STABILITY ANALYSIS ON BOUNDARY LAYER FLOW IN NANOFLUID OVER A FLAT SURFACE UNDER VARIOUS EFFECTS

NAJWA BINTI MOHD NAJIB

IPM 2019 2

STABILITY ANALYSIS ON BOUNDARY LAYER FLOW IN NANOFLUID OVER A FLAT SURFACE UNDER VARIOUS EFFECTS

By

NAJWA BINTI MOHD NAJIB

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfillment of the Requirements for the Degree of Doctor of Philosophy

April 2019

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfillment of the requirement for the degree of Doctor of Philosophy

STABILITY ANALYSIS ON BOUNDARY LAYER FLOW IN NANOFLUID OVER A FLAT SURFACE UNDER VARIOUS EFFECTS

By

NAJWA BINTI MOHD NAJIB

April 2019

Chairman: Norfifah binti Bachok @ Lati, PhD
Institute: Institute for Mathematical Research

Consideration of steady boundary layer flow, heat and mass transfer filled with nanofluids over a moving, stretching or shrinking surfaces are investigated numerically. The models used for solving nanofluids problems in this thesis are Buongiorno's model and Tiwari and Das model. The governing partial differential equations corresponded to the boundary conditions are transformed into ordinary differential equations using a suitable similarity transformation. The stability analysis is derived by introducing the partial differential equations in unsteady case. These equations are then solved by using bvp4c function.

The numerical results of skin friction, heat and mass transfer coefficient as well as velocity, temperature and concentration profiles for both models are presented in tables and graphs with respect to the governing parameters, namely, moving parameter, stretching or shrinking parameter, suction parameter, first order slip and second order slip parameters, Brownian motion parameter, thermophoresis parameter, nanoparticles volume fraction, types of nanoparticles, Soret number, Dufour number and Biot number. Comparison of results with the previous studies is done to validate the present results. It is found that the behavior of the flow, heat and mass transfer are influenced by the corresponded parameters. Since all problems posses dual solutions, the stability analysis is performed to verify which solutions are stable and physically realizable.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

ANALISIS KESTABILAN DI ATAS ALIRAN LAPISAN SEMPADAN DALAM NANOBENDALIR TERHADAP PERMUKAAN RATA DI BAWAH BEBERAPA KESAN

Oleh

NAJWA BINTI MOHD NAJIB

April 2019

Pengerusi: Norfifah binti Bachok @ Lati, PhD
Institut: Institut Penyelidikan Matematik

Pertimbangan aliran lapisan sempadan yang mantap, pemindahan haba dan jisim yang diisi dengan bendalir nano di atas permukaan bergerak, meregang atau mengecut diselidiki secara berangka. Model yang digunakan untuk menyelesaikan masalah nanobendalir dalam tesis ini adalah model Buongiorno dan model Tiwari dan Das. Persamaan pembezaan separa menakluk dengan syarat sempadan dijelmakan kepada persamaan pembezaan biasa menggunakan penjelmaan keserupaan yang sesuai. Analisis kestabilan diperoleh dengan memperkenalkan persamaan pembezaan separa dalam kes aliran tak mantap. Persamaan ini seterusnya diselesaikan dengan menggunakan fungsi bvp4c.

Keputusan berangka untuk pekali geseran kulit, pemindahan haba dan jisim serta profil halaju, suhu dan kepekatan bagi kedua-dua model ditunjukkan dalam bentuk jadual dan graf terhadap parameter-parameter yang menakluk, iaitu parameter nisbah halaju, parameter regangan atau kecutan, parameter sedutan, parameter gelinciran peringkat pertama dan kedua, parameter gerakan Brownian, parameter termoforesis, pecahan isipadu nanozarah, jenis nanozarah, nombor Soret, nombor Dufour dan nombor Biot. Perbandingan keputusan dengan kajian terdahulu dibuat bagi mengesahkan keputusan kajian. Didapati bahawa ciri-ciri aliran, pemindahan haba dan pemindahan jisim dipengaruhi oleh parameter-parameter menakluk tersebut. Oleh kerana semua masalah mempunyai penyelesaian dual, analisis kestabilan dilakukan untuk mengesahkan penyelesaian mana yang stabil dan boleh direalisasikan secara fizikal.

ACKNOWLEDGEMENTS

In the name of Allah, the Most Gracious, the Most Merciful. Alhamdulillah, first of all I would like to thank my creator, Allah S.W.T for granting me with His blessing and ability to complete my thesis for doctoral research.

I would like to express my deep sense of respects, gratitude and appreciation to my beloved supervisor, Assoc. Prof. Dr. Norfifah Bachok for her guidance, advices, supervision and helps from the beginning until I have completed this research timely. Many thanks goes to my co-supervisors, Prof. Dr. Norihan Md. Arifin and Dr. Fadzilah Md. Ali for their support and advices. During these three years, I have learned a lot from this research with many experiences and challenges. Thanks to my beloved supervisor and co-supervisor who gave an opportunity for me to explore and learn a new thing as well as giving me positive comments, ideas and suggestions in order to improve this research.

I would like to express my sincere appreciation to the lecturers and staffs of the Institute for Mathematical Research (INSPEM) who always assisted me during my difficult time. Next, these acknowledgement would not be completed without mentioning my main financial support during my studies, KPT.

For my beloved family; I am extremely grateful to my husband, Mohd Hafizi who always support, encourage and been there for me when I had some problems and need someone to talk with. His support always gave me strength and motivation to complete this research. To my son and daughter, Hami and Hani thanks for being such great kids and sorry because of lack time spent together. To my parents and my mother-in-law (ayoh, mok and ma) and siblings, thank you for all your prayers and support during my study. Lastly, an immeasurable gratitude goes to my dear friends and colleagues, especially friends who are under the same supervision of mine for sharing valuable knowledge and always give support to me. Thank you.

This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfillment of the requirement for the degree of Doctor of Philosophy. The members of the Supervisory Committee were as follows:

Norfifah binti Bachok @ Lati, PhD
Assosiate Professor
Faculty of Science
Universiti Putra Malaysia
(Chairperson)

Norihan binti Md. Arifin, PhD
Professor
Faculty of Science
Universiti Putra Malaysia
(Member)

Fadzilah binti Md. Ali, PhD
Senior Lecturer
Faculty of Science
Universiti Putra Malaysia
(Member)

ROBIAH BINTI YUNUS, PhD
Professor and Dean
School of Graduate Studies
Universiti Putra Malaysia

Date:

Declaration by graduate student

I hereby confirm that:

- this thesis is my original work;
- quotations, illustrations and citations have been duly referenced;
- this thesis has not been submitted previously or concurrently for any other degree at any other institutions;
- intellectual property from the thesis and copyright of thesis are fully-owned by Universiti Putra Malaysia, as according to the Universiti Putra Malaysia (Research) Rules 2012;
- written permission must be obtained from supervisor and the office of Deputy Vice-Chancellor (Research and Innovation) before thesis is published (in the form of written, printed or in electronic form) including books, journals, modules, proceedings, popular writings, seminar papers, manuscripts, posters, reports, lecture notes, learning modules or any other materials as stated in the Universiti Putra Malaysia (Research) Rules 2012;
- there is no plagiarism or data falsification/fabrication in the thesis, and scholarly integrity is upheld as according to the Universiti Putra Malaysia (Graduate Studies) Rules 2003 (Revision 2012-2013) and the Universiti Putra Malaysia (Research) Rules 2012. The thesis has undergone plagiarism detection software.

