UNIVERSITI PUTRA MALAYSIA

DIRECT ONE-STEP BLOCK METHODS FOR SOLVING THIRD AND FOURTH ORDER ORDINARY DIFFERENTIAL EQUATIONS

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By

EHAB HASAN ABDULRAHMAN

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirements for the Degree of Master of Science

July 2018

DEDICATIONS

To all of my love;
Parents & Maaly
Muhammed & Narjis
Ghasaq, Ali, Ahmed

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Master of Science

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July 2018

Chairman : **Zanariah Abdul Majid, PhD**
Faculty : **Institute for Mathematical Research**

One-step block methods are presented in this research work to solve initial value problems (IVPs) of general third and fourth order ordinary differential equations (ODEs). These methods are utilized to solve general third and fourth order ODEs using constant step size. The methods will simultaneously obtain the approximation solutions at two and three points in a block. The general third order and fourth order ODEs are solved directly. Most of existing literatures have used IVPs to reduce problems in first order ODE systems. However, the approach in the current research is more efficient than the common technique involving first order equations. This research defines the order of the derived two-point and the three-point one-step block methods. In addition, the block method adopts Lagrange's interpolation formulae to compute the integration coefficients. Notably, a new code is developed to solve the IVPs of third order and fourth order ODEs using constant step size. In the numerical results, the performance of the developed methods generated better results in terms of the total number of steps, maximum error, and total function calls compared with existing methods. In conclusion, the proposed direct one-step block methods in this thesis are appropriate for solving third and fourth order ODEs.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Master Sains

KAEDAH LANGSUNG BLOK SATU-LANGKAH BAGI MENYELESAIKAN PERSAMAAN PEMBEZAAN BIASA PERINGKAT KETIGA DAN KEEMPAT

Oleh

EHAB HASAN ABDULRAHMAN

Julai 2018

Pengerusi : **Zanariah Abdul Majid, PhD**
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Kaedah blok satu-langkah dibentangkan di dalam tesis ini untuk menyelesaikan masalah nilai awal (MNA) persamaan pembezaan biasa (PPB) peringkat ketiga dan keempat. Kaedah-kaedah ini digunakan untuk menyelesaikan PPB peringkat ketiga dan keempat menggunakan saiz langkah malar. Kaedah tersebut akan mendapatkan penyelesaian penghampiran pada masa yang sama di dua dan tiga titik dalam satu blok. PPB peringkat ketiga dan peringkat keempat diselesaikan secara langsung. Kebanyakan bahan rujukan yang sedia ada akan menurunkan masalah dalam sistem PPB peringkat pertama. Walau bagaimanapun, pendekatan di dalam penyelidikan ini adalah lebih cekap daripada teknik biasa yang melibatkan persamaan peringkat pertama. Penyelidikan ini mendefinisikan peringkat kaedah blok satu-langkah dua-titik dan tiga-titik yang diterbitkan. Tambahan lagi, formula kaedah blok melibatkan formula interpolasi Lagrange untuk mengira pekali-pekali kamiran. Selanjutnya, algoritma dibangunkan untuk menyelesaikan MNA PPB peringkat ketiga dan keempat menggunakan saiz langkah yang malar. Di dalam keputusan berangka, prestasi kaedah-kaedah yang dibangunkan menjana hasil yang lebih baik dari segi jumlah bilangan langkah, ralat maksimum, dan jumlah panggilan fungsi berbanding dengan kaedah yang sedia ada. Sebagai kesimpulan, cadangan kaedah blok langsung satu-langkah di dalam tesis ini adalah sesuai untuk menyelesaikan PPB peringkat ketiga dan keempat.

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This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfilment of the requirement for the degree of Master of Science. The members of the Supervisory Committee were as follows:

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LIST OF ABBREVIATIONS

IVPs	Initial Value Problems
ODE	Ordinary Differential Equations
LMM	Linear Multistep Method
ABM	Adam-Bashforth Moulton

CHAPTER 1

INTRODUCTION

1.1 Background

Researchers aim to transform mathematics into a practical field as opposed to theories and formulas. They employ a numerical solution of ordinary differential equations (ODEs) as a noteworthy approach to this goal. At present, applied mathematics is generally utilized in the sciences and engineering fields, such as in fluid dynamics, and in electrical and other areas. Formulations and concepts of differential equations are generally fundamental and involve a specific problem that can be classified as a one-step method and multi-step method. In an example of a one-step method, implicit Runge-Kutta method used a value from only from one previous point to indicate the solutions. A multi-step method or a method such as the Adam model formula defines the solutions attributed to more than one previous p oint. Therefore, any acceptable method can be used to obtain the acceptable approximate solution. The fundamental concept of the block method is to simultaneously obtain the approximate solutions at several points in a block. This approach can avoid lengthy calculation, thereby minimizing computational work. This block method also reduces computational time, thus enabling the method to be more competitive.

