



***CHAOS SYNCHRONIZATION OF LOGISTICS, QI AND  
MAXWELL-BLOCH SYSTEMS FOR CHAOS-BASED COMMUNICATION***

**NUR AISYAH BINTI ABDUL FATAF**

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**By**

**NUR AISYAH BINTI ABDUL FATAF**

**Thesis Submitted to the School of Graduate Studies, Universiti Putra  
Malaysia, in Fulfillment of the Requirements for the Doctor of Philosophy**

**September 2018**

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## **DEDICATIONS**

**To all of my love**

**Ibu Ayah**

**Hubby Sallehuddin, Hannah, Akid**

**Hadi, Hidayah, Aziz, Hafeez, Arif, Solehah, Muaz**



Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfillment  
of the requirement for the degree of Doctor of Philosophy

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September 2018

**Chairman: Mohamad Rushdan bin Md Said, PhD**  
**Institute: Institute for Mathematical Research**

Since chaos synchronization has become a very important topic regarding applications in communication, various different communication schemes have been proposed by many researchers. In most of the cases, chaos based communication research are based on synchronization of chaotic systems. The signal at the transmitter and receiver are synchronized in order to use the output in communication model. In this thesis, we investigate chaos based communication by using non-synchronized states of chaotic/hyperchaotic systems. The message can be transmitted even if the transmitter and the receiver are in non-synchronized states.

At the beginning of the thesis, we investigate the synchronization between discrete chaotic systems using techniques of nonlinear controller. The choice of the technique is due to the effectiveness of the synchronization scheme in terms of time and error. After achieving synchronization, we then apply the method for communication. We implement the chaotic masking communication scheme technique to demonstrate the effectiveness of the proposed scheme.

In the next part, we examine an adaptive synchronization of hyperchaotic Qi-System by using nonlinear controller. Based on the analysis, the system loses its information during the synchronization due to the effect of the nonlinear controller. Besides, the complexity also decreases from a single system to the coupled systems. Hence, we propose and redefine the chaotic masking scheme with non-synchronized coupled systems for communication. The analysis shows that the proposed scheme is successfully implemented and very robust.

Finally, we explore the dynamics of a 5D model governed by the Maxwell-Bloch equations and investigate the synchronization of the systems by using unidirectional linear coupling. We also identify the non-synchronized region and utilize the region to implement a new communication scheme by using chaos shift key technique. The analysis shows the effectiveness of the proposed scheme.



Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

**PENYEGERAKAN KEKALUTAN BAGI SISTEM LOGISTIK, QI DAN  
MAXWELL-BLOCH UNTUK KOMUNIKASI BERASASKAN KALUT**

Oleh

**NUR AISYAH BINTI ABDUL FATAF**

September 2018

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Oleh kerana penyegerakan kekalutan telah menjadi topik yang sangat penting dalam aplikasi komunikasi, pelbagai skim komunikasi yang berbeza telah dicadangkan oleh ramai penyelidik. Dalam kebanyakan kes, penyelidikan komunikasi berdasarkan kekalutan adalah berdasarkan penyegerakan sistem kekalutan. Isyarat pemancar dan penerima disegerakkan untuk menggunakan output dalam model komunikasi. Oleh itu, dalam tesis ini, kami menyiasat komunikasi berasaskan kekalutan dengan menggunakan sistem kekalutan tidak disegerakkan. Mesej boleh dihantar jika pemancar dan penerima berada dalam keadaan tidak disegerakkan.

Pada permulaan tesis ini, kami menyelidik penyegerakan antara sistem diskret kalut menggunakan teknik pengawal tak linear. Teknik ini dipilih kerana keberkesanan skema penyegerakan dari segi masa dan ralat. Selepas mencapai penyegerakan, kami menggunakannya untuk berkomunikasi. Kami melaksanakan teknik komunikasi berpelindung kalut untuk menunjukkan keberkesanan skim yang telah dicadangkan.

