MAGNETOHYDRODYNAMICS BOUNDARY LAYER FLOW OF NON-NEWTONIAN FLUID OVER A PERMEABLE SHRINKING SURFACE

RUSYA IRYANTI BINTI YAHAYA

IPM 2019 11

MAGNETOHYDRODYNAMICS BOUNDARY LAYER FLOW OF NON-NEWTONIAN FLUID OVER A PERMEABLE SHRINKING SURFACE

By

RUSYA IRYANTI BINTI YAHAYA

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirements for the Degree of Master of Science

March 2019

DEDICATIONS

To my beloved
parents and family.
Thank you for everything.

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment
of the requirement for the degree of Master of Science

MAGNETOHYDRODYNAMICS BOUNDARY LAYER FLOW OF NON-NEWTONIAN FLUID OVER A PERMEABLE SHRINKING SURFACE

By

RUSYA IRYANTI BINTI YAHAYA

March 2019

Chairman : **Norihan Md Arifin, PhD**
Institute : **Institute For Mathematical Research**

Non-Newtonian fluids are fluids whose viscosities cannot be described by Newton's law of viscosity and are dependent on the shear rate. The magnetohydrodynamics (MHD) boundary layer flow of three types of non-Newtonian fluids, namely, the Carreau fluid, Casson fluid and micropolar fluid are studied in this thesis. These fluids flow over a permeable shrinking surface, with different boundary conditions considered for each fluid. The MHD flow of Carreau fluid over a non-linearly shrinking sheet with thermal radiation and convective boundary condition is studied as the first problem. Then, the MHD flow of Casson fluid near a stagnation point on a linearly shrinking sheet is considered as the second problem. The effects of slip and homogeneous-heterogeneous reactions are studied in this problem. Meanwhile, the third problem discussed the effects of thermal radiation on the MHD flow of micropolar fluid over an exponentially shrinking sheet. The governing partial differential equations of these problems are transformed into ordinary differential equations using the similarity transformations. Then, these equations are solved along the boundary conditions using a numerical method called the shooting method, with the computations done in the Maple software. The effects of various parameters on the flow, concentration and thermal fields of the fluids are discussed and presented in tables and graphs. At some values of the parameters, dual solutions are obtained. Therefore, stability analysis is performed to determine the significance of these solutions to the problems. The smallest eigenvalues for the first and second solutions are computed using the bvp4c solver in MATLAB. The first solution is found to have positive smallest eigenvalues, while the second solution has negative smallest eigenvalues. Thus, the first solution is stable and significant, whereas the second solution is unstable and less significant to the problems. The presence of a magnetic field and suction are observed to boost the velocity of the fluids but causes the temperature of

the fluids to drop. Besides that, the increase in these parameters enhances the concentration of reactants in the second problem and the microrotation of micropolar fluid in the third problem.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Master Sains

ALIRAN LAPISAN SEMPADAN MAGNETOHIDRODINAMIK BAGI BENDALIR BUKAN NEWTONAN TERHADAP PERMUKAAN TELAP MENGECUT

Oleh

RUSYA IRYANTI BINTI YAHAYA

Mac 2019

Pengerusi : Norihan Md Arifin, Ph.D.
Institut : Institut Penyelidikan Matematik

Bendalir bukan Newtonan ialah bendalir yang mana kelikatannya tidak dapat diperihalkan dengan hukum kelikatan Newton dan bergantung kepada kadar ricih. Aliran lapisan sempadan magnetohidrodinamik (MHD) bagi tiga jenis bendalir bukan Newtonan iaitu bendalir Carreau, bendalir Casson dan bendalir mikropolar telah dikaji di dalam tesis ini. Bendalir-bendalir ini mengalir terhadap permukaan telap mengecut dengan syarat-syarat sempadan yang berbeza bagi setiap bendalir. Aliran MHD bendalir Carreau terhadap helaian mengecut secara tak linear dengan radiasi terma dan syarat sempadan olakan telah dikaji sebagai masalah yang pertama. Kemudian, aliran MHD bendalir Casson berhampiran titik genangan di helaian mengecut secara linear telah dipertimbangkan sebagai masalah yang kedua. Dalam masalah ini, kesan gelincir dan tindak balas homogen-heterogen telah dikaji. Sementara itu, kesan radiasi terma pada aliran MHD bendalir mikropolar terhadap helaian mengecut secara eksponen telah dibincangkan sebagai masalah yang ketiga. Persamaan menakluk bagi masalah-masalah ini yang berbentuk persamaan pembezaan separa telah dijelmakan kepada persamaan pembezaan biasa dengan menggunakan penjelmaan keserupaan. Kemudian, persamaan ini diselesaikan bersama-sama syarat sempadan dengan menggunakan sebuah kaedah berangka yang dipanggil kaedah tembakan dengan pengiraannya dilakukan dalam perisian Maple. Kesan pelbagai parameter terhadap medan aliran, kepekatan dan terma bendalir telah dibincangkan dan ditunjukkan dalam bentuk jadual dan graf. Pada sesetengah nilai parameter, penyelesaian dual diperolehi. Oleh itu, analisis kestabilan telah dilakukan bagi menentukan kepentingan penyelesaian tersebut kepada masalah-masalah ini. Nilai-nilai eigen terkecil bagi penyelesaian pertama dan kedua telah dikira menggunakan solver bvp4c di MATLAB. Penyelesaian pertama didapati mempunyai nilai-nilai eigen terkecil positif, sementara penyelesaian kedua mempunyai nilai-nilai eigen terkecil negatif.

Oleh itu, penyelesaian pertama adalah stabil dan penting, manakala penyelesaian kedua adalah tidak stabil dan kurang penting kepada masalah-masalah ini. Kehadiran medan magnet dan sedutan didapati membantu meningkatkan halaju bendalir tetapi menyebabkan suhu bendalir menurun. Selain itu, peningkatan kedua-dua parameter ini telah meningkatkan kepekatan bahan tindak balas dalam masalah kedua dan putaran mikro bendalir mikropolar dalam masalah ketiga.

ACKNOWLEDGEMENTS

In the name of Allah, the Most Gracious, the Most Merciful

Alhamdulillah, all praises to Allah for giving me the strength and His blessings, with His permission I was able to complete this thesis. Special gratitude to my supervisor, Prof. Dr. Norihan Md Arifin for her guidance and continuous support **throughout my study. Her valuable comments and suggestions on my research works have contributed to the success of this thesis. Thank you for providing me with ideas, explanations and enlightenment whenever I ran into trouble or has questions on my research. I would also like to thank my co-supervisor, Dr. Siti Suzilliana Putri Mohamed Isa for her advice, comments and suggestions throughout the writing process of this thesis. Again, thanks to both my supervisor and co-supervisor for giving me the opportunity to gain new knowledge and experience by allowing me to participate in various workshops and conferences throughout my study period.**

Next, I would like to express my appreciation to the staffs of the Institute for Mathematical Research (INSPEM) and School of Graduate Studies (SGS), UPM for their assistance, services and facilities. My acknowledgement also goes to the Ministry of Education (MOE), Malaysia, for providing me the financial support through the **MyBrainSc scholarship. Thanks to these parties, I had a pleasant study journey.**

Finally, my deepest gratitude to my parents, Mr. Yahaya Bin Saman and Mrs. Rusmah Binti Adnan, as well as my siblings for their endless support, prayers and encouragements. Thank you so much for everything. Also, to my friends, it has been nice to know all of you. Thank you for all your help and for making my study in UPM more fun and enjoyable. Thank you.

This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfilment of the requirement for the degree of Master of Science. The **members of the Supervisory Committee were as follows:**

Norihan Md Arifin, PhD
Professor
Faculty of Science
Universiti Putra Malaysia
(Chairperson)

Siti Suzilliana Putri Mohamed Isa, PhD
Senior Lecturer
Centre of Foundation Studies For Agricultural Science
Universiti Putra Malaysia
(Member)

ROBIAH BINTI YUNUS, PhD
Professor and Dean
School of Graduate Studies
Universiti Putra Malaysia

Date:

Declaration by graduate student

I hereby confirm that:

- **this thesis is my original work**
- **quotations, illustrations and citations have been duly referenced**
- **this thesis has not been submitted previously or concurrently for any other degree at any other institutions**
- **intellectual property from the thesis and copyright of thesis are fully-owned by Universiti Putra Malaysia, as according to the Universiti Putra Malaysia (Research) Rules 2012**
- written permission must be obtained from supervisor and the office of Deputy **Vice-Chancellor (Research and Innovation) before thesis is published (in the form of written, printed or in electronic form) including books, journals, modules, proceedings, popular writings, seminar papers, manuscripts, posters, reports, lecture notes, learning modules or any other materials as stated in the Universiti Putra Malaysia (Research) Rules 2012**
- there is no plagiarism or data falsification/fabrication in the thesis, and scholarly **integrity is upheld as according to the Universiti Putra Malaysia (Graduate Studies) Rules 2003 (Revision 2012-2013) and the Universiti Putra Malaysia (Research) Rules 2012. The thesis has undergone plagiarism detection software.**

Signature:________________________ **Date:**________________________

Name and Matric No: Rusya Iryanti Binti Yahaya, GS49044

Declaration by Members of Supervisory Committee

This is to confirm that:

