STABILITY ANALYSIS ON BOUNDARY LAYER FLOW AND HEAT TRANSFER OVER A PERMEABLE SURFACE IN PRESENCE OF THERMAL RADIATION

SHAHIRAH BINTI ABU BAKAR

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By

SHAHIRAH BINTI ABU BAKAR

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfillment of the Requirements for the Doctor of Philosophy

July 2018

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DEDICATIONS

To my adorable daughters, Hana and Hadirah;
and my other half, Zul.

You will forever be my always.

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfillment of the requirement for the degree of Doctor of Philosophy

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July 2018

Chairman: Professor Norihan Md Arifin, PhD
Institute: Institute for Mathematical Research

The study of stability analysis on boundary layer flow and heat transfer over a permeable surface in presence of thermal radiation is numerically studied in this thesis. The aim of this thesis is to analyze the following five problems, which are: (i) forced convection stagnation point slip flow in a Darcy porous medium towards a shrinking sheet in presence of thermal radiation and suction; (ii) forced convection flow over a permeable stretching sheet with variable thickness in presence of free stream, magnetic field and thermal radiation; (iii) mixed convection boundary layer flow over a permeable surface embedded in a porous medium saturated by a nanofluid with thermal radiation, MHD (magnetohydrodynamics) and heat generation; (iv) mixed convection boundary layer flow over a permeable vertical cylinder embedded in a porous medium saturated by a nanofluid with thermal radiation; and (v) unsteady mixed convection stagnation point flow over a permeable moving surface along the flow impingement direction with thermal radiation. Similarity transformation is used in all problems to reduce the governing system of partial differential equations into a system of ordinary differential equations, which is numerically solved using shooting method and bvp4c function. The programming codes for the shooting method are built using MAPLE software, while the programming codes for the bvp4c function are built using MATLAB software. The characteristics of the reduced skin friction coefficient and Nusselt number, together with velocity and temperature profiles, for various existence of parameters are discussed and analyzed in details. It is observed that the reduced skin friction coefficient increases with the increasing of permeability and thermal radiation parameter, where the boundary layer thickness and velocity gradient are seen to be affected by those parameters. Further, the problems possessed dual solutions for a certain range of parameters, in which we performed a stability analysis by solving the linear eigenvalue problems to identify which of the two solution is stable. Our analysis reveals that the first solution (upper branch) is stable and physically realizable, while the second solution (lower branch) is unstable.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

ANALISIS KESTABILAN BAGI ALIRAN LAPISAN SEMPADAN DAN PEMINDAHAN HABA YANG MELALUI PERMUKAAN TELAP DENGAN KEHADIRAN RADIASI TERMAL

Oleh

SHAHIRAH BINTI ABU BAKAR

Julai 2018

Pengerusi: Professor Norihan Md Arifin, PhD
Institut: Institut Penyelidikan Matematik

Kajian tentang analisis kestabilan bagi aliran lapisan sempadan dan pemindahan haba yang melalui permukaan telap dengan kehadiran radiasi termal telah dikaji secara berangka di dalam tesis ini. Tujuan utama tesis ini adalah untuk menganalisa lima masalah seperti yang berikut, iaitu: (i) olakan paksa dengan aliran gelinciran titik genangan di dalam medium berliang Darcy pada lapisan mengecut dengan kehadiran radiasi termal dan sedutan; (ii) olakan paksa yang melalui permukaan meregang telap dan ketebalan pembolehubah dengan kehadiran aliran bebas, medan magnet dan radiasi termal; (iii) aliran lapisan sempadan olakan campuran pada permukaan telap yang tertanam di dalam medium berliang dan tepu dengan bendalir nano serta radiasi termal, MHD (magnet-hidrodinamik) dan penjanaan haba; (iv) aliran lapisan sempadan olakan campuran yang melalui silinder menegak telap yang tertanam di dalam medium berliang dan tepu dengan bendalir nano serta radiasi termal; dan (v) olakan campuran tak mantap dengan aliran titik genangan yang melalui permukaan bergerak telap bersama-sama dengan arah aliran pelepasan serta radiasi termal. Penjelmaan keserupaan telah digunakan di dalam kesemua masalah untuk menjelmakan sistem persamaan pembezaan separa kepada sistem persamaan pembezaan biasa, dan diselesaikan secara berangka dengan kaedah tembakan dan fungsi bvp4c. Kod atur cara bagi kaedah tembakan dibina menggunakan perisian MAPLE, manakala kod atur cara bagi fungsi bvp4c pula dibina menggunakan perisian MATLAB. Sifat-sifat bagi pekali geseran kulit dan nombor Nusselt, juga dengan profil halaju dan suhu, untuk pelbagai kewujudan parameter telah dianalisa dan dibincangkan secara terperinci. Kajian mendapati bahawa pekali geseran kulit meningkat dengan peningkatan parameter kebolehtelapan dan radiasi termal, di mana ketebalan lapisan sempadan dan halaju kecerunan juga kelihatan terkesan dengan parameter-parameter yang dinyatakan. Di samping itu, kajian mendapati penyelesaian dual diperoleh bagi