Di bahagian seterusnya, kami juga menyelidik penyegerakan penyesuaian kekalutan hiper Sistem-Qi dengan menggunakan pengawal tak linear. Berdasarkan analisis yang telah dilakukan, sistem ini kehilangan maklumatnya semasa penyegerakan disebabkan oleh kesan pengawal tak linear. Selain itu, kerumitan juga berkurangan daripada sistem tunggal kepada sistem berganda. Oleh itu, kami mencadangkan dan menakrifkan semula skim komunikasi berpelindung kalut menggunakan sistem tak selaras. Analisis menunjukkan bahawa skim yang dicadangkan berjaya dilaksanakan dan mantap.

Di samping itu, kami juga meneroka dinamik model 5D yang dikawal oleh persamaan Maxwell-Bloch dan menyelidik penyegerakan sistem dengan menggunakan pengawal linear satu arah. Kami juga mengenal pasti rantau tak diseragakkan dan menggunakan rantau ini untuk melaksanakan skim komunikasi dengan menggunakan teknik kunci pengalihan kalut. Analisis menunjukkan keberkesanan skim yang dicadangkan.





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**Last but not least, I wish to express my sincerest appreciation and words of gratitude to everyone who has contributed and supported me, directly or indirectly towards the completion of this amazing journey my friends, my colleagues in UPNM.**

I certify that a Thesis Examination Committee has met on 5 September 2018 to conduct the final examination of Nur Aisyah Binti Abdul Fataf on her thesis entitled **Chaos Synchronization of Logistics, Qi and Maxwell-Bloch Systems for Chaos-Based Communication** in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U. (A) 106] 15 March 1998. The Committee recommends that the student be awarded the Doctor of Philosophy.

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## LIST OF ABBREVIATIONS

<b>CS</b>	<b>Complete Synchronization</b>
<b>GS</b>	<b>Generalized Synchronization</b>
<b>PS</b>	<b>Phase Synchronization</b>
<b>CSK</b>	<b>Chaos Shift Key</b>
<b>RP</b>	<b>Recurrence Plot</b>
<b>WRPE</b>	<b>Weighted Recurrence Plot Entropy</b>
<b>EO</b>	<b>Electro Optic</b>
<b>CI-DCSK</b>	<b>Carrier Index Differential Chaos Shift Key</b>
<b>LE</b>	<b>Lyapunov Exponents</b>
<b>CLE</b>	<b>Conditional Lyapunov Exponents</b>
<b>TLE</b>	<b>Transverse Lyapunov Exponents</b>
<b>CE</b>	<b>Communication Error</b>
<b>MSE</b>	<b>Mean Synchronization Error</b>
<b>SNR</b>	<b>Signal to Noise Ratio</b>
<b>AWGN</b>	<b>Additive White Gaussian Noise</b>
<b>BER</b>	<b>Bit Error Rate</b>
<b>ODE</b>	<b>Ordinary Differential Equation</b>
<b>PDE</b>	<b>Partial Differential Equation</b>
<b>LLE</b>	<b>Largest Lyapunov Exponent</b>
<b>x(t)</b>	<b>Gaussian Noise</b>

# CHAPTER 1

## INTRODUCTION

### 1.1 Dynamics of Chaotic Systems

**Chaos is one of the fascinating phenomena that can be observed in nature. Among the examples of chaotic events in nature are the weather and climate, dynamics of satellites in the solar system, time evolution of the magnetic field of celestial bodies, and population growth in ecology while the examples of chaotic events in laboratory are the electrical circuits, lasers, chemical reactions, fluid dynamics, mechanical systems, and magneto-mechanical devices.**

Besides, chaos can be observed in various physical and real world applications such as electronics and communications, plasma and lasers, biology and ecological applications. The nonlinear structure can be widely varied with respect to the parameters associated with the dynamics of chaotic system. It has also been observed that any chaotic model is a source of pseudo random numbers which can be implemented easily using digital signal processor or field-programmable gate array outputs.

Since 1990, communication and signal processing applications of chaos were roughly established as an area of permanent interest after the theories of chaos synchronization and chaos control were worked out in more details. As in present, permanent research with commercially viable engineering solutions was represented in sound engineering applications of quasi random sequence generation, modelling of communication channels using chaos, chaotic cryptography, digital image encoding, and chaotic transport phenomena in complex networks (Kocarev et al., 2009).