- **the research conducted and the writing of this thesis was under our supervision**
- **supervision responsibilities as stated in the Universiti Putra Malaysia (Graduate Studies) Rules 2003 (Revision 2012-2013) are adhered to.**

Signature: ______________________________
Name of Chairman of Supervisory Committee: ______________________________

Signature: ______________________________
Name of Member of Supervisory Committee: ______________________________

TABLE OF CONTENTS

Page

CHAPTER

LIST OF TABLES

Table **Page**

LIST OF FIGURES

LIST OF ABBREVIATIONS

MHD	Magnetohydrodynamics
bvp4c	Boundary value problem with fourth-order accuracy
a	Acceleration
a_1 a_2 a^{**} B_0 c v_0	Constants
b r	Constants related to shrinking speed
B	Strength of magnetic field
c_p	Specific heat
C_{r1} C_{r2}	Concentration of chemical species R_1 and R_2, respectively
C_{r0}	Uniform concentration of chemical species R_1
C_f	Local skin friction coefficient
D_{R_1} D_{R_2}	Diffusion coefficients for chemical species R_1 and R_2, respectively
F^*	Force
G^*	Slope
h_f	Convective heat transfer coefficient
j	Microinertia per unit mass
k_c k_s	Rate constants
k^*	Mean absorption coefficient
K	Strength of homogeneous reaction
K_s	Strength of heterogeneous reaction
K^*	Material parameter
L	Characteristic length
L^*	Velocity slip length parameter
m	Mass
m_x	Wall couple stress
M	Magnetic parameter
M_x	Local couple stress
M^*	Momentum
n	Power law index
n^*	Constant for concentration of microelements
N	Microrotation or angular velocity
N_R	Radiation parameter
Nu_x	Local Nusselt number
O	Initial point
O^*	Target point
p	Pressure
p_y	Yield stress
Pr	Prandtl number
q_r	Radiative heat flux
q_w	Wall heat transfer
R	Gas constant
R_1 R_2	Chemical species

R_m	**Magnetic Reynolds number**
Re	**Reynolds number**
s	Strength of stagnation flow
S	**Suction or mass transfer parameter**
Sc	**Schmidt number**
t	**Time**
T	**Fluid temperature**
T_f	Convective fluid temperature
T_w	**Temperature of the sheet**
T_0	**Rate of temperature increment along the sheet**
T	**Free stream temperature**
u v	**Velocity components in x− and y−directions, respectively**
u_e	Velocity of external flow
u_w	**Variable wall velocity or velocity of the sheet**
u_0	**Initial velocity**
U	**Characteristic velocity**
v_w	**Mass transfer velocity**
v^*	**Velocity**
We	**Weissenberg number**

Greek symbols

b	**Casson parameter**
d	Ratio of diffusion coefficients
d_l	**Boundary layer thickness**
d_t	**Thermal boundary layer thickness**
h	**Similarity variable**
G	**Material constant called relaxation time**
g	**Unknown eigenvalue**
k	**Vortex viscosity**
k^*	**Thermal conductivity**
l	**Shrinking parameter**
m	**Dynamic viscosity**
m_B	**Plastic dynamic viscosity**
m_0	**Permeability of free space**
n	**Kinematic viscosity**
p_c	**Critical value for the product of the component of the deformation rate with itself**
y	**Stream function**
r	**Density**
s	**Electrical conductivity**
s^*	**Stefan-Boltzmann constant**
t	**Shear stress**
t_w	**Wall shear stress**

t^*	**Dimensionless time variable**
q	**Dimensionless temperature**
q_w	**Temperature ratio parameter**
u	**Velocity slip parameter**
j	**Biot number**
x	**Magnetic diffusivity**
z	**Spin gradient viscosity**

Subscript

w	**Condition at the wall or surface of the sheet**
	Free stream condition

Superscript

0	**Differentiation with respect to h**

CHAPTER 1

INTRODUCTION

This chapter contains the introduction to some of the terms used in this thesis. A brief history and definitions of these terms are given to enhance the understanding of **the studied problems.**

1.1 Fluid

Fluid is any substance that deforms continuously under applied shear stress. Shear stress is tangential stress that acts along the surface of a body and is defined as **the ratio of applied force to the cross-sectional area of the body. The inability of** fluid to resist stress no matter how small the amount is, causes the fluid to deform continuously and flow. According to Çengel and Turner (2004), when a fluid is at **rest, the only stress that acts towards the surface of the body is the normal stress** called pressure. The supporting walls of the fluid, for example, a liquid container **helps to eliminate the shear stress. When the walls are removed, shear develops and** the fluid flows. The examples of fluids are liquids, gases and plasmas.

There are two types of fluid flow namely the laminar flow and the turbulent flow, as illustrated in Figure 1.1 and Figure 1.2 respectively. Laminar flow commonly occurs when a fluid with high viscosity flows slowly in a relatively small channel. In laminar flow, the fluid particles move in an orderly manner and parallel to the wall of the flow channel. The layers of the fluid slide parallel to each other without any disturbance or interference. However, the laminar flow can be transformed into a turbulent flow and vice versa by changing certain conditions, for example, the flow area.

The transition from laminar to turbulent flow was studied by Osborne Reynolds in **1880s. A dimensionless parameter called the Reynolds number was introduced to** identify the fluid flow as laminar or turbulent (Barnes, 2000). The Reynolds number can be defined as the ratio of inertial forces to viscous forces, and it depends on the characteristic length and the properties of the fluid such as the velocity, density, and viscosity. At low Reynolds number, the fluid flow is laminar. However, as the Reynolds number exceeds a certain value, the flow becomes turbulent.

Turbulent flow occurs when a fluid with high velocity flows rapidly through a large diameter channel. The flow is irregular and chaotic as the fluid layers slide past each other randomly and get mixed together. A fluid that is in turbulent flow contains high kinetic energy. Once the energy is dissipated, the flow becomes laminar again.

If the fluid properties of the flow, for example, the velocity, temperature, density and pressure does not depend on time, the flow is said to be steady. Otherwise, the flow is unsteady. The study on the flow of fluid is called fluid dynamics.

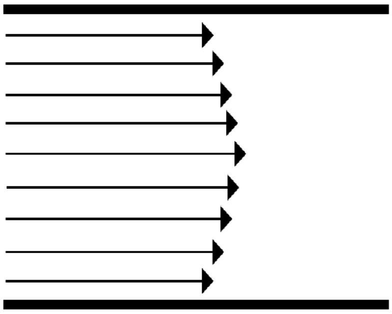

Figure 1.1. Laminar flow

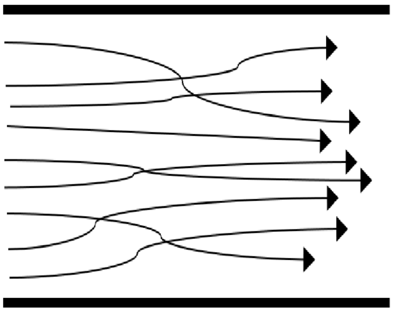

Figure 1.2. Turbulent flow

1.2 Non-Newtonian Fluid

Non-Newtonian fluid is fluid that does not obey Newton's law of viscosity. New**ton s law of viscosity states that the shear stress, t** between adjacent fluid layers is **directly proportional to the velocity gradients, dV dn between the two layers. This relationship can be written as follows:**

$$t \quad dV \; dn$$

where the velocity gradient dV dn represents the shear rate. The ratio of shear stress to shear rate is constant at a given temperature and pressure. The proportionality constant is defined as the coefficient of viscosity, **m and becomes**

$$t \quad m(dV \; dn):$$

A fluid that obeys this law is known as the Newtonian fluid. The viscosity of the Newtonian fluid is independent of the shear rate and constant under all flow conditions with constant temperature and pressure. The examples of Newtonian fluid are **water and organic solvents.**

On the other hand, the viscosity of non-Newtonian fluid depends on the shear rate **and in some instances, is time-dependent. According to Levenspiel (2014), there** are three classes of non-Newtonian fluids. The first class of non-Newtonian fluids is time-independent fluids or also known as the generalized Newtonian fluids (Chhabra, 2010). For this class of fluids, the shear rate only depends on the shear stress, if and only if the temperature of the fluids remains constant (Collyer, 1973). The main types of fluids that belong to this class are Bingham plastic fluids, pseudoplastic fluids and dilatant fluids. Bingham plastic fluid, for example, toothpaste exhibits yield stress **where an enough shear stress is needed to overcome the yield stress and makes the** fluid flow. Once the fluid flow, it behaves as a Newtonian fluid with the shear rate proportional to the shear stress. Next, the pseudoplastic fluid is also known as shear-thinning fluid where the apparent viscosity of this fluid decreases with increasing shear rate, for example in a paint. Conversely, the dilatant fluid is known as shear-thickening fluid where the apparent viscosity of this fluid increases with increasing shear rate, for example in a quicksand.

The next class of non-Newtonian fluid is time-dependent fluids where the behaviour depends on the history of treatment of the fluids. This class of fluids is divided into two subgroups, namely the thixotropic fluids and the rheopectic fluids. At a constant shear rate, the viscosity of a thixotropic fluid decreases with time while for rheopectic fluid, the viscosity increases with time. The examples for the thixotropic fluid are drilling fluids and printing inks while for the rheopectic fluid is bentonite clay.

