UNIVERSITI PUTRA MALAYSIA

# *STABILITY ANALYSIS OF DUAL SOLUTIONS FOR BOUNDARY LAYER STAGNATION POINT FLOW OVER A SHRINKING SHEET WITH SUCTION*

NURUL SYUHADA BINTI ISMAIL

IPM 2018 10

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By

NURUL SYUHADA BINTI ISMAIL

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfillment of the Requirements for the Degree of Doctor of Philosophy

July 2018

## DEDICATIONS

*To all of my love;*

*Mak & Abah*
*siblings, lecturers and friends*

*-thank you for everthing-*

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfillment of the requirement for the degree of Doctor of Philosophy

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**NURUL SYUHADA BINTI ISMAIL**

**July 2018**

**Chairman: Norihan binti Md. Arifin, PhD**
**Institute: Institute for Mathematical Research**

At the surface of the object in the flow field, there exist stagnation points when the fluid is brought to rest effected from the object. This stagnation region experiences the highest pressure. This thesis studies some problems in stagnation point region by considering five problem in different situation. The five problems considered are stagnation point flow over exponentially shrinking sheet, stagnation point flow over shrinking sheet in homogeneoues heterogeneous reactions, MHD stagnation point flow over shrinking sheet, MHD stagnation point flow over shrinking sheet in nanofluid and unsteady MHD stagnation point flow over shrinking sheet. Shrinking sheet and suction parameter is considered in all the problems. The partial differential equations for each problem are first transformed into similarity equations in ordinary differential equations form by similarity transformations. Then, the equation obtained are then solved numerically by using the bvp4c function and shooting method. We used commercially available software which is Maple to generate the shooting technique where Runge-Kutta method together with Newton-Raphson method is involved. Meanwhile bvp4c function is used in MATLAB. Comparisons with existing solutions in literature for specific cases have been made and the present results show an excellent agreement from previous work. It is found that dual solutions exist for a certain range of shrinking and suction parameter for all problems. Therefore, stability analysis is performed to determine the stable solutions by using the bvp4c function. This analysis concludes that, only the first solution is stable and physically significant while the second solution is unstable.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

**ANALISIS KESTABILAN BAGI PENYELESAIAN DUAL UNTUK ALIRAN LAPISAN SEMPADAN TITIK GENANGAN TERHADAP PERMUKAAN MENGECUT DENGAN SEDUTAN**

Oleh

**NURUL SYUHADA BINTI ISMAIL**

**Julai 2018**

**Pengerusi: Norihan binti Md. Arifin, PhD**
**Institut: Institut Penyelidikan Matematik**

Di permukaan objek di medan aliran, terdapat keberadaan titik genangan apabila bendalir terkesan untuk berehat hasil daripada objek tersebut. Titik genangan ini mengalami tekanan tertinggi. Tesis ini bercadang untuk menyelesaikan masalah di kawasan titik genangan dan ini dilakukan dengan mempertimbangkan lima masalah yang berbeza dengan situasi yang berlainan. Lima masalah yang ditakrifkan ialah aliran titik genangan terhadap permukaan mengecut secara mendadak, aliran-aliran genangan terhadap permukaan mengecut dalam reaksi heterogen - homogen, aliran titik genangan MHD terhadap permukaan mengecut, aliran genangan MHD terhadap permukaan mengecut di nanobendalir dan aliran titik genangan MHD yang tak mantap terhadap permukaan mengecut. Sebagai catatan, parameter pengecutan dan penyedutan dipertimbangkan dalam semua masalah. Persamaan terbitan separa untuk setiap masalah diubah menjadi persamaan kesamaan dalam bentuk persamaan pembezaan biasa melalui penjelmaan keserupaan. Kemudian, persamaan yang diperoleh diselesaikan secara berangka dengan menggunakan fungsi bvp4c dan kaedah tembakan. Kami menggunakan perisian yang tersedia secara komersial iaitu Maple untuk menghasilkan kaedah tembakan di mana kaedah Runge-Kutta bersama-sama dengan Newton-Raphson juga terlibat. Sementara itu, fungsi bvp4c digunakan di MATLAB. Perbandingan dengan penyelesaian yang sedia ada dalam kesusasteraan untuk kes-kes tertentu telah dibuat dan hasilnya menunjukkan perbandingan yang baik daripada kajian sebelumnya. Adalah didapati bahawa penyelesaian dual telah wujud bagi sesetengah julat paramater ketakmantapan, parameter pengecutan dan parameter sedutan untuk semua masalah. Oleh itu, analisis kestabilan dilakukan untuk menentukan kestabilan penyelesaian yang diperoleh dengan menggunakan fungsi bvp4c. Kajian ini menyimpulkan bahawa, hanya penyelesaian pertama yang stabil dan penting secara fizikal sementara penyelesaian kedua tidak stabil.