Chaos theory does not have proper definitions. The dynamics of a chaotic system should be deterministic and nonlinear. Chaotic system are not analytically solvable and the solutions or trajectories of the system remain bounded within the phase space. This unstable state has a strong dependence on the values of the parameters.

Chaos is highly sensitive to initial condition. Small differences in initial condition yield widely diverging outcomes for such dynamical systems. In detail, the main characteristics and manifestations of chaos, as well as the transitions from more regular to chaotic solutions will be discussed. Among the main characteristics of a nonlinear dynamical system that exhibits the deterministic chaos for any given values of the parameters are as follows:

## 1. Sensitive dependence on initial conditions

One of the important property of chaotic dynamics is the sensitive dependence on initial conditions (butterfly effect). It is due to infinitesimal changes in the initial condition, which leads to completely different trajectories. The butterfly effect describes how a small change in one state of a deterministic nonlinear system can result in large differences in a later state. The butterfly effect can be properly defined as follows:

**Definition 1.1 (Sensitive Dependence on Initial Conditions)** (Solev et al., 2011). The map  $f : X \rightarrow X$  is said to exhibit sensitive dependence on initial conditions if there exists  $d > 0$  such that, for any  $x \in X$  and any neighborhood  $s$  of  $x$ , there exists  $y \in s$  and  $n \geq 0$  such that  $|f^n(x) - f^n(y)| > d$ .

For a given map  $f$ , this means that for a given point  $x$ , there exist at least one arbitrarily close point whose image after  $n$  iterations will differ by  $d$  from the image of  $x$ . As an example, if we have two points  $x, y \in A = (0, 1)$  and that  $y = x + d_0$  holds, where  $d_0 > 0$  and it will be our initial separation.

Then, for the separation  $d_m$  after  $m$  iterations of the map, we can express it as  $d_m = |j^m d_0|$ , from which it is obvious that  $d_m > d_0$ , because of  $|m| > 1$ , where  $m$  the multiplier of the fixed point. However, we can think of the sensitivity to initial conditions from different view which we can assume that a point  $x_0$  belongs to a sub-interval of  $A$  with has a diameter of  $2^k$ . Based on the action of the map, after  $k$  iterations, the sub-interval will be mapped onto the entire state space of the map (referring to the whole interval  $A$ ) and the state of the system  $x_k$  could be anywhere in  $A$ . As for  $x_n$ , with  $n \geq k$  we are absolutely uncertain about the location of  $x_n$  although at the beginning we know that the initial point is within a small sub-interval (Solev et al., 2011).

Lyapunov exponents (LE) can be used to quantify this sensitivity to initial conditions. Similarly to the discussion above, assuming we have some initial conditions  $x_0 + d_0$ , where  $d_0$  is very small. If we can express the separation of  $d_n$  after  $n$  iterations as  $|d_n| \approx e^{l n} |d_0|$ , then  $l$  is a Lyapunov exponent. For any one-dimensional map there is only one Lyapunov exponent that can be expressed as follows:

$$l = \lim_{n \rightarrow \infty} \frac{1}{n} \ln \frac{|d_n|}{|d_0|} = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{i=0}^{n-1} \ln |(f^i)'(x_0)| \quad (1.1.1)$$

Sensitivity to initial conditions (and chaotic motion in general) is associated with a strictly positive Lyapunov exponent because the distance between the nearby points grows exponentially. If we try to compute the Lyapunov exponent of the tent map, we easily arrive at  $l = \log 2$ . In essence, Lyapunov exponents give us a measure of the divergence of infinitesimally close orbits (Solev et al., 2011).

## 2. Irregular motion in phase space

This characteristic is illustrated by very complex or sometimes described as the noise-like patterns of oscillations of the solutions within a bounded and compact sets. Chaos exhibits the quality of particularity as such complex oscillations are fully reproducible for the same numeric precision in the parameter values and initial conditions. Additionally, such quasi-stochastic behavior can be qualified by the specific character of the associated measures and invariant densities (Solev et al., 2011).

