EFFICIENCY OF 4253HT SMOOTHERS IN EXTRACTING SIGNAL FROM NOISE AND THEIR APPLICATIONS IN FORECASTING

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By

NURUL NISA' BINTI KHAIROL AZMI

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirements for the Degree of Doctor of Philosophy

February 2019

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DEDICATIONS

This research is wholeheartedly dedicated to my parents, Khairol Azmi and Zainab who gave me strength to survive this journey and for their constant moral and emotional support. To my beloved husband Khairul Anwar for his continuous love, patience and kindness. To my siblings, Ayong, Ana, Hasni, Ija and Muizz for their advice and encouragement. To my little princess, Auni: You are the best thing that ever happened in my life.

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Doctor of Philosophy

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February 2019

Chairman : **Mohd Bakri Adam, PhD**
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Compound smoother is a non-linear smoothing technique that has the ability to reduce heavy noise from signal and at the same time, is resistant to sudden changes and impulse in a data series. The compound smoother of 4253HT has been studied and modified in the algorithm, specifically to estimate the middle point of running median for even span size by applying the following types of means; geometric, harmonic, quadratic and contraharmonic.

The stability of running median of even span with modification toward the positive and negative block pulse were discussed. The modified 4253HT using harmonic mean works best in preserving edge at sudden changes point from down to upward and negative block pulse. Modified 4253HT using contraharmonic mean on the other hand, has been found to preserve edge of upward point and positive block pulse. The combination of modified 4253HT using harmonic and contraharmonic means adaptively, produce a new smoother with more resistance to block pulse and better preservation of the edge.

The performance of the modified compound smoothers was assessed via simulation. The signal of sinusoidal and special functions; Doppler, HeavySine, Bumps and Block was generated with non-Gaussian noise added that produced high volatility and disturbed by outliers. The performance were measured by regression coefficient, Estimated Integrated Mean Square Error (EIMSE) and variance reduction. The 4253HT has the ability to capture the signal from heavy noise data. In general, 4253HT performs best at smaller frequency and the recovery of signal from heavy

noise at high frequency using 4253HT is fairly good. This is asserted by the smooth value which was close to the signal, indicating its capability to extract signal from highly fluctuating noise.

The modified 4253HT using adaptive mean showed the most effective, compared to others, in extracting low, moderate and high frequency of sinusoidal signal from the noise with 10%, 25%, 50% and 75% contaminated normal distribution. The Doppler, Block, Bumps and Heavy Sine signal show the modified 4253HT using adaptive mean also managed to recover those signal from noise better than other modified 4253HT and the existing one. The extracted signal was then used for better forecasting which was facilitated by seasonal Holt-Winters, ARAR and seasonal ARIMA algorithm.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

KEEFISYENAN PELICIN 4253HT DALAM MENGEKSTRAK ISYARAT DARIPADA HINGAR DAN APLIKASINYA DALAM PERAMALAN

Oleh

NURUL NISA' BINTI KHAIROL AZMI

Februari 2019

Pengerusi : **Mohd Bakri Adam, PhD**
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Pelicin kompaun adalah teknik pelicinan bukan linear yang mempunyai kebolehan untuk mengurangkan hingar daripada isyarat dan pada masa yang sama mempunyai ketahanan terhadap perubahan yang secara tiba-tiba dan impulsif di dalam data. Pelicin kompaun 4253HT telah dikaji dan diubahsuai dalam algoritmanya terutama dalam menganggar titik tengah pelicin median bagi jangka bersaiz genap dengan menggunakan jenis purata yang berikut: geometrik, harmonik, kuadratik dan kontra harmonik. Kestabilan bagi median jangka bersaiz genap terhadap blok impulsif positif dan negatif dibincangkan. 4253HT yg dimodifikasi menggunakan purata harmonik paling baik dalam memelihara titik yang berubah dari rendah ke tinggi dan blok impulsif yang negatif. Manakala, 4253HT yg dimodifikasi menggunakan purata kontra harmonik berjaya memelihara titik yang berubah dari tinggi ke rendah dan blok impulsif yang positif. Gabungan 4253HT yg dimodifikasi menggunakan purata harmonik dan kontra harmonik secara adaptif menghasilkan pelicin yang lebih tinggi ketahanannya terhadap blok impulsif dan memelihara titik hujung dengan lebih baik.

Prestasi pelicin kompaun yang diubahsuai dinilai melalui simulasi. Isyarat sinusoidal dan beberapa fungsi istimewa seperti *Doppler, Block, Bumps* dan *Heavy Sine* telah dijana dengan hingar bukan Gaussian ditambah yang menghasilkan volatiliti yang tinggi dan diganggu oleh unsur luaran. Prestasi diukur oleh koefisyen regresi, jangkaan integrasi purata ralat kuasa dua dan penurunan varians. 4253HT berprestasi baik pada frekuensi yang rendah dan mengembalikan isyarat dari hingar yang tinggi pada frekuensi yang lebih tinggi menggunakan 4253HT adalah agak

baik memandangkan nilai yang licin menghampiri nilai isyarat yang mana menunjukkan kebolehan mengasingkan isyarat daripada hingar yang bervolatiliti tinggi.

Modifikasi 4253HT mengunakan purata adaptif adalah yang paling efektif berbanding yang lain dalam mengekstrak isyarat sinusoidal berfrekuensi rendah, sederhana dan tinggi daripada 10%, 25%, 50% and 75% hingar tercemar. Isyarat *Doppler, Block, Bumps* dan *Heavy Sine* menunjukkan modifikasi 4253HT mengunakan purata adaptif berkeupayaan untuk mengembalikan semula isyarat tersebut lebih baik berbanding berbanding modifikasi 4253HT dan yang sedia ada. Isyarat yang telah diekstrak kemudiannya digunakan untuk peramalan menggunakan algoritma Holt Winter bermusim, ARAR dan ARIMA bermusim.