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This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfilment of the requirement for the degree of Doctor of Philosophy. The members of the Supervisory Committee were as follows:

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Date:

**Declaration by Members of Supervisory Committee**

This is to confirm that:

- the research conducted and the writing of this thesis was under our supervision;
- supervision responsibilities as stated in the Universiti Putra Malaysia (Graduate Studies) Rules 2003 (Revision 2012-2013) are adhered to.

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Name of
Chairman of
Supervisory
Committee: ______________________________

Signature: ______________________________

Name of
Member of
Supervisory
Committee: ______________________________

## TABLE OF CONTENTS

# LIST OF TABLES

## LIST OF FIGURES

# LIST OF ABBREVIATIONS

| | |
|---|---|
| $c_p$ | specific heat at a constant temperature |
| $C_f$ | skin friction coefficient |
| $Pr$ | Prandtl number |
| $Nu$ | Nusselt number |
| $Re$ | Reynolds number |
| $Sh$ | Sherwood number |
| $Sc$ | Schmidt number |
| $Ec$ | Eckert number |
| $Le$ | Lewis number |
| $\tau_w$ | wall shear stress |
| $q_w$ | surface heat flux |
| $k$ | thermal conductivity |
| $T$ | fluid temperature |
| $T_w$ | wall temperature |
| $T_\infty$ | ambient temperature |
| $T_0$ | characteristic temperature of the sheet |
| $C$ | fractional nanoparticles |
| $C_w$ | fractional nanoparticles at the surface |
| $C_\infty$ | free stream fractional nanoparticles |
| $U_\infty$ | free stream velocity |
| $u, v$ | fluid velocity component $x$ and $y$ direction respectively |
| $u_w$ | shrinking/stretching sheet velocity |
| $v_w$ | velocity of the mass flux |
| $U_e$ | velocity of the inviscid fluid |
| $U_w$ | constant characteristic velocity of the sheet |
| $D_A, D_E$ | diffusion coefficients |
| $a, b$ | concentrations of the chemical species |
| $B_0$ | magnetic field |
| $R_s$ | velocity slip parameter |
| $J_s$ | thermal slip parameter |
| $s$ | suction parameter |
| $M$ | magnetic parameter |
| $Q$ | heat generation parameter |
| $R_d$ | thermal radiation parameter |
| $K_s$ | heterogeneous reaction |
| $K$ | homogeneous reaction |
| $Nb$ | Brownian motion parameter |
| $Nt$ | thermophoresis parameter |

**Greek Symbols**

| | |
|---|---|
| $\theta$ | dimensionless temperature |
| $\beta$ | thermal expansion coefficient |
| $\nu$ | kinematic viscosity of the fluid |
| $\mu$ | dynamic viscosity of the fluid |
| $\rho$ | density of the fluid |
| $\psi$ | stream function |
| $\tau_w$ | shear stress at the surface |
| $\eta$ | similarity variable |
| $\lambda$ | stretching or shrinking parameter |
| $\chi$ | unsteadiness parameter |

**Subscripts**

| | |
|---|---|
| $w$ | conditions at the surface |
| $\infty$ | ambient/free stream condition |

**Superscript**

| | |
|---|---|
| $'$ | differentiation with respect to $\eta$ |

# CHAPTER 1

# INTRODUCTION

## 1.1 Introduction

This thesis concerned with the mathematical theory of fluid flows. Fluid mechanics is one of the major areas for the application of mathematics and has obvious practical applications in many important disciplines such as aeronautics, meteorology, geophysical fluid mechanics, biofluid mechanics, and many others. Using a general continuum mechanical approach, we will first derive the governing equations (the famous Navier-Stokes equations) from first principle. We will then apply these equations to a variety of practical problems and examine the appropriate simplifications and solution strategies.

From a historical perspective, by the mid-1800s, the Navier-Stokes equation was known, but couldn't be solved except for flows of very simple geometries. Until in year 1904, Ludwig Prandtl (1875-1953) had introduced the boundary layer approximation. The boundary layer concept had become the workhorse of engineering fluid mechanics throughout most of the 1900s. The detailed explanation about boundary layer will be discussed in the Section 1.2. Instead of boundary layer concept, there are also a few of general introductions in fluid dynamics that discuss on stagnation point, types of fluid flow, types of fluid, type of boundary conditions, dimensionless parameters and parameters that involved in this study. Other than basic introduction about fluid dynamics, the importance of the research, objectives and scopes of study, and thesis organization for the whole research are also discussed.

## 1.2 Boundary layer Theory

To solve the problems that are related to the boundary layer, having a basic understanding about boundary layer theory is really important. A major revolution in fluid mechanics occurred in 1904 when Ludwig Prandtl had introduced the boundary layer approximation. His analysis simplifies the complicated Navier-Stokes and energy equations and makes it possible to obtain solutions for the problems that involve in many applications. Prandtl's idea was to divide the flow into two regions. First, an outer flow region that is inviscid and is described by the Euler equations. The second one is an inner flow region called a boundary layer, which is a very thin layer region of flow near a solid wall where viscous forces and rotationality cannot be brushed aside. This region obeys Navier-Stokes equation but could be reduced to much simple form, called boundary layer equation (Yunus and Cimbala (2006)).The boundary layer interpretation is valid only for the portions of the surface for which the main flow remains attached, that is unseparated.