Signature:__________________________Date:__________________________

Name and Matric No: Najwa binti Mohd Najib, GS43729

Declaration by Members of Supervisory Committee

This is to confirm that:

- the research conducted and the writing of this thesis was under our supervision;
- supervision responsibilities as stated in the Universiti Putra Malaysia (Graduate Studies) Rules 2003 (Revision 2012-2013) are adhered to.

Signature: ______________________
Name of
Chairman of
Supervisory
Committee: ______________________

Signature: ______________________
Name of
Member of
Supervisory
Committee: ______________________

Signature: ______________________
Name of
Member of
Supervisory
Committee: ______________________

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF ABBREVIATIONS

a, b	constants
A, B	constants
d	boundary layer thickness
Al_2O_3	Alumina
Bi	Biot number
Cf_x	skin friction coefficient
c_p	specific heat at constant pressure
C	concentration
C_w	surface concentration
c_s	concentration susceptibility
C_∞	free stream concentration
Cu	Copper
D_B	Brownian diffusion coefficient
D_m	diffusion coefficient
D_T	thermophoresis diffusion coefficient
Du	Dufour number
h_f	convective heat transfer coefficient
k	thermal conductivity
K_n	Knudsen number
K_T	thermal diffusion ratio
L	length characteristics
Le	Lewis number
Nb	Brownian motion parameter
Nt	thermophoresis parameter
Nu_x	local Nusselt number
p	pressure
Pr	Prandtl number
q_m	surface mass flux
q_w	surface heat flux
Re_x	local Reynolds number
s	suction parameter
Sh_x	local Sherwood number
Sr	Soret number
t	time
T	fluid temperature
T_m	mean fluid temperature
T_w	surface temperature
T_∞	free stream temperature
TiO_2	Titania
U_w	velocity at the boundary
U_{slip}	slip velocity at the boundary
u, v	velocity component along x-axis and y-axis
V	velocity vector
x, y	Cartesian coordinate

Greek Symbols

α	thermal diffusivity
η	similarity variables
ψ	stream function
θ	dimensionless temperature
ϕ	dimensionless concentration
ε	stretching or shrinking parameter
λ	velocity ratio parameter or moving parameter
μ	dynamic viscosity
ν	kinematic viscosity
ρ	fluid density
ρc_f	volumetric heat capacity of fluid
ρc_p	volumetric heat capacity
τ	dimensionless time variable
τ_w	surface shear stress
φ	nanoparticles volume fraction parameter
σ	first order slip parameter
δ	second order slip parameter
γ	eigenvalue parameter
Γ	momentum accomodation coefficient
∇	Laplace operation
ω	molecular mean free path
Ω	ratio of volumetric heat capacity and the base fluid volumetric heat capasity

Subscripts

c	critical value
f	fluid
nf	nanofluid
s	solid nanoparticles
w	condition at the wall
∞	condition at infinity

Superscript

$'$	differentiation with respect to η

CHAPTER 1

INTRODUCTION

1.1 Introduction

This thesis focuses on the mathematical model that resembles reality by using mathematical language without experimenting. Additionally, the model can be fixed, modified or sometimes might be use in research or being reference to build a good model. Throughout this chapter, the main terms or keywords used in this thesis will be elaborated. The problem statements, objectives, scopes, significance of study and the outline of the thesis are also discussed in this chapter.

1.2 Boundary Layer Flow

Boundary layer flow refers to a thin layer of viscous fluid adjacent to a solid surface with a moving stream in which (within it boundary layer thickness) the velocity varies from zero at the wall up to the velocity at the boundary or also called as the free stream velocity, refer Figure 1.1. The concept of boundary layer flow was introduced by Ludwig Prandtl in 1904 that triggering revolution the understanding and analysis of fluid dynamics. The main idea proposed by Prandtl is that the flow can be divided into two parts which are inviscid flow at the main region and the thin layer adjacent to the solid surface or known as a boundary layer, Anderson (2005). At the boundary layer, the friction force has to be considered whereas the friction force outside the boundary layer are very small and can be neglected, Schlichting (1979). According to Prandtl, the Navier-Stokes equations can be simplified using the boundary layer concept. Furthermore, he claimed that the significant of viscosity effect on fluid flow for Reynolds number are higher, $Re >> 1$.

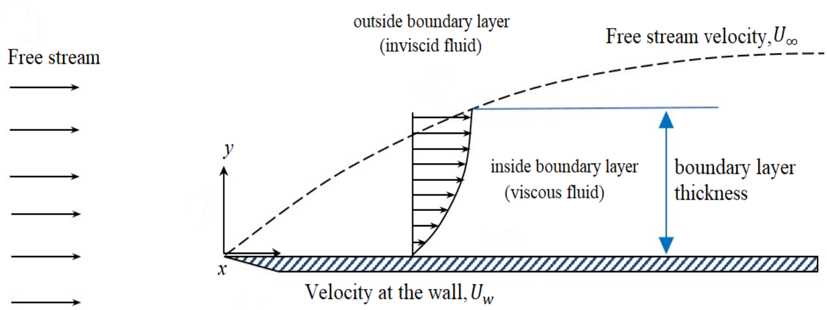

Figure 1.1: The physical model of boundary layer flow
(https://en.wikipedia.org/wiki/Boundary layer thickness)

Prandtl number Pr is a dimensionless number represents the ratio of momentum diffusivity (kinematic viscosity) to the thermal diffusivity. Pr is defined as

$$Pr = \frac{\mu c_\rho}{k} = \frac{\nu}{\alpha} = \frac{\text{viscous diffusion rate}}{\text{thermal diffusion rate}}, \tag{1.2.1}$$

where μ is the dynamic viscosity, c_ρ is the specific heat, ρ is density, ν is the kinematic viscosity and α is thermal diffusivity. If the fluid has higher viscosity then Pr is greater and hence the heat transfer rate will be less convective. Pr obviously influences the thermal boundary layer thickness and heat transfer depending on the fluid properties.

- Gases: Pr = 0.7 to 1
- Water: Pr = 1 to 10
- Liquid metals: Pr = 0.001 to 0.03

1.3 Stagnation Point Flow

Stagnation point can be described as a point in a flow region where the local fluid velocity is zero. At the plate surface ($x = y = 0$) there exist a point called stagnation point where the fluids come to the rest by the plate.

Stagnation point flow is the motion of fluid near the stagnation point where the fluid pressure, heat transfer and rates of mass deposition are highest. The free stream is divided into half after its coming through the stagnation point. Along the dividing streamline, the fluid moves towards the plate, refer Figure 1.2.

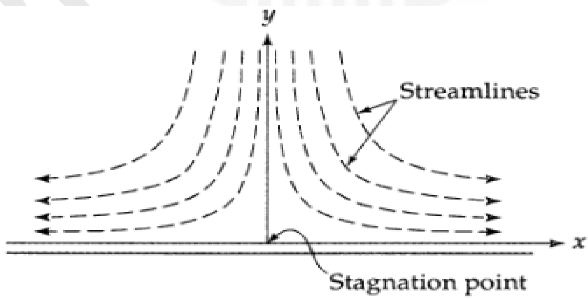

Figure 1.2: Stagnation point flow
(https://www.transtutors.com)

1.4 Heat Transfer

Heat transfer can be defined as a movement of heat from higher temperature towards lower temperature across the boundary layer system to its surroundings. The phenomena of heat transfer in boundary layer can be referred when the plate or surface of the boundary layer is heated from the bottom. Then the heat flows from the bottom of the plate towards the surroundings till reach the same temperature, at which the plate and its surroundings are said to be in thermal equilibrium.