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This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfilment of the requirement for the degree of Doctor of Philosophy. The members of the Supervisory Committee were as follows:

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INTRODUCTION

1.1 Smoothing

Smoothing is more of a curve fitting whereby the main purpose is tracing the trend from a set of data series blurred by noise. In a data series, trends provide the direction to choose appropriate method of estimation. Smoothing data series does not necessarily have to be well fitted, but most importantly it has an ability to reduce noise so that overall picture regarding global behavior of data series can be captured. The pattern extracted from smoothing process is able to provides some guideline on a suitable modelling estimation for forecasting purpose. Smoothing does not only helps in curve fitting but also very useful in determining future values by eliminating non-well behave noise.

Smoothing by definition varies according to the fields of interest. Some studies, use the term filtering to refer to smoothing for example Ataman et al. (1981), Bovik et al. (1983), Gabbouj et al. (1992), Zeng (1994) and Miao and Jiang (2013). In order to avoid any confusion, the term smoothing is used consistently throughout this research. The main concern of smoothing is to capture underlying pattern by removing unwanted noise from the data series. Thus, it is appropriate if a data series to be regarded as a mixture of smooth and rough component, whereby

$$\text{data} = \text{smooth} + \text{rough}. \tag{1.1}$$

Data can also be referred as a trend mixed with noise, true value with measurement error or regional trend mixed with local deviation. Since the ”smooth” is intended to be smooth, the points are shown connected in a sequence. The main purpose of smoothing is to reduce noise in a data series so that the smoothed values produce a good signal of deterministic trend and shape of the distribution. Hence, the objectives of smoothing can be extended to more specific forms are as follows;

1. to determine a suitable model that fitted the dependent variable with variable of interest,
2. to reveal any obscure patterns,
3. to eliminate significant spikes or outliers and
4. to examine patterns in the noise.

Smoothing is very useful prior to conducting further analysis which requires assumptions and specific shape of distribution. The application of smoothing is not limited to one-dimensional data as it can be employed in image processing analysis where smoothing the data involves two-dimensional algorithm.

The most crucial part in smoothing process is measuring the smoothness of data series. Nonetheless, it is very subjective in practice to identify the sufficiency of disturbing noise that has been eliminated. Anderson and Chiraratananon (1971) state that variance after smoothing should be lesser than the original sequence considering that random errors have been reduced.

1.2 Basic Definitions in Smoothing

Smoothing in this study focuses on its operation in temporal data. Some terminologies related to smoothing temporal data are described for further understanding.

1.2.1 Temporal Data

Let **X** be a temporal data and defined as doubly infinite numerical sequence, Mallows (1980);

$$\mathbf{X} = \{X_{-N}, \ldots, X_{-3}, X_{-2}, X_{-1}, X_0, X_1, X_1, X_2, X_3, \ldots, X_N\} \tag{1.2}$$

where X_t is the observation at time t, $t = 0, \pm 1, \pm 2, \ldots, \pm N$ of a time varying random variable of phenomenon **X**.

The analysis of temporal data usually starts by observing the graph to foresee the general trend and later decide on appropriate method of data analysis. Temporal data analysis is very useful to trace pattern for forecasting purpose. Hence, reducing noise from a data series is an essential way to discover the general trend or curve before further analysis can be conducted. Common approaches are transformation or differencing. The most prominent is by smoothing a data series using a smoother.

When a sequence of **X** is observed, two main components can be extracted, signal and noise. Both components are described in the subsequent parts of this chapter.

1.2.2 Signal

In smoothing, a signal is assumed as a smooth continuous curve or general pattern that can be described by plotting graph.

Definition 1.1 *Anderson and Chirarattananon (1971): A signal can be defined as the most useful relationship between the time varying random variable generating the time series and time.*

Each data point is similar and at the same level as the neighboring values. The points also need to be consistent if there are any changes in the direction of the data series. A signal can be taken as a deterministic function whereby it is commonly disturbed by noise either Gaussian or non-Gaussian.

1.2.3 Noise

Commonly, a data series is masked by noise that makes the process of estimation complicated. In general, noise can be defined as follows;

Definition 1.2 *Jankowitz (2007): **Noise** is a component in a data series that interferes the detection of signal.*

Generally, noise is randomly distributed either with known distribution or some cases, with no specific distribution.

Definition 1.3 *Jankowitz (2007) : **Gaussian noise** is obtained by generating an independent, identically and random observations from Gaussian distribution.*

Definition 1.4 *Jankowitz (2007) : **Non-Gaussian** is any noise that is not generated by normal distribution. The example of non-Gaussian noise includes noise generated by long tailed distribution and the one obtained in the forms of extreme data points or several consecutive extreme points or block pulse.*

Non-Gaussian noise is not considered well behave due to their tremendous effects on the pattern. **Heavy noise** is another form of non-Gaussian noise producing high volatility in a data series.

In real situation, extracting unknown signal from heavy noise is difficult to handle. Similarly, measuring successful smoothing can also be challenging as it can be very subjective under condition where signal has a non-deterministic pattern. Hence, simulated signal added with noise are generated and evaluated in order to identify the effectiveness of the smoother in extracting signal from noise, Jankowitz (2007). The process of extracting the signal from distracting noise can be done by employing a smoother.

1.2.4 Smoother

Smoother is an important tool in statistical analysis that helps a lot in directing researchers for further decision making.

Definition 1.5 *Rohwer (2005): Smoother S is a notation that operates $\mathbf{X}$ to bring out a new component of $S_i(\boldsymbol{X}_t)$ that produces smoothed value at time t and i represents the number of smoothing algorithm being applied.*

A good smoother S, has an ability to extract signal from noise.