### 1.2.1 Types of Boundary Layer

The concept of boundary layers is central to the understanding of convection heat and mass transfer between a surface and a fluid flowing past it. For flow over any surface, there will always exist a velocity boundary layer and hence surface friction. Likewise, a thermal boundary layer, and hence convection heat transfer, will always exist if the surface and free stream temperatures differ. Similarly, a concentration boundary layer and convection mass transfer will exist if the surface concentration of a species differs from its free stream concentration. Thus, velocity, thermal, and concentration boundary layers are described, and their relationships to the friction coefficient, convection heat transfer coefficient, and convection mass transfer coefficient are introduced (Bergman and Incropera (2011))

#### 1.2.1.1 The Velocity Boundary Layer

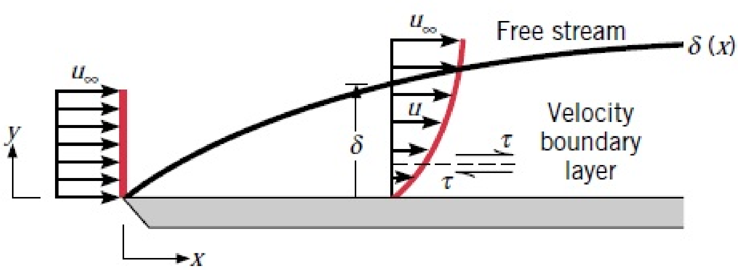


**Figure 1.1: Velocity boundary layer development on a flat plate [Bergman and Incropera (2011)]**

Velocity boundary layer develops when there is fluid flow over a surface. When fluid particles make contact with the surface, they assume zero velocity. These particles then act to retard the motion of particles in the adjoining fluid layer which is responding to retard the motion of particles in the next layer, and so on until at a distance $y = \delta$ the surface, the effect becomes negligible. With increasing distance $y$ from the surface, the $x$ velocity component of the fluid $u$ must then increase until it approaches the free stream value $u_\infty$. The quantity $\delta$ is termed the boundary layer thickness and it is typically defined as the value of y for which $u = 0.99u_\infty$.

The boundary layer velocity profile refers to the manner in which $u$ varies with $y$ through the boundary layer. Because it pertains to the fluid velocity, the foregoing boundary layer may be referred to more specifically as the velocity boundary layer. It develops whenever there is fluid flow over a surface and it is of fundamental importance to problems involving convection transport. In a velocity boundary layer, the velocity gradient at the surface depends on the distance $x$ from the leading edge of the plate. Therefore, the surface shear stress and friction coefficient also depend on $x$ (Bergman and Incropera (2011)).

#### 1.2.1.2 The Thermal Boundary Layer

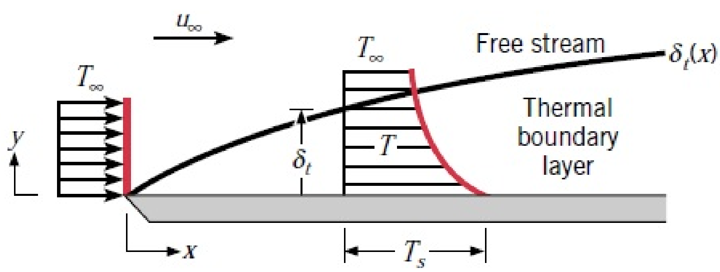


**Figure 1.2: Thermal boundary layer development on an isothermal flat plate [Bergman and Incropera (2011)].**

A thermal boundary layer develops when a fluid at specified temperature flows over a surface that is at a different temperature. At the leading edge the temperature profile is uniform, with $T(y) = T_\infty$. The region of the fluid in which these temperature gradients exist is the thermal boundary layer, and its thickness $\delta_t$ is typically defined as the value of $y$ for which the ratio $[(T_s - T)/(T_s - T_\infty)] = 0.99$. With increasing distance from the leading edge, the effects of heat transfer penetrate further into the free stream and the thermal boundary layer grows.

The thickness of the boundary layer increases in the flow direction since the effects of heat transfer are felt at greater distances from the surface towards further down stream. The convection heat transfer rate over places along the surface is directly related to the temperature gradient at that location. Therefore, the shape of the temperature profile in the thermal boundary layer dictates the convection heat transfer between a solid surface and the fluid flowing over (Cengel and Ghajar (2011)).