sesetengah julat parameter, di mana analisis kestabilan dijalankan dengan menyelesaikan masalah nilai eigen linear untuk mengenal pasti penyelesaian yang stabil di antara dua penyelesaian. Hasil analisis kemudiannya mendapati bahawa penyelesaian pertama (cabang atas) adalah stabil dan boleh direalisasikan secara fizikal, manakala penyelesaian kedua (cabang bawah) adalah tidak stabil.

ACKNOWLEDGEMENTS

Bismillahirrahmanirrahim.

In the name of Allah, the most Gracious and the most Merciful.

In the name of Allah the Almighty who give me the enlightenment, truth, knowledge and with regards to Prophet Muhammad S.A.W. for guiding us to the straight path. Praise be to Allah S.W.T., Lord of the World. Alhamdulillah, for His blessings showered upon me for making the writing of this thesis a successful one. O Allah, grant me the blessing of my success so that I can remain grateful to you.

First and foremost, I would like to express my greatest appreciation for the assistance and guidance received throughout the journey of this thesis. I acknowledge my respected supervisor, Prof. Dr. Norihan Md Arifin, and co-supervisors, Dr. Fadzilah Md Ali and Assoc. Prof. Dr. Norfifah Bachok, for their valuable guidances and encouraged me which are beyond descriptions and words during the course of my study. Not forget to mention Prof. Dr. Ioan Pop from Babeş-Bolyai University, Cluj-Napoca, Romania, for his guidances and positive comments throughout my study journey.

Million thanks to my other half, Mr. Muhammad Zulkifli who had given his endless unconditional love, valuable motivations, maximum supports and strive together through thick and thin. I acknowledge my parents who are also helping me to climb until reaching the peak, especially during the hardest times.

Nevertheless, I extend my sincere appreciation to the Institute for Mathematical Research (INSPEM) for providing me with a very good research equipments and environments. Last but not least, I would like to acknowledge Ministry of Higher Education (MOHE) and School of Graduate Studies (SGS) for the financial support throughout my research course.

May Allah bless us all.

This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfilment of the requirement for the degree of Doctor of Philosophy. The members of the Supervisory Committee were as follows:

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LIST OF ABBREVIATIONS

BVP	Boundary value problem
IVP	Initial value problem
MHD	Magnetohydrodynamics
ODEs	Ordinary differential equations
PDEs	Partial differential equations
bvp4c	boundary value problem with fourth-order accuracy
a	Straining rate parameter
A_s	Constant value for variable thickness sheet
b	Characteristic of temperature
B_0	Magnetic field
C_f	Skin friction coefficient
C_p	Specific heat at constant pressure
d	Radius of a cylinder
D_1	Thermal slip factor
e	Exponent
$e^{-\eta}$	Internal heat generation parameter
f	Dimensionless stream function
f_0	Stability of dimensionless stream function
F_0	Small relative to f_0
g	Gravity acceleration
Gr_x	Grashof number
G_0	Small relative to θ_0
k	Fluid thermal conductivity
$k*$	Mean absorption coefficient
k_1	Porous medium permeability parameter
K	Porous medium permeability
L_1	Velocity slip factor
m	Velocity power index
M	Magnetic parameter
n	Plane or axisymmetric flow
Nu_x	Nusselt number
Pe_x	Péclet number for porous medium
Pr	Prandtl number
q_w	Heat flux from the surface of the plate
q'''	Internal heat generation
r	Cylindrical coordinates measured along the cylinder axis
Rd	Radiation parameter
Re_x	Local Reynolds number
S	Suction/injection parameter
t	Time
T	Temperature of the fluid
T_w	Temperature of the sheet
T_0	Constant for heated/cooled cylinder

T_∞	Free stream temperature
T^4	Linear function of temperature
u_e	Free stream of the stagnation point flow
u, v	Velocity components along x-, y- directions
U_f	Constant for stretching velocity of the sheet
U_w	Stretching velocity of the sheet
U_0	Constant for free stream velocity
U_∞	Free stream velocity
v_w	Moving velocity of the body along y-axis