1.2.5 Span of Smoother

In this research, smoothers work based on running span or window size. Let the observations in a window be as follows $\mathbf{W} = \left\{ X_{t-\frac{k-1}{2}}, \ldots, X_t, \ldots, X_{t+\frac{k-1}{2}} \right\}$ where $k = 2u+1$ and $u = 1, 2, \ldots, n$. For example, if $u = 1$, the observations in a span are $\mathbf{W} = \{X_{t-1}, X_t, X_{t+1}\}$ which makes the span size to be equivalent to three.

This is only applicable for window of odd size. For even span size, $k = 2u$. For example, if $u = 2$, the observations in a span are $\mathbf{W} = \{X_{t-2}, X_{t-1}, X_t, X_{t+1}\}$ which makes the span size to be equivalent to four.

1.3 Types of Smoothers

Generally, a smoother can be classified into linear and non-linear. Most of the linear approaches involve parameter estimation and work best in removing Gaussian noise from the signal. On the other hand, non-linear is more flexible in treating non-normal noise in a data series.

1.3.1 Linear Smoothers

Smoothing techniques which initially evolve from linear to non-linear are actually based on suitability of the data. Linear smoother is very popular, easy to understand theoretically that it comes handy upon implementation. Linear smoothers can also be defined as a linear function of a data series. Generally, a linear smoother can be expressed as follows;

Definition 1.6 *Rohwer (2005) : A sequence* $\{Y_t\}$ *is the (discrete) convolute of sequence* $\{X_t\}$ *and* β *if and only if*

$$Y_t = \sum_{j=-u}^{u} \beta_j X_{t-j}. \tag{1.3}$$

The simplest linear smoothing algorithm is moving average where the smoothed value is replaced by the mean of three neighboring values in a span. For example, the output of moving average of span size three, $\{Y_t\}$ on a sequence of $\{X_t\}$ produces the following;

$$Y_t = \frac{X_{t-1} + X_t + X_{t+1}}{3} \tag{1.4}$$

where the weightage of all values in a span is equal. The longer the span size, the smoother the data series is. However, too long span size cause the lost of important information in a data series.

A linear smoothing with weighted function gives different effects to the smoothed values. The weight takes into consideration the distance from neighboring values. For example, the output of weighted moving average of span size five, $\{Y_t\}$ can be expressed as follows;

$$Y_t = \frac{X_{t-2} + 2X_{t-1} 3X_t + 2X_{t+1} + X_{t+2}}{9}. \tag{1.5}$$

The closer the neighboring values are to the smoothed value, the higher weighted is assigned. If the smoothing data series at first pass is not sufficient to reduce the disturbing noise, multiple times of smoothing can be applied to the data series. Smoothing a data series using algorithm in Equation (1.4) twice, produces equivalent algorithm presented in Equation (1.5). Since the algorithm of linear smoother is in the function of mean, the major drawback is its resistance to sharp changes or outliers.

There are many versions of extended linear smoother such as kernel smoothers, local polynomial smoothers, spline and kringing. The linear approach is theoretically easy to understand and widely applicable in various fields. Commonly, in linear smoothing, certain assumptions such as data must be independent and identically normally distributed need to be satisfied. However, in most real life situation, the assumptions require beyond just the recruitment of linear smoother.

1.3.2 Non-linear Smoother

Non-linear smoothers are designed to compensate the weakness of linear approach. Non-linear smoothers work very well in the existence of outliers that meet the criteria of resistance, robustness, extracting non-Gaussian noise, preservation of information edge and the recovery of the details of a signal. Although seems theoretically difficult, analysing non-linear smoothers seems to be applicably effective in real practices.

One of the most prominent non-linear smoothers is a smoother that is based on order statistic. Median smoother is part of order statistic smoother (OSS) where the output is the middle point which comes from an ordered sequence. Median smoother is one of the categories that exhibit the feature of being robust to outliers, Tukey (1977). Generally, the output of OSS on the length k in a sequence $\{X_t\}$ for n is odd can be expressed as follows, Bovik et al. (1983);

$$Y_t = \text{OSS}\left(\{X_i\}_{i=t-k}^{t+k}\right)\sum_{t=i}^{k}\alpha_i X_{(t)} \tag{1.6}$$

where $X_{(t)}$ is the order statistics of $X_{t-\frac{k+1}{2}},\ldots,X_t,\ldots,X_{t+\frac{k+1}{2}}$ and $k=2u+1$. The α_i are constants and applicable to different types of order statistic such as median smoother, maximum and minimum smoother and α-trimmed mean smoother. The median smoother is a function Y_t with coefficients;

$$\alpha_i = \begin{cases} 1; & i=\frac{k+1}{2}, \\ 0; & \text{otherwise.} \end{cases} \tag{1.7}$$

For maximum smoother, the coefficients is defined as;

$$\alpha_i = \begin{cases} 0; & i=1,\ldots,k-1 \\ 1; & i=k. \end{cases} \tag{1.8}$$

As a generalization, all of coefficients are constrained to be zero except for the i-th which is set to be unity, Bovik et al. (1983).

Rabiner et al. (1975) made a comparison of the performance of Hanning or weighted moving average with running median and found that running median is superior than Hanning in smoothing several waveforms such as log input energy of a speech signal, zero-crossing rate, and pitch period. Ever since then, median smoother has evolve into various of modification to accommodate the shortcoming.

1.4 Compound Smoother

One of the acknowledged types of non-linear smoother is a compound smoother. Compound smoother is a multiple combination of running median or median smoother, weighted moving average, splitting and re-smoothing a rough algorithm. The following sub sections elaborate some elements that are used commonly in compound smoother.

1.4.1 Median Smoother

Median smoother works based on window or span size. The output of median smoother is obtained by arranging the sequence $\mathbf{X}$ in ascending order, $\left\{X_{(t-u)},\ldots,X_{(t)},\ldots,X_{(t+u)}\right\}$ and taking the middle point, $X_{(t)}$ as the output.