Finally, the third class of non-Newtonian fluid is viscoelastic fluids. According to Irgens (2014), viscoelastic materials can be classified either as solids or fluids. The reason is that the materials exhibit both the elastic properties of solids and the flow behaviour of fluids. These fluids will deform and flow when subjected to stress. Examples of viscoelastic fluids are gels and bitumen.

In this thesis, the modelled fluids in the problems are Carreau fluid, Casson fluid and micropolar fluid. Carreau fluid and Casson fluid are time-independent non-Newtonian fluids. Carreau fluid has the behaviour of Newtonian fluid at a low shear rate and as power-law fluid at a high shear rate. Power-law fluid is characterized by the power-law index, n. At n 1, the fluid has the behaviour of a Newtonian fluid, when n 1, the fluid behaves as shear-thinning fluid and when n 1, the fluid behaves as shear-thickening fluid. Meanwhile, Casson fluid is a shear-thinning fluid that exhibits yield stress. The fluid has an infinite viscosity at zero rate of shear, no flow occurs below the yield stress and zero viscosity at an infinite rate of shear. As for micropolar fluid, it is a class of non-Newtonian fluids described by the micropolar theories. Lukaszewicz (1999) stated that micropolar fluid is fluid with microstructures. The microstructures have their own internal angular momentum that is independent of the fluid motion.

1.3 Boundary Layer

A boundary layer is a thin layer of viscous fluid, formed near the surface of an object that is in contact with the moving stream of a fluid. Boundary layer theory was first introduced by Ludwig Prandtl, in a paper titled On the Motion of a Fluid with Very Small Viscosity". This paper was first presented at the Third International Congress of Mathematicians in 1904 at Heidelberg and then published in the year after (Tani, 1977). Generally, this paper is concerned with the flow and behaviour of a fluid with small viscosity near the wall of a solid boundary. It was found that the smaller the viscosity, the thinner the boundary layer.

The formation of the boundary layer is as illustrated in Figure 1.3. In this figure, a fluid is assumed to flow with an initial velocity, **u_0 towards a stationary solid boundary located along the x**-axis. As the fluid flows, the fluid molecules adjacent to the **solid boundary adheres to the solid surface and becomes stationary, provided that a no-slip condition is assumed. This occurs due to the frictional force between the** solid surface and the fluid. The viscosity or the resistance of fluid towards flow causes the fluid molecules near the solid surface to slow down as it collides with the stationary fluid molecules attached on the surface of the solid boundary. The velocity of the fluid decreases and this effect persisted throughout the region near the solid boundary. At some distance, the flow of the fluid is not affected by the **collisions anymore. This region is known as the free stream region where the effect** of viscosity is negligible, and the fluid velocity is the same as the initial velocity, **u_0. Meanwhile, the region at which the velocity gradient ranges from zero to 99 of the free stream velocity is known as the boundary layer. The thickness of the boundary layer is given by d_l.**

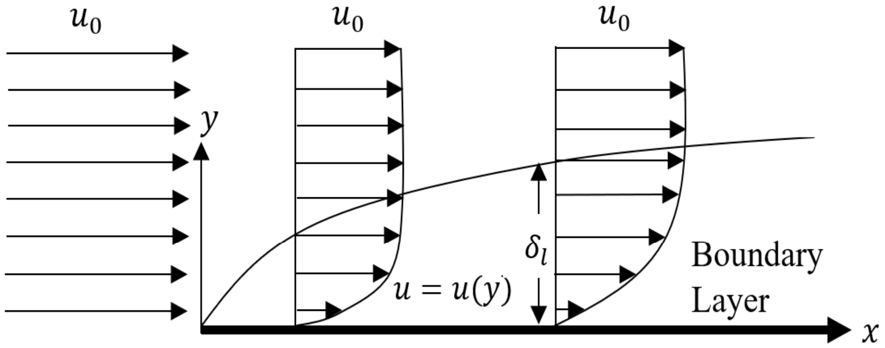

Figure 1.3. The formation of boundary layer

In Figure 1.4, the formation of the thermal boundary layer is shown. Now, the fluid is assumed to flow with a temperature of **T , while the solid boundary has a temperature of T_w. Since the temperature T_w T , heat is transferred from the solid surface to** the fluid and causes the fluid temperature to increase. This effect persists throughout the fluid layers due to heat transfer. However, the farther the fluid layer from the **solid surface, the smaller the amount of heat transferred to the layer. Thus, at some** distance from the solid boundary, the temperature of the fluid does not change and is **equal to the free stream temperature, T . Therefore, the thermal boundary layer can** be defined as the region from the solid surface to the point at which the temperature **is 99 of the free stream temperature. The thickness of the thermal boundary layer is given by d_t.**

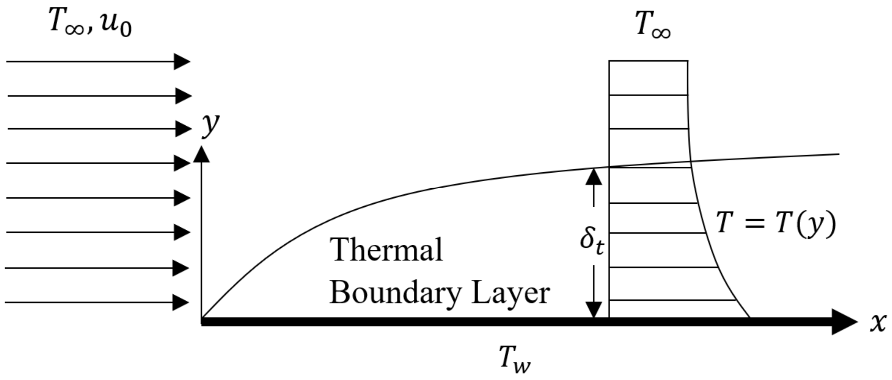

Figure 1.4. The formation of thermal boundary layer

1.4 Stagnation Point

Stagnation point exists on the surface of a solid boundary. At the stagnation point, the fluid flow is brought to rest by the solid boundary which causes the fluid velocity to become zero. Besides that, the pressure, heat transfer and mass deposition rates are said to be the highest at this point (Wang, 2008). According to Drazin and Riley (2006), the stagnation point is the point where the fluid stream attaches to or separates from the solid boundary. As a steady viscous fluid with a velocity of $\mathbf{u_e}$ flows towards a stationary solid boundary, the fluid stream then hits a point on the surface of the boundary and is brought to rest by it. This point is known as the stagnation point. The fluid then divides about the stagnation streamline, as shown in Figure 1.5. Above the stagnation streamline, the fluid flows to the higher region of the solid boundary while below the stagnation streamline, the fluid flows to the lower region of the solid boundary. The flow of fluid near the stagnation point is called the stagnation point flow (Batchelor, 2000).

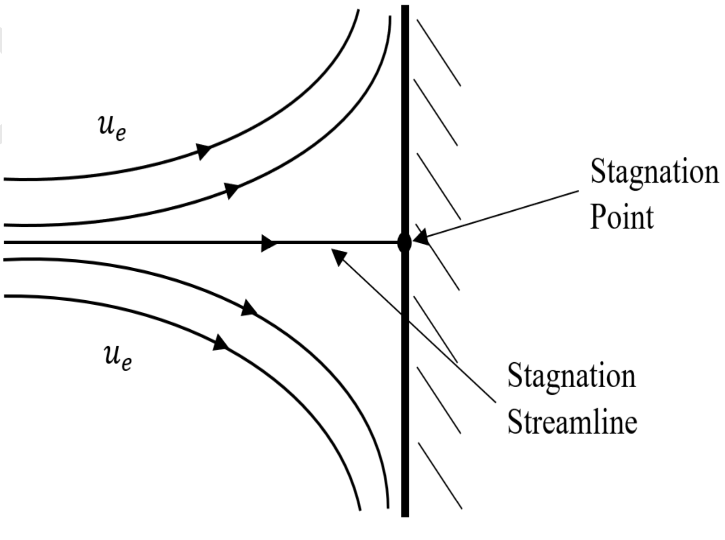

Figure 1.5. Stagnation point

1.5 Types of Effects

In the study of boundary layer flow, various types of effects such as magnetohy**drodynamics, thermal radiation, chemical reactions and slip are considered. The** influence of these effects on the flow, thermal and concentration fields of the fluids **can be analysed by introducing these effects into the governing equations or boundary** conditions of the flow problem. Some of the effects considered in this thesis are **magnetohydrodynamics, thermal radiation, homogeneous-heterogeneous reactions and slip. A brief explanation about these effects is given in the following subsections.**

1.5.1 Magnetohydrodynamics

Magnetohydrodynamics (MHD) studies the flow of an electrically conducting fluid in the presence of a magnetic field. According to Roberts (1967), MHD is also known as hydromagnetics or magneto-fluid dynamics. The study on MHD flow was established during the late 1930s or early 1940s. In 1942, an important finding called **the Alfvén wave was made, which led to the growth of MHD. According to Cramer** (2001), this finding proved that electromagnetic waves can propagate in conducting fluids. Some of the examples on the applications of MHD are in thermonuclear **fusion, electromagnetic pump and electromagnetic stirring.**

In MHD, the flow of conducting fluid across the magnetic field creates a potential difference that causes electric currents to flow. According to Davidson (2001), the induced electric currents will give rise to a second magnetic field called the induced magnetic field. Then, this leads to the change such that the fluid appears to ‘drag’ the magnetic field lines along with it. Besides that, the interaction will also give rise to Lorentz forces that inhibit the relative movement of the magnetic field and the fluid.

