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GENERALIZATIONS OF v-LINDELÖF GENERALIZED TOPOLOGICAL SPACES

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By

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DEDICATIONS

To all of my love;
my mother & my husband & my kids

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Doctor of Philosophy

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MARIAM M. ABUAGE

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Chair: Professor Adem Kılıçman, PhD
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A significant contribution to the theory of generalized open sets was made by Császár (1997), he introduced the concept of generalized neighborhood systems and generalized topological spaces. Further, he showed that the fundamental definitions and the major part of numerous statements and structures in the set topology can be formulated by replacing topology with the generalized topology. During this work, we introduce two kinds of ν-separations axioms in generalized topological spaces, which are generated by ν-regular open sets; namely, almost G-regular and G-semiregular. Therefore, properties and characterization are introduced. Relation among theses generalized topological spaces and some other ν-separations axioms are considered. These two kinds of ν-separations axioms are essential to relies some results in our work.

We define three types of ν-Lindelöf generalized topological spaces. Namely; nearly ν-Lindelöf, almost ν-Lindelöf and weakly ν-Lindelöf (briefly. $n\nu$-Lindelöf, $a\nu$-Lindelöf and $w\nu$-Lindelöf). Some properties and characterizations of these three generalizations of ν-Lindelöf generalized topological spaces are given. The relations among them are studied and some counterexamples are shown in order to prove that the studies of generalizations are proper generalizations of ν-Lindelöf generalized topological spaces. Subspaces and subsets of these generalized topological spaces are studied. We show that some subsets of these generalized topological spaces inherit these covering properties and some others they do not. Moreover, G-semiregular property on these spaces is studied to establish that all of these properties are G-semiregular properties on the contrary of ν-Lindelöf property which is not a G-semiregular property.

Mappings and generalized continuous functions are also studied on these generalizations and we prove that these properties are generalized topological properties. Relations and some properties of many decompositions of generalized continuity that recently defined and studied are given. Counterexamples are also given to establish the relations among these generalizations of generalized continuity. We show that some proper mappings preserve these generalized topological properties such as (δ,δ')-continuity preserves $n\nu$-Lindelöf property. $\theta(\nu,\mu)$-continuity preserves $a\nu$-Lindelöf property. Almost (ν,μ)-continuity preserves $w\nu$-Lindelöf property. Moreover, we give some conditions on the functions or on the generalized topological spaces to prove that weak forms of generalized continuity preserve some of these covering properties under these conditions.

The product property on these generalizations is also studied. We show that these topological properties are not preserved by product, even under a finite product. Some conditions are given on these generalizations to prove that these properties are preserved by finite product under these conditions. We show that, in weak $P-G$-spaces, finite product of $n\nu$-Lindelöf generalized topological spaces is $n\nu$-Lindelöf and finite product of $w\nu$-Lindelöf generalized topological spaces is $a\nu$-Lindelöf.

Using the notions of generalized topology and hereditary classes, in order to we define some of generalizations of $\nu\mathscr{H}$-Lindelöf, namely; $n\nu\mathscr{H}$-Lindelöf, $a\nu\mathscr{H}$-Lindelöf and $w\nu\mathscr{H}$-Lindelöf hereditary generalized topological spaces. Moreover, we investigate basic properties of the concepts, the relation among them, their relation to known concepts and their preservation by functions properties.

Soft generalized topological spaces played an important role in recently years. Some basic definitions and important results related to soft generalized topology on an initial soft set are given, the concept of soft ν-Lindelöf soft generalized topological spaces is introduced. Basic properties and relation between ν-Lindelöf spaces in generalized topological spaces and soft ν-Lindelöf soft generalized topological spaces are showed. We can say that a soft ν-Lindelöf soft generalized topological spaces gives a parametrized family of ν-Lindelöf generalized topological spaces on the initial universe.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

PENGITLAKAN RUANG TOPOLOGI TERITLAK ν-LINDELÖF

Oleh

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April 2018

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Sumbangan besar kepada teori set terbuka teritlak dibuat oleh Császár (1997), beliau memperkenalkan konsep sistem kejiranan teritlak dan ruang topologi teritlak. Selanjutnya, beliau menunjukkan bahawa definisi asas dan sebahagian besar daripada banyak pernyataan dan struktur dalam topologi set dapat dirumuskan dengan menggantikan topologi dengan topologi teritlak. Kajian ini akan memperkenalkan dua jenis aksiom ν-pemisahan dalam ruang topologi teritlak, yang dijana oleh set ν-sekata; iaitu, hampir G-sekata dan G-semisekata. Oleh itu, sifat-sifat dan ciri-ciri mereka diperkenalkan. Hubungan di antara ruang topologi teritlak dan beberapa aksiom ν-pemisahan yang lain akan dipertimbangkan. Kedua-dua jenis aksiom ν-pemisahan ini sangat penting kerana akan mempengaruhi hasil kajian ini.