There are three modes of heat transfer such as conduction, convection and radiation. The description of each modes are describe below:

- Conduction or also known as diffusion is the transfer of heat between two solid bodies through direct contact. For example, the heat burners on the stoves will conduct heat energy to the bottom of a pan and thus the pan conducts heat to its contents as illustrated in Figure 1.3. Another example related to our study is when one end of the metal such as copper is heated then, the other end of the copper metal also get heated due to diffusion of heat transfer from the heated side.

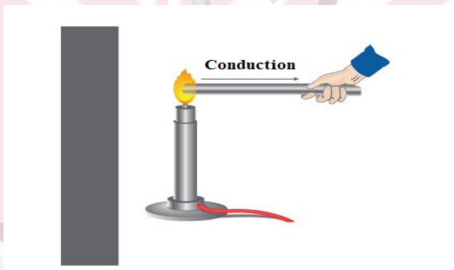

Figure 1.3: Conduction heat transfer
(http://swtechfire.com/fundamentals-of-fire-engineering)

- Convection describes as a transfer of heat between the solid surface and the liquid, see Figure 1.4. There are two types of convection which are natural and force convection. The natural convection occurs when molecules or particles at the bottom of a cooking vessel rise and warm while cooler and heavier particles sink. Hence, a circulation will occur that evenly distributed heat throughout the substances. One of the examples of this situation can be found when heating the water in the pot where the heated water from the bottom of the pot will boil and circulate in the pot. The force convection happens when the streams and currents in the fluid are induced by the external forces such as fans, stirrers, and pumps. The examples of this situation including stirring liquid in a pot or uses a fan in the exhaust system of the oven to blow hot air over and around the food.

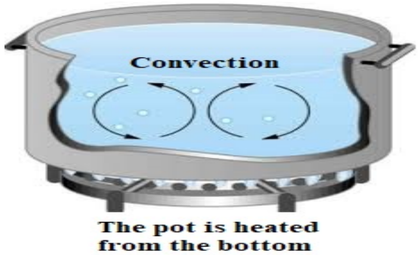

Figure 1.4: Convection heat transfer
(https://www.jobilize.com)

- Radiation is the term used when two bodies are at different temperatures and separated by distance, refer Figure 1.5. Differ from convection and conduction, there is no medium to transfer the radiation heat. The radiation heat transfer occurs because of the electromagnetic waves that exist in the atmosphere. The natural phenomena of radiation is the heat of the sun coming on the earth.

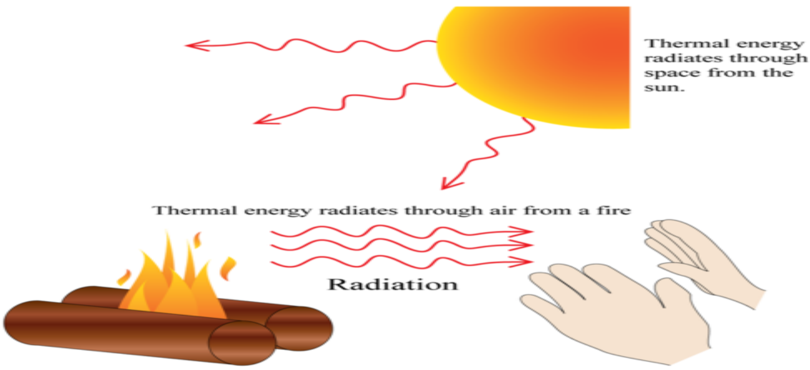

Figure 1.5: Radiation heat transfer
(https://www.ck12.org)

However, only convection heat transfer is taken into consideration in this thesis.

1.5 Mass Transfer

Mass transfer can be described as a net movement of mass (commonly in stream, phrase, fraction or components) from one location to another location. In our related study, the term mass transfer also known as diffusive and convective transport of chemical species within physical system (boundary layer flow). The simplest case of mass transfer can occur in a medium at rest in which the force is driven by the concentration differences in adjacent regions of the medium and the mechanism called as molecular diffusion. Technically the movement of mass transfer will diffuse from higher concentration towards the lower concentration due to mass flux.

The applications of mass transfer processes in nature are numerous such as the the evaporation of water from a pond to the atmosphere, the purification of blood in the kidneys and liver. Some industrial processes involve mass transfer can be found in emulsification process, distillation of alcohol, separation of chemical components in distillation columns, adsorbers in activated carbon beds and also in liquid-liquid extraction which can be refered in Figure 1.6. The mass transfer process is also significant in an industrial cooling tower where there also involved heat transfer in this process. The cooling tower combines heat transfer to mass transfer by allowing hot water to flow directly into the air. Then the water is cooled by expelling its contents in water vapor form.

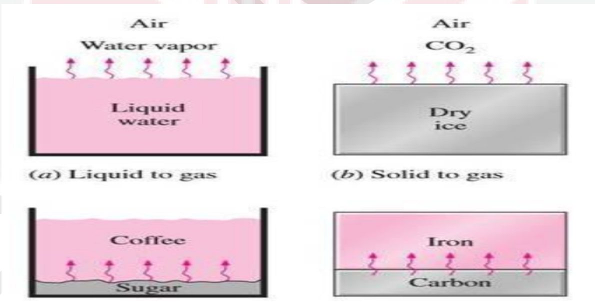

Figure 1.6: Mass transfer processes
(http://folk.uio.no/ravi/cutn/pmat/4.diffusion+ficks.pdf)

1.6 Nanofluid

Nanofluid is a fluid containing micrometer or nanometer sized particles which diameter of particles in between 0 to 100nm size. The nanoparticles used in nanofluids are commonly made of metals, oxides, carbides or carbon nanotubes whereas the common base fluid namely water, ethylene glycol and oil. However, the mentioned common base fluid have poor capability to enhance the thermal conductivity of the fluid. Hence, an alternative solution has been engineered by suspending nanoparticles into the base fluid. This is because, the collision of nanoparticles in base fluid are the best technique to enhance thermal conductivity of the fluid. According to Choi (1995) by adding a small portion ($< 1\%$ volume fraction) of nanoparticles into the base fluid will enhance the thermal conductivity twice. This is because the nanometer sized nanoparticles acts like a fluid molecules Khanafer et al. (2003). The example of cross section structure of nanofluid is displayed in Figure 1.7. The chemical and physical properties of nanoparticles lead to increase in thermal conductivity of the fluid itself and also increase in heat transfer rate, Das et al. (2008). The nanoparticles can be found in various form such as in sphere, rod or cubic and can be scattered individually. In this thesis only nanoparticles in sphere form is taken into consideration.

There are a few models constructed by researchers to determine the effectiveness of thermal conductivity of nanofluid theoretically such Khanafer et al. (2003), Buongiorno (2006), Tiwari and Das (2007), Nield and Kuznetsov (2009) and Kuznetsov (2010). However, only Buongiorno (2006) and Tiwari and Das (2007) model will be discussed throughout this thesis.