One of the important dimensionless parameter in MHD is the magnetic Reynolds number, R_m which can be defined as

$$R_m \quad \frac{UL}{x} \quad ULm_0 s$$

where U is the characteristic velocity, L is the characteristic length and x $\frac{1}{m_0 s}$ is the magnetic diffusivity with m_0 as the permeability of free space and s as the electrical conductivity. The magnetic Reynolds number gives an estimation of the relative effects of magnetic induction to magnetic diffusion. If R_m is very large, then the diffusion of the magnetic field can be neglected. The magnetic field lines become ‘frozen’ into the conducting fluid and move along with the fluid. Meanwhile, when **R_m is very small, the magnetic induction can be neglected as the magnetic diffusion** dominates over the induction of the magnetic field.

1.5.2 Heat Transfer

Heat is a form of energy that can be transferred from one medium to another, provided that there is a temperature difference between the mediums. The heat energy is transferred from a medium with a higher temperature to the ones with a lower temperature. This process continues until an equilibrium is achieved within these mediums. As an example, consider a situation where a warm drink is placed in a refrigerator, the warm drink then cools down. The reason is that heat from the warm drink is transferred to the cold surroundings of the refrigerator. The transfer of heat continues until both mediums reach the same temperature.

The study on the exchange of heat energy between mediums that occurs due to a temperature difference is known as the heat transfer. The rate of heat transfer is defined as the amount of heat being transferred per unit time, and it depends on the magnitude of the temperature gradient. The temperature gradient or the rate of change of temperature is the difference in temperature per unit length. The increase in temperature gradient will increase the rate of heat transfer.

Heat can be transferred from one medium to another by conduction, convection and radiation. Conduction takes place in solids and fluids. In conduction, the transfer of energy occurs within a substance or across substances which are in physical contact. The energy is transferred from the more energetic particles to the less energetic particles through the particles interaction. In fluids, conduction may occur through collisions and diffusions of the molecules during the random motion. Whereas in solids, conduction occurs through the vibrations of the molecules and the energy transported by the free electrons. The rate of heat conduction depends on the temperature difference between the mediums. Besides that, it also depends on the geometry of the medium, for example, the thickness and the material of the medium.

The next mode of heat transfer is convection, which involves both solids and fluids. In convection, the transfer of energy occurs between a solid surface and an adjacent moving fluid through the bulk motion of the fluid. As an example, consider a cold air blown towards a hot block. First, the heat is transferred from the surface of the block to the adjacent air layer through conduction. Then, convection occurs within the air where the heated air near the surface moves up while the cold air sinks down. Thus, both the effects of conduction and fluid motion are involved in convection. The faster the fluid motion, the higher the rate of convection heat transfer. If the fluid is not in motion or static, then the only form of heat transfer between the fluid and the solid surface is conduction. Besides that, the heat transfer within a fluid that causes phase change is also considered as convection, for example, boiling that involves the transition from a liquid phase to the gas phase. There are two types of convection which are free convection and forced convection. Free convection or natural convection is solely due to the density difference caused by the temperature difference in the fluid. The hot and less dense fluid moves up while the cold and denser fluid moves down, and this brings about the bulk fluid movement. If the fluid movement is due to external force, for example, from fan, pump, wind or compressor, then the convection is called forced convection. The combination of free convection and forced convection which significantly and simultaneously contribute to the heat transfer is called mixed convection (Dawood et al., 2015).

Radiation is the energy that is emitted by a matter in the form of electromagnetic waves or photons. The emission occurs due to the molecular and atomic agitation associated with the internal energy of the matter (Siegel and Howell, 1992). Radiation that is related to heat transfer and temperature is known as thermal radiation. Thermal radiation can occur through solids and fluids. Any matter with a temperature above absolute zero emits thermal radiation. The absence of medium does not hinder thermal radiation because the photons which transport the emitted energy can travel through a vacuum. The importance of thermal radiation is for solar energy technologies and combustion chamber.

Finally, all modes of heat transfer can occur in a fluid, but not all can happen simultaneously. A static fluid may have heat transfer through conduction and radiation while a moving fluid may have convection and radiation. Thus, convection and conduction may occur in fluid but not simultaneously.

1.5.3 Homogeneous-Heterogeneous Reactions

Any chemical reactions with reactants and products that are in the same phase (solid, liquid or gaseous) are known as homogeneous reaction, for example, the reaction between liquids. In fluid flow, this reaction usually occurs in the bulk of the fluid. Meanwhile, a heterogeneous reaction is any chemical reactions with different phases of reactants, such as the reaction between solid and liquid, and also the reaction on catalytic surfaces. In fluid flow over a solid boundary, the heterogeneous reaction usually occurs at the surface of the solid boundary.

1.5.4 Slip Boundary Condition

No-slip and slip boundary conditions are commonly considered in the flow velocity of a fluid. In the case of no-slip boundary condition or also known as the no-velocity-offset boundary condition, the fluid layer that is in contact with the solid boundary is assumed to have equal velocity as the boundary (Rapp, 2017). Thus, the relative movement between the boundary and the fluid layer is zero. However, there is relative movement between the boundary and the fluid layer in the slip boundary condition, or also known as the velocity-offset boundary condition. Therefore, the velocity of the fluid layer is unequal to the boundary. The slip boundary condition

can be written as:

$$u_w = L^* \frac{\partial u}{\partial y}\bigg|_w$$

where L^* is the slip length which describes the hypothetical distance inside the boundary at which the flow velocity would reach the velocity of the boundary, as **shown in Figure 1.6.**

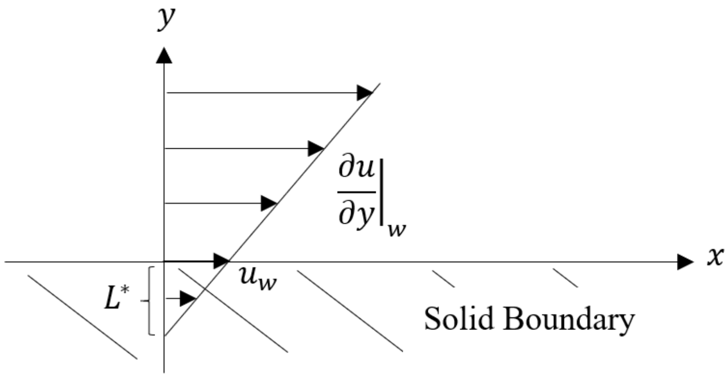

Figure 1.6. Slip boundary condition

1.6 Shrinking Surface

In 1961, Sakiadis (1961a) did a study on the boundary layer flow due to a moving continuous solid surface. This flow usually occurred in the manufacturing of polymer sheet **where the sheet is being issued continuously from a slit. In this process, the sheet is sometimes stretched to obtain the desired thickness, and this process needs to be controlled. In the study by Sakiadis (1961b), the continuous surface was considered to move with a constant velocity of $u_w = u_0$. In addition, the surface can also be considered moving with a variable velocity, $u_w = u_w(x)$. The variable wall velocity can also be called the stretching or shrinking wall velocity, depending on the sign of $u_w(x)$. The sign of $u_w(x)$ is positive for stretching surface while negative for shrinking surface.**

The first study on fluid flow towards a contracting or shrinking surface was by Wang **(1990). Then, Miklavčic** and Wang (2006) considered the flow over a shrinking sheet where the sheet velocity is towards a fixed point. In this study, the vorticity of the shrinking sheet is found to be unconfined within the boundary layer. Therefore, the flow on a shrinking sheet is unlikely to exist, unless an adequate amount of suction is imposed on the surface of the sheet. The imposition of suction causes the fluid near **the sheet to be sucked out through the sheet surface. Thus, the sheet surface must be porous or permeable to enable suction.**

1.7 Stability Analysis

Stability analysis is a method used in determining the stability of solutions. In a problem with more than one solution, stability analysis is carried out to identify which solutions are stable and unstable. The stable solution is physically meaningful and realisable in practice, while the unstable solution is insignificant to the problem.

Stability analysis was first performed by Merkin (1986). In the study, two solutions were obtained and divided into the upper branch solution and the lower branch solution. Before conducting the stability analysis, the problem was first considered as an unsteady problem or also known as the time-dependent problem. Then, a linear eigenvalue problem was introduced, with g as the unknown eigenvalue. The smallest value of g was computed using numerical computation. The results revealed that the upper branch solutions had positive values of g, while the lower branch solutions had negative values of g. A conclusion was made that the upper branch solution was stable while the lower branch solution was unstable.

Therefore, stability analysis is important especially for problems with more than one solution. This analysis helps to identify the stable solution which is significant to the problem.