#### 1.2.1.3 The Concentration Boundary Layer

A concentration boundary layer is similar to the velocity and thermal boundary layers. It is the region of the fluid in which concentration gradients exist, and its thickness $\delta_c$ is typically defined as the value of y for which $[(C_{A,s} - C_A)/(C_{A,s} - C_{A,\infty})] = 0.99$. With increasing distance from the leading edge, the effects of species transfer penetrate further into the free stream and the concentration boundary layer grows. Species transfer by convection between the surface and the free stream fluid is determined by conditions in the boundary layer and we are interested in determining the rate at which this transfer occurs. Therefore, conditions in the concentration boundary layer which strongly influence the surface concentration gradient also influence

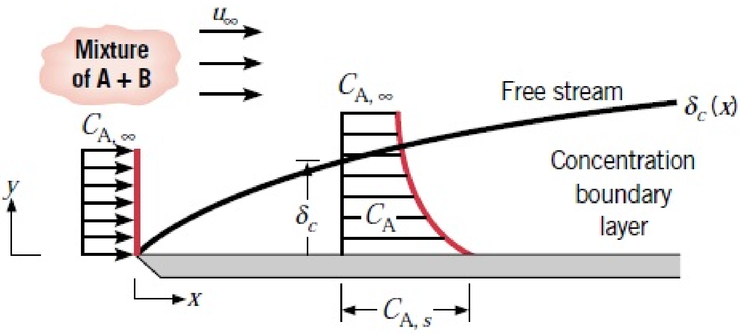


**Figure 1.3: Species concentration boundary layer development on a flat plate [Bergman and Incropera (2011)].**

the convection mass transfer coefficient and hence the rate of species transfer in the boundary layer (Bergman and Incropera (2011)).

### 1.2.2 Boundary layer Stagnation Point Flow

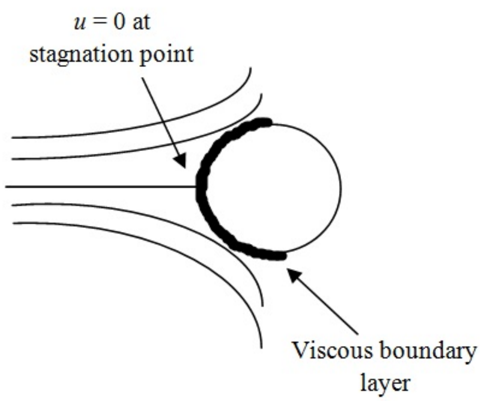


**Figure 1.4: Psychical model for flow at Stagnation point**

When the course collides with solid object and the constant speed at the stagnation point is zero as shown in Figure 1.4, the stagnation point take place. Highest point of the degree of the pressure, maximum values of heat transfer and mass deposition happen in the stagnation area. Hiemenz (1911) was the first who analyzed the stable flow in the proximity of a stagnation level. Generally, these courses are broadly used in the product formation procedures in manufacturing. The aerodynamics extrusion of plastic layers, the partition sheet throughout product operating conveyors, cooling of nuclear reactors and many hydrodynamic procedures, fabric and paper sectors, the cooling of an unlimited metallic panel, cooling of electronic appliances by ventilators and blood movement issues (Batool and Ashraf (2013)).

## 1.3 Typed of Fluid Flow

### 1.3.1 Compressible and Incompressible Flow

A flow can be classified as compressible or incompressible depending on the level of variation of density during flow. A flow is said to be incompressible if the density remains nearly constant throughout. Therefore, the volume of every portion of fluid remains unchanged over the course of its motion when the flow ( or the fluid ) is incompressible.

### 1.3.2 Steady and Unsteady Flow

The term steady implies no change at a point with time. During steady flow, the fluid properties can change from point to point within a device, but at any fixed point they remain constant. Steady flow conditions can be closely approximated by devices that are intended for continuous operation such as turbines, pumps, boilers, condensers and refrigeration systems (fluid mechanics). Unsteady or non-steady flow is the flow where its properties depends on time.

## 1.4 Types of Fluid

### 1.4.1 Viscous Fluid

A viscous fluid is one which resists movement or the movement of an object through the fluid. All fluids, liquid, gas or plasma have some measure of viscosity which can be compared using mathematical formulas or direct measurements of movement. Though all fluids have viscosity, a viscous fluid in the everyday sense of the term is one that has a high level of viscosity. These types of fluid may move slowly or not at all, depending on how viscous they are.

The type of matter a fluid is made of is the main determiner of how viscous it is, though other factors including temperature will also affect viscosity. In general, liquids will become less viscous as their temperature rises while gases will become more viscous with an increase in temperature. Gases become more viscous when they are heated because the atoms in the gas move more rapidly as temperature rises resulting in more collisions between atoms and thus more resistance. Pressure also can affect viscosity, though this is not generally seen in liquids because unlike gaseous matter, liquid matter is very difficult to compress.