Greek symbols

δ	Velocity slip parameter
ε	Dimensionless porosity of porous medium
η	Similarity variable
γ	Unknown eigenvalue
λ	Mixed convection parameter
μ	Dynamic viscosity
ν	Kinematic viscosity
ω	Sheet thickness parameter
ϕ	Velocity parameter
ψ	Stream function
ρ	Fluid density
σ	Electrical conductivity
τ	Dimensionless variable for time
τ_w	Shear stress at the plate surface
θ	Non-dimensionless temperature
θ_0	Stability of non-dimensionless temperature
υ	Thermal expansion coefficient
φ	Nanoparticle volume fraction parameter
ς	Stefan-Boltzmann constant
ξ	Curvature parameter

Subscripts

f	Fluid
nf	Nanofluid
s	Solid
w	Condition at the surface of the sheet
∞	Condition at infinity

CHAPTER 1

INTRODUCTION

1.1 Convective Heat Transfer

Heat transfer is a discipline that considers the conversion, generation and exchange of thermal energy between physical systems. The thermal energy is commonly known as heat in fluid dynamics area. Heat transfer is a result from three different phenomenas, namely radiation, conduction and convection, as illustrated with a simple explanation in Figure 1.1. Radiation and conduction are depending on temperature difference, while convection is depending on temperature difference and mass transport on the fluid. The dominant contribution for heat transfer is due to the bulk, or motion of fluid particles. The process of heat transfer is one of considerable interests in daily human life and engineering industries. One of a good model that can represent the heat transfer in daily life is when our body produced heat by a continuous metabolism of nutrients in order to provides energy. Thus, our human body must maintain a consistent internal temperature to keep a continuous health bodily function. Therefore, excess heat must be dissipated from our body to keep it from overheating. This can be seen for an example, when we used to wear a breathable and light cloth during daytime, while in night we used to wear a thicker cloth to control the heat from our body. On the other hand, heat transfer is also considerably important in technology systems due to its applications in designing cooling devices, nuclear plant components, engine coolant radiator used in an automobile, just to name a few.

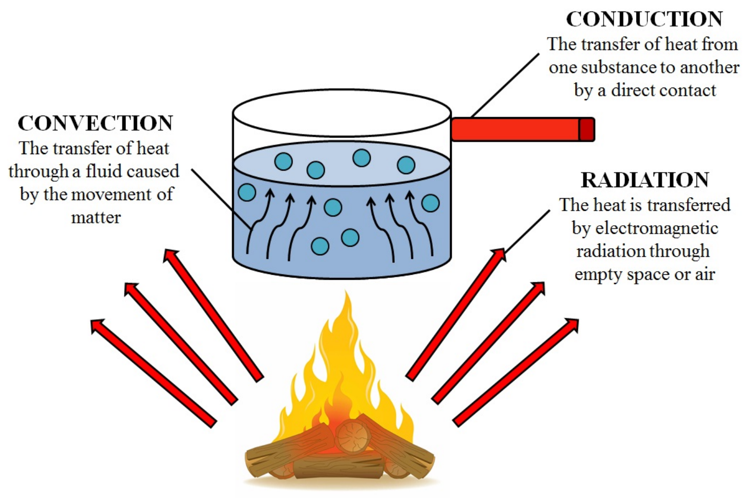
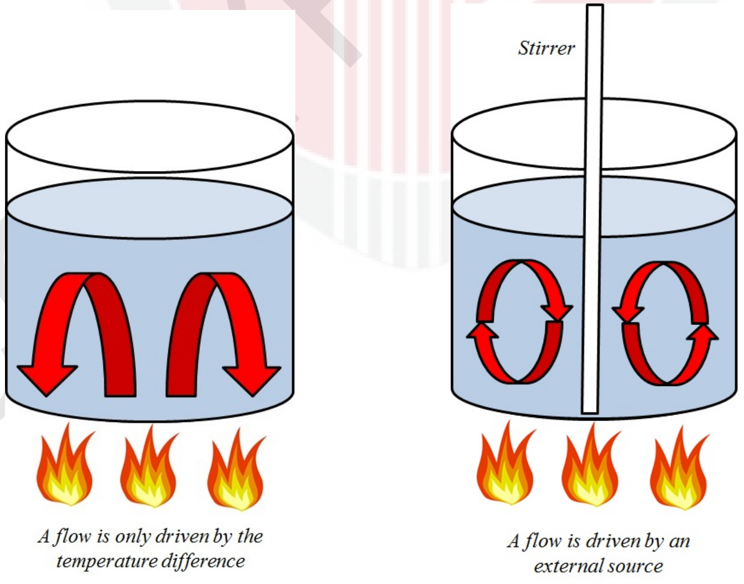