For illustration, consider the following conditions. For odd span size, let $\{\tilde{X}_{t,k}\}$ denotes as the output of median smoother in a sequence of X at index t and window size k, where $k = 2u+1$ and $u = 1,2,\ldots,\frac{k-1}{2}$ where $u \in \mathbb{Z}$. For even span size, let $\{\tilde{X}_{t,k}\}$ denotes as output of median smoother in a sequence of X at index t and window size k, where $k = 2u$ and $u = 1,2,\ldots,\frac{k}{2}$ where $u \in \mathbb{Z}$.

1.4.1.1 Median Smoother Span Size Two

The output of median smoother of $k = 2$ which is a mean of two values in a window of size two can be denoted as follows;

$$\begin{aligned} \tilde{X}_{t,2} &= \text{median}(X_t, X_{t+1}) && (1.9)\\ &= \text{mean}(X_t, X_{t+1}) && (1.10)\\ &= \frac{1}{2}\left(X_t + X_{t+1}\right). && (1.11) \end{aligned}$$

Let $\{X_t\} = \{X_1, X_2, \ldots, X_n\}$ represents a set of data series with discrete index at t where, $t = 1,2,\ldots,n$. Table 1.1 depicts the algorithm of median smoother with span size two. The points at the end of a data series is lost in the computation. There are many approaches to remedy the missing points at both ends. In this case however, the end point is appended with actual values, $\hat{X}_n = X_n$. The missing end points are discussed further in Section 1.4.4.

Table 1.1: Algorithm of median smoother of window size two

X_t	X_1	X_2	X_3	$\dots$	X_{n-1}	X_n
operation	$\frac{X_1+X_2}{2}$	$\frac{X_2+X_3}{2}$	$\frac{X_3+X_4}{2}$	$\dots$	$\frac{X_{n-1}+X_n}{2}$	X_n
output	$\tilde{X}_1$	$\tilde{X}_2$	$\tilde{X}_3$	$\dots$	$\tilde{X}_{n-1}$	$\tilde{X}_n$

Median smoother of span size two is equivalent to moving average of span size two. The order of observations is not important. Hence, the median smoother of window size two shares the same properties of linear smoother and not being robust to outliers.

1.4.1.2 Median Smoother Span Size Three

The output of median smoother of window size three, $\{\tilde{X}_{t,3}\}$ on a sequence of $\{X\} = \{X_{t-u}, \dots, X_t, \dots, X_{t+u}\}$ where $u = 1, 2\dots, \frac{k+1}{2}$ and $u \in \mathbb{Z}$ is denotes as the following equation;

$$\tilde{X}_{t,3} = \text{median}(X_{t-1}, X_t, X_{t+1}). \tag{1.12}$$

The output of median smoother of span size three, $\{\tilde{X}_{t,3}\}$ is produced by the following algorithm;

1. Sort the following sequence X_{t-1}, X_t, X_{t+1}.
2. The sorted sequence is $X_{(t-1)}, X_{(t)}, X_{(t+1)}$.
3. From the sorted sequence, an output of median smoother, $\tilde{X}_{t,3} = X_{(t)}$.

Table 1.2 shows the algorithm of median smoother for window size three.

1.4.1.3 Median Smoother of Span Size Four

Median smoother of window size four, $\{\tilde{X}_{t,4}\}$ on a sequence of $\{X\} = \{X_{t-u}, \dots, X_t, \dots, X_{t+u}\}$ where $u = 1, 2\dots, \frac{k}{2}$ and $u \in \mathbb{Z}$, can be denoted as the following equation;

$$\tilde{X}_{t,4} = \text{median}(X_{t-2}, X_{t-1}, X_t, X_{t+1}). \tag{1.13}$$

Let say, the observations in a sequence are $X_{t-2}, X_{t-1}, X_t, X_{t+1}$. By sorting the observations in a sequence, together they become $X^*_{(t-2)}, X^*_{(t-1)}, X^*_{(t)}, X^*_{(t+1)}$. From this sequence, the median value is an arithmetic mean of observations at $(t-1)$ and (t), $\tilde{X}_{t,4} = \frac{X^*_{(t-1)} + X^*_{(t)}}{2}$. In summary, the output of median smoother, $\hat{X}_{t,4}$ is obtained by the following algorithms;

1. Sort the following sequence $X_{t-2}, X_{t-1}, X_t, X_{t+1}$.

2. The sorted sequence is $X^*_{(t-2)}, X^*_{(t-1)}, X^*_{(t)}, X^*_{(t+1)}$.

3. From the sorted sequence, an output of median smoother, $\hat{X}_{t,4} = \frac{X^*_{(t-1)} + X_{(t)}}{2}^*$.

Table 1.3 shows the algorithm of median smoother for window size four.

1.4.1.4 Median Smoother of Span Size Five

Median smoother of span size five, $\{\tilde{X}_{t,5}\}$ on a sequence of $\{X\} = \{X_{t-u}, \ldots, X_t, \ldots, X_{t+u}\}$ where $u = 1, 2 \ldots, \frac{k}{2}$ and and $u \in \mathbb{Z}$ is denotes as;

$$\tilde{X}_{t,5} = \text{median}(X_{t-2}, X_{t-1}, X_t, X_{t+1}, X_{t+2}). \quad (1.14)$$

The output of median smoother with a span size five is produced by the following algorithms;

1. Sort the following sequence $X_{t-2}, X_{t-1}, X_t, X_{t+1}, X_{t+2}$.

2. The sorted sequence is $X_{(t-2)}, X_{(t-1)}, X_{(t)}, X_{(t+1)}, X_{(t+2)}$.

3. From the sorted sequence, $\tilde{X}_{t,5} = X_{(t)}$.

Table 1.4 shows the algorithm of median smoother for window size five.