An extremely viscous fluid may have properties that make it behave more like a solid than a liquid. Butter is an example of a fluid with a high viscosity. Though butter does flow at room temperature, it is so resistant to movement that it is difficult to perceive it as a fluid. Heating butter will cause it to become noticeably less viscous. Glass is also a liquid. When glass cools and hardens into a solid-like state, its viscosity approaches infinity, meaning that it no longer flows at all.

### 1.4.2 Nanofluids

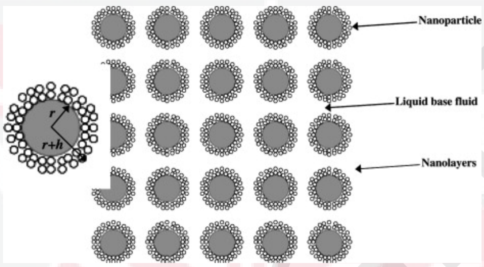


**Figure 1.5: Psychical model for Nanofluid [Kakaç and Pramuanjaroenkij (2009)]**

As shown in Figure 1.5, a current type of thermal conducting fluids known as nanofluids comprise of minimal number of nanosized particles (typically less than 100nm) which are steadily and consistently suspended in a fluid firstly used by Choi and Eastman (1995). By including nanoparticle into the basis mixture, the transfer attributes, flow and thermal transport capacity of the fluids can be improved and the thermal conductivity of the basic solution incidentally expanded which is recognized as the leading barrier in heat transfer performance (Zaimi et al. (2014)). Since they are adequately minor to act to fluid molecules, they can run evenly via micro passages in the absence of blocking (Khanafer et al. (2003)). Due to their various technical and biomedical implementations, countless researches on nanofluids are being carried out by scientists and engineers. For examples, food and drink, cancer therapy, vehicle cooling, paper and printing and textiles, transformer cooling, oil and gas and electronics cooling and detergency(Uddin et al. (2012)).

## 1.5 Heat Transfer

Heat transfer is thermal energy in transit due to a spatial temperature difference. The basic requirement for heat transfer is the presence of a temperature difference. There can be no net heat transfer between two mediums that are at the same temperature. The larger the temperature gradient, the higher the rate of heat transfer. The literature of heat transfer generally recognizes three distinct modes heat transmission that are

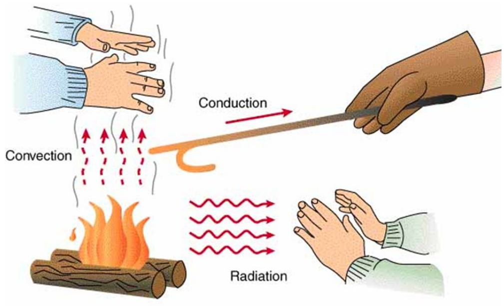


**Figure 1.6: Psychical model Type of Heat Transfer**

conduction, convection and radiation. The term conduction is used to refer to the heat transfer that will occur across the medium when a temperature gradient exists in a stationary medium either a solid or a fluid. In contrast, the heat transfer that will occur between a surface and a moving fluid when they are at different temperatures is explained by the term convection. Energy in the form of electromagnetic waves is emitted by the surfaces of finite temperature. Therefore, radiation between two surfaces at different temperatures produces net heat transfer in the absence of an intervening medium.

Heat transfer plays a major role in the design of many other devices such as car radiators, various components of power plants and even spacecraft. Heat transfer is important not only in engineered systems but also in nature. Temperature regulates and triggers biological responses in all living systems and ultimately marks the boundary between sickness and health. Two common examples include hypothermia, which results from excessive cooling of the human body, and heat stroke, which is triggered in warm, humid environments (Bergman and Incropera (2011)) .

## 1.6 Mass Transfer

Many significant heat transfer problems encountered in practice involve mass transfer. Mass transfer requires the presence of two regions at different chemical compositions and mass transfer refers to the movement of a chemical species from a high concentration region toward a lower concentration one relative to the other chemical species present in the medium. Mass transfer can also occur in liquids and solids as well as in gases as shown in Figure 1.7. Another factor that influences that diffusion process is the molecular spacing. The larger the spacing, in general, the higher the diffusion rate. Therefore, the diffusion rate are typically much higher in gases than they are in liquids and much higher in liquids then in solids.

Mass transfer is the basis for many biological and chemical processes. Biological

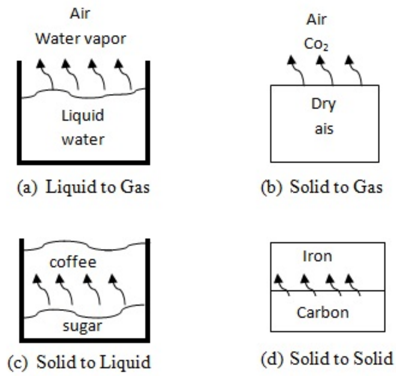


**Figure 1.7: Some examples of mass transfer that involve a liquid and/or a solid**

processes include the oxygenation of blood and the transport of ions across membranes within the kidney. Chemical processes include the chemical vapor deposition (CVD) of silane ($SiH_4$) onto a silicon wafer, the doping of a silicon wafer to form a semiconducting thin film, the aeration of waste water, and the purification of ores and isotopes (Welty et al. (2009)).