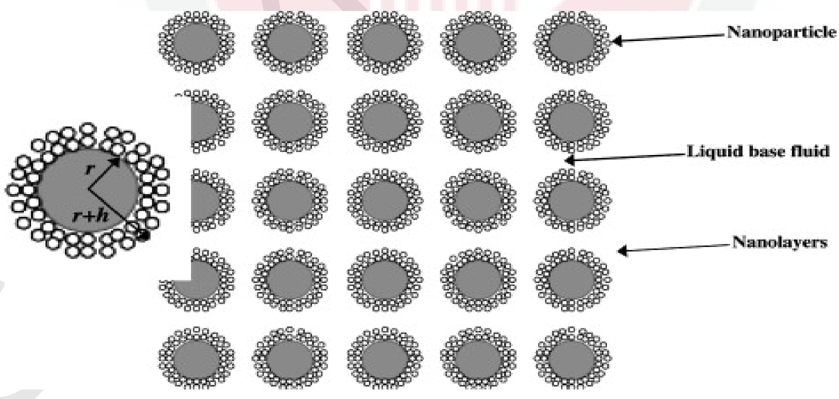

Figure 1.7: The cross-section of nanofluids
(https://aip.scitation.org/doi/pdf/10.1063/1.5018569)

1.6.1 Buongiorno Model

Buongiorno model is a two phase model where slip velocity between the base fluid and nanoparticles are not equal to zero. This condition happens due to several factors such as gravity, friction between the base fluid and nanoparticles, Brownian motion and thermophoresis. Buongiorno (2006) proposed seven slip mechanisms which results in relative velocity between the nanoparticles and the base fluid. These are inertia, Brownian motion, thermophoresis, diffusiophoresis, Magnus effect, fluid drainage, and gravity. Out of these seven mechanisms only Brownian motion and thermophoresis are important in solving nanofluid problems using Buongiorno model. These two mechanisms play an important role in heat and mass transfer and can be described as below:

- Brownian motion: A random motion of particles within the base fluid. The Brownian motion only exists when the particles are very small and can vanishes when the particles size becomes larger, Prasher et al. (2006). According to Malvandi et al. (2016), the smaller nanoparticles are able to accumulate at the heated wall and enhance the heat transfer rate whereas the larger nanoparticles depletes at the heated wall and at once prevents considerable enhancement in heat transfer rate.

- Thermophoresis: A diffusion of particles under the effect of a temperature gradient which means higher energy particles at the higher temperature produce a greater momentum than the particles at the lower temperature. Besides that, the effects of thermophoresis also can be found in mass transfer. According to Bahiraei (2017), the larger particles results in non-uniform concentration distribution. Therefore, smaller particles are very good medium for enhancing of heat and mass transfer.

1.6.2 Tiwari and Das Model

Tiwari and Das (2007) model is a single phase model where the fluid and the particles are assumed to be in thermal equilibrium and move with the same velocity with no slip condition is applied. This model considers the viscosity model introduced by Brinkman (1952) and Maxwell-Garnet thermal conductivity. The nanoparticles volume fraction play an important role in this model to increase the heat transfer rate. The effectiveness of thermal conductivity in nanofluid depends on the increment in the nanoparticles volume fraction, Karthikeyan et al. (2008). Jang and Choi (2007) reported that small amount of nanoparticles volume fraction is sufficient to enhance the conventional heat transfer.

1.7 Slip Velocity

The first order slip or partial slip condition proposed by Navier in 1823 is defined as

$$u(x,y) = A\frac{\partial u}{\partial y}, \tag{1.7.1}$$

where u is the fluid velocity, A is the slip length, $\frac{\partial u}{\partial y}$ is the shear stress at the boundary and y is the coordinates tangential to the surface. The velocity component normal to the surface is naturally zero as mass cannot penetrate an impermeable solid surface, see Figure 1.8.

Wu (2008) has introduced the new slip condition named second order velocity slip. This model was formulated from the kinetic theory under the deeper physical consideration between gas molecules and walls to improve the first order slip model (Maxwell slip model), second order slip model, 1.5 order slip model. This is because, the prediction of mentioned slip models start to deviate from linearized Boltzmann solution when the Knudsen number becomes greater than 1. Therefore, the improved second order slip model proposed by Wu (2008) is convenient to apply and also gives reliable predictions at high Knudsen number.

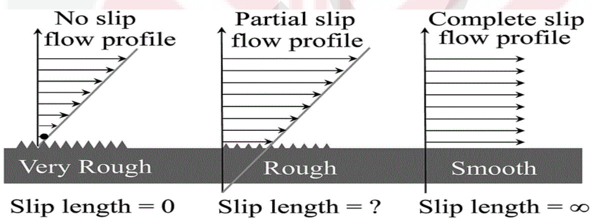

Figure 1.8: The slip length along flat plate
(https://pubs.rsc.org/en/content/getauthorversionpdf/C5AY00574D)

1.8 Soret and Dufour Effects

Soret effect or also known as thermodiffusion is a diffusion of particles from higher temperature towards lower temperature due to mass flux. Dufour effect (diffusion-thermo) is a reverse phenomenon of Soret effect where the particles are diffused from higher concentration to the lower concentration due to energy flux. According to Kafoussias and Williams (1995) and Bhattacharyya et al. (2014), both Soret and Dufour effects were formulated from the kinetic theory of gases by Chapman and Cowling (1952). The necessary formulae was derived in details to describe these two effects for monatomic gases and polyatomic gas mixture by Hirshfelder et al. (1954).

1.9 Thermal Convective Boundary Condition

The thermal convective boundary condition is described when the bottom surface of the plate is heated by convection from a hot fluid, see Aziz (2009). The parameter involving in this convective boundary condition is Biot number, which was introduced by French physicist Jean Baptiste Biot in 1804. He analyzed the interaction between conduction in a solid and convection at its surface. In addition, Biot number also can be defined as the ratio of the convective heat resistance within the boundary layer to the convective heat transfer resistance across the boundary layer. According to Isa et al. (2017), the convective heat transfer involves in engineering procedures, namely, nuclear plants, gas turbines and storage of thermal energy.Such processes obtain high temperature which flow is subject to the convective boundary condition.

1.10 Stability Analysis

Stability analysis is an analysis to validate which solutions are a stable solution and physically realizable by determining the smallest eigenvalue γ. Based on the literature review, the stability analysis was introduced by Merkin (1985) in order to determine the stability analysis of the solutions since they realized that there exist dual or more solutions in boundary layer flow. The stable solution defined when there exist an initial decay that does not interrupt the boundary layer flow. Meanwhile, the unstable solution is taken when there exist an early growth or disruption in the boundary layer flow.

1.11 Problem Statements

The problems regarding the boundary layer flow over a moving plate as well as stretching or shrinking surface in nanofluid in the presence of second order velocity slip at the boundary are getting more interest by the authors. Besides that, the performing of stability analysis in each study gained attraction among the researchers because they can know which solutions are stable and physically realizable. Hence, some of the statements that can be made are:

1. What are the parameters contributing to the existance of dual solutions?
2. What are the parameters contribute to the expansion of the solutions obtained?
3. Whether first or second solution is a stable solution?
4. How does the nanofluid affect the skin friction, heat and mass transfer coefficient?
5. How does the presence of slip parameters, Biot, Soret as well as Dufour number affect the skin friction, heat and mass transfer coefficient?