1.8 Problem Statement

The non-Newtonian fluids have various applications in industrial and engineering processes. Hence, attracts researchers to performed multiple studies on the boundary layer flow of this type of fluids, especially the Carreau fluid, Casson fluid and micropolar fluid. In these studies, a variety of conditions are considered in the fluid flow, for example, the slip effect, suction, thermal radiation, mixed convection and convective boundary condition. However, the following issues are found:

1. Only certain combinations of the conditions are discussed in one s study, for example, the combination of suction and thermal radiation. Other conditions may be overlooked or left out by the researchers for others to study.
2. In consequences, a literature gap is created.

In response to this problem, we will discuss the effects of some of the overlooked conditions to the flow of Carreau fluid, Casson fluid and micropolar fluid, with the hope of filling the existing literature gap. These conditions will be introduced as additional parameters in the governing equations and boundary conditions of the fluids. Then, a numerical method called the shooting method will be used to solve these equations subject to the boundary conditions. In this thesis, stability analysis will be carried out to determine the stability and significance of the solutions to the problems. Thus, eases the process of choosing a physically meaningful solution, where the effects of the additional parameters on the fluids can be analysed by observing the behaviour of this solution.

1.9 Objectives and Scope of Study

The objective of this thesis is to analyse the mathematical model of non-Newtonian fluid for the following problems:

1. MHD flow of Carreau fluid over a non-linearly shrinking sheet with suction, thermal radiation and convective boundary condition.
2. MHD flow of Casson fluid over a linearly shrinking sheet with slip, suction and homogeneous-heterogeneous reactions.
3. MHD flow of micropolar fluid over an exponentially shrinking sheet with suction and thermal radiation.

The first problem extends the study by Khan et al. (2016b) to the case of a non-linearly shrinking sheet with suction, while the second problem extends the study by Sheikh and Abbas (2017) to the case of MHD flow over a linearly shrinking sheet. In the third problem, the study by Aurangzaib et al. (2016b) is extended to the case of MHD flow over an exponentially shrinking sheet with thermal radiation. In addition, stability analysis will be done in these problems.

The scope of the study is limited to the steady, incompressible and two-dimensional MHD fluid flow over a permeable shrinking sheet. Besides that, the magnetic Reynolds number in each of the problems is assumed to be very small. This assumption is made to neglect the effect of the induced magnetic field.

1.10 Outline of the Thesis

There are seven chapters in this thesis. The first chapter, Chapter 1 is the introduction. In this chapter, some brief definition and history are given for the reader to have a better understanding of the terms used in this thesis. Besides that, the objectives and the thesis outline are also described in the first chapter to provide the reader with some ideas about the thesis.

Chapter 2 comprises the literature reviews related to the studied problems. The previous studies done by other researchers on Carreau fluid, Casson fluid and micropolar fluid are summarised and discussed in this chapter. Then, the general mathematical formulation of the studied problems and the numerical methods used in these problems are explained in Chapter 3.

Next, Chapter 4 to Chapter 6 present a detailed discussion on the three problems listed in the previous section. Chapter 4 discussed the MHD boundary layer flow of Carreau fluid over a non-linearly shrinking sheet in the presence of thermal radiation, suction and convective boundary condition. Meanwhile, Chapter 5 explained the MHD flow of Casson fluid over a linearly shrinking sheet with slip, suction and homogeneous-heterogeneous reactions. The MHD boundary layer flow of micropolar fluid over an exponentially shrinking sheet with suction and thermal radiation is discussed in Chapter 6. In general, these chapters contain an introduction section, followed by the mathematical analysis and stability analysis sections, the results and discussion section and finally, the conclusions section. The summary of this thesis is made in the last chapter, Chapter 7 and some ideas on the possible future work are included in this chapter.

REFERENCES

Abd El-Aziz, M. and Afify, A. A. (2018). Influences of slip velocity and induced magnetic field on MHD stagnation-point flow and heat transfer of casson fluid over a stretching sheet. Mathematical Problems in Engineering, 2018.

Akbar, N. S. and Nadeem, S. (2014). Carreau fluid model for blood flow through a tapered artery with a stenosis. Ain Shams Engineering Journal, 5(4):1307 1316.

Akbar, N. S., Nadeem, S., Haq, R. U., and Ye, S. (2014). MHD stagnation point flow of carreau fluid toward a permeable shrinking sheet: Dual solutions. Ain Shams Engineering Journal, 5(4):1233 1239.

Ali, F. M., Nazar, R., Arifin, N. M., and Pop, I. (2013). Dual solutions in MHD flow on a nonlinear porous shrinking sheet in a viscous fluid. Boundary Value Problems, 2013(1):32.

Anderson, J. D. (1995). Computational Fluid Dynamics: The Basics with Applications. McGraw-Hill international editions. McGraw-Hill, New York.

Arifin, N. M., Yusof, S. N., and Ismail, N. S. (2017). Slip effects on MHD stagnation-point flow of carreau fluid past a permeable shrinking sheet. MATTER: International Journal of Science and Technology, 3(2).

Ashraf, M. B., Hayat, T., and Alsaedi, A. (2017). Mixed convection flow of casson fluid over a stretching sheet with convective boundary conditions and hall effect. Boundary Value Problems, 2017(1):137.

Aurangzaib, Bhattacharyya, K., and Shafie, S. (2016a). Effect of partial slip on an unsteady MHD mixed convection stagnation-point flow of a micropolar fluid towards a permeable shrinking sheet. Alexandria Engineering Journal, 55(2):1285 1293.

Aurangzaib, Uddin, M. S., Bhattacharyya, K., and Shafie, S. (2016b). Micropolar fluid flow and heat transfer over an exponentially permeable shrinking sheet. Propulsion and Power Research, 5(4):310 317.

Awaludin, I. S., Weidman, P. D., and Ishak, A. (2016). Stability analysis of stagnation-point flow over a stretching/shrinking sheet. AIP Advances, 6(4).

Baag, S., Mishra, S. R., Dash, G. C., and Acharya, M. R. (2017). Numerical investigation on MHD micropolar fluid flow toward a stagnation point on a vertical surface with heat source and chemical reaction. Journal of King Saud University - Engineering Sciences, 29(1):75 83.

Barnes, H. A. (2000). A Handbook of Elementary Rheology. University of Wales, Institute of Non-Newtonian Fluid Mechanics, Aberystwyth.

Batchelor, G. K. (2000). An Introduction to Fluid Dynamics. Cambridge Mathematical Library. Cambridge University Press, Cambridge.

Bhattacharyya, K. (2011a). Boundary layer flow and heat transfer over an exponentially shrinking sheet. Chinese Physics Letters, 28(7):074701.

Bhattacharyya, K. (2011b). Dual solutions in boundary layer stagnation-point flow and mass transfer with chemical reaction past a stretching/shrinking sheet. International Communications in Heat and Mass Transfer, 38(7):917 922.

Bhattacharyya, K. (2013a). Boundary layer stagnation-point flow of casson fluid and heat transfer towards a shrinking/stretching sheet. Frontiers in Heat and Mass Transfer (FHMT), 4(2).

Bhattacharyya, K. (2013b). MHD stagnation-point flow of casson fluid and heat transfer over a stretching sheet with thermal radiation. Journal of Thermodynamics, 2013.

Bhattacharyya, K., Hayat, T., and Alsaedi, A. (2013). Analytic solution for magnetohydrodynamic boundary layer flow of casson fluid over a stretching/shrinking sheet with wall mass transfer. Chinese Physics B, 22(2):024702.

Bhattacharyya, K., Mukhopadhyay, S., Layek, G. C., and Pop, I. (2012). Effects of thermal radiation on micropolar fluid flow and heat transfer over a porous shrinking sheet. International Journal of Heat and Mass Transfer, 55(11):2945 2952.

Bhattacharyya, K. and Pop, I. (2011). MHD boundary layer flow due to an exponentially shrinking sheet. Magnetohydrodynamics, 47:337 344.

Bhattacharyya, K., Uddin, M. S., and Layek, G. C. (2016). Exact solution for thermal boundary layer in casson fluid flow over permeable shrinking sheet with variable wall temperature and thermal radiation. Alexandria Engineering Journal, 55(2):1703 1712.

Bilal, S., Shafqatullah, Alshomrani, A. S., Malik, M. Y., Kausar, N., Khan, F., and Rehman, K. U. (2018). Analysis of carreau fluid in the presence of thermal stratification and magnetic field effect. Results in Physics, 10:118 125.

Brenner, H. and Nadim, A. (1996). The lorentz reciprocal theorem for micropolar fluids. In Kuiken, H. K., editor, The Centenary of a Paper on Slow Viscous Flow by the Physicist H.A. Lorentz, pages 169 176. Springer Netherlands, Dordrecht.

Carreau, P. J. (1968). Rheological Equations from Molecular Network Theories. PhD thesis, University Of Wisconsin.

engel, Y. A. and Turner, R. H. (2004). *Fundamentals of Thermal-fluid Sciences.* McGraw-Hill series in mechanical engineering. McGraw-Hill Companies, Boston.

Chaudhary, M. A. and Merkin, J. H. (1995). A simple isothermal model for homogeneous-heterogeneous reactions in boundary-layer flow. I equal diffusivities. Fluid Dynamics Research, 16(6):311 333.