Kita mentakrif tiga jenis ruang topologi teritlak ν-Lindelöf iaitu; dekat ν-Lindelöf, hampir ν-Lindelöf dan secara lemah ν-Lindelöf (dikenali sebagai $n\nu$-Lindelöf, $a\nu$-Lindelöf dan $w\nu$ -Lindelöf). Sesetengah sifat dan ciri untuk ketiga-tiga pengitlakan ruang topologi teritlak ν-Lindelöf telah diberikan. Hubungan antara mereka telah dikaji dan sesetengah contoh penyangkal diberikan untuk membuktikan bahawa kajian pengitlakan ini adalah pengitlakan yang sewajarnya bagi ruang topologi teritlak ν-Lindelöf. Subruang dan subset bagi ruang topologi teritlak ini telah dikaji. Kajian menunjukkan bahawa beberapa subset ruang teritlak ini mewarisi ciri-ciri tudungan dan sebahagian lagi tidak. Tambahan pula, sifat G-semisekata pada ruang ini telah dikaji untuk menentukan bahawa semua sifat ini adalah ciri-ciri G-semisekata yang bercanggah dengan sifat ν-Lindelöf yang merupakan bukan sifat G-semisekata.

Pemetaan dan fungsi selanjar teritlak juga telah dikaji dan kajian membuktikan bahawa sifat-sifat ini adalah sifat-sifat topologi teritlak. Hubungan dan beberapa sifat

penguraian keselanjaran teritlak yang baru-baru ini telah ditakrif dan dikaji adalah diberi. Contoh penyangkal juga telah diberikan untuk membangunkan hubungan di antara pengitlakan keselanjaran teritlak. Kajian menunjukkan bahawa beberapa pemetaan wajar telah mengekalkan sifat-sifat topologi teritlak seperti keselanjaran-(δ,δ') yang mengekalkan sifat $n\nu$-Lindelöf. Keselanjaran-$\theta(\nu,\mu)$ mengekalkan sifat $a\nu$-Lindelöf. Hampir keselanjaran-(ν,μ) mengekalkan sifat $w\nu$-Lindelöf. Selain itu, kajian memberikan beberapa syarat pada fungsi atau pada ruang topologi teritlak untuk membuktikan bahawa bentuk-bentuk lemah keselanjaran teritlak, memelihara sebahagian daripada sifat tudungan di bawah syarat-syarat ini.

Ciri-ciri hasildarab pada pengitlakan ini juga telah dikaji. Kajian menunjukkan bahawa sifat topologi ini tidak dikekalkan oleh hasildarab, walaupun di bawah hasildarab terhingga. Beberapa syarat telah diberikan pada pengitlakan ini untuk memastikan bahawa sifat-sifat ini dikekalkan oleh hasildarab terhingga. Kajian menunjukkan bahawa, dalam ruang $P-G$ lemah, hasildarab terhingga ruang topologi teritlak $n\nu$-Lindelöf adalah $n\nu$-Lindelöf dan hasildarab terhingga ruang topologi teritlak $w\nu$-Lindeöf adalah $a\nu$-Lindelöf.

Menggunakan tanggapan topologi teritlak dan kelas keturunan, kita mentakrifkan beberapa pengitlakan $\nu\mathscr{H}$-Lindelöf, iaitu; ruang topologi teritlak keturunan $n\nu\mathscr{H}$-Lindelöf, $a\nu\mathscr{H}$-Lindelöf dan $w\nu\mathscr{H}$-Lindelöf. Selain itu, kita mengkaji sifat asas konsep-konsep tersebut,, hubungan di antara mereka, hubungan mereka dengan konsep yang diketahui dan pengekalan mereka oleh sifat fungsi.

Ruang topologi teritlak lembut memainkan peranan penting sejak kebelakangan ini. Beberapa takrifan asas dan hasil penting yang berkaitan dengan topologi teritlak lembut pada set lembut awalan telah diberikan, konsep ruang topologi teritlak lembut ν-Lindelof lembut diperkenalkan. Sifat asas dan hubungan antara ruang ν-Lindelö

