini dibuktikan dalam haus, lesu dan penghauslakuran. Kajian asas ini berasaskan EHL sesuhu, keadaan mantap, dan sentuhan garis halus. Penggelekan bagi dua buah penggelek silinder telah dianggarkan oleh sebuah penggelek dan satu satah.

EHL penggelekan keras berhubung dengan unsur sentuhan melawanbentuk yang terdiri daripada bahan modulus kenyal tinggi seperti logam. Masalahnya ialah untuk mencari satu penyelesaian, yang menyesuaikan persamaan hidrodinamik



yang diwakili oleh persamaan Reynolds, dan persamaan kenyal, sementara pada waktu yang sama membenarkan perubahan sifat-sifat bahan pelincir dengan tekanan. Regim yang terhasil adalah tak-lelurus.

Satu penyelesaian hibrid telah digunakan bagi menyelesaikan masalah kekenyalan menggunakan BEM; dan menyelesaikan persamaan Reynolds bagi tekanan menggunakan kaedah perbezaan terhingga (FDM) dalam penyelesaian terganding sepenuhnya. BEM secara dasarnya mengandungi penjelmaan bagi persamaan kebezaan separa yang menerangkan kelakuan bagi pembolehubah yang tidak diketahui di dalam dan di atas sempadan domain kepada persamaan kamiran berkaitan dengan nilai-nilai sempadan, dan penyelesaian berangka bagi persamaan ini. Persamaan kamiran sempadan ini telah dirumuskan bagi kekenyalan dan diselesaikan menggunakan BEM. Persamaan hidrodinamik telah diselesaikan menggunakan teknik lelaran Newton-Raphson. Penyelesaian tertumpu memberikan tekanan dan ketebalan filem pelincir.

Keputusan-keputusan keseluruhan dalam melaksanakan aturcara hibrid BEM-FDM memberikan persetujuan sepenuhnya jika dibandingkan dengan aturcara menggunakan FDM, sementara menghasilkan pengurangan dalam masa CPU. Keputusan-keputusan ini juga menawarkan persetujuan yang baik dengan kerjakerja berangka yang lain yang telah diterbitkan. Ini mengesahkan penggunaan kaedah yang telah dibangunkan. Untuk memanfaatkan sepenuhnya kerja penyelidikan ini, aturcara yang telah dibangunkan memerlukan tambahan dalam model dengan memasukkan kelakuan tak-Newtonian bagi pelincir dan kesan haba.



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I certify that an Examination Committee met on 11th April 2001 to conduct the final examination of Mahmoud Hassan Onsa on his Doctor of Philosophy thesis entitled "Elastohydrodynamic Analysis of Rolling Line Contact Using Boundary Element Method" in accordance with Universiti Pertanian Malaysia (Higher Degree) Act 1980 and Universiti Pertanian Malaysia (Higher Degree) Regulations 1981. The Committee recommends that the candidate be awarded the relevant degree. Members of the examination committee are as follows:

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DECLARATION

I hereby declare that the thesis is based on my original work except for quotations and citations, which have been duly acknowledged. I also declare that it has not been previously or concurrently submitted for any other degree at UPM or other institutions.

MAHMOUD HASSAN ONSA

Date: 17 April 2001



TABLE OF CONTENTS

| DEDICATION | ii |
|------------------|------|
| ABSTRACT | iii |
| ABSTRAK | v |
| ACKNOWLEDGEMENTS | vii |
| APPROVAL | ix |
| DECLARATION | xi |
| LIST OF TABLES | XV |
| LIST OF FIGURES | xvii |
| NOMENCLATURE | XX |