Chhabra, R. P. (2010). Non-newtonian fluids: An introduction. In Krishnan, J. M., Deshpande, A. P., and Kumar, P. B. S., editors, Rheology of Complex Fluids, pages 3 34. Springer New York, New York.

Chhabra, R. P. and Uhlherr, P. H. T. (1980). Creeping motion of spheres through shear-thinning elastic fluids described by the carreau viscosity equation. **Rheologica Acta, 19(2):187–195.**

Collyer, A. A. (1973). Time independent fluids. **Physics Education, 8(5):333–338.**

Cramer, N. F. (2001). The Physics of Alfvén Waves. Wiley-VCH, Berlin, 1st edition.

Crane, L. J. (1970). Flow past a stretching plate. Zeitschrift für angewandte Mathematik und Physik ZAMP, 21(4):645–647.

Das, K. (2012). Slip effects on MHD mixed convection stagnation point flow of a micropolar fluid towards a shrinking vertical sheet. **Computers & Mathematics with Applications, 63(1):255–267.**

Das, K. and Sarkar, A. (2016). Effect of melting on an MHD micropolar fluid flow **toward a shrinking sheet with thermal radiation. Journal of Applied Mechanics and Technical Physics, 57(4):681–689.**

Davidson, P. A. (2001). An Introduction to Magnetohydrodynamics. Cambridge Texts in Applied Mathematics. Cambridge University Press, Cambridge.

Dawood, H. K., Mohammed, H. A., Sidik, N. A. C., Munisamy, K. M., and Wahid, M. A. (2015). Forced, natural and mixed-convection heat transfer and fluid flow **in annulus: A review. International Communications in Heat and Mass Transfer, 62:45–57.**

Drazin, P. G. and Riley, N. (2006). *The Navier-Stokes Equations: A Classification of* **Flows and Exact Solutions. London Mathematical Society Lecture Note Series. Cambridge University Press, Cambridge.**

Dutta, S. and Hazarika, G. C. (2017). Effects of variable viscosity and thermal conductivity on the MHD flow of micropolar fluid on a continuous moving surface. **International Journal of Computer Applications, 170(9):46–53.**

Eringen, A. C. (1966). Theory of micropolar fluids. **Journal of Mathematics and Mechanics, 16(1):1–18.**

Fang, T. (2008). Boundary layer flow over a shrinking sheet with power-law velocity. **International Journal of Heat and Mass Transfer, 51(25):5838–5843.**

Fang, T. and Zhang, J. (2010). Thermal boundary layers over a shrinking sheet: an analytical solution. Acta Mechanica, 209(3):325–343.

Farooq, M., Anzar, Q. U. A., Hayat, T., Khan, M. I., and Anjum, A. (2017). Local similar solution of MHD stagnation point flow in carreau fluid over a non-linear stretched surface with double stratified medium. **Results in Physics, 7:3078–3089.**

Fauzi, N. F., Ahmad, S., and Pop, I. (2015). Stagnation point flow and heat transfer **over a nonlinear shrinking sheet with slip effects. Alexandria Engineering Journal, 54(4):929–934.**

Ghosh, S. and Mukhopadhyay, S. (2017). Viscous flow due to an exponentially shrinking permeable sheet in nanofluid in presence of slip. International Journal for Computational Methods in Engineering Science and Mechanics, 18(6):309 317.

Hafidzuddin, E. H., Nazar, R., Arifin, N. M., and Pop, I. (2015). Numerical solutions of boundary layer flow over an exponentially stretching/shrinking sheet with generalized slip velocity. International Journal of Mathematical, Computational, Physical, Electrical and Computer Engineering, 9(4):244 249.

Hafidzuddin, E. H., Nazar, R., Arifin, N. M., and Pop, I. (2016). Boundary layer flow and heat transfer over a permeable exponentially stretching/shrinking sheet with generalized slip velocity. Journal of Applied Fluid Mechanics, 9(4):2025 2036.

Haldar, S., Mukhopadhyay, S., and Layek, G. C. (2017). Dual solutions of casson fluid flows over a power-law stretching sheet. Journal of Applied Mechanics and Technical Physics, 58(4):629 634.

Haldar, S., Mukhopadhyay, S., and Layek, G. C. (2018). Flow and heat transfer of casson fluid over an exponentially shrinking permeable sheet in presence of exponentially moving free stream with convective boundary condition. Mechanics of Advanced Materials and Structures, 0(0):1 7.

Hamid, R. A., Nazar, R., and Pop, I. (2017). Boundary layer flow of a dusty fluid over a permeable shrinking surface. International Journal of Numerical Methods for Heat Fluid Flow, 27(4):758 772.

Hansen, A. G. (1967). Generalized similarity analysis of partial differential equations. In Ames, W. F., editor, Nonlinear Partial Differential Equations, pages 1 17. Academic Press.

Harris, S. D., Ingham, D. B., and Pop, I. (2009). Mixed convection boundary-layer flow near the stagnation point on a vertical surface in a porous medium: Brinkman model with slip. Transport in Porous Media, 77(2):267 285.

Hashim and Khan, M. (2017). Critical values in flow patterns of magneto-carreau fluid over a circular cylinder with diffusion species: Multiple solutions. Journal of the Taiwan Institute of Chemical Engineers, 77(Supplement C):282 292.

Hashim, Khan, M., and Alshomrani, A. S. (2017). Numerical simulation for flow and heat transfer to carreau fluid with magnetic field effect: Dual nature study. Journal of Magnetism and Magnetic Materials, 443(Supplement C):13 21.

Hashim, Khan, M., Alshomrani, A. S., and Haq, R. U. (2018a). Investigation of dual solutions in flow of a non-newtonian fluid with homogeneous-heterogeneous reactions: Critical points. European Journal of Mechanics - B/Fluids, 68:30 38.

Hashim, Khan, M., Sharjeel, and Khan, U. (2018b). Stability analysis in the transient flow of carreau fluid with non-linear radiative heat transfer and nanomaterials: Critical points. Journal of Molecular Liquids, 272:787 800.

Hayat, T., Asad, S., Mustafa, M., and Alsaedi, A. (2014). Boundary layer flow of carreau fluid over a convectively heated stretching sheet. Applied Mathematics and Computation, 246:12 22.

Hayat, T., Khan, M. I., Waqas, M., and Alsaedi, A. (2017a). Mathematical modeling of non-newtonian fluid with chemical aspects: A new formulation and results by numerical technique. Colloids and Surfaces A: Physicochemical and Engineering Aspects, 518:263 272.

Hayat, T., Qayyum, S., Waqas, M., and Ahmed, B. (2017b). Influence of thermal radiation and chemical reaction in mixed convection stagnation point flow of carreau fluid. Results in Physics, 7:4058 4064.

Hayat, T., Saif, R. S., Ellahi, R., Muhammad, T., and Ahmad, B. (2017c). Numerical study of boundary-layer flow due to a nonlinear curved stretching sheet with convective heat and mass conditions. Results in Physics, 7:2601 2606.

Hayat, T., Sajjad, R., Ellahi, R., Alsaedi, A., and Muhammad, T. (2017d). Homogeneous-heterogeneous reactions in MHD flow of micropolar fluid by a curved stretching surface. Journal of Molecular Liquids, 240:209 220.

Hayat, T., Ullah, I., Ahmad, B., and Alsaedi, A. (2017e). Radiative flow of carreau liquid in presence of newtonian heating and chemical reaction. Results in Physics, 7:715 722.

Hiemenz, K. (1911). Die grenzschicht an einem inden gleichförmigen flüssigkeitsstrom eingetauchten geraden kreiszylinder. Dingler s Polytech. J., 326:321 324.

Ibrahim, W. (2017). MHD boundary layer flow and heat transfer of micropolar fluid past a stretching sheet with second order slip. Journal of the Brazilian Society of Mechanical Sciences and Engineering, 39(3):791 799.

Irgens, F. (2014). Rheology and Non-Newtonian Fluids. Springer International Publishing, New York, 1st edition.

Isa, S. S. P. M., Arifin, N. M., Nazar, R., Bachok, N., Ali, F. M., and Pop, I. (2017). MHD mixed convection boundary layer flow of a casson fluid bounded by permeable shrinking sheet with exponential variation. Scientia Iranica, 24(2):637 647.

Ishak, A. (2014). Flow and heat transfer over a shrinking sheet: A stability analysis. International Journal of Mathematical and Computational Sciences, 8(5):902 906.

Kameswaran, P. K., Shaw, S., and Sibanda, P. (2014). Dual solutions of casson fluid flow over a stretching or shrinking sheet. Sadhana, 39(6):1573 1583.

Kamran, M. (2018). Heat source/sink and newtonian heating effects on convective micropolar fluid flow over a stretching/shrinking sheet with slip flow model. International Journal of Heat and Technology, 36(2):473 482.

Kamran, M. and Wiwatanapataphee, B. (2018). Chemical reaction and newtonian heating effects on steady convection flow of a micropolar fluid with second order slip at the boundary. European Journal of Mechanics - B/Fluids, 71:138 150.

Keimanesh, R. and Aghanajafi, C. (2017). The effect of temperature-dependent viscosity and thermal conductivity on micropolar fluid over a stretching sheet. *Tehnički vjesnik*, 24(2):371 378.