## 1.7 Types of Boundary Conditions

### 1.7.1 Stretching and Shrinking Sheet

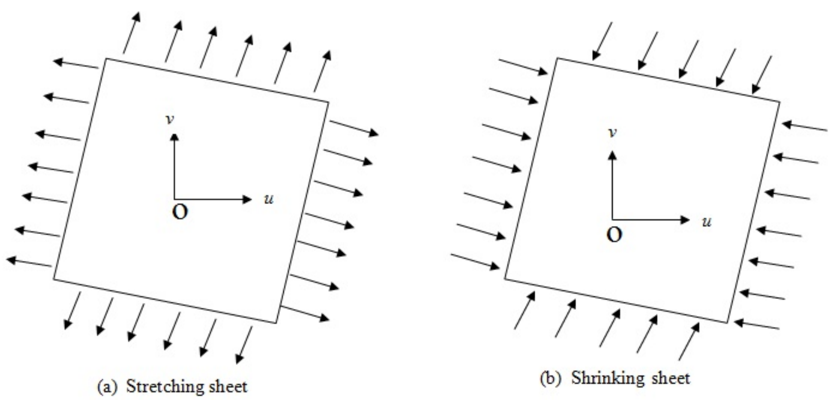


**Figure 1.8: Psychical model and coordinate system for flow towards shrinking sheet**

As shown in Figure 1.8(a), when the velocity on the boundary shuts off a fixed point, stretching sheet occurs. The boundary layer flow due to a stretching surface is important in extrusion processes such as metal sheet extrusion, polymer extrusion and other industrial processes. As depicted in Figure 1.8(b), the movement of the velocity at the boundary towards a fixed point generates shrinking sheet. There are two conditions for the flow of a shrinking sheet to exist, namely whether a stagnation flow is considered (Wang (2008)) to maintain the velocity of shrinking sheet in the boundary layer or a sufficient suction is added on the boundary (Miklavcic and Wang (2006)). Shrinking issue is applicable in the research of the environmental management strategies, shrink swell behavior and the capillary effects in smaller pores and the hydraulic properties of agricultural clay soils which are important for agricultural development (Batool and Ashraf (2013)).

### 1.7.2 Suction

Suction is one of the methods of boundary layer control, which have the aim of reducing drag on bodies in an external flow or of reducing losses of energy in channels. This method was suggested by L. Prandtl in 1904 as one of the means of preventing or ”delaying” boundary layer separation. Suction is applied in practice for increasing the efficiency of diffusers with high compression ratio of the working fluid (with large convergence angles) by means of delaying early separation of the boundary layer. Boundary layer suction through slots near the trailing edge is used for increasing lift and decreasing drag of aerofoils operating at large incidence angles. Suction is also an effective means of the boundary layer laminarization, which decreases friction losses

### 1.7.3 Slip and No Slip

All experimental observations indicate that a fluid in motion comes to a complete stop at the surface and assumes a zero velocity relative to the surface. That is, a fluid in direct contact with a solid to the surface due to viscous effects and there is no slip known as the no slip condition. However, the flow velocity at the solid wall is non-zero in the presence of slip flow. The fluids that exhibit boundary slip have important technological uses like in the polishing the synthetic heart valves and internal cavities.

## 1.8 Magnetohydrodynamics (MHD) Fluid Flow

In reference to the fact that the rate of cooling can be controlled by the application of magnetic field, the study of magnetohydrodynamics (MHD) flow an electrically conducting fluid is of considerable interest in metallurgical and metal working processes. In metallurgical processes, the process of drawing the strips in an electrically conducting fluid subject to a magnetic field is able to controls the rates of cooling and stretching of the strips in order to obtain a final product with desired characteristics.

## 1.9 Dimensionless Parameters

In convection studies, it is common practice to non-dimensionalize the governing equations and combine the variables, which group together into dimensionless numbers in order to reduce the number of total variables (Cengel (2003)). All of the foregoing dimensionless parameters have physical interpretations that relate to conditions in the flow, not only for boundary layers but also for other flow types as well (Bergman and Incropera (2011)). The parameters that involve in this study are:

1. **Prandtl number** : Provides a measure of the relative effectiveness of momentum and energy transport by diffusion in the velocity and thermal boundary layers, respectively. From this interpretation, it follows that the value of Pr strongly influences the relative growth of the velocity and thermal boundary layers (Bergman and Incropera (2011)). It is named after Ludwig Prandtl, who introduced the concept of boundary layer in 1904. The Prandtl numbers of fluids range from less than 0.01 for liquid metals to more 100,000 for heavy oils. The Prandtl numbers of gases are about 1, which indicates that both momentum and heat dissipate through the fluid at about the same rate. Heat diffuses very quickly in liquid metal ($Pr \leq 1$) and very slowly in oils ($Pr \geq 1$) relative to momentum (Cengel and Ghajar (2011)).