Table 1.2: Algorithm of median smoother of window size three

X_t	X_1	X_2	X_3	$\cdots$	X_{n-1}	X_n
operation (sorting in window)	X_1	$X_{(1)},X_{(2)},X_{(3)}$	$X_{(2)},X_{(3)},X_{(4)}$	$\cdots$	$X_{(n-2)},X_{(n-1)},X_{(n)}$	X_n
output	$\tilde{X}_1 = X_1$	$\tilde{X}_2 = X_{(2)}$	$\tilde{X}_3 = X_{(3)}$	$\cdots$	$\tilde{X}_{n-1} = X_{(n-1)}$	$\tilde{X}_n$

Table 1.3: Algorithm of median smoother of window size four

X_t	X_1	X_2	X_3	X_4	$\cdots$	X_{n-2}	X_{n-1}	X_n
operation (sorting in window)	X_1	X_2	$X_{(1)},X_{(2)},X_{(3)},X_{(4)}$	$X_{(2)},X_{(3)},X_{(4)},X_{(5)}$	$\cdots$ $\cdots$	$X_{(n-3)},X_{(n-2)},X_{(n-1)},X_{(n)}$	X_{n-1}	X_n X_n
output	$\tilde{X}_1 = X_1$	$\tilde{X}_2 = X_2$	$\tilde{X}_3 = \frac{X_{(2)}+X_{(3)}}{2}$	$\tilde{X}_4 = \frac{X_{(3)}+X_{(4)}}{2}$	$\cdots$	$\tilde{X}_{n-2} = \frac{X_{(n-2)}+X_{(n-1)}}{2}$	$\tilde{X}_{n-1} = X_{(n-1)}$	$\tilde{X}_n$

Table 1.4: Algorithm of median smoother of window size five

X_t	X_1	X_2	X_3	X_4	$\cdots$	X_{n-2}	X_{n-1}	X_n
operation (sorting in window)	X_1	X_2	$X_{(1)},X_{(2)},X_{(3)},X_{(4)},X_{(5)}$	$X_{(2)},X_{(3)},X_{(4)},X_{(5)},X_{(6)}$	$\cdots$ $\cdots$	$X_{(n-4)},X_{(n-3)},X_{(n-2)},X_{(n-1)},X_{(n)}$	X_{n-1}	X_n X_n
output	$\tilde{X}_1 = X_1$	$\tilde{X}_2 = X_2$	$\tilde{X}_3 = X_{(3)}$	$\tilde{X}_4 = X_{(4)}$	$\cdots$	$\tilde{X}_{n-2} = X_{(n-2)}$	$\tilde{X}_{n-1} = X_{(n-1)}$	$\tilde{X}_n$

1.4.1.5 Generalization of Median Smoother for k Span Size

Median smoother of odd span size, the output is obtained by arranging the observations in a running window and choosing the middle point as the output. Median smoother of odd window size $\tilde{X}_{t,k}$ is as follows;

$$\tilde{X}_{t,k} = \text{median}(X_{t-\frac{k-1}{2}}, \ldots, X_t, \ldots, X_{t+\frac{k-1}{2}}) \tag{1.15}$$

where $k = 2u+1$ and $u = 1, 2, \ldots, \frac{k+1}{2}$ where $u \in \mathbb{Z}$. This equation is only applicable to median smoother of odd window size where $k \geq 3$.

For even window size, the output of median smoother is obtained by averaging two middle points of data, arranged in ascending order, $X^*_{(t)}$ and $X^*_{(t+1)}$. The output of median smoother, $\tilde{X}_{t,k}$ with $k = 2u$ can be expressed as follows;

$$\tilde{X}_{t,k} = \text{median}\left(X_{t-\frac{k-2}{2}}, \ldots, X_t, \ldots, X_{t+\frac{k}{2}}\right) \tag{1.16}$$

where the middle point is computed by $\frac{X^*_{(t-1)}+X^*_{(t)}}{2}$ and $\{X^*\}$ represents the observations that have been ordered in a window.

In general, the median smoother is obtained by the following steps;

1. Select the window size.

2. Arrange the observations in a window in ascending order.

3. Choose the middle point as the output of median smoother. For odd window size, the middle point is a single true value in the sequence. For even window size, the middle point is shared by two consecutive arranged points and computed using arithmetic mean.

Median smoother of window size not more that five is preferable. Median smoother of even window size take into consideration the neighboring values and hence edges are not totally destroyed.

Commonly, median smoother of even span two and four is combined to get a centered median (Section 3.2). Therefore, the notation used to describe the algorithm of median smoother of window size four followed by median smoother of window size two is 42. It is very rare to use median smoother of window size greater than seven. Hence, notation 42 does not mean that the length of window is 42.

1.4.2 Repeated Median Smoother

Repeated median smoother refers to applying a smoother of equal span size on the same data repeatedly. It is denoted as R. For example, 3R can be defined as smoothing a data series by running median with window of size three, then is re-smoothed using running median of span size three again over the data that has been smoothed before. For example, let the output $\{Y_t\}$ produced by running median of span size three on $\{X_t\} = \{X_{t-u}, \ldots, X_{t+u}\}$ to be expressed as follows;

$$Y_t = \text{median}(X_{t-1}, X_t, X_{t+1}). \tag{1.17}$$

The 3R smoother, $\{V_t\}$ is obtained by smoothing $\{Y_t\} = \{Y_{t-u}, \ldots, Y_t, \ldots, Y_{t+1}\}$ using;

$$V_t = \text{median}(Y_{t-1}, Y_t, Y_{t+1}). \tag{1.18}$$

If the smoothed values become root after smoothing on the first pass, repeated running median is not necessary.