Figure 1.1: Types of heat transfer

Based on physical science, convective is the collections of molecules concerted movement within fluids, rheids or gases. Convection is also commonly known as the dominance of heat transfer form in liquids and gases. Convection has no definite or particular shape, and it cannot occurs in solids since the diffusion of significance or bulk current flows are unable to take place in solids. However, in certain case of heat transfer, heat diffusion can take place in solids but is referred separately. Convection can be distinguished into two types, namely convection heat transfer and convection mass transfer. Convection heat transfer refers to the combinations of heat transported by advection and conduction. In a simple concept, advection is a larger scale motion of fluid currents, while conduction is a Brownian motion of individual particles. However, in certain cases, some researchers considered convection based on advective phenomena solely like transport equations. The process of heat transfer is highly emphasizing in temperature and heat flows. Both of temperature and heat flows represent the thermal and energy movements, equally, from one region to another. The rate of heat transfer in a particular direction is depending on the temperature gradient magnitude, where the greater the magnitude is, then the larger heat transfer rate would be. A simple everyday model that can described the convection at its best is boiling water, where the heat is passes into the pot from a burner and will heated the water at the bottom. Then, the convection will transfer the heat from the bottom to the cooler water at the above. This hot water rises due to its less dense, and cooler water that has higher density moves down to replace it. This continuous movement will causing a circular motion until a stable thermal equilibrium is achieved, see Ishak (2005).



(a) Natural convection

(b) Forced convection

Figure 1.2: Types of convections

Convection may happens in all scale of fluids, which there are various of circumstances required to lead the arise of different types of convection. Convection heat transfer takes place in two conditions, or can be divided into two types; namely natural convection (free convection) and forced convection, as shown in a simple model in Figure 1.2. Meanwhile, the combination of natural and forced convections will resulting in mixed convection.

Natural or free convection occurs when fluid motion is caused by buoyancy forces that happens due to variations of temperature in the fluid. The fluid motion does not involved in any external or internal factor but only affected by differences of fluid densities. The differences of fluid densities may resulting from temperature gradients, concentrations or composition. Natural convection can occurs between fluid and solid surfaces when there are different temperatures exist and contacted within each other as explained by Rudramoorthy and Mayilsamy (2006). Natural convection can be more rapid or more likely to occur if there are three conditions happen, which are (i) a greater variation of densities between two fluids; (ii) gravity that resulting a larger acceleration to drives the convection; or (iii) larger distance through the convecting medium. In a contrary, natural convection is less rapid or more unlikely to happen if (i) the diffusion is increases which will diffusing away the thermal gradient; or (ii) the fluid is more viscous.

Forced convection occurs when a fluid is forced by an external source or outside factor to flow over a surface and creating an artificially induced convection current. External sources may be classified as by using a fan, pump, suction device, etc. These uses of external sources will provide a high velocity fluid, where it can results in a decreased of thermal resistance across the boundary layer from the fluid to the heated surface. Forced convection is commonly used to increase the rate of heat transfer because it is more likely to transport significant amounts of heat energy efficiently and produce quicker result than natural convection.

Another type of mechanism for heat transfer that discussed in this thesis is mixed convection. In any forced convection situations, some amount of natural convection is always occur whenever there is a presence of gravitational force. The effect of natural convection is not negligible and it acts naturally with forced convection in transferring the heat together, where this mechanism is referred as mixed convection. This is also can be defined as when buoyant and pressure forces interact between each other. The amount of convection form that contributes to the heat transfer is determined by the condition of temperature, flow, geometry or orientation. Mixed convection is commonly used in a very high power output devices that operate at extremely high temperatures where the forced convection is not enough to dissipate the required heat. Example of these processes is in transporting molten metal, where engineers can determine a necessary fluid flow velocity to produce the desired temperature distribution and prevent part of the systems from failing.

1.2 Boundary Layer

In early 1800s, the Navier-Stokes equations, named after Claude-Louis Navier and George Gabriel Stokes, described the motion of viscous fluid substances. At the same time, several analysis solutions for Euler methods and ideal fluid equations for complex geometric flow were successfully reported by other mathematicians. These equations arise by applying Newton's second law to fluid motion. However, Cengel and Cimbala (2006) explained that these equations are only specified for simplified geometric flow and the obtained results are mostly physically not in correlation. Cued to above, mathematicians and researches believed that the only method that can be used in fluid flow study is by experimental method.