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF ABBREVIATIONS

$\mathbb{R}$	The set of real numbers
$\mathbb{N}$	The set of natural numbers
$\mathscr{X}_{\mathscr{G}}$	Non-empty set
τ	Ordinary topology
$(\mathscr{X}_{\mathscr{G}}, \tau)$	topological space $\mathscr{X}_{\mathscr{G}}$
$P(\mathscr{X}_{\mathscr{G}})$	The power set of $\mathscr{X}_{\mathscr{G}}$
$\mathscr{G}\mathscr{T}$	Generalized Topology
$\mathscr{G}\mathscr{T}\mathscr{S}$	Generalize Topological Space
$\mathscr{Q}\mathscr{T}\mathscr{S}$	Quasi- Topological Space
ν	Generalized topologies on $\mathscr{X}_{\mathscr{G}}$
$(\mathscr{X}_{\mathscr{G}}, \nu)$	Generalized Topological Spaces on ν
$(\mathscr{X}_{\mathscr{G}}, \nu_{\delta})$	The ν-semiregularization generalized topology of $(\mathscr{X}_{\mathscr{G}}, \nu)$
$\nu(A)$	The induced generalized topology on A
$i_{\nu}(A)$	ν-interior of A
$c_{\nu}(A)$	ν-closure of A
$\mathscr{X}_{\mathscr{G}} \backslash A$	The Complement of A in $\mathscr{X}_{\mathscr{G}}$
$\mathscr{H}$	Hereditary Class
$\mathscr{H}\mathscr{G}\mathscr{T}\mathscr{S}$	Hereditary Generalized Topological Spaces
$s\mathscr{G}\mathscr{T}$	soft Generalized Topology
$s\mathscr{G}\mathscr{T}\mathscr{S}$	soft Generalized Topological Space
$s\mathscr{Q}\mathscr{T}\mathscr{S}$	soft Quasi-Topological Space
$(\mathscr{S}_A, \nu)$	Soft Generalized Topological Space $\mathscr{S}_A$
$\widehat{A}$	A-universal soft set
$\Delta, \Omega, \Gamma, N$	Index sets

CHAPTER 1

INTRODUCTION

It is well-known that a large number of studies is devoted to the study of classes of subsets of a topological space, containing the class of open sets, and possessing properties more or less similar to those of open sets. E.g. Levine (1963) has introduced semi-open sets, Njàstad (1965) α-open sets and El-Monsef et al. (1983) β-open sets. A further group of generalized open sets is obtained with the help of another principle; we mention locally closed sets (Bourbaki, 1966), $\mathscr{A}$-sets (Tong, 1986) and $\mathscr{B}$-sets (Tong, 1989). Recently, a significant contribution to the theory of open sets, was extended by Császár (1997). Especially, he defined some basic operators and concepts of generalized neighborhood systems and generalized topological spaces. In many cases, generalized open sets have the property that the arbitrary union of them belong to the same class of sets; this property is postulated in the concept of generalized topology in the sense of Császár (2002). More precisely, let us say that a non-empty set $\mathscr{X}_{\mathscr{G}}$, $P(\mathscr{X}_{\mathscr{G}})$ denotes the power set of $\mathscr{X}_{\mathscr{G}}$ and ν be a non-empty family of $P(\mathscr{X}_{\mathscr{G}})$. The symbol ν implies a generalized topology on $\mathscr{X}_{\mathscr{G}}$ if the empty set $\emptyset \in \nu$ and $\mathscr{U}_\gamma \in \nu$ where $\gamma \in \Omega$ implies $\bigcup_{\gamma\in\Omega} \mathscr{U}_\gamma \in \nu$. The pair $(\mathscr{X}_{\mathscr{G}}, \nu)$ is called generalized topological space.

Generalized topological spaces that we are going to discuss is one of the research interest recently in the area of topology. It should be also noted that at present, there are several hundred works dedicated to the investigation of generalized topological spaces, most of them deal with the theory itself but very few deal with applications.

1.1 Historical Remarks

A lot of attention has been made to study properties of covering in topological spaces, which include open and different kind of generalized open sets. The idea of Lindelöf property came from studying compactness property. Since compactness is a very important property in topology and analysis, mathematicians have studied it widely. In compact spaces we deal with open covers to admit finite subcovers. After that compactness was generalized to countable compactness and sequentially compactness. Since the relationship between compactness and Lindelöfness are very strong, where every compact space is Lindelöf but not the converse, many properties of compact spaces were generalized onto Lindelöf spaces. Thus mathematicians first studied some generalizations of compact spaces such as: paracompactness, nearly compactness, weakly compactness etc. Mathematicians called all these concepts covering properties. They generalized these concepts again to new covering properties such as nearly Lindelöf, almost Lindelöf, weakly Lindelöf, para-Lindelöf and other generalizations.

Many authors have been introduced the generalizations of Lindelöf space separately for many reasons and according to the sets that they are interested in such as Balasubramanian (1982), Mashhour et al. (1984), Cammaroto and Santoro (1996), Fawakhreh and Kılıçman (2001) and Noiri and Popa (2010). Moreover, in a few last years the generalization of Lindelöf spaces have been extended and generalized to bitopological setting as in (Kılıçman and Salleh, 2007), (Kılıçman and Salleh, 2009) and (Salleh and Kılıçman, 2013).