Khan, I., Rehman, K. U., Malik, M. Y., and Shafquatullah (2018a). A computational analysis on homogeneous-heterogeneous mechanism in carreau fluid flow. Results in Physics, 8:1028 1033.

Khan, I., Ullah, S., Malik, M. Y., and Hussain, A. (2018b). Numerical analysis of mhd carreau fluid flow over a stretching cylinder with homogenous-heterogeneous reactions. Results in Physics, 9:1141 1147.

Khan, M. and Azam, M. (2016). Unsteady boundary layer flow of carreau fluid over a permeable stretching surface. Results in Physics, 6:1168 1174.

Khan, M. and Hashim (2015). Boundary layer flow and heat transfer to carreau fluid over a nonlinear stretching sheet. AIP Advances, 5(10):107203.

Khan, M., Hashim, and Alshomrani, A. S. (2016a). MHD stagnation-point flow of a carreau fluid and heat transfer in the presence of convective boundary conditions. PLOS ONE, 11(6):1 22.

Khan, M., Hashim, Hussain, M., and Azam, M. (2016b). Magnetohydrodynamic flow of carreau fluid over a convectively heated surface in the presence of non-linear radiation. Journal of Magnetism and Magnetic Materials, 412:63 68.

Khan, M., Malik, M. Y., Salahuddin, T., and Khan, I. (2017a). Numerical modeling of carreau fluid due to variable thicked surface. Results in Physics, 7:2384 2390.

Khan, M., Sardar, H., and Gulzar, M. M. (2018c). On radiative heat transfer in stagnation point flow of mhd carreau fluid over a stretched surface. Results in Physics, 8:524 531.

Khan, M., Sardar, H., Gulzar, M. M., and Alshomrani, A. S. (2018d). On multiple solutions of non-newtonian carreau fluid flow over an inclined shrinking sheet. Results in Physics, 8:926 932.

Khan, M. I., Hayat, T., Khan, M. I., and Alsaedi, A. (2017b). A modified homogeneous-heterogeneous reactions for MHD stagnation flow with viscous dissipation and joule heating. International Journal of Heat and Mass Transfer, 113(Supplement C):310 317.

Khan, M. I., Waqas, M., Hayat, T., and Alsaedi, A. (2017c). A comparative study of casson fluid with homogeneous-heterogeneous reactions. Journal of Colloid and Interface Science, 498(Supplement C):85 90.

Khan, M. I., Waqas, M., Hayat, T., and Alsaedi, A. (2017d). Magnetohydrodynamic (MHD) stagnation point flow of casson fluid over a stretched surface with homogeneous-heterogeneous reactions. Journal of Theoretical and Computational Chemistry, 16(03):1750022.

Khan, Z. H., Qasim, M., Ishfaq, N., and Khan, W. A. (2017e). Dual solutions of MHD boundary layer flow of a micropolar fluid with weak concentration over a stretching/shrinking sheet. Communications in Theoretical Physics, 67(4):449.

Kosarineia, A. and Sharhani, S. (2018). Second law analysis of magneto-micropolar fluid flow between parallel porous plates. Journal of Thermal Science and Engineering Applications, page 041017.

Kumar, K. G., Archana, M., Gireesha, B. J., Krishanamurthy, M. R., and Rudraswamy, N. G. (2018). Cross diffusion effect on MHD mixed convection flow of nonlinear radiative heat and mass transfer of casson fluid over a vertical plate. Results in Physics, 8:694 701.

Kumar, K. G., Gireesha, B. J., Rudraswamy, N. G., and Manjunatha, S. (2017). Radiative heat transfers of carreau fluid flow over a stretching sheet with fluid particle suspension and temperature jump. Results in Physics, 7:3976 3983.

Levenspiel, O. (2014). Engineering Flow and Heat Exchange. Springer US, New York, 3rd edition.

Lok, Y. Y., Ishak, A., and Pop, I. (2018). Oblique stagnation slip flow of a micropolar fluid towards a stretching/shrinking surface: A stability analysis. Chinese Journal of Physics.

Lukaszewicz, G. (1999). Micropolar Fluids: Theory and Applications. Birkh user Basel.

Mabood, F., Ibrahim, S. M., Rashidi, M. M., Shadloo, M. S., and Lorenzini, G. (2016). Non-uniform heat source/sink and soret effects on MHD non-darcian convective flow past a stretching sheet in a micropolar fluid with radiation. International Journal of Heat and Mass Transfer, 93:674 682.

Machireddy, G. R. and Naramgari, S. (2016). Heat and mass transfer in radiative MHD carreau fluid with cross diffusion. Ain Shams Engineering Journal.

Mahapatra, T. R. and Nandy, S. K. (2011). Stability analysis of dual solutions in stagnation-point flow and heat transfer over a power-law shrinking surface. International Journal of Nonlinear Science, 12(1):86 94.

Mahapatra, T. R., Nandy, S. K., and Gupta, A. S. (2010). Dual solution of MHD stagnation-point flow towards a stretching surface. Engineering, 2(04):299 305.

Mahmood, A., Chen, B., and Ghaffari, A. (2016). Hydromagnetic hiemenz flow of micropolar fluid over a nonlinearly stretching/shrinking sheet: Dual solutions by using chebyshev spectral newton iterative scheme. Journal of Magnetism and Magnetic Materials, 416:329 334.

Mahmoud, M. A. A. and Waheed, S. E. (2012). MHD flow and heat transfer of a micropolar fluid over a stretching surface with heat generation (absorption) and slip velocity. Journal of the Egyptian Mathematical Society, 20(1):20 27.

Mandal, I. C. and Mukhopadhyay, S. (2013). Heat transfer analysis for fluid flow over an exponentially stretching porous sheet with surface heat flux in porous medium. Ain Shams Engineering Journal, 4(1):103 110.

Mandal, I. C. and Mukhopadhyay, S. (2018). Nonlinear convection in micropolar fluid flow past an exponentially stretching sheet in an exponentially moving stream with thermal radiation. Mechanics of Advanced Materials and Structures, pages 1 7.

Mansur, S., Ishak, A., and Pop, I. (2015). The magnetohydrodynamic stagnation point flow of a nanofluid over a stretching/shrinking sheet with suction. PLOS ONE, 10(3):e0117733.

Meade, D. B., Haran, B. S., and White, R. E. (1996). The shooting technique for the solution of two-point boundary value problems. MapleTech, 3:85 93.

Medikare, M., Joga, S., and Chidem, K. K. (2016). MHD stagnation point flow of a casson fluid over a nonlinearly stretching sheet with viscous dissipation. American Journal of Computational Mathematics, 6(01):37.

Merkin, J. H. (1980). Mixed convection boundary layer flow on a vertical surface in a saturated porous medium. Journal of Engineering Mathematics, 14(4):301 313.

Merkin, J. H. (1986). On dual solutions occurring in mixed convection in a porous medium. Journal of Engineering Mathematics, 20(2):171 179.

Merkin, J. H. (1996). A model for isothermal homogeneous-heterogeneous reactions in boundary-layer flow. Mathematical and Computer Modelling, 24(8):125 136.

Miklavčic, M. and Wang, C. (2006). Viscous flow due to a shrinking sheet. Quarterly of Applied Mathematics, 64(2):283 290.

Mishra, S. R., Khan, I., Al-mdallal, Q. M., and Asifa, T. (2018). Free convective micropolar fluid flow and heat transfer over a shrinking sheet with heat source. Case Studies in Thermal Engineering, 11:113 119.

Mukhopadhyay, S. (2013a). Casson fluid flow and heat transfer over a nonlinearly stretching surface. Chinese Physics B, 22(7):074701.

Mukhopadhyay, S. (2013b). Slip effects on MHD boundary layer flow over an exponentially stretching sheet with suction/blowing and thermal radiation. Ain Shams Engineering Journal, 4(3):485 491.

Na, T. Y. (1979). Computational Methods in Engineering Boundary Value Problems. Mathematics in science and engineering : a series of monographs and textbooks. Academic Press.

Nadeem, S., Haq, R. U., and Lee, C. (2012). MHD flow of a casson fluid over an exponentially shrinking sheet. Scientia Iranica, 19(6):1550 1553.

Nadeem, S. and Hussain, A. (2009). MHD flow of a viscous fluid on a nonlinear porous shrinking sheet with homotopy analysis method. Applied Mathematics and Mechanics, 30(12):1569 1578.

Naganthran, K. and Nazar, R. (2016). Stability analysis of MHD stagnation-point flow towards a permeable stretching/shrinking surface in a carreau fluid. AIP Conference Proceedings, 1750(1):030031.

Postelnicu, A. and Pop, I. (2011). Falkner-skan boundary layer flow of a power-law fluid past a stretching wedge. Applied Mathematics and Computation, 217(9):4359 4368.

Pramanik, S. (2014). Casson fluid flow and heat transfer past an exponentially porous stretching surface in presence of thermal radiation. Ain Shams Engineering Journal, 5(1):205 212.

Praveena, D., Varma, S. V. K., Veeresh, C., and Raju, M. C. (2017). Chemical reaction and heat source effect on MHD free convection flow of a micropolar fluid through a porous medium over a semi-infinite moving plate with constant heat and mass flux. Journal of Information and Optimization Sciences, 38(8):1325 1345.