Later on, the theory of boundary layer was first defined by Ludwig Prandtl in 1904, where this concept is to describe the shallow fluid domain that adjoins the solid wall bathed by the flow, as explained by Anderson (2005). In Prandtl's paper, he simplified the fluid flow equations by dividing the flow field into two areas, where (i) the flow field that inside the boundary layer is dominated by viscosity and creating the majority of drag; while (ii) the flow field that outside the boundary layer can neglecting the viscosity without significant effects on the solution. This finding allows a total solution for the flow in both areas, where it is later becomes a significant simplification of full Navier-Stokes equations.

In general, boundary layer is a fluid layer in the immediate vicinity of a bounding surface where the viscosity effects are significant. The respective equations can be allowed to simplified in the field of flow outside the boundary layer which will resulting the heat transfer existence in boundary layer. In the field of heat transfer, a thermal boundary layer is possibly occurs and this can relate to a fact that a surface can have multiple types of boundary layers simultaneously, as illustrated in Figure 1.3. The velocity boundary layer thickness is defined as the distance point from the solid body at which the velocity of viscous flow is too near from the free stream velocity. Meanwhile, the thickness of thermal boundary layer can be explained as the gap, or distance, from the solid body where the temperature is too near to the temperature found from an inviscid solution. The ratio of these both thicknesses is determined by Prandtl number Pr, where the thicker or thinner of thermal boundary layer is depending on the value of Pr number. If the Prandtl number is less than 1, the thermal boundary layer is said to be thicker than velocity boundary layer, while if the Prandtl number is more than 1, then the boundary layer of thermal is thinner than the boundary layer of velocity. If the Prandtl number is 1, thus the two boundary layers are said to having the same thicknesses.

The boundary layer flow regime can be differentiated into two types, namely laminar and turbulent. Laminar boundary layer flow has a very smooth flow, where it creates less drag of skin friction, but it is commonly unstable. At first, boundary layer flow begins as a smooth laminar flow, where then the laminar boundary layer

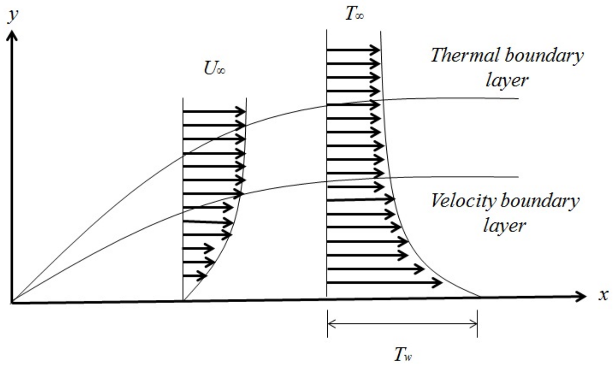


Figure 1.3: Velocity and thermal boundary layers on a flat plate

thickness enhances as the flow continues back from the leading edge. In a contrary, turbulent boundary layer flow is not as smooth as laminar flow since it contains swirls. The starting point of turbulent flow is coming from a breaking transition of smooth laminar flow at some distance back from the leading edge. To make a simple explanation, boundary layer flow is starting as a laminar flow, where the boundary layer becomes thicker and undergoes transition to turbulent flow and finally continues to develop along the body surface.

1.3 Permeable Surface (Suction and Injection)

The main reason of suction or injection (blowing) through the bounding surface of a fluid is to affect the heat transfer rate from the surface and significantly change the flow field. Suction is a method of physically increase the heat transfer and skin friction coefficients while injection acts to decrease both coefficients, see Al-Sanea (2004). Flows with permeable surface have some interesting features; for example, every streamline will start at the inner sphere and ends up at the outer sphere for the case of smaller suction, while if the suction effect is larger, the breaking point of secondary flow is obliterated.

Historically, the idea of suction has been used by Ludwig Prandtl in 1904 where he considered the suction technique in a circular cylinder, see Schlichting et al. (1955) and Van Den Berg (2012). His flow visualization convincingly showed that the boundary layer adhered to the suction side of the cylinder over a considerably larger portion of its surface. The slow flow in boundary layer is taken up by suction, and energized by rotation on the same matter. This force acting on a fluid caused by the difference of internal and external pressure regions, and tends to convert the fluid flow from a higher pressure to a lower pressure region. The gradient and ambient of pressures in this region will propel towards the lower pressure area, and produce a partial vacuum by the air removal in order to force fluid into a vacant space of procure adhesion. This method is considered seriously since then and been as one of important application in technology industries nowadays as the usage of suction; e.g. suction in pump or fan, is to reduce the act of pressure to create a necessary force.