CHAPTER

| 1 | INTE | RODUCTION | 1 |
|---|------|--|----|
| | 1.1 | Overview | 1 |
| | 1.2 | Lubrication Regimes | 4 |
| | 1.3 | The Elastohydrodynamic Lubrication (EHL) | 5 |
| | 1.4 | Types of Rolling Contacts | 7 |
| | 1.5 | Analysis of EHL Problem | 8 |
| | 1.6 | Boundary Element Methods | 10 |
| | 1.7 | Objectives | 11 |
| | 1.8 | Significance of this Study | 12 |
| | 1.9 | Structure of the Thesis | 12 |
| 2 | LITE | RATURE REVIEW | 14 |
| | 2.1 | Introduction | 14 |
| | 2.2 | Experimental EHL Film Thickness Measurements | 14 |
| | 2.3 | Theoretical EHL | 20 |
| | 2.4 | Numerical Solutions to EHL Lubrication | 24 |
| | | 2.4.1 Newton-Raphson Technique | 25 |
| | | 2.4.2 Multi-level or Multi-grid Techniques | 26 |
| | | 2.4.3 Finite Difference Method | 27 |
| | | 2.4.4 Finite Element Method | 28 |
| | | 2.4.5 Boundary Element Method | 29 |
| | 2.5 | BEM and the Contact Problem | 32 |
| | 2.6 | Line Contact EHL | 36 |
| | 2.7 | Isothermal and Thermal Elastohydrodynamic Lubrication | 36 |
| | 2.8 | Closure | 38 |
| 3 | THEO | DRY OF ELASTOHYDRODYNAMICS OF HARD ROLLING | 39 |
| | 3.1 | Introduction | 39 |
| | 3.2 | Reynolds Equation | 40 |
| | | 3.2.1 Assumptions Made for Reynolds Equation and Their | |
| | | Justifications | 41 |
| | | 3.2.2 Simplifications of Reynolds Equation | 42 |
| | 3.3 | Hard Rolling versus Soft Rolling EHL | 43 |
| | 3.4 | Hertzian Pressure Distribution | 44 |



| | 3.5 | Rolling Contact Geometry | 47 |
|---|------------|---|-----|
| | 3.6 | Rolling Contact Representation | 49 |
| | 3.7 | Lubricant Film Geometry | 53 |
| | 3.8 | Elastic Deformation Equation | 54 |
| | 3.9 | Lubricant Properties | 55 |
| | ••• | 3.9.1 Barus Pressure-Viscosity Relation | 55 |
| | | 3.9.2 Roelands Pressure-Viscosity Relation | 56 |
| | | 3.9.3 Mass Density Pressure Relation | 59 |
| | 3 10 | Dimensionless Parameters | 60 |
| | 3.11 | Closure | 63 |
| 4 | NUM | IERICAL SOLUTION | 64 |
| | 4.1 | Introduction | 64 |
| | 4.2 | Hybrid Solution | 65 |
| | 4.3 | BEM Solution | 66 |
| | | 4.3.1 Boundary Integral Equation Formulation | 67 |
| | | 4.3.2 Boundary Integral Equation | 68 |
| | | 4.3.3 Quadratic Isoparametric Elements | 69 |
| | | 4.3.4 Shape Functions | 69 |
| | 4.4 | Elasticity Equation Model I (BEM) | 71 |
| | 4.5 | Reynolds Equation | 72 |
| | | 4.5.1 Dirac Delta Function and the Fundamental Solution | 73 |
| | | 4.5.2 Discretization of Reynolds Equation | 74 |
| | | 4.5.3 Calculation of the Pressure Gradient | 76 |
| | 4.6 | Initial Film Thickness and Pressure Distribution | 78 |
| | 4.7 | Elasticity Equation Model II (FDM) | 79 |
| | 4.8 | Coupled Solution of the Elasticity and Reynolds Equations | 84 |
| | | 4.8.1 Partial Coupling | 84 |
| | | 4.8.2 Full Coupling | 86 |
| | | 4.8.3 Special Partial Coupling | 86 |
| | 4.9 | Newton-Raphson Technique | 86 |
| | 4.10 | Fluid Properties Functions | 90 |
| | | 4.10.1 Mass Density Pressure Relation | 90 |
| | | 4.10.2 Viscosity Pressure Relation | 90 |
| | 4.11 | Grid Generation | 91 |
| | | 4.11.1 Constant Element-size Mesh | 92 |
| | | 4.11.2 Multi-interval Mesh | 92 |
| | | 4.11.3 Adaptive Element-size Mesh | 93 |
| | 4.12 | Closure | 94 |
| 5 | COM | PUTER PROGRAM AND RESULTS | 95 |
| | 5.1 | Introduction | 95 |
| | 5.2 | Working Environment | 96 |
| | 5.3 | Working with Personal Computers | 97 |
| | | 5.3.1 Test I | 97 |
| | | 5.3.2 Test II | 98 |
| | . . | 5.3.3 Test III | 99 |
| | 5.4 | Main Program Layout | 100 |
| | | 5.4.1 Input Data | 101 |