2. **Nusselt number** : Provides a measure of the convection heat transfer occurring at the surface. The Nusselt number is named after Wilhelm Nusselt, who made significant contributions to convective heat transfer in the first half of the twentieth century. The larger the Nusselt number, the more effective the convection (Cengel and Ghajar (2011)).

3. **Reynolds number** : After exhaustive experiments in the 1880s, Osborn Reynolds discovered that the flow regime depends mainly on the ratio of the inertia forces to viscous forces in the fluid (Cengel and Ghajar (2011)). We should also expect the magnitude of the Reynolds number to influence the velocity boundary layer thickness . With increasing Re at a fixed location on a surface, we expect viscous forces to become less influential relative to inertia forces. Hence the effects of viscosity do not penetrate as far into the free stream, and the value of diminishes (Bergman and Incropera (2011)).

4. **Eckert number** : Named after Ernst R. G. Eckert that provides a measure of the kinetic energy of the flow relative to the enthalpy difference across the thermal boundary layer. It plays an important role in high-speed flows for which viscous dissipation is significant (Bergman and Incropera (2011)).

5. **Schmidt number** : Provides a measure of the relative effectiveness of momentum and mass transport by diffusion in the velocity and concentration boundary layers (Bergman and Incropera (2011)). It was named after the German engineer Ernst Heinrich Wilhelm Schmidt (1892 – 1975). Schmidt number is the mass transfer equivalent of Prandtl Number. For gases, Sc and Pr have similar values (0.7) and this is used as the basis for simple heat and mass transfer.

6. **Lewis number** : It is named after Warren K. Lewis (1882 – 1975), the Lewis number is a measure of the relative thermal and concentration boundary layer thicknesses. It is used to characterize fluid flows where there is simultaneous heat and mass transfer (Cohen (2007)).

7. **Sherwood number** : Represents the ratio of the convective mass transfer to the rate of diffusive mass transport (Heldman (2003)) and is named in honor of Thomas Kilgore Sherwood. It is particularly valuable in situations where the Reynolds number and Schmidt number are readily available.

8. **Coefficient of friction** : A coefficient of friction is a value that shows the relationship between the force of friction between two objects and the normal reaction between the objects that are involved. The coefficient of friction depends on the objects that are causing friction. A value of 0 means there is no friction at all between the objects. A value of 1 means the frictional force is equal to the normal force.

## 1.10 Heat Generation

In the problems dealing with chemical reactions and those concerned with dissociating fluids, the research of heat generation or absorption in moving fluids is crucial. Temperature distribution may be modified by possible heat generation effects, which may influence particle deposition and distribution rate; therefore, the particle deposition and distribution rate in the conductor wafers.

## 1.11 Thermal Radiation

The emission by the hot walls and working fluid cause the thermal radiation within such systems to take place. At great operating temperature the existence of thermal radiation modifies the thermal boundary layer structure and the thermal radiation effects are relatively significant when the difference between the sheet and the ambient temperature is large. The importance of the heat transfer analysis of boundary layer flow with radiation can also be seen in space vehicle re-entry, electrical power generation, solar power technology, astrophysical flows and more manufactural sectors.

## 1.12 Viscous Dissipation

In a viscous fluid flow, the viscosity of the fluid will change the motion of the fluid (kinetic energy) into internal energy of the fluid by taking the energy from it. This refers to the process of heating up the fluid. The process is called as dissipation or viscous dissipation and it is partly irreversible. Viscous dissipation modifies the temperature distribution through a key role playing like an energy source which directs to influence heat transfer rate.

### 1.13 Objectives and Scope of Study

The objectives of this study are to analyse the mathematical formulation to obtain the numerical solutions and perform the stability analysis on the dual solutions for the following :

1. The stagnation-point flow and heat transfer over an exponentially shrinking sheet in the presence of heat generation.

2. The stagnation-point flow over a shrinking sheet with homogeneous heterogeneous reactions. The parameter involve in this problem are suction parameter $s$, homogeneous reaction $K$, heterogeneous reaction $K_s$ and Schmidt number $Sc$.

3. The MHD stagnation-point flow and heat transfer over a shrinking sheet in the presence of heat generation where velocity and thermal slips will be considered at the boundary.

4. MHD stagnation-point flow and heat transfer over a shrinking sheet in the presence of viscous dissipation and thermal radiation in nanofluids where velocity slips will be considered at the boundary.

5. Unsteady MHD stagnation-point flow and heat transfer over a shrinking sheet in the presence of viscous dissipation and thermal radiation.