On the other hand, injection, or blowing, is an act when the pressure energy of a fluid is converted to a velocity energy which creates a low pressure zone and entrains a suction fluid. The process of injection begins when an injection passed into the fluid by an injector, the mixed fluid expands and recompresses by converting the velocity energy to pressure energy. Instead, fluid under a higher pressure is converted into a high velocity gradient and creates a low pressure at that point. The pressure energy then is converted to a kinetic energy, which then will performed vice versa at the diffuser outlet when the mixed fluid expands in the divergent diffuser. The application of injection in various engineering and science technologies has become quite importance due to their adaptability and simplicity of relative such as in steam jet cooling systems, oil recovery processes, boiling water nuclear reactors, thermal power stations, and such. Hence, in this preceding study, the focus is on the important and relevant of suction method on the flow field.

1.4 Thermal Radiation

Heat can be transferred from one place to another by three common methods, which are (i) conduction in solids; (ii) convection of fluids; and (iii) radiation through anything that allows radiation to pass. In heat transfer study, both conduction and convection require matter to transferring the heat. However, radiation does not require any intervening source between the heat and the heated object. For a simple example, we can feel the heat from the Sun even though we are not touching it. This is because heat can be transferred through an empty space by a process named thermal radiation. Thermal radiation is an electromagnetic radiation that generated by the thermal motion of charged particles in matter and moves in a speed of light, in which all matter that has temperature greater than zero emits thermal radiation. There are four properties that characterized thermal radiation, which are:

(i) Thermal radiation emitted by a body that contains any temperature consists of a wide range of frequencies.

(ii) The dominance of emitted radiation frequency range will enhance to higher frequencies as the temperature of the emitter increases.

(iii) The total amount of all radiation frequencies increases steeply as the temperature rises. The amount of heat energy emitted by a surface grows as T^4, where T is the absolute temperature of the body.

(iv) The electromagnetic radiation rate that emitted at a given frequency is proportional to the absorption rate that it would be experience by the source. This can be simplify as a surface that absorbs more red light will also thermally radiates more red light.

The rate of thermal radiation or absorption is depending upon the nature of the surface. Objects with a characteristic of good emitter is also a good absorber as well. Absorptivity, reflectivity and emissivity of all bodies are depend on the radiation wavelength. The temperature itself determines the distribution of wavelength of the electromagnetic radiation. An application that can be related is a blackened surface is a good absorber, since the blackbody is the one who absorbs all the radiant energy that falls on it. If the same surface is silvered, it becomes poor emitter and a poor absorber. This is one of the fact that color is also plays a main role to become a good emitter. The lighter colors, including white, and metallic substances absorb less illuminating light, and thus resulting in less heat. Otherwise, color can make small difference as regards heat transfer between an object at daily temperatures and its surroundings, since the wavelengths emitted are nowhere near the visible spectrum and far from the infrared.

1.5 Dimensionless Numbers

Dimensionless numbers in fluid dynamics are a set of dimensionless quantities that have an important role in the fluids behaviors. The ratio of effective diffusivities is analyzed by the classical numbers in mass, momentum and energy transport phenomenas. The dimensionless numbers give the relative strengths of different phenomena in viscosity, inertia, mass transport and conductive heat transport. Some of the dimensionless numbers that have been used in this preceding study are Reynolds number Re, Prandtl number Pr, Nusselt number Nu and Péclet number, Pe. The definition and details of these four dimensionless numbers are well discussed in Section 1.5.1, 1.5.2, 1.5.3 and 1.5.4, respectively.

1.5.1 Reynolds Number

The Reynolds number is the ratio of inertial forces to viscous forces and is a convenient parameter to evaluate if the flow condition is laminar or turbulent, where it is subjected to the movement of internal relative due to different fluid velocities. It can be interpreted that the fluid particles is sufficient enough to keep in line when the viscous forces are dominant, and this flow is said to be a laminar flow. When the fluid is flowing faster and inertial forces dominate over the viscous forces, which tend to produce chaotic swirls, eddies and vortices, then the flow is in turbulent state. The concept was introduced by Sir George Stokes in 1851, but the Reynolds number was named after Osborne Reynolds, see Çengel and Cimbala (2006), who popularized its use since 1883. The Reynolds number is defined as:

$$Re_x = \frac{\text{Inertia forces}}{\text{Viscous forces}} = \frac{U_\infty L}{\nu}, \tag{1.1}$$

where U_∞ is free stream velocity, L is characteristic linear dimension and $\nu = \frac{\mu}{\rho}$ is fluid kinematic viscosity, in which ρ is fluid density and μ is dynamic viscosity of the fluid. Reynolds number can be defined for several different situations where a fluid is in relative motion to a surface. The Reynolds number regime can be differentiated into three situations and are also called as critical Reynolds numbers, which are (i) if the Reynolds number is less than 2000, the flow is laminar; (ii) if the Reynolds number is between 2000 and 4000, the flow is unstable and sometimes referred to a transitional flow; and (iii) if the Reynolds number is greater than 3500, the flow is turbulent. In addition, it is worth to know that most of fluid systems in nuclear facilities operate with turbulent flow.