1.4.3 Splitting

Splitting in compound smoother is denoted as 'S'. Splitting works when a data sequence is divided into separate pieces in the middle of width-2 peak or trough. Each end is smoothed separately with the end-value rule and then the parts are glued together, Jankowitz (2007). Mathematically, splitting is generated in the following steps. Let $\{Y_t\}$ be the output of the following sequence $\{X_t\} = \{X_{t-u}, \ldots, X_{t+u}\}$ with;

$$\begin{aligned} Y_{t-1} &= \text{median}(X_{t-2}, X_{t-1}, 3X_{t-2} - 2X_{t-3}) \\ Y_t &= \text{median}(3X_{t+1} - 2X_{t+2}, X_t, X_{t+1}). \end{aligned} \tag{1.19}$$

Hence, the output of splitting, $\{W_t\}$ on a sequence of $\{Y_t\}$ is as follows;

$$W_t = \text{median}(Y_{t-1}, Y_t, Y_{t+1}). \tag{1.20}$$

Splitting is useful when dealing with bumps conveniently in a data series. The process of splitting is done after applying 3R smoother.

1.4.4 End Points

If both end points are not taken into account in the algorithm, the computation of one point of smooth value will took a long span size. For example, a smoothed value using compound smoother 4253HT required 25 observations in a window, Conradie et al. (2009). A running median of size 42 can cause the lost of two observations at both ends points. If the missing points are ignored, the next process of smoothing can result in the lost of other values in a data series.

Apart from minimising the probability of losing values at both ends, substituting end points will also reduce the complexity of the algorithm. There are several approaches to dealing with the end points. Nevertheless, there are no specific guideline on how to determine the best approach to this.

Let **Y** be denoted as the output of a median smoother on a sequence of $\{X_t\} = \{X_{t-u}, \ldots, X_t, \ldots, X_{t+u}\}$. The computations of end points according to Jankowitz (2007) are;

1. **Rule of replicating end values**

   The most common practice is to add extra point at both ends of data. One instance to do this, is to generate the value before X_1 as X_0 and value after X_n as X_{n+1}.

2. **Copy on end value rule**

   The lost smoothed values at both ends are appended with actual observations. Let Y_t is the smoothed value in a sequence. The Y_1 can be replaced by X_1 and Y_n by X_n.

3. **Step-down end value rule**

   This can be done via window of smaller size. For example, missing end points in a median smoother of span size three are computed by using median smoother with a span size two.

4. **Omit end value rule**

Remove the end values and only make use the available smoothed values.

5. **Extrapolation end value rule**

Tukey (1977) introduces a rule that extrapolates the end values. The technique applies recent smoothed values obtained from a linear extrapolation to construct another observation beyond the end of a sequence. The second and third end values which cannot be smoothed are copied using the actual observations in a sequence. A $0-th$ observation, $\hat{Y}_0$ is estimated by linearly extrapolating the second and third smoothed. Let Y_2 and Y_3 be the smoothed values and for equally spaced data with t-spacing $\triangle t$. The slope at beginning of end values is

$$\frac{Y_3 - Y_2}{\triangle t}. \tag{1.21}$$

The $\hat{Y}_0$, is extrapolate by computing the difference between the second smoothed values and two equally space of slope. It can be estimated as follows;

$$\begin{aligned}\hat{Y}_0 =& Y_2 - 2\triangle t \frac{(Y_3 - Y_2)}{\triangle t} \\ =& 3Y_2 - 2Y_3.\end{aligned} \tag{1.22}$$

Likewise for the last value of a sequence of n observations, $\hat{Y}_{n+1}$ is estimated as;

$$\hat{Y}_{n+1} = 3Y_{n-1} - 2Y_{n-2}. \tag{1.23}$$

The beginning point of end value is computed by using the median of observations extrapolated values with second and third smoothed values. Thus, the Y_1 can be estimated as;

$$Y_1 = \text{median}(\hat{Y}_0, Y_1, Y_2) \tag{1.24}$$

and the last smoothed value is

$$Y_n = \text{median}(\hat{Y}_{n+1}, Y_n, Y_{n-1}). \tag{1.25}$$

1.4.5 Hanning

Hanning is another name for running weighted average. Hanning plays an important role in making the data cleaner and smooth. However, Hanning is not resistant to outliers. Henceforth, this means Hanning is much affected by the existence of occasionally significant spikes. This agrees with Velleman and Hoaglin (1981) indication that Hanning is only to be applied after the removal of outliers by running median.

The sum of weight assigned to each sequence must be equal to one. There are many versions of determine the weightage, for example, actuaries used Spencer's 15-Point Moving Average to smooth trading volume, Kenney and Keeping (1962). The symmetric weight coefficients are $\frac{3}{320}, \frac{6}{320}, \frac{5}{320}, \frac{3}{320}, \frac{21}{320}, \frac{46}{320}, \frac{67}{320}, \frac{74}{320}$. Let h_t represents the output of Hanning on a sequence of $\{X\} = \{X_{t-u}, \ldots, X_t, \ldots, X_{t+u}\}$. The output of Hanning can be expressed as follows;

$$\begin{aligned} h_t = & \frac{3}{320}X_{t-7} + \frac{6}{320}X_{t-6} + \frac{5}{320}X_{t-5} + \frac{3}{320}X_{t-4} + \frac{21}{320}X_{t-3} + \\ & \frac{46}{320}X_{t-2} + \frac{67}{320}X_{t-1} + \frac{74}{320}X_t + \frac{67}{320}X_{t+1} + \frac{46}{320}X_{t+2} + \\ & \frac{21}{320}X_{t+3} + \frac{3}{320}X_{t+4} + \frac{5}{320}X_{t+5} + \frac{6}{320}X_{t+6} + \frac{3}{320}X_{t+7}. \end{aligned} \tag{1.26}$$

Pekárová et al. (2003) applied Henderson's 5-point Moving Average with $\frac{1}{16}, \frac{1}{4}, \frac{3}{8}, \frac{1}{4}, \frac{1}{16}$ for smoothing the discharge time of selected rivers and denoted the output of Hanning as follows;

$$h_t = \frac{1}{16}X_{t-2} + \frac{1}{4}X_{t-1} + \frac{3}{8}X_t + \frac{1}{4}X_{t+1} + \frac{1}{16}X_{t+2}. \tag{1.27}$$

Tukey (1977) used a symmetric coefficients of form $\frac{1}{4}, \frac{1}{2}, \frac{1}{4}$ of running weighted average as a smoother which according to Mills (1991) produces a nice curvature after the stabilization of outliers via running median smoother.