Initial value problems (IVPs) are also included in the differential equation. In those parts of the IVPs, the solution of advantage is determined by specifying the values of all the solution components at two point and three-point as well as a direction of integration. In particular, differential equations for third order and four order ODEs were established with emphasis to propose two-point and three-point one- step block method directly which consist of order three and four for solving initial value problems. Mainly, researchers focused on the derivation of one step block method and multistep methods with the constant coefficients for solving general ordinary differential equations directly based on Lagrange interpolation formula, B-spline and Newton-Gregory backward interpolation. The explanation of the definition can be referred in Burden and Faires (1993).

1.2 The Initial Value Problem

The initial value problems (IVPs) for a system of s first order ordinary differential equations are defined by

$$y' = f(x,y) \qquad y(a) = \eta. \tag{1.1}$$

where $y(x) = [y_1(x), y_2(x), \ldots, y_s(x)]^T$.

and $\eta = [\eta_1, \ldots, \eta_s]^T$ is the vector of the initial condition.

1.3 Linear Multistep Method

Consider the initial value problem for a single first-order ordinary differential as in equation (1.1). With the solution in $a \leq x \leq b$ and assume that $f(x,y)$ satisfies the *Lipschitz condition*. The following equality

$$x_n = a + nh, \qquad n = 0, 1, 2, \ldots$$

where h is a constant step size denoted the theoretical solution. Let y_n be approximation to the theoretical solution at $(x_n, y(x_n))$. So, $f_n = f(x_n, y_n)$. The computation of the sequence $\{y_n\}$ takes the form of a linear relationship between y_{n+j}, f_{n+j} ; $j = 0, 1, 2, ..., k$.

Therefore, the equation is called as a linear multistep method of k-step or a linear k-step method. The general form of the linear multistep method is as follows:

$$\sum_{j=0}^{k} \alpha_j y_{n+j} = h \sum_{j=0}^{k} \beta_j f_{n+j}. \tag{1.2}$$

α_j and β_j are constants, $\alpha_k \neq 0$ and both α_0 and β_0 are not zero. Usually, we assume that $\alpha_k = 1$. If $\beta_k = 0$, then the method is said to be explicit and implicit otherwise.

1.4 Objectives of the Thesis

This research aims to investigate the performance of the proposed direct one-step block method for solving third order and fourth order ODEs. The objectives of this research are outlined as follows:

1) To derive the formulae of two-point and three-point one-step block methods for solving the general third order and fourth order ODEs.

2) To determine the order and stability analysis of the proposed block methods.

3) To construct a code for each proposed method by implementing the block methods using constant step size.

4) To compare the numerical results with the existing methods.

1.5 Scope of the Study

This research focuses on solving third order and fourth order ODEs without reducing a system of first order ODEs. The proposed method involves the derivation of two-point and three-point one-step block methods based on the closest point in a block. During the implementation of the block method, the iteration process will only involve the simple iteration for convergent. Furthermore, this research aims to show that the proposed method has zero stability. All the programs for the proposed methods were written in C language and implemented using the constant step size.

1.6 Outline of the Thesis

Chapter 1 provides a brief introduction to the field of numerical analysis. The objectives and the scope of the study are also discussed.

Chapter 2 presents the review of relevant literature and relates the same to the concept of the numerical solution in this research. This chapter also describes the preliminary theory of the included numerical methods and explains the review of previous works related to this study.

Chapter 3 discusses the derivation of the two-point one-step block methods using the constant step size technique, including the numerical results and resulting analysis. The requisite conditions to determine the order of the methods is described, and stability analysis is also demonstrated.

Chapter 4 focuses on the derivation of the three-point one-step block method ODE using constant step size. The order of method is discussed. Discussion of the numerical results and the performance of the methods are also shown.

Chapter 5 provides a summary of this thesis and discusses directions for future work.

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