1.12 Objectives and Scopes

The objectives of this thesis are to provide the mathematical formulation, create an algorithm and solve computationally using bvp4c function in Matlab software for the five problems below:

- Boundary layer flow of nanofluids over a moving surface in a flowing fluid in the presence of second order slip using Buongiorno model and perform the stability analysis.
- Flow and heat transfer of nanofluids over stretching or shrinking surface in the presence of second order slip and thermal convective boundary condition using Buongiorno model and perform the stability analysis.
- Stagnation point flow in nanofluids over stretching or shrinking surface in the presence of second order slip, Soret and Dufour effects using Buongiorno model and perform the stability analysis.
- Boundary layer flow, heat and mass transfer of nanofluids over a moving surface in the presence of Soret and Dufour effects using Tiwari and Das model and perform the stability analysis.
- Stagnation point flow in nanofluid over stretching or shrinking surface in the presence of second order slip, Soret and Dufour effects using Tiwari and Das model and perform the stability analysis.

The scopes of this thesis are to focus on a steady two-dimensional laminar flow towards the horizontal surface with various boundary conditions such as second order velocity slip and thermal convective boundary condition as well as with Soret and Dufour effects at the heat and mass transfer over a moving plate and stretching or shrinking surface in nanofluids by using Buongiorno model (2006) and Tiwari and Das model (2007). The steady state refers to when all governing parameters involved namely, temperature, pressure and the flow rates are not changing due to time.

1.13 Significance of Study

The boundary layer characteristics on a moving surface is an important type of flow occurring in many industrial and technological processes. The examples of practical application of a continuous moving surface shall be referred in the boundary layer along a liquid film in condensation processes and a polymer sheet or filament extruded continuously from a dye, or a long thread traveling between a feed roll and a wind-up roll on conveyor belts, Patil et al. (2009). The applications of stretching or shrinking sheets in industrial fields such as paper production, hot rolling, wire drawing, glass-fiber production and aerodynamic extrusion. The cooling of a long metallic wire in a bath (an electrolyte) is one of the physical situation belonging to stretching or shrinking sheets category. Flow due to a stretching surface can be found in glass blowing, continuous casting, and spinning of fibers. During its manufacturing process, a stretched sheet interacts with the ambient fluid thermally and mechanically. The final products are depending on the effectiveness of heat transfer and cooling system in order to achieve top quality.

Many engineering and industrial processes involve heat transfer by means of a flowing fluid in either laminar or turbulent flow. A decrease in thermal resistance of heat transfer in the fluids would definitely benefit many of these applications. As known, nanofluids have the potential to reduce thermal resistance in industrial processes such as electronics, medical, food and manufacturing would benefit from such improved heat transfer, see Shateyi and Prakash (2014). Moreover, the applications of nanofluids include hybrid-powered engines, fuel cells, pharmaceutical applications, refrigeration, food processing industry and automotives, see Ramzan and Yousaf (2015).

The no-slip condition is no longer applicable in some cases such as in the macroscopically physical phenomenon in fluid mechanics. Based on the experiment by Cottin-Bizonne et al. (2005), the empirical non-slip boundary condition may break down depending on the fluid properties and the interfacial roughness. Therefore, the slip effect should be taken into consideration which leads to the requirement of a slip boundary condition, Khader (2014). There are many Newtonian and non-Newtonian fluids which will involve slip regime like particulate fluid and rarefied gases. The typical examples are gas flows inside nanotubes, and air lubrication of head-disk interface of disk drives. In addition, wall slip readily occurs for an array of complex fluid such as emulsions, suspensions, foams, and polymer solutions, see Sharma and Ishak (2016). Some applications involving slip in fluid mechanics can be detected in flow of rarefied gases at low Knudsen number (Sharipov and Seleznez (1998)), flow past superhydrophobic microsurfaces (Choi and Kim (2006) and Ng and Wang (2009)) and flow against rough surfaces (Wang (2003)). Also, the fluids that exhibit boundary slip have important technological applications such as in the polishing of artificial heart valves and internal cavities, see Khader (2014).

According to Bonner and Sundelof (1984) and Platten (2006), the role of the Soret effect has been evoked in biological systems, namely as mass transport across biological membranes induced by small thermal gradient in living matter where thermodiffusion could assume a sizable magnitude for an ensemble of cells with the dimension of an organ or a tumor. Thermodiffusion (Soret effect) has numerous industrial applications such as optimum oil recovery from hydrocarbon reservoirs, fabrication of semiconductor devices in olten metal and semiconductor mixtures as well as separation of species such as polymers, manipulation of macromolecules such as DNA, see Eslamian (2011). The application involves both Soret and Dufour effects are important in areas such as hydrology, petrology, and geosciences.

In addition, the convective heat transfer becomes very important in processes involving high temperature such as gas turbines, nuclear plants and thermal energy storage, see Fenuga et al. (2018). These processes obtain high temperature, which the flow is subjected to the convective boundary condition. Lastly, the significant of analyzing the stability analysis is to determine which solutions are stable and at the same time physically applicable. Therefore, as a mathematicians, we will convince the engineers to choose the stable solution rather than unstable solution when they want to do an experiment.

1.14 Outline of the Thesis

There are nine chapters in this thesis including this chapter. Chapter 1 is discussing the research background, problem statements, objectives, scopes and significance of study. The literature reviews on the studied problems are presented in Chapter 2. Chapter 3 discusses the derivation of the Buongiorno's model and Tiwari and Das model that have been used in this study to solve the nanofluid problems. The numerical method on how to apply the ordinary differential equations into the bvp4c function is explained in details in Chapter 3.

Chapter 4 until 6 consider the stability analysis and mathematical modeling on boundary layer flow, heat and mass transfer in nanofluid using Buongiorno's model (2006) over a moving plate as well as stretching or shrinking surface. In particular, Chapter 4 discusses the effects of second order slip at the boundary on the moving plate in the nanofluid. Next, Chapter 5 studies the effects of second order velocity slip together with the presence of thermal convective boundary conditions on the boundary layer flow over a stretching or shrinking surface. Effects of Soret and Dufour on energy and concentration equation are studied in the presence of second order velocity slip immersed in nanofluid is considered in Chapter 6. In these three chapters, the effects of velocity ratio parameter between the plate and the free stream, stretching or shrinking parameter, Brownian motion and thermophoresis parameter towards the flow behavior, heat and mass transfer is studied.

Chapter 7 and 8 discuss the stability analysis and mathematical modeling on boundary layer flow, heat and mass transfer in nanofluid using Tiwari and Das model (2007) over a moving plate and also stretching or shrinking surface. Both problems discuss how the Soret and Dufour effects affected the heat transfer and mass transfer at the surface. In addition, the presence of second order velocity slip at the wall is studied in Chapter 8. In these two chapters, the effects of velocity ratio parameter between the plate and the free stream, stretching or shrinking parameter and also the nanoparticle volume fraction towards the flow behavior, heat and mass transfer is studied.

Lastly, conclusions of all problems and future work are presented in Chapter 9.

REFERENCES

Abdelhafez, T. A. (1985). Skin friction and heat transfer on a continuous flat surface moving in a parallel free stream. *International Journal of Heat and Mass Transfer*, 28(6):1234–1237.