1.5.2 Prandtl Number

The Prandtl number is defined as a dimensionless number approximating the ratio of momentum diffusivity to thermal diffusivity. Prandtl number is also one of a fundamental of fluid behavior that acts together with velocity and pressure. Prandtl number is significant since its efficiency in determined the thermal conductivity of gases at high temperatures and can be expressed as:

$$Pr = \frac{\text{Momentum diffusivity}}{\text{Thermal diffusivity}} = \frac{C_p\mu}{k} = \frac{\mu/\rho}{k/(\rho C_p)} = \frac{\nu}{\alpha}, \tag{1.2}$$

where C_p is specific heat, k is thermal conductivity of the fluid, ν is fluid kinematic viscosity and α is thermal diffusivity. The relative of the velocity boundary layer thickness to the boundary layer of thermal is shows by the momentum ratio molecular diffusivity of heat. This means that the smaller value of Prandtl number, $Pr \leq 1$, defines that the thermal diffusivity is dominating. Otherwise, if Prandtl number shows value greater than 1, $Pr \geq 1$, it is defines that momentum diffusivity is dominating the behavior. The value of Prandtl number for water is between 1 until 10 and significantly depending on temperature. The types of fluids and its value of Prandtl number is physically shown in Figure 1.4.

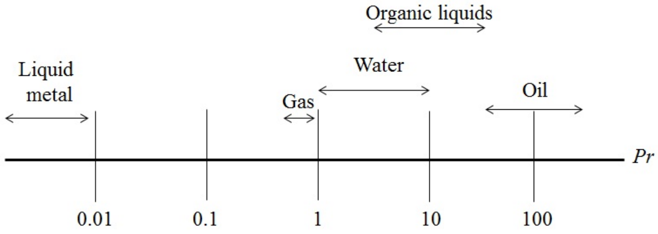


Figure 1.4: Values of Prandtl number on fluids

1.5.3 Nusselt Number

At a boundary surface within a fluid, the Nusselt number is a ratio of convective to conductive heat transfer across the boundary. Named after Wilhelm Nusselt, this dimensionless number is measured with a motionless fluid but still consider under the same condition as heat convection. Nusselt number can be defined as:

$$Nu_x = \frac{\text{Convective heat transfer}}{\text{Conductive heat transfer}} = \frac{hL}{k}, \tag{1.3}$$

where h, L and k are convective heat transfer coefficient of the flow, characteristic length and fluid thermal conductivity, respectively. The selection of characteristic length L, should be ally with the boundary layer thickness direction. At the same time, the fluid thermal conductivity is typically evaluated at the temperature, where it is considered as the mean average of of the wall surface and bulk fluid of temperature. The characteristic of Nusselt number is said to be in laminar flow if the Nusselt number shows close to one. Otherwise, a larger value of Nusselt number corresponds to more active convection, and the flow is in turbulent state.

1.5.4 Péclet Number

In the context of thermal fluids, the Péclet number corresponds to the product of Reynolds and Prandtl numbers. It is defined to be the rate of advection ratio of a physical quantity by the diffusion rate flow of the same quantity driven by an appropriate gradient. The Péclet number is defined as:

$$Pe_x = \frac{\text{Heat transport by convection}}{\text{Heat transport by conduction}} = \frac{Lu}{k/(\rho C_p)} = \frac{Lu}{\alpha}, \tag{1.4}$$

where L, u, k, ρ, C_p and α are characteristic length, local velocity flow, fluid thermal conductivity, fluid density, specific heat at a constant rate and thermal diffusivity, respectively. The usage of Péclet number in engineering process is widely used, since the flow dependency is diminished and leads the variables to create a one way property in the flow. Thus, simpler computational models can be adopted when modeling certain situations with higher value of Péclet number. Péclet number is depending on heat capacity, density, characteristic length, velocity and heat transfer coefficient in order to evaluate the possible value of Péclet number.