## 3. Qualitative change of the character of the solutions

This characteristic is described by one or more subsequent bifurcations. As we evolve the system in time, the structural changes of the phase set to which chaotic solutions converge. Such attractors namely the chaotic attractors sometimes do not resemble at all the topological structure of other solutions, for example the period orbits. This is the result of a global structural change of the phase space. Subsets which are compact and simply connected, together with certain parameter ranges undergo series of non-smooth changes in their geometry and topology, mainly caused by the subsequent stretching and folding cascades. Then, such attractors are also called strange attractors due to their specific geometry and self-similar structure on the different time-scales (Solev et al., 2011).

## 4. Ergodicity

This characteristic is described when a trajectory in phase space comes arbitrarily close to its earlier states. It essentially reflects that the system eventually is confined to a spatial object which is a set of points being called as an attractor. The density of such points is time invariant and this property is important in cryptography (Solev et al., 2011).

## 5. Mixing

This characteristic is characterized by a system in which a small interval of initial conditions gets spread over the full phase space in its asymptotic evolution. In a chaotic system, an arbitrary interval of initial conditions spreads over the part (attractor) of the phase space to which the trajectory asymptotically confines. Thus any region gets into every other region of the spatial attractor of phase space (Solev et al., 2011).

### 1.2 Synchronization Between Chaotic Systems

Synchronization means the trajectories of one of the systems will converge to the same values as the other systems and they will remain in steps with each other. The first studies about the topic of chaotic synchronization was introduced by Yamada and Fujisaka (1983). Later, the synchronization of chaotic oscillations between two nonlinear systems has been reported especially in different fields of engineering since the first prediction of chaos synchronization back then by Pecora and Carroll in 1990. In the suggested method of chaos synchronization, two indistinguishable nonlinear systems namely the transmitter and receiver systems which exhibit



chaotic dynamics are prepared. Based on the receiver system, it is divided into two sub-systems (driving and response systems) and the receiver consists of one of the sub-systems (response system).

As discussed and mentioned previously, chaos is very sensitive to the initial condition of a system. Hence, without a signal transmission from the transmitter to the receiver, identical waveform is impossible to be produced as outputs from the two systems. But when a chaotic output from the driving system in the transmitter is sent to the receiver, the receiver synchronizes with the transmitted signal under certain conditions of the system parameters. Thus, chaos synchronization is realized here and the transmitter must be a chaotic system. However, the receiver itself may or may not be a chaotic system without the transmission of a chaotic signal from the transmitter (Pecora and Carroll, 1991). Note that chaos synchronization can be attained as far as the conditional Lyapunov exponents (CLE) between the transmitter and receiver systems show negative values.

The studies on synchronization between dynamical systems has been conducted actively since the last three decades. The research on this phenomena became significant because it can be fitted into various branches of science and engineering. The applications of this area can be implemented in the field of control theory, secure communications, ecological and biological processes, complex and any other kind of networks. Different types of synchronization schemes can be observed in coupled chaotic systems, such as the Complete Synchronization (CS) has been carried out by Maa et al. (2011), the Generalized Synchronization (GS) has been conducted by Dmitriev et al. (2009) while the Phase Synchronization (PS) is studied by Mukherjee et al. (2014).

The motivation for the study of chaotic synchronization relies on its numerous potential applications since the applications of chaotic synchronization range from living systems applications to the non-living systems applications. Among the examples of applications of chaotic synchronization to living systems include synchronization in neurobiology (Gonzalez-Miranda, 2004) and chemical reactions among pancreatic cells (Mosekilde et al., 2002). Meanwhile, the examples of applications of chaotic synchronization to non-living systems include synchronization in earth sciences (Gonzalez-Miranda, 2004) and synchronization of chaotic electrochemical oscillators (Kiss and Hudson, 2003).

The principles of chaotic synchronization is illustrated by the general block diagram presented in Figure 1.1. Based on Figure 1.1, the master chaotic system transmits one or more of its signals to the slave system. In general, slave system is entirely different from the master system and note that the slave system is also another chaotic system. However, this slave system may or may not synchronize with the master system and it depends on the nature of the master signal that being supplied to the slave system.