Acheson, D. J. (1990). *Elementary Fluid Dynamics*. New York: Oxford University Press, Inc.

Afridi, M. I. and Qasim, M. (2018). Entropy generation and heat transfer in boundary layer flow over a thin needle moving in a parallel stream in the presence of nonlinear rosseland radiation. *International Journal of Thermal Sciences*, 123:117–128.

Afzal, N., Badaruddin, A., and Elgarvi, A. A. (1993). Momentum and transport on a continuous flat surface moving in a parallel stream. *International Journal of Heat and Mass Transfer*, 36:3399–3403.

Ahmed, A., Siddique, J. I., and Sagheer, M. (2018). Dual solutions in a boundary layer flow of a power law fluid over a moving permeable flat plate with thermal radiation, viscous dissipation and heat generation/absorption. *Fluids*, 3(6):1–16.

Alam, M. S. and Rahman, M. M. (2006). Dufour and soret effects on mixed convection flow past a vertical porous flat plate with variable suction. *Nonlinear Analysis: Modelling and Control*, 11(1):3–12.

Aleng, N. L., Bachok, N., and Arifin, N. M. (2014). Boundary layer flow of a nanofluid and heat transfer over an exponentially shrinking sheet: Copper-water. *Mathematical and Computational Methods in Science and Engineering*, 21:137–142.

Anderson, J. D. (2005). Ludwig prandtl's boundary layer. *Physics Today*, 58(12):42–48.

Arpaci, V. S. and Larsen, P. S. (1984). *Convection Heat Transfer*. New Jersey: Prentice-Hall, Inc.

Aziz, A. (2009). A similarity solution for laminar thermal boundary layer over a flat plate with a convective surface boundary condition. *Communications in Nonlinear Science and Numerical Simulation*, 14:1064–1068.

Bachok, N., Ishak, A., and Pop, I. (2010). Boundary-layer flow of nanofluids over a moving surface in a flowing fluid. *International Journal of Thermal Sciences*, 49(9):1663–1668.

Bachok, N., Ishak, A., and Pop, I. (2011). Stagnation-point flow over a stretching/shrinking sheet in a nanofluid. *Nanoscale Research Letters*, 6:1–10.

Bachok, N., Ishak, A., and Pop, I. (2012). Flow and heat transfer characteristics on a moving plate in a nanofluid. *International Journal of Heat and Mass Transfer*, 55:642–648.

Bachok, N., Ishak, A., and Pop, I. (2013). Mixed convection boundary layer flow over a moving vertical flat plate in an external fluid flow with viscous dissipation effect. *Plos One*, 8(4):e60766–6.

Bachok, N., Najib, N., Arifin, N. M., and Senu, N. (2016). Stability of dual solutions in boundary layer flow and heat transfer on a moving plate in a copper-water nanofluid with slip effect. *WSEAS Transactions on Fluid Mechanics*, 11:151–158.

Bahiraei, M. (2017). Impact of thermophoresis on nanoparticle distribution in nanofluids. *Results in Physics*, 7:136–138.

Bhattacharyya, K., Layek, G. C., and Seth, G. S. (2014). Soret and dufour effects on convective heat and mass transfer in stagnation-point flow towards a shrinking surface. *Physica Scripta*, 89:1–10.

Bonner, F. J. and Sundelof, L. O. (1984). Thermal diffusion as a mechanism for biological transport. *Z. Naturforsch*, 39:656–661.

Brinkman, H. C. (1952). The viscosity of concentrated suspensions and solutions. *Journal of Chemical Physics*, 20:571–584.

Buongiorno, J. (2006). Convective transport in nanofluids. *ASME Journal of Heat Transfer*, 128:240–250.

Cengel, Y. and Cimbala, J. (2006). *Fluid Mechanics: Fundamental and Applications*. New York: McGraw-Hill Companies, Inc.

Chapman, S. and Cowling, T. G. (1952). *The Mathematical Theory of Non-uniform Gases*. Cambridge: Cambridge University Press,.

Chen, C. K. and Char, M. I. (1988). Heat transfer of a continuous stretching surface with suction or blowing. *Journal of Mathematical Analysis and Applications*, 135:568–580.

Choi, C. H. and Kim, C. J. (2006). Large slip of aqueous liquid flow over a nanoengineered superhydrophobic surface. *Physical Review Letters*, 96:066001.

Choi, S. U. S. (1995). Enhancing thermal conductivity of fluids with nanoparticles. *American Society of Mechanical Engineers, Fluid Engineering Division*, 231:99–105.

Choi, S. U. S. (1998). Nanofluid technology: Current status and future research. *The Second Korean-American Scientists and Engineers Association Research Trend Study Project Review and the Korea-U.S. Technical Conference on Strategic Technologies*, 1998.

Cottin-Bizonne, C., Cross, B., Steinberger, A., and Charlaix, E. (2005). Boundary slip on smooth hydrophobic surfaces: intrinsic effects and possible artifacts. *Physical Review Letters*, 94:56–102.

Crane, L. J. (1970). Flow past a stretching plate. *Zeitschrift fr angewandte Mathematik und Physik*, 21:645–647.

Das, S. K., Choi, S. U. S., Yu, W., and Pradeep, T. (2008). *Nanofluids: Science and Technology*. New Jersey: John Wiley & Son Inc.

Dzulkifli, N. F., Bachok, N., Pop, I., Yacob, N. A., Arifin, N. M., and Rosali, H. (2017a). Soret and dufour effects on unsteady boundary layer flow and heat transfer of nanofluid over a stretching/shrinking sheet: A stability analysis. *Journal of Chemical Engineering & Process Technology*, 8(3):1000336–9.

Dzulkifli, N. F., Bachok, N., Pop, I., Yacob, N. A., Arifin, N. M., and Rosali, H. (2017b). Stability of partial slip, soret and dufour effects on unsteady boundary layer flow and heat transfer in copper-water nanofluid over a stretching/shrinking sheet. *IOP Conference Series: Journal of Physics*, 890:012031–7.

Elbade, N. T., El-Saka, A. G., and Fouad, A. (2004). Thermal-diffusion and diffusion-thermo effects on mixed free-forced convection and mass transfer boundary layer flow for non-newtonian fluid with temperature dependent viscosity. *Applied Mathematics and Computation*, 152:867–883.

Eslamian, M. (2011). Advances in thermodiffusion and thermophoresis (soret effect) in liquid mixtures. *Frontiers Heat and Mass Transfer*, 2:1–20.

Fang, T. and Aziz, A. (2010). Viscous flow with second-order slip velocity over a stretching sheet. *Z. Naturforsch.*, 65a:10871092.

Fang, T., Yao, S., Zhang, J., and Aziz, A. (2010). Viscous flow over a shrinking sheet with a second-order slip flow model. *Communications in Nonlinear Science and Numerical Simulation*, 15:1831–1842.

Fang, T. and Zhang, J. (2009). Closed-form exact solution of mhd viscous flow over a shrinking sheet. *Communications in Nonlinear Science and Numerical Simulation*, 14:28532857.

Fenuga, O. J., Abiala, I. O., and Salawu, S. O. (2018). Analysis of thermal boundary layer flow over a vertical plate with electrical conductivity and convective surface boundary conditions. *Physical Science International Journal*, 17(2):1–9.

Gupta, P. S. and Gupta, A. S. (1977). Heat and mass transfer on a stretching sheet with suction or blowing. *The Canadian Journal of Chemical Engineering*, 55:744–746.