The Hanning is computed according to the following expression;

$$h_t = \frac{1}{4}X_{t-1} + \frac{1}{2}X_t + \frac{1}{4}X_{t+1}. \tag{1.28}$$

Hanning provides a smooth and gentle curve after outliers being eliminated from a data series.

1.4.6 Re-smooth the Rough (Twice)

Smoothing repeatedly causes a data series to be overly smoothed that some important feature might be left out during the process. For example, median smoother occasionally eliminates Gaussian noise and Hanning tends to pull tops down and bottoms up. Re-smoothing the rough helps to recover the important features that has been eliminated while being smoothing repeatedly.

Tukey (1977) introduces an effective method to recover the over smoothed values by applying the smoothing algorithm to the rough. This process is called re-roughing and later will be appended with the smoothed values. If the algorithm used for re-roughing the rough is similar to the algorithm used for smoothing a data series, it is then shall be written as ”twice” and denoted as T.

For example, let $\{e_t\}$ be denoted as rough, obtained by subtracting the actual values, $\{X_t\}$ and smoothed values computed from median smoother of span size three, $\{\tilde{X}_{t,3}\}$. The rough is re-smoothed by using median smoother of span size three and written as $\tilde{e}_{t,3} = \text{median}(e_{t-1}, e_t, e_{t+1})$. The re-smoothed rough is then added back to the smoothed values and expressed as the followings;

$$X_t = \tilde{X}_{t,3} + \tilde{e}_{t,3}. \tag{1.29}$$

Equation (1.29) is denoted as 3T. The number ”3” represents median smoother of window size three and ”T” is an expression of re-smoothed the rough operation.

1.4.7 Notation in Compound Smoother

The following notations describe the common process in the compound smoother;

- Numeric term (eg. 2,3,4,5) represents the span size of a median smoother.
- R denotes the running median of same span size performed repeatedly.
- H is a short notation for Hanning or weighted running average.
- S refers to splitting.
- T indicates ”Twice” as in the case of re-smoothing the rough and adding it back to the smoothed values in a sequence.

Compound smoother works by combining different types of algorithms. Tukey (1977) come up with various types of compound smoothers such as 3R, 53H, 53HT, 3RSSH and 3RSSHT. Velleman (1980) then extended the Tukey's ideas of running median for even span size in the compound smoother namely, 4253H and 4253HT. The 3R denotes the repeated median smoother of span size three while 53H represents the algorithm of running median of span size five, followed by running median of span size three and Hanning. The numbers in a compound smoother are basically related to span size of the running median. The letters in a compound smoother in contrast, represent the specific operation in the smoothing.

1.5 Exploratory Data Analysis (EDA)

In statistical analysis, the estimation of the smoothed values is the most crucial part. There are three main approaches to estimation including classical, Bayesian and Exploratory Data Analysis (EDA).

Classical analysis puts emphasis on parameter estimation subjected to known distribution. The interpretation is objective and procedure of estimation is well established. Bayesian methods are designed for rational incorporation of prior information into the process of statistical analysis, Gelman et al. (2014).

Behrens (1997) mention that EDA, plays a major role in pattern identification in a data. In EDA, attempts are made to identify the major features of a data set of interest and to generate ideas for further investigation, Cox (2017). Graphical analysis in EDA is to explore patterns prior to conducting further analysis in formal statistical approach. EDA leads the researcher to determine the right path for statistical modeling and analysis.

Since EDA is designed to illuminate underlying pattern in noisy data so that underlying data structure not be obscured or completely hidden in the process, Ellison (1993). The results revealed by EDA, provides several options of appropriate statistical models that suit to the pattern being distinguished.

Even though classical approach is well established in theory and implementation, in most real cases some required assumptions that failed to be full filled. One kind of robustness particularly valuable in a data is resistance to observations drawn from distributions which are longer tailed than the Gaussian and particularly to outliers, isolated value which fall apart from the main body of data, Wainer (1976). EDA is appropriate when there is a large amount of variability in a data.

Nonetheless, Gelman (2004) asserts that there is no specific statistical rules or assumptions for the distribution in EDA. In conventional approach, the distribution of a data needs to be first identified before the analysis of parametric can be conducted. However, for some end users of statistical tools, the assumptions for underlying data are ignored, leading to results that are less reliable. Since the focus of this study is compound smoother, EDA is hence, the appropriate approached.

1.6 Smoother as a Forecasting Tools

Forecasting is one of the important tools that help in decision making. The forecasting technique should be able to deal with seasonality, high volatility and sudden changes in a data series. The existing of data with high variation and complexity has driven forecasters to develop many forecasting techniques.

In statistics, forecasting is based on how the past data behave over the time period on the average. The future values are expected to continue to occur based on what had happened before. It is usually difficult to make a prediction from raw data since the trend is not so obvious and mixed up with the unexpected event and distorted by random error. Hence, raw data need to be massage first before applying any forecasting technique and it is a norm in time series analysis.

Smoothing is one of statistical technique that can be employed to reduce the effect of non-well behaved noise for forecasting purpose. Many researchers applied the smoother to raw data before determining the forecast model so that the uncertainty can be minimized. The smoothed data should be reasonable to represent the original data, Montgomery et al. (2015). Hence, choosing the appropriate type of smoother plays an important role so that the general features of the data is mantained and do not eliminate well-behaved noise excessively.