| | | 5.4.2 Calculation of the Initial Pressure and Elastic | |
|------------|--------|---|-----|
| | | Deformation | 103 |
| | | 5.4.3 Calculation of the Active Coordinate Range | 104 |
| | | 5.4.4 Relaxation Criteria | 104 |
| | | 5.4.5 Newton-Raphson Iteration | 104 |
| | | 5.4.6 Solution of the Elasticity Equation using BEM | 104 |
| | 5.5 | Graphical Presentation | 105 |
| | 5.6 | Calculation of the CPU Time | 105 |
| | 5.7 | Effect of Number of Elements | 106 |
| | | 5.7.1 Constant Element-size Mesh | 107 |
| | | 5.7.2 Multi-interval Constant Element-size Mesh | 111 |
| | 5.8 | Effect of Viscosity Model | 113 |
| | 5.9 | Effect of Rolling Speed | 114 |
| | 5.10 | Numerical Problems Faced in the Solution | 116 |
| | | 5.9.1 Cavitations | 116 |
| | | 5.9.2 Stability | 117 |
| | 5.11 | Closure | 119 |
| 6 | COM | PARISONS AND DISCUSSIONS | 120 |
| | 6.1 | Introduction | 120 |
| | 6.2 | Experimental Work | 120 |
| | 6.3 | Comparisons between BEM-FDM Hybrid Model and FDM | |
| | | Solutions | 121 |
| | 6.4 | Comparison with Houpert and Hamrock Results | 125 |
| | 6.5 | Comparison with Hughes et al. Results | 127 |
| | 6.6 | Comparison with Lehtovaara Results | 129 |
| | 6.7 | Closure | 132 |
| 7 | CON | CLUSIONS AND RECOMMENDATIONS FOR FURTHER | |
| | WOR | К | 133 |
| | 7.1 | Conclusions | 133 |
| | 7.2 | Recommendations for Further Work | 134 |
| REFERENCES | | 136 | |
| APPE | NDIC | ES | 146 |
| A | Elasti | c Deformation Equation | 147 |
| В | Exam | ple of a Matlab Program for Plotting | 152 |
| VITA | | | 154 |



LIST OF TABLES

| Table | | Page |
|-------|--|------|
| 2.1 | Dimensionless parameters definitions and typical ranges | 23 |
| 2.2 | Typical range for the EHL hard rolling parameters (Gohar, 1988) | 24 |
| 2.3 | Calculation parameters and results for line contact (Gohar and Kuhn, 1994) | 34 |
| 3.1 | Example of typical values for Hertzian half width | 46 |
| 3.2 | Typical range for the EHL hard rolling parameters (Gohar, 1988) | 58 |
| 4.1 | Elasticity equation dimensionless parameters | 80 |
| 5.1 | Squaring 1.0000001 by itself twenty-seven times | 97 |
| 5.2 | Machine precisions for some microcomputers | 99 |
| 5.3 | Accuracy and speed of various working environments | 100 |
| 5.4 | Parameters of hard rolling used for the variation of the mesh configuration | 106 |
| 5.5 | The effect of mesh size in the calculation of the magnitude of the pressure spike and the maximum central pressure | 108 |
| 5.6 | Example of four interval constant-element-size mesh | 111 |
| 5.7 | Parameters of hard rolling used for the variation of the viscosity model | 113 |
| 5.8 | Parameters used for calculation of pressure and film thickness | 115 |
| 6.1 | Dimensional and dimensionless parameters for comparison between the BEM and FDM | 122 |
| 6.2 | Dimensional and dimensionless parameters for comparison between the BEM and FDM | 123 |
| 6.3 | Dimensionless variables for comparison with Houpert and Hamrock (1986) | 126 |
| 6.4 | Geometric and material parameters for comparison with Hughes et al. (1999) | 128 |
| 6.5 | Characteristic variables used by Lehtovaara (1998) | 130 |

| 6.6 | Example of calculation of the dimensionless parameters L and M | |
|-----|--|-----|
| | used by Lehtovaara (1998) | 131 |



LIST OF FIGURES

| Figur | e | Page |
|-------|---|------|
| 1.1 | Divisions of tribology from tribology to elastohydrodynamic lubricatio | n 3 |
| 1.2 | Generation of pressure by hydrodynamic wedge action (Summer-Smith, 1994) | 5 |
| 1.3 | Lubricated non-conformal surfaces of high-elastic-modulus material | 7 |
| 1.4 | Pure rolling contact of roller bearing | 8 |
| 1.5 | Rolling and sliding contact in two meshing spur gears (Wallach and Chowenhill, 1989) | 8 |
| 2.1 | Asperity contact in EHL | 16 |
| 2.2 | a. EHL line contact pressure distribution. b. EHL line contact film thickness for calculation parameter in Table 2.3 (Gohar and Kuhn, 1994) | 35 |
| 3.1 | Parameters of hydrodynamic lubrication | 42 |
| 3.2 | Hertzian contact and pressure distribution of two cylinders | 45 |
| 3.3 | Roller bearing as a representation of direct internal and external rolling | 47 |
| 3.4 | Profile of two involute gears teeth close to their contact | 48 |
| 3.5 | Rolling contact and the equivalent cylinder of reduced radius R and plane | 49 |
| 3.6 | External rolling cylinders representation | 50 |
| 3.7 | Internal rolling cylinders representation | 53 |
| 3.8 | Distortion terms of a distorted roller | 54 |
| 3.9 | Barus viscosity-pressure relation | 56 |
| 3.10 | Roelands viscosity-pressure relations for a typical lubricant | 58 |
| 3.11 | Ratio of η_B to η_R variation with pressure | 59 |
| 3.12 | Variation of density with pressure using equation [3.22] and Hirano et al. (1985) results | 60 |
| 4.1 | Quadratic isoparametric element | 70 |