Hansen, A. G. (1964). *Similarity Analysis of Boundary Value Problems in Engineering*. New Jersey: Prentice-Hall, Inc.

Harris, S. D., Ingham, D. B., and Pop, I. (2009). Mixed convection boundary-layer flow near the stagnation point on a vertical surface in a porous medium: Brinkman model with slip. *Transport Porous Media*, 77:267–285.

Hirshfelder, J. O., Curtis, C. F., and Bird, R. B. (1954). *Molecular Theory of Gases and Liquids*. New York: Wiley,.

Ibrahim, W., Shankar, B., and Nandeppanavar, M. M. (2013). Mhd stagnation point flow and heat transfer due to nanofluid towards a stretching sheet. *International Journal of Heat and Mass Transfer*, 56:1–9.

Isa, S. S. P. M., Arifin, N. M., Nazar, R., Bachok, N., and Ali, F. (2017). The effect of convective boundary condition on mhd mixed convection boundary layer flow over an exponentially stretching vertical sheet. *IOP Conference Series: Journal of Physics*, 949:012016–14.

Jahan, S., Hamzah, S., Nazar, R., and Pop, I. (2017). Flow and heat transfer past a permeable nonlinearly stretching/shrinking sheet in a nanofluid: A revised model with stability analysis. *Journal of Molecular Liquids*, 233:211–221.

Jang, S. P. and Choi, S. U. S. (2007). Effects of various parameters on nanofluid thermal conductivity. *Journal of Heat Transfer*, 129:617–623.

Kafoussias, N. G. and Williams, E. W. (1995). Thermal-diffusion and diffusion-thermo effects on mixed free-forced convection and mass transfer boundary layer flow with temperature dependent viscosity. *International Journal of Engineering Sciences*, 33(9):1369–1384.

Kandasamy, R., Muhaimin, I., and Mohamad, R. (2013). Thermophoresis and brownian motion effects on mhd boundary-layer flow of a nanofluid in the presence of thermal stratification due to solar radiation. *International Journal of Mechanical Sciences*, 70:146–154.

Kardri, M. A., Bachok, N., Arifin, N. M., and Ali, F. M. (2017a). Boundary layer flow and heat transfer over a stretching cylinder in a copper-water nanofluid. *AIP Conference Proceedings*, 1795:020012–8.

Kardri, M. A., Bachok, N., Arifin, N. M., and Ali, F. M. (2017b). Second-order velocity slip with axisymmetric stagnation point flow and heat transfer due to a stretching vertical plate in a copper-water nanofluid. *IOP Conference Series: Journal of Physics*, 890:012022–7.

Karthikeyan, N. R., Philip, J., and Raj, B. (2008). Effect of clustering on the thermal conductivity of nanofluids. *Material Chemistry and Physics*, 109:50–55.

Kays, W. M. and Crawford, M. E. (1993). *Convective Heat and Mass Transfer*. New York: McGraw-Hill Companies, Inc.

Khader, M. M. (2014). Laguerre collocation method for the flow and heat transfer due to a permeable stretching surface embedded in a porous medium with a second order slip and viscous dissipation. *Applied Mathematics and Computation*, 243:503–513.

Khan, W. A. and Pop, I. (2010). Boundary layer flow of a nanofluid past a stretching sheet. *International Journal of Heat and Mass Transfer*, 53:2477–2483.

Khan, W. A. and Pop, I. (2012). Boundary layer flow past a stretching surface in a porous medium saturated by a nanofluid: Brinkman-forchheimer model. *Plos One*, 7(10):e47031–6.

Khanafer, K., Vafai, K., and Lightstone, M. (2003). Bouyancy-driven heat transfer enhancement in a two-dimensional enclosure utilizing nanofluids. *International Journal of Heat and Mass Transfer*, 46:3639–3653.

Kleinstreuer, C. (2010). *Modern Fluid Dynamics*. New York: Springer,.

Kuznetsov, A. V. (2010). The onset of nanofluid bioconvection in a suspension containing both nanoparticles and gyrotactic microorganisms. *International Communications in Heat and Mass Transfer*, 37:1421–1425.

Kuznetsov, A. V. and Nield, D. A. (2010). Natural convective boundary-layer flow of a nanofluid past a vertical plate. *International Journal of Thermal Science*, 49(2):243–247.

Kuznetsov, A. V. and Nield, D. A. (2013). The cheng-minkowycz problem for natural convective boundary layer flow in a porous medium saturated by a nanofluid: a revised model. *International Journal of Heat and Mass Transfer*, 65:682–685.

Kuznetsov, A. V. and Nield, D. A. (2014). Natural convective boundary-layer flow of a nanofluid past a vertical plate: A revised model. *International Journal of Thermal Science*, 77:126–129.

Maiga, S. E. B., Palm, S. J., Nguyen, C. T., Roy, G., and Galanis, N. (2005). Heat transfer enhancement by using nanaofluids in forced convection flows. *International Journal of Heat and Fluid Flow*, 26:530–546.

Makinde, O. D. and Aziz, A. (2011). Boundary layer flow of a nanofluid past a stretching sheet with a convective boundary conditions. *International Journal of Thermal Science*, 50:1326–1332.

Malvandi, A., Heysiattalab, S., and Ganji, D. D. (2016). Thermophoresis and brownian motion effects on heat transfer enhancement at film boiling of nanofluids over a vertical cylinder. *Journal of Molecular Liquids*, 216:503–509.

Mansur, S. and Ishak, A. (2016). Unsteady boundary layer flow of a nanofluid over a stretching/shrinking sheet with a convective boundary condition. *Journal of the Egyptian Mathematical Society*, 24(4):650–655.

Mansur, S., Ishak, A., and Pop, I. (2015). Stagnation-point flow towards a stretching/shrinking sheet in a nanofluid using buongiornos model. *Journal od Process Mechanical Engineering*, 231(2):172–180.

Merill, K., Beauchesne, M., Previte, J., Paullet, J., and Weidman, P. (2006). Final steady flow near a stagnation point on a vertical surface in a porous medium. *International Journal of Heat and Mass Transfer*, 49:4681–4686.

Merkin, J. H. (1985). On dual solutions occuring in mixed convection in a porous medium. *Journal of Engineering Mathematics*, 20:171–179.

Miklavcic, M. and Wang, C. Y. (2006). Viscous flow due to a shrinking sheet. *Quarterly of Applied Mathematics*, 64:283–290.

Mukhopadhyay, S. and Anderson, H. I. (2009). Effects of slip and heat transfer analysis of flow over an unsteady stretching surface. *Heat Mass Transfer*, 45:1447–1452.

Nadeem, S., Ul-Haq, R., and Khan, Z. H. (2014). Heat transfer analysis of water-based nanofluid over an exponentially stretching sheet. *Alexandria Engineering Journal*, 53:219–224.

Nakayama, A. (1995). *PC-Aided Numerical Heat Transfer and Convective Flow*. Florida CRC Press, Inc.

Ng, C. O. and Wang, C. Y. (2009). Stokes shear flow over a grating: implications for superhydrophobic slip. *Physics of Fluids*, 21:013602.

Nield, D. A. and Kuznetsov, A. V. (2009). The cheng-minkowycz problem for natural convective boundary layer fow in a porous medium saturated by a nanofluid. *International Journal of Heat and Mass Transfer*, 52:5792–5795.