The most popular technique is a family of exponential smoothing. Exponential smoothing method is known as a non-parametric approach since no assumptions pertaining the distribution was required. It also accomodate with forecasting technique once the equation model was determined. This method is widely applicable in various field and still relevant at the present period(Rendon-Sanchez and de Menezes (2019), Elias and Nashat (2019), Hartomo et al. (2019), Tran et al. (2019), Suppalakpanya et al. (2019) and many more).

Compound smoother is also popular option that widely employed in revealing potential pattern exists in the data series for forecasting purpose. Hourcade and Nadaud (2010) performed energy forecast by smoothing the data first using linear regression and compound smoother 4253HT. The Tukey smoother able to detect the two-period major movement in the data series and allowing a quick assesment to degree of linearity of the data. Sargent and Bedford (2010) also employed the compound smoother for forecasting one-step ahead Australian Football League (AFL) player performance. The result found that exponentially smoothing a Tukey-smoothed series has delivered a significantly smaller average forecast error than using an unsmoothed series. Therefore, it is vital to extend the purpose of smoothing to forecasting the data that heavily disturbed by noise.

1.7 Problem Statement

Linear smoother is optimal to eliminate Gaussian noise and track trends that are common in practice, Bernholt et al. (2006). However, noise of high volatility tends to mask the general picture of a data series. The existence of non well-behaved noise violated the assumptions of linear model. Usually, least square estimation which is well known for its poor performance in the presence of outliers or long-tailed distribution data is used.

According to Venetsanopoulos and Pitas (1990), linear smoothers also have a high tendency to blur important features and lack of the ability to remove impulsive noise. Not only that, linear smoothers are highly vulnerable to outliers and could not deal well with nonlinearity in a data series. Blurry edge which leads to the lost of important information is actually due to the sudden changes in a series, Bernholt et al. (2006).

Due to its ability to remove non-Gaussian noise from a data series, median smoother is usually the favored smoothing tools. Unfortunately, median smoother tends to over smoothed a data series since it eliminates Gaussian noise too.

One of established types of median smoothers which have been widely employed in various area settings is compound smoother. Compound smoother is known as a powerful tool to smooth a data series without excessively disrupting the details of a data series.

Despite this good traits, the compound smoother does not respond well to oscillated trend, Tóthmérész and Erdei (1995) and Jin and Xu (2013). The number observations of compound smoother should be at least seven, otherwise it will converge to constant root, Janosky et al. (1997). The Velleman's compound smoother as indicated by Sargent and Bedford (2010) has been revised when possible combinations of multiple step of running median, Hanning and re-smooth the rough are tested out. Improvement on the existing compound smoother in comparison has yet to be explored.

1.8 Research Objectives

Since there is an opportunity for improvement in compound smoother, some modifications to running median of span size 42 is suggested in this study. The existing study only focuses on noise with long tailed distribution. The pattern with small portion of contaminated can easily be observed with naked eyes. Unfortunately, for data with high fluctuation, the signal might mix up with heavy noise, making it hard to capture any possible trends. In this research, the performance of smoothers in highly volatile data is compared and evaluated. This research provides some values added to the existing study and also motivates future research to expand the idea this study addresses for a better solution. Guided by the earlier discussion, the purposes of study are summarized as follows;

1. To modify existing compound smoothers

2. To determine the stability of modified compound smoothers towards block pulse.

3. To evaluate the performance of modified compound smoother via simulation procedure with higher percentage of contaminated normal noise for sinusoidal, Doppler, Bumps, Blocks and Heavy Sine function.

4. To formulate a strategy of forecasting by extracting deterministic components in data series.

5. To apply the proposed modified smoother to financial, environment and agriculture data.

1.9 Thesis Outline

There are seven main chapters in this thesis. Chapter 2 presents the evolution of smoothing techniques involving median and compound smoothers. Some of the deterministic properties of smoothers are also explored. The advantages and shortcomings of existing method are discussed while assessing the opportunity for improvement.

In Chapter 3, the components and process of compound smoothers including median smoother, Hanning, splitting and twice are described in details. In doing so, the stability of compound smoother towards block pulse are presented. The simulation procedure to generate the signal of sinusoidal, Doppler, Bumps, Blocks and Heavy Sine with noise is presented and explained in the same chapter. Chapter 3 also lists out the measures on how the smoothers successfully recover signal from noise with outliers and high volatility.

Chapter 4 focuses on the modification of existing compound smoother namely 4253HT. Some adjustments in computing the middle point of output median smoother are highlighted. Different types of means are appended such as geometric, quadratic, harmonic and contra harmonic means. In this chapter, comparison between the stability of modified compound smoother are presented in the way they each deal with impulse, block pulse and edge. The results of simulation, guided by procedure elaborated in Chapter 3 are then compared and discussed.

Chapter 5 extends the modification by adaptively assigning the different types of adjustments to 4253HT according to changes in the data series. The stability of 4253HT, modified through adaptive mean are discussed in term of how it behaves on positive and negative block pulse size two, three and four. The performance was compared with the best modification in Chapter 4 via simulation procedure as explained in Chapter 3.

In Chapter 6, a strategy for forecasting is introduced by using deterministic trend and noise obtained from smoothing process. Three established methods of forecasting; Holts-Winters, Seasonal ARIMA and ARAR algorithms are applied by taking into consideration their ability to forecast times series with the existence of trend and seasonality. A data series that is equipped with linear trend, regular fluctuations and noise of high volatility are generated. Some comparisons are done to prove that forecasting using smoothed values is better than forecasting using original data.

Some applications of modified compound smoother are also discussed in Chapter 7. In this research, data refer to the daily price index of a bank in Malaysia that issues sukuk, the amount of daily rainfall at Universiti Malaya, Kuala Lumpur station for the year 2006, daily average of temperature recorded at Petaling Jaya station from 1/1/2009 to 31/12/2011 and total production of crude palm oil in Malaysia from 1990 to 2010. The output of original and modified compound smoothers are compared.

The last chapter concludes the findings of the research, highlights the contributions and discusses future opportunity for research in other areas.

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