| 4.2 | Co-ordinates system | 72 |
|------|---|-----|
| 4.3 | Dirac delta function $\delta(x-a)$ | 74 |
| 4.4 | Calculation of the elastic deformation | 81 |
| 4.5 | Geometric interpretation of Newton Raphson technique | 87 |
| 4.6 | Non-converging situation of Newton Raphson technique | 87 |
| 4.7 | Program flow chart | 89 |
| 4.8 | Nodes of the grid for a roller and plane presentation | 91 |
| 5.1 | Program squaring | 98 |
| 5.2 | Program accuracy | 98 |
| 5.3 | Program accuspeed | 99 |
| 5.4 | Detailed program structure | 102 |
| 5.5 | Elements and nodes for solution of the elasticity equation using hybrid model | 105 |
| 5.6 | Results of pressure and film thickness of not converged solution, (50 elements) | 108 |
| 5.7 | Pressure distribution and film thickness using 80 and 420 constant element-size meshes | 109 |
| 5.8 | Pressure distribution and film thickness using four constant element-size meshes | 110 |
| 5.9 | Pressure distribution and film thickness using four multi-interval constant element-size meshes | 112 |
| 5.10 | Pressure and film thickness using Barus and Roelands viscosity models | 114 |
| 5.11 | Dimensionless pressure distribution for rolling speeds equal to 0.0, 4, 14 and 24 m/s | 115 |
| 5.12 | Dimensionless film thickness for rolling speeds 0.0, 4, 14 and 24 m/s | 116 |
| 5.13 | Unstable behaviour happening near the pressure spike position | 118 |
| 6.1 | Comparison between BEM-FDM hybrid and FDM solutions for test data of Table 6.1 | 122 |



| 6.2 | Comparison between hybrid BEM-FDM and FDM models (Onsa et al., 2000a) | 123 |
|------|---|-----|
| 6.3 | Comparison between BEM-FDM hybrid and FDM solutions for test data of Table 6.2 | 124 |
| 6.4a | Comparison of pressure distribution with Houpert and Hamrock (1986) results using dimensionless variables in Table 6.3 | 126 |
| 6.4b | Comparison of film thickness with Houpert and Hamrock (1986) results using dimensionless variable in Table 6.3 | 127 |
| 6.5 | Comparison between the BEM-FDM solution and Hughes et al. (1999) results for 0.5 GPa maximum pressure | 129 |
| 6.6 | Comparison between the BEM-FDM solution and Hughes et al. (1999) results for 1.6 GPa maximum pressure | 129 |
| 6.7 | Lehtovaara (1998) pressure distribution for dimensionless material parameter (L = 10) and dimensionless load parameter (M = 2 (a), M = 5(b), M = 20(c) and M = 100(d) | 131 |
| 6.8 | Lehtovaara (1998) fluid film thickness for dimensionless material parameter (L = 10) and dimensionless load parameter (M = 2 (a), M = 5(b), M = 20(c) and M = 100(d) | 132 |
| A.1 | Plane cylindrical and Cartesian co-ordinates used for line-load, w acting on the boundary surface of elastic half-space, Tripp (1985) | 149 |
| A.2 | Line strip load in Cartesian co-ordinates | 151 |



NOMENCLATURE

- a_{ij} Weighting factor used in dP/dX formula at node i
- *b* Hertzian half width
- BEM Boundary element method
- C₁ Weighting factor used to integrate P
- CPU Centeral processing unit
- *E* Modulus of elasticity
- *E'* Reduced modulus of elasticity
- EHL Elastohydrodynamic lubrication
- FDM Finite differance method
- FEM Finite element method
- G Dinensionless material property
- $G(X, \zeta)$ Weighting factor
- G_y G matrix
- *h* Film thickness
- *h*' Reynolds equation term = $\rho h^3 / 12\eta$
- h_{min} Minimum film thickness
- *H* Dimensionless film thickness
- H_{min} Dimensionless minimum film thickness
- *H_c* Dimensionless central film thickness
- H_y H matrix
- i, j, k Indexes
- L Dimensionless material parameter, $L = G (2U)^{1/4}$
- M Dimensionless load parameter, $M = W (2U)^{-1/2}$
- *N* Number of nodes