Nield, D. A. and Kuznetsov, A. V. (2014). Thermal instability in a porous medium layer saturated by a nanofluid: A revised model. *International Journal of Heat and Mass Transfer*, 68:211–214.

Noghrehabadi, A., Pourrajab, R., and Ghalambaz, M. (2013). Flow and heat transfer of nanofluids over stretching sheet taking into account partial slip and thermal convective boundary conditions. *Heat Mass Transfer*, 49:1357–1366.

Omar, N. S., Bachok, N., and Arifin, N. M. (2015). Stagnation point flow over a stretching or shrinking cylinder in a copper-water nanofluid. *Indian Journal of Science and Technology*, 8(31):1–7.

Oztop, H. F. and Abu-Nada, E. (2008). Numerical study of natural convection in partially heated rectangular enclosures filled with nanofluids. *International Journal of Heat and Fluid Flow*, 29:1326–1336.

Pal, D. and Mandal, G. (2017). Double diffusive magnetohydrodynamic heat and mass transfer of nanofluids over a nonlinear stretching/shrinking sheet with viscous-ohmic dissipation and thermal radiation. *Propulsion and Power Research*, 6(1):58–39.

Patil, P. M., Roy, S., and Chamkha, A. J. (2009). Double diffusive mixed convection flow over a moving vertical plate in the presence of internal heat generation and a chemical reaction. *Turkish Journal of Engineering and Environmental Sciences*, 33:193–205.

Pavlov, K. B. (1974). Magnetohydrodynamic flow of an incompressible viscous fluid caused by the deformation of a plane surface. *Magn. Gidrod.*, 10:146–148.

Platten, J. K. (2006). The soret effect: A review of recent experimental results. *Journal of Applied Mechanics*, 73:5–15.

Pohlhausen, K. (1921). Zur naherungsweisen integration der differentialgleichung der laminaren grenzschicht. *Zeitscher. f. angew. Math. u. Mech.*, 1:115–120, 257–261.

Polidari, G., Fohanno, S., and Nguyen, C. T. (2007). A note on heat transfer modelling of newtonian nanofluids in laminar free convection. *International Journal of Thermal Sciences*, 46:739–744.

Pop, I. and Na, T. Y. (1996). Unsteady flow past a stretching sheet. *Mechanics Research Communications*, 23:413–422.

Prasher, R., Bhattacharya, P., and Phelan, P. E. (2006). Brownian-motion-based convective-conductive model ot the effective thermal conductivity of nanofluids. *Journal of Heat Transfer*, 128:588–595.

Ramzan, M. and Yousaf, F. (2015). Boundary layer flow of three-dimensional viscoelastic nanofluid past a bi-directional stretching sheet with newtonian heating. *AIP Advances*, 5:057132–15.

Rosali, H., Ishak, A., Nazar, R., and Pop, I. (2016). Mixed convection boundary layer flow pasta vertical cone embedded in a porous medium subjected to a convective boundary condition. *Propulsion and Power Research*, 5(2):118–122.

Rosca, A. V. and Pop, I. (2013). Flow and heat transfer over a vertical permeable stretching/shrinking sheet with a second order slip. *International Journal of Heat and Mass Transfer*, 60:355–364.

Rosca, N. C., Rosca, A. V., and Pop, I. (2016). Semi-analytical solution for the flow of a nanofluid over a permeable stretching/shrinking sheet with velocity slip using buongiornos mathematical model. *European Journal of Mechanics B/Fluids*, 58:39–49.

Sakiadis, B. C. (1961). Boundary-layer behaviour on continous solid surfaces: I. boundary-layer equations for two-dimensional and axisymmetric flow. *AIChe Journal*, 7:26–28.

Sarif, N. M., Salleh, M. Z., and Nazar, R. (2016). Mixed convection flow over a horizontal circular cylinder in a viscous fluid at the lower stagnation point with convective boundary conditions. *ScienceAsia*, 42S:5–10.

Schlichting, H. (1979). *Boundary Layer Theory*. New York: McGraw-Hill Companies, Inc.

Shampine, L., Kierzenka, J., and Reichelt, M. (2000). *Solving boundary value problems for ordinary differential equations in Matlab with bvp4c*.

Sharipov, F. and Seleznez, V. (1998). Data on internal rarefied gas flows. *Journal of Physical and Chemical Reference Data*, 27:657–706.

Sharma, R. and Ishak, A. (2014). Second order slip flow of cu-water nanofluid over a stretching sheet with heat transfer. *WSEAS Transactions on Fluid Mechanics*, 9:26–33.

Sharma, R. and Ishak, A. (2016). Stagnation point flow of a micropolar fluid over a stretching/shrinking sheet with second-order velocity slip. *Journal of Aerospace Engineering*, 04016025:1–8.

Shateyi, S. and Prakash, J. (2014). A new numerical approach for mhd laminar boundary layer flow and heat transfer of nanofluids over a moving surface in the presence of thermal radiation. *Boundary Value Problems*, 2014(2):1–12.

Singh, G. and Chamkha, A. J. (2013). Dual solutions for second-order slip flow and heat transfer on a vertical permeable shrinking sheet. *Ain Shams Engineering Journal*, 4:911917.

Tiwari, R. K. and Das, M. K. (2007). Heat transfer augmentation in a two-sided lid-driven differentially heated square cavity utilizing nanofluids. *International Journal of Heat and Mass Transfer*, 50:2002–2018.

Tsou, F. K., Sparrow, E. M., and Goldstein, R. J. (1967). Flow and heat transfer in the boundary layer on a continuous moving surface. *International Journal of Heat and Mass Transfer*, 10:219–235.

Uddin, M. J., Khan, W. A., and Ismail, A. I. (2012). Mhd free convective boundary layer flow of a nanofluid past a flat vertical plate with newtonian heating boundary condition. *Plos One*, 7(11):e49499–8.

Ullah, I., Khan, I., and Shafie, S. (2017). Soret and dufour effects on unsteady mixed convection slip flow of casson fluid over a nonlinearly stretching sheet with convective boundary condition. *Sientific Reports*, 7:1113–19.

Wang, C. Y. (2003). Flow over a surface with parallel grooves. *Physics of Fluids*, 15:1114–1121.

Wang, C. Y. (2008). Stagnation flow towards a shrinking sheet. *International Journal of Non-Linear Mechanics*, 43(5):377–382.

Weidman, D., Kubitschek, D. G., and Davis, A. M. J. (2006). The effect of transpiration on self-similar boundary layer flow over a moving surface. *International Journal of Engineering Science*, 44:730–737.

Wu, L. (2008). A slip model for rarefied gas flows at arbitrary knudsen number. *Applied Physics Letter*, 93:253103–3.

Wu, L. (2016). Mass transfer induced slip effect on viscous gas flows above a shrinking/stretching sheet. *International Journal of Heat and Mass Transfer*, 93:17–22.

Yacob, N. A., Ishak, A., and Pop, I. (2011). Falkner-skan problem for a static or moving wedge in nanofluids. *International Journal of Thermal Sciences*, 50:133–139.

Zaimi, K., Ishak, A., and Pop, I. (2014). Flow past a permeable stretching/shrinking sheet in a nanofluid using two-phase model. *Plos One*, 9(11):e111743–6.