1.2 Problem Statement

In general topology, generalizations of Lindelöf space that depend on open covers and regularly open covers was the important topic in the term between 1959 and 1996. Furthermore, these generalizations have been extended to bitopological setting by many researchers and authors. Recently, generalizations of properties of covering have been done to generalized topological spaces, the earlier generalizations to generalized covering properties are ν-compact and ν-Lindelöf generalized topological spaces for instant, (see. (Sarsak, 2013a) and (Thomas and John, 2012)). In this work, we define the notions of generalizations of ν-Lindelöf generalized topological spaces.

1.3 Research Objectives

Our objectives are:

- To define and introduce two kinds of ν-separations axioms in generalized topological spaces, which generated by ν-regular open sets namely; almost G-regular and G-semiregular.
- To study three types of generalizations of ν-Lindelöf space in generalized topological spaces, which depend on ν-open covers namely; nearly ν-Lindelöf, almost ν-Lindelöf, weakly ν-Lindelöf generalized topological spaces and find relation among them.
- To investigate mappings and some generalized continuity on theses generalizations of ν-Lindelöf generalized topological space.
- To show the preservation of generalized ν-Lindelöf spaces under product generalized topology.
- To study generalization of ν-Lindelöf with respect to a hereditary classes and find relation between these hereditary generalized topological spaces and the other types of classes.
- To generalize the covering properties into soft generalized topological spaces.

1.4 Scope of the Study

This study will focus on three kind of generalizations of ν-Lindelöf, which independent on ν-regular open cover. Further, the relations between covering properties and ν-separations axioms play an important role in generalizations of ν-Lindelöf spaces, since Sarsak (2010) studied weak separation axioms in generalized topological spaces. Also in the same year Min (2010a) defined G-regular space in generalized topological spaces and showed some basic properties for interior and closure operators defined by δ-open and δ-closed sets. We will introduce we introduce other ν-separations axioms generated by ν-regular sets namely; almost G-regular and G-semiregular generalized topological spaces. Some properties and relations among them and other ν-separations axioms will be showed. Mappings of generalized topology are considered the most researched points in generalized topological spaces, we will show the effect of some mappings on these generalizations of ν-Lindelöf. Császár (2008b) showed that the definition of the product of topologies is generalized in such a way that topologies are replaced by generalized topologies in the sense of Császár (2002), we are investigated that ν-Lindelöf and its generalizations are not closed under finite product. Moreover, provide some necessary conditions for these generalized spaces to be preserved under finite product. Császár (2007a) defined a nonempty class of subsets of a nonempty set, called hereditary class and studied modification of generalized topology via hereditary classes. We will study and define the generalizations of ν-Lindelöf with respect to hereditary classes, their relation to known concepts and their preservation by functions properties. Thomas and John (2014) defined soft generalized topology on an initial soft set and saw that soft generalized topology gives a parametrized family of generalized topologies on the initial universe, some properties covering in soft generalized topological spaces are studied such as, soft soft $n\nu$-compact and soft ν-Lindelöf soft generalized topological spaces.

1.5 Organization of the Thesis

This thesis consists of nine chapters. The first chapter point out to some classes of subsets of topological spaces and describe the concept of generalized open sets, the objectives of the study are listed. The second chapter is literature review, where we list all the definitions and state all the results, which are essential for the development of the thesis, as basic concepts of generalized open sets, ν-separations axioms and generalizations of covering properties in generalized topological spaces.

In the next six chapters, we present our works. The third chapter defines some separations axioms generated by ν-regular sets, which are important to fulfilment our results in the rest of chapters. The fourth chapter defines the generalizations of ν-Lindelöf in generalized topological spaces, namely; nearly ν-Lindelöf, almost ν-Lindelöf and weakly ν-Lindelöf. Also, the relation among them and their characterization are investigated. The fifth chapter shows the effect of some mappings on the generalizations of ν-Lindelöf, such as almost (ν,μ)-continuous and (δ,δ')-continuous function. The sixth chapter investigates the preservation of ν-Lindelöf and its generalizations under product property and we provide some necessary conditions for these generalizations to be preserved. In seventh chapter, we define and study the generalizations of ν-Lindelöf with respect to hereditary classes. Moreover, some of our results in last two chapters consider special case of some results in this chapter. The eight chapter introduces some generalizations of covering properties in soft generalized topological spaces such as soft ν-Lindelöf and soft ν-paracompact soft generalized topological spaces.

Lastly, the chapter nine devotes to some open problems that we encountered during this work.

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