- P Dimensionless pressure = p/p_H
- p Pressure
- p_H Hertzian pressure
- R, R' Reduced radius
- R_1 Radius of roller 1
- R_2 Radius of roller 2

 R_x , R_y Effective radius in x and y direction

- t Traction
- T_y Traction matrix
- U Average rolling velocity
- U Dimensionless speed parameter
- u Displacement, velocity
- U₁ Roller 1 velocity
- U₂ Roller 2 velocity
- U_y Displacement matrix
- W Dimensionless load
- w Load per unit length
- *X* Dimensionless coordinate
- x x-coordinate along rolling direction
- x, y, z x, y, z coordinates
- *X_{end}* Coordinate X at end of calculation zone (exit)
- X_N Coordinate X at node N



Greek Symbols

- α Roelands pressure-viscosity
- ε Infinitesimal distance from a point
- δ Elastic deformation

 $\delta(x-a)$ Dirac delta function

- δ_{ij} Dirac delta function
- **Γ** Boundary
- η Viscosity
- η_{o} Viscosity at standard temperature and pressure

 η_1, η_2 Transformation coordinate system

- ρ Density
- ζ_o Point of under consideration
- ζ Any point of calculation

$$\varphi$$
 Reynolds equation term = $\frac{d}{dx} [\rho h U]$

Subscripts

- *1*, *2* Rolling body 1 and 2
- H, h Hertzian
- i, j, k Indices
- min Minimum value
- N Node number N

Note: other symbols are defined in the text.



CHAPTER 1

INTRODUCTION

1.1 Overview

The main aim of this research is to use an efficient and fast numerical method for the solution of the challenging problem of the elastohydrodynamic of hard rolling line contact. The boundary element method (BEM) is utilised to achieve this goal. The investigations lead to two numerical models; one uses BEM and the other uses the finite difference method (FDM). The novel contribution of this study to the field of hard rolling elastohydrodynamic lubrication (EHL) is to incorporate the BEM to the numerical solution of the problem.

EHL is a form of fluid film lubrication where the elastic deformation of the lubricated bodies becomes significant. The elastic deformation together with the hydrodynamic behaviour of the lubricant film determine the regime. The EHL study describes the separation using a lubricant film between two elastic machine elements loaded against each other in a relative motion. The lubricant film is capable of reducing friction and wear between surfaces by the mechanism of separating the contact surfaces. Nevertheless, the elastic deformation is the prominent characteristic of EHL, as the materials under contact can have high modulus of elasticity that denote the hard EHL, as well as a low modulus value which in turn designates the soft EHL. Since the EHL of hard rolling is the dominant mode of lubrication in many critical, highly stressed machine elements, the study of the stress concentration and deformation is important to predict the performance and the life expectancy of the machine element. In these machine



elements such as gears, cams and followers and bearings, most of the failures are manifested in wear, fatigue and scuffing.

EHL is a sort of fluid film lubrication, which is related to the hydrodynamic lubrication by the common hydrodynamic action. The Hydrodynamic lubrication is generally characterized geometrically by the conformal contact, and its applications in journal and thrust bearings are well developed. The confirmation of theoretical prediction through experiments has led to satisfactory design procedures. The conformity between the bearing components enables a substantial load to be carried at a relatively small lubricant film pressure. EHL however, is characterized by a non-conformal contact, in which the load is carried out by a small lubricant foot print area at relatively high pressure.

The concepts of lubrication come to exist in the prehistoric period. When human invented the wheel and drove it in an axle, they found that it ran easier and faster when lubricated. It is likely that even in prehistory, men were interested in two aspects of friction. Firstly, man is interested in its effect in generating heat and then producing fire, and secondly, its effect in reducing the motion of his first cart. It is known that in the Pheronic Egypt, lubrication was used to facilitate the movement of sledges carrying large statues and building blocks of stone. Cowls Encyclopaedia of Science and Technology mentions that the chariots found in the Egyptian tombs have traces of lubricant, which upon analysis proved to be a mutton or beef tallow. Although lubrication is an old subject, it is still advancing with the advances in the machinery and industrial technologies, and even advances rapidly with the need to conserve energy and reduce wear.