On a different view, it is possible to design a controller at the slave side which will enforce synchronization if the master-slave system does not synchronize accordingly for a given master signal(s).

There are various kinds of controller for synchronization such as adaptive controller (Wang et al., 2004), periodic parametric perturbation (Astakhov et al., 1997), variable structure control (Yin et al., 2002), backstepping control (Lu and Zhang, 2001) and fuzzy control (Tanaka et al., 1998). Hence, it is possible to design a communication system based on the principles of chaotic synchronization once the master-slave synchronization has been achieved.

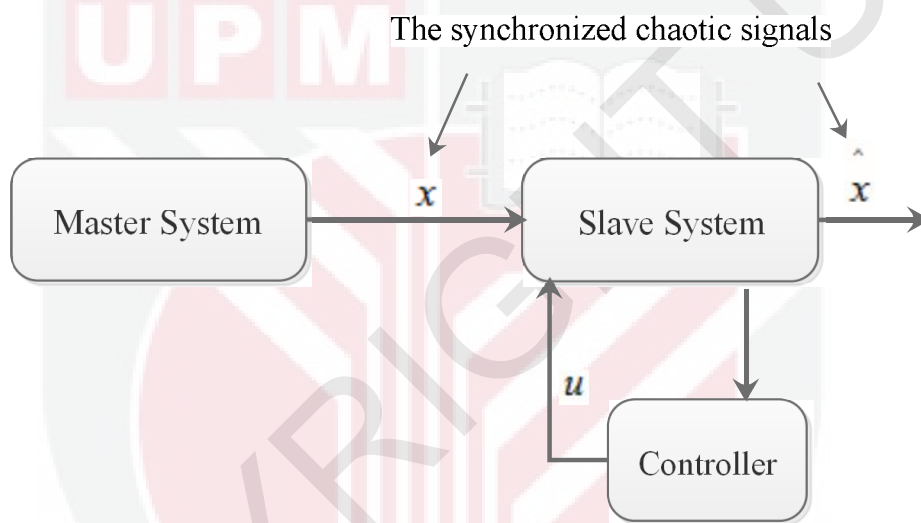


Figure 1.1: General block diagram illustrating the principles of chaotic synchronization, where  $x$  denotes the master signal and  $\hat{x}$  the slave signal. (Source: Jovic, 2011)

### 1.3 Chaos Synchronization Based Communication

In designing the chaos-based communication, there exist two approaches to be considered namely the analogue-based and the digital-based. According to the technique of chaos synchronization by Pecora and Carroll (1990), most analogue chaos-based communication are secure communication schemes designed for noisy channels. Chaotic systems are very significant for communication. Theoretically, chaotic systems can produce infinite numbers of chaotic signals which are non-periodic as compared to pseudo random signals which are limited in number and are periodic. Due to this property, chaotic systems become one of the interest for research in communication. There are several ways for the information to be transmitted in chaos-synchronization-based communication including the following:

### 1. Chaotic Masking

The chaotic masking technique is important as the analogue message signal, which is denoted by  $m(t)$ , is added to the output of the chaos generator, which is  $x(t)$ , within the transmitter. The details of this technique will be discussed further in Chapter 2.

### 2. Chaotic Modulation

As mentioned earlier, chaotic masking is primarily used for analogue transmission but chaotic parameter modulation is used for transmission of binary information. Unlike the chaotic masking, chaotic modulation on the other hand incorporates the message into the dynamical equations producing the chaotic carrier (Jovic, 2011).

### 3. Chaotic Switching or Chaos Shift Keying

According to Dedieu et al. (1993), chaotic switching or chaos shift keying (CSK) is mainly used for choosing the carrier signal from two or more different chaotic attractors by utilizing the use of a binary message signal. This technique will be discussed further in chapter 2.