Qasim, M. and Noreen, S. (2014). Heat transfer in the boundary layer flow of a casson fluid over a permeable shrinking sheet with viscous dissipation. The European Physical Journal Plus, 129(1):7.

Raju, C. S. K., Sandeep, N., and Saleem, S. (2016a). Effects of induced magnetic field and homogeneous-heterogeneous reactions on stagnation flow of a casson fluid. Engineering Science and Technology, an International Journal, 19(2):875 887.

Raju, C. S. K., Sandeep, N., Sugunamma, V., Babu, M. J., and Reddy, J. V. R. (2016b). Heat and mass transfer in magnetohydrodynamic casson fluid over an exponentially permeable stretching surface. Engineering Science and Technology, an International Journal, 19(1):45 52.

Ramesh, G. K., Prasannakumara, B. C., Gireesha, B., and Rashidi, M. M. (2016). Casson fluid flow near the stagnation point over a stretching sheet with variable thickness and radiation. Journal of Applied Fluid Mechanics, 9(3):1115 1122.

Rana, S., Mehmood, R., and Akbar, N. S. (2016). Mixed convective oblique flow of a casson fluid with partial slip, internal heating and homogeneous-heterogeneous reactions. Journal of Molecular Liquids, 222:1010 1019.

Rapp, B. E. (2017). Chapter 9 - fluids. In *Microfluidics: Modelling, Mechanics and* Mathematics, Micro and Nano Technologies, pages 243 263. Elsevier, Oxford.

Rashidi, M. M., Mohimanian pour, S. A., and Abbasbandy, S. (2011). Analytic approximate solutions for heat transfer of a micropolar fluid through a porous medium with radiation. Communications in Nonlinear Science and Numerical Simulation, 16(4):1874 1889.

Reddy, A., Bala, P., and Suneetha, S. (2017). Effects of homogeneous-heterogeneous chemical reaction and slip velocity on MHD stagnation flow of a micropolar fluid over a permeable stretching/shrinking surface embedded in a porous medium. Frontiers in Heat and Mass Transfer (FHMT), 8.

Reddy, G. V. R. and Krishna, Y. H. (2018). Soret and dufour effects on MHD micropolar fluid flow over a linearly stretching sheet, through a non-darcy porous medium. International Journal of Applied Mechanics and Engineering, 23(2).

Reddy, P. B. A. (2016). Magnetohydrodynamic flow of a casson fluid over an exponentially inclined permeable stretching surface with thermal radiation and chemical reaction. Ain Shams Engineering Journal, 7(2):593 602.

Reddy, P. B. A. and Suneetha, S. (2018). Impact of cattaneo-christov heat flux in the casson fluid flow over a stretching surface with aligned magnetic field and homogeneous-heterogeneous chemical reaction. Frontiers in Heat and Mass Transfer (FHMT), 10(7).

Rehman, K. U., Alshomrani, A. S., and Malik, M. Y. (2018). Carreau fluid flow in a thermally stratified medium with heat generation/absorption effects. Case Studies in Thermal Engineering, 12:16 25.

Rehman, K. U., Malik, A. A., Malik, M. Y., Sandeep, N., and Saba, N. U. (2017a). Numerical study of double stratification in casson fluid flow in the presence of mixed convection and chemical reaction. Results in Physics, 7:2997 3006.

Rehman, K. U., Qaiser, A., Malik, M. Y., and Ali, U. (2017b). Numerical communication for MHD thermally stratified dual convection flow of casson fluid yields by stretching cylinder. Chinese Journal of Physics, 55(4):1605 1614.

Roberts, P. H. (1967). An Introduction to Magnetohydrodynamics. Longmans, London.

Rosca, A. V. and Pop, I. (2013). Flow and heat transfer over a vertical permeable stretching/shrinking sheet with a second order slip. International Journal of Heat and Mass Transfer, 60(Supplement C):355 364.

Roşca, N. C. and Pop, I. (2014). Boundary layer flow past a permeable shrinking sheet in a micropolar fluid with a second order slip flow model. European Journal of Mechanics - B/Fluids, 48:115 122.

Rosali, H., Ishak, A., and Pop, I. (2012). Micropolar fluid flow towards a stretching/shrinking sheet in a porous medium with suction. International Communications in Heat and Mass Transfer, 39(6):826 829.

Rout, P. K., Sahoo, S. N., Dash, G. C., and Mishra, S. R. (2016). Chemical reaction effect on mhd free convection flow in a micropolar fluid. Alexandria Engineering Journal, 55(3):2967 2973.

Sakiadis, B. C. (1961a). Boundary layer behavior on continuous solid surfaces: I. boundary layer equations for two-dimensional and axisymmetric flow. AIChE Journal, 7(1):26 28.

Sakiadis, B. C. (1961b). Boundary-layer behavior on continuous solid surfaces: II. the boundary layer on a continuous flat surface. **AIChE Journal, 7(2):221 225.**

Shah, R. A., Abbas, T., Idrees, M., and Ullah, M. (2017). MHD carreau fluid slip flow over a porous stretching sheet with viscous dissipation and variable thermal **conductivity. Boundary Value Problems.**

Sharma, R., Ishak, A., and Pop, I. (2016). Stagnation point flow of a micropolar fluid over a stretching/shrinking sheet with second-order velocity slip. **Journal of Aerospace Engineering, 29(5):04016025.**

Sheikh, M. and Abbas, Z. (2017). Homogeneous-heterogeneous reactions in stag-nation point flow of casson fluid due to a stretching/shrinking sheet with uniform **suction and slip effects. Ain Shams Engineering Journal, 8(3):467 474.**

Sheikh, N. A., Ali, F., Khan, I., Saqib, M., and Khan, A. (2017). MHD flow of micropolar fluid over an oscillating vertical plate embedded in porous media with **constant temperature and concentration. Mathematical Problems in Engineering, 2017:1 20.**

Siegel, R. and Howell, J. R. (1992). Thermal Radiation Heat Transfer. Hemisphere Pub. Corp., Washington, DC, 3rd edition.

Soid, S. K., Ishak, A., and Pop, I. (2017). Unsteady MHD flow and heat transfer over **a shrinking sheet with ohmic heating. Chinese Journal of Physics, 55(4):1626 1636.**

Srinivasacharya, D. and Upendar, M. (2013). Effect of double stratification on MHD free convection in a micropolar fluid. **Journal of the Egyptian Mathematical Society, 21(3):370 378.**

Takhar, H. S. and Soundalgekar, V. M. (1986). Boundary layer flow of a micropolar fluid – non-similar solution. **Mechanics Research Communications, 13(5):255 263.**

Tani, I. (1977). History of boundary layer theory. *Annual review of fluid mechanics*, **9(1):87 111.**

Turkyilmazoglu, M. (2016). Flow of a micropolar fluid due to a porous stretching **sheet and heat transfer. International Journal of Non-Linear Mechanics, 83:59 64.**

Umeshaiah, M., Prasannakumara, B. C., Krishnamurthy, M. R., Rudraswamy, N. G., and Gireesha, B. J. (2017). Nonlinear radiative heat transfer to carreau fluid over **a nonlinear stretching sheet in a porous medium in the presence of non-uniform heat source/sink and viscous dissipation. Frontiers in Heat and Mass Transfer (FHMT), 9(1).**

Versteeg, H. K. and Malalasekera, W. (2007). An Introduction to Computational Fluid Dynamics: The Finite Volume Method. Pearson Education Limited, Harlow, 2nd edition.

Wang, C. Y. (1990). Liquid film on an unsteady stretching surface. **Quarterly of Applied Mathematics, 48(4):601 610.**

Wang, C. Y. (2008). Stagnation flow towards a shrinking sheet. **International Journal of Non-Linear Mechanics, 43(5):377 382.**

Waqas, M., Alsaedi, A., Shehzad, S. A., Hayat, T., and Asghar, S. (2017). Mixed convective stagnation point flow of carreau fluid with variable properties. **Journal of the Brazilian Society of Mechanical Sciences and Engineering, 39(8):3005 3017.**

Weidman, P. D., Kubitschek, D. G., and Davis, A. M. J. (2006). The effect of transpiration on self-similar boundary layer flow over moving surfaces. **International Journal of Engineering Science, 44(11):730 737.**

Yacob, N. A. and Ishak, A. (2012). Micropolar fluid flow over a shrinking sheet. **Meccanica, 47(2):293 299.**

Yian, L. Y., Ishak, A., and Pop, I. (2011). MHD stagnation point flow with suction **towards a shrinking sheet. Sains Malaysiana, 40(10):1179 1186.**

Zaib, A., Bhattacharyya, K., Uddin, M., and Shafie, S. (2016). Dual solutions of non-newtonian casson fluid flow and heat transfer over an exponentially perme**able shrinking sheet with viscous dissipation. Modelling and Simulation in Engineering, 2016.**

Zheng, L., Niu, J., Zhang, X., and Ma, L. (2012). Dual solutions for flow and radiative heat transfer of a micropolar fluid over stretching/shrinking sheet. **International Journal of Heat and Mass Transfer, 55(25):7577 7586.**