1.6 Problem Statement

In this study, we performed an investigation to evaluate any possible information on fluid flow as well as the characteristics of its corresponding fluid flow and heat transfer. In addition, we will examine the governing equations of mass and momentum balance together with the appropriate similarity variables on its fundamental transformation. Furthermore, several parameters or additional characters are considered in our respective system of boundary layer, either on the governing equations or boundary conditions, in order to evaluate the significant effects and consequences regarding on the additional parameters. In this thesis, we study and observe the steady and unsteady state of convections, together with stretching or shrinking parameter, permeable or impermeable, moving or stationary effect, as well as the existence of thermal radiation effects.

1.7 Objectives of Study

The objectives of this present study are to construct mathematical model for each problem, prepare the analysis and formulation for each mathematical model constructed, numerically solve the mathematical modeling by a similarity transformation, and perform a stability analysis on dual solutions obtained, for the following five problems in this preceding thesis, which are:

(i) Forced convection boundary layer stagnation point slip flow in a Darcy porous medium towards a shrinking sheet in presence of thermal radiation and suction.

(ii) Forced convection flow over a permeable stretching sheet with variable thickness in presence of free stream, magnetic field and thermal radiation.

(iii) Mixed convection boundary layer flow over a permeable surface embedded in a porous medium saturated by a nanofluid with thermal radiation, MHD and heat generation.

(iv) Mixed convection boundary layer flow along a permeable vertical cylinder embedded in a porous medium saturated by a nanofluid with thermal radiation.

(v) Unsteady mixed convection stagnation point flow over a permeable moving surface along the flow impingement direction with thermal radiation.

1.8 Scope of Study

The scope of study is limited to the respective problems consisting of steady and unsteady states, two dimensional forced and mixed convections, immersed in viscous, electrically and incompressible conducting fluid with the presence of thermal radiation, together with the permeable wall in boundary conditions. It is worth to know that the major study scope is to mainly focus on the effects of thermal radiation on boundary layer flow and transfer of heat towards a permeable surface, where several other parameters and convections are considered in respective problems.

1.9 Thesis Outline

This thesis is divided into nine chapters including introduction and conclusion. Chapter 1 is the preliminary chapter consisting the general concept of convective heat transfer which is divided into two categories, namely natural and forced convection, followed by the introduction of boundary layer, permeable surface, thermal radiation and dimensionless numbers. Four kinds of dimensionless numbers are also considered in this chapter, which are Reynolds number, Prandtl number, Nusselt number and Péclet number. The application of convective heat transfer in industry, main objective and scope of this study are also highlighted in Chapter 1.

Chapter 2 reviews the pioneering studies on the onset of boundary layer flow and heat transfer by experimentally and numerically. We also highlighted the investigators who explored the onset of free and forced convection flow of a boundary layer over a permeable surface (suction and injection) with thermal radiation, followed by the onset of mixed convection flow of a heat transfer and boundary layer with the effects of thermal radiation over a permeable surface. Chapter 3 explains the methodology for five following problems, where the three respective methods are shooting technique, bvp4c function and stability analysis. These three are the major methods that we used in order to solve our corresponding five problems.

Later, the first problem of this thesis is discussed in details in Chapter 4. Chapter 4 starts with the introduction, followed by the mathematical formulation, stability analysis, results and discussion, and ends with conclusions. This problem analyzed the effects of suction and thermal radiation parameter on the flow field. Since we noticed that two different branches are obtained for a certain parameters range, we then performed an analysis of stability. The process detail of similarity transformation and stability analysis are briefly defined in Chapter 4.

Chapter 5 considers the problem of magnetic field, thermal radiation and free stream effects on forced convection over a permeable stretching sheet with variable thickness. Next, the problem of mixed convection flow of a boundary layer with internal heat generation over a permeable surface embedded in a porous medium filled with a nanofluid with MHD and thermal radiation is analyzed in Chapter 6. The derivation process and stability analysis for this model are defined in details in this chapter.

Chapter 7 briefly explains the problem of mixed convection boundary layer flow along a permeable vertical cylinder embedded in a porous medium saturated with a nanofluid with the effects of thermal radiation. This chapter considers a vertical cylinder, which is a different model with the previous problems. In this chapter, our model is relating to d, where d is radius, since it is one of a properties of a cylinder. On the other hand, the process of similarity transformation and stability analysis for this model are just similar with the previous problems.

The final problem, which is the unsteady mixed convection stagnation point flow over a permeable moving surface along the flow impingement direction with thermal radiation effects is physically considered in Chapter 8. This problems discussed the unsteady state of the governing model in both momentum and energy equations. The major difference between steady and unsteady state is when we consider time in our governing model. We also included several considerable research that relating to unsteady state of boundary layer.

The final chapter, namely Chapter 9, contains the summary of present research and some possible further research.

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