## 1.4 Motivation of the Research

Chaos is a source of instabilities. As we all know, truly random number is almost impossible to generate. Chaotic system can generate infinite series of pseudo random numbers, which is very useful in classical cryptography. Of late it has been observed that, in most of the communication channel or network the dynamics becomes chaotic. For example the optical fibre, the internet network, the wireless communication are all chaotic in nature. In this thesis, we choose chaotic/hyperchaotic discrete and continuous models, those can be effective as chaos based communication models. This motivates us to design the communication schemes based on non-synchronized states of chaotic systems and study its effectiveness. We observe that, synchronization is not necessary in a chaos based communication model. The transmitter and receiver can successfully communicate messages even in the fully non synchronization states. This research can increase the domain of communication using chaotic model, as it include the non synchronization states also.

## 1.5 Scope and Limitation of the Research

In this thesis, we focus on the three main systems namely Logistic, Qi and Maxwell-Bloch systems. We investigate the synchronization and communication of each systems.

In the first part of the thesis, we study the synchronization between two discrete

logistic maps. We derive the suitable controller by using unidirectional adaptive coupling in the response system. We also implemented a communication scheme based on synchronization between two logistic maps.

In the next part, we investigate the synchronization between two continuous Qi systems with linear and nonlinear coupling respectively. We also investigate the conditions for the non-synchronized manifold. We measure and compare the complexity of single and coupled Qi systems respectively in both synchronized and non-synchronized states. We propose the chaotic masking communication scheme in non-synchronized states for transmitting analogue signals.

Finally, we explore the dynamics of a semiconductor laser by considering a higher dimensional chaotic model governed by the Maxwell-Bloch equations. We investigate the hyperchaotic nature of the system by using Lyapunov exponents. Unidirectional linear coupling is effective for synchronization between two laser models. We propose a communication scheme by using chaos shift key where the systems are in non synchronization state.

## 1.6 Problem Statements

The problem statements of the research are as follows:

1. A communication scheme can be designed once chaotic synchronization is achieved by using a suitable controller. Thus, we aim to find out a suitable controller such that chaos synchronization between two discrete chaotic systems can be utilized as a satisfactory communication scheme.
2. Synchronization is an obvious phenomena that can be achieved between chaotic or hyperchaotic systems. However, due to the design of a controller, synchronization might be not achieved. The corresponding state is called a non-synchronized state. Complexity is an effective measure to describe the fluctuation in error dynamics. Thus, our purpose is to measure the complexity of a single and coupled chaotic or hyperchaotic systems in synchronized and non-synchronized states.
3. There are several ways of transmitting information using chaos synchronization-based communication. Thus, our intention is to find suitable techniques for the realization of chaos synchronization based communication.

## 1.7 Research Objectives

The objectives of the research are as follows:

1. To propose a new chaos based communication technique between two discrete/continuous chaotic systems by using unidirectional adaptive coupling.
2. To measure and hence compare the complexity of synchronized and non-synchronized states of continuous hyperchaotic systems by using Weighted Recurrence Plot Entropy (WRPE).
3. To design a novel communication scheme when the systems are in non-synchronized state.
4. To refine the application of communication using chaos synchronization by chaotic masking and chaos shift key techniques.

## 1.8 Research Flow Activities

The research flow activities of the thesis is illustrated in Figure 1.2 below. In Chapter 3, we propose chaos synchronization scheme between discrete systems by using nonlinear adaptive coupling. Next, we apply the proposed scheme to communication by using chaotic masking technique.

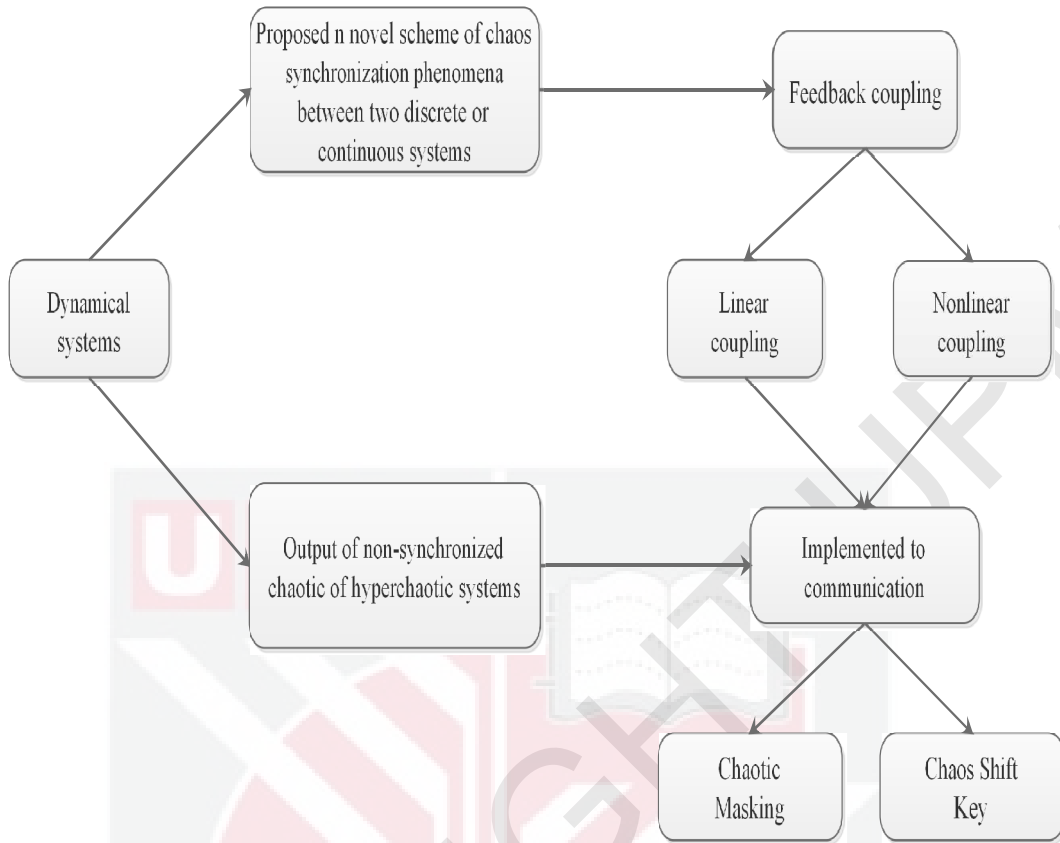
We also investigate the complexity of single and coupled systems that will be discussed in Chapter 4. We observe the complexity can be decreased from single system to coupled systems by using a nonlinear controller. In addition, the complexity in coupled systems is higher in non-synchronized state rather than in synchronized state. Hence, we propose a novel scheme of continuous coupled systems in communication by using the output of non-synchronized states.

## 1.9 Thesis Outline

This thesis is organized as follows.

In Chapter 1, we present an introduction that guide readers to the motivation of this research. We also highlight the problem statements, motivation, scope and limitation, objectives and flow of activities on this research.

Chapter 2 provides review on previous works done related to the topic of synchronization of chaotic and hyperchaotic systems that will be used throughout this thesis.



**Figure 1.2: General block diagram of the research flow activities.**

**The work presented in Chapter 3 focuses on the synchronization phenomenon between two discrete chaotic systems. A scheme for synchronization between two discrete maps with adaptive coupling has been studied analytically. The scheme can be successfully implemented for generalized synchronization between two chaotic maps. We also demonstrate a communication scheme based on the synchronization between two Logistic maps.**

**In Chapters 4 and 5, we focus on the synchronized and non-synchronized states between continuous hyperchaotic systems. To be more specific, we check whether the complexity of non-synchronized states is higher in comparison to the same in synchronized states. Is it important for chaos based communication scheme that the corresponding complexity of the systems should be high enough. It is a challenge to design a communication scheme when the error dynamic is fluctuating and not converging to zero. Hence, in Chapter 4, we propose and redefine the chaotic masking scheme for communication with coupled non-synchronized systems, where the error dynamics are completely fluctuating and non-synchronized.**

**The semiconductor laser is the most effective experimental realization for optical communication. We investigate the dynamics of a 5D laser model which is both**

chaotic and hyperchaotic for certain set of parameter values. The system has rich dynamics in term of complexity that can be effective for communication. In Chapter 5, we propose the a form of chaos shift key (CSK), which can be applicable for coupled systems in a non-synchronized state.

Finally, in Chapter 6, we summarize all the contributions made out in this thesis, along with some potential future works.



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