The scope of this study is limited to the problems of stagnation point boundary layer flows for steady and unsteady, incompressible and two dimensional towards shrinking sheets with suction in viscous fluids or nanofluid. Slip effect at the boundary condition is also considered in this study. The governing partial differential equations for each problem considered are transformed into the ordinary differential equation by using similarity transformation. We used commercially Maple software to obtain the numerical result by shooting method and MATLAB software by bvp4c function. For all the problems, stability analysis are performed for the dual solutions obtained.

### 1.14 Thesis Organization

This thesis consists of nine chapters. The first chapter start with the background of fluid dynamics where the inauguration of boundary layer theory begins. The understanding about the boundary layer theory is the most important part to solve any kind of problems related to boundary layer. In solving boundary layer problem, there are some significant comprehensions that should be emphasized, like the type of fluid flow, type of fluid, stagnation point flow, type of boundary conditions, dimensionless parameter and also the parameters that involve in this study. The scope and objective of the study and the organization of the thesis are also included in this chapter.

The summaries of the previous studies that were carried out by the various researchers which are related to the scope of study are included in literature review in chapter 2. This chapter has been divided into seven parts which began with the introduction to the chapter. This study had examined five problems, because of that, the literature review was divided into five parts which refer to the first until the fifth problem. While the last part will discuss on the literature review for stability analysis.

Chapter 3 will discuss on the methodology and numerical method which are divided into 4 parts. Firstly, initiated with introduction, followed by boundary layer equation and the last section is the numerical method. Section 3.2 will deliberate on the derivation of the basic of boundary layer equation at the stagnation point over shrinking sheet with suction. Then, continue with derivation of stability analysis in Section 3.3. The last part of this chapter is Section 3.4 that confers about the numerical method to obtain numerical solutions for every problem stated in this study.

Next, all of the five problems in this study are given in Chapter 4 until Chapter 8 where every chapter is divided into five parts. For the first section, the section begins with introduction of this study. Then, mathematical formulation of the problem and the stability analysis are deliberated in the second section and the third section. Results and discussions obtained from this study are presented in section four.The fifth section is the conclusion section.

Finally, the conclusion for the whole problems study will be summarized in Chapter 9. This chapter will also give the suggestions for improvements in the future studies.

## REFERENCES


Abbas, Z., Sheikh, M., and Pop, I. (2015). Stagnation-point flow of a hydromagnetic viscous fluid over stretching/shrinking sheet with generalized slip condition in the presence of homogeneous–heterogeneous reactions. *Journal of the Taiwan Institute of Chemical Engineers*, 55:69–75.

Andersson, H. (1992). Mhd flow of a viscoelastic fluid past a stretching surface. *Acta Mechanica*, 95(1):227–230.

Andersson, H. (1995). An exact solution of the navier-stokes equations for magnetohydrodynamic flow. *Acta Mechanica*, 113(1-4):241–244.

Andersson, H., Bech, K., and Dandapat, B. (1992). Magnetohydrodynamic flow of a power-law fluid over a stretching sheet. *International Journal of Non-Linear Mechanics*, 27(6):929–936.

Bachok, N., Ishak, A., Nazar, R., and Pop, I. (2010). Flow and heat transfer at a general three-dimensional stagnation point in a nanofluid. *Physica B: Condensed Matter*, 405(24):4914–4918.

Bachok, N., Ishak, A., and Pop, I. (2011). On the stagnation-point flow towards a stretching sheet with homogeneous–heterogeneous reactions effects. *Communications in Nonlinear Science and Numerical Simulation*, 16(11):4296–4302.

Bachok, N., Ishak, A., and Pop, I. (2012a). Boundary layer stagnation-point flow and heat transfer over an exponentially stretching/shrinking sheet in a nanofluid. *International Journal of Heat and Mass Transfer*, 55(25):8122–8128.

Bachok, N., Ishak, A., and Pop, I. (2012b). Unsteady boundary-layer flow and heat transfer of a nanofluid over a permeable stretching/shrinking sheet. *International Journal of Heat and Mass Transfer*, 55(7):2102–2109.

Batool, K. and Ashraf, M. (2013). Stagnation point flow and heat transfer of a magneto-micropolar fluid towards a shrinking sheet with mass transfer and chemical reaction. *Journal of Mechanics*, 29(3):411–422.

Bergman, T. L. and Incropera, F. P. (2011). *Fundamentals of heat and mass transfer*. John Wiley & Sons.

Bhattacharyya, K. (2011a). Boundary layer flow and heat transfer over an exponentially shrinking sheet. *Chinese Physics Letters*, 28(7):074701.

Bhattacharyya, K. (2011b). Dual solutions in unsteady stagnation-point flow over a shrinking sheet. *Chinese Physics Letters*, 28(8):084702.

Bhattacharyya, K. and Layek, G. (2011). Effects of suction/blowing on steady boundary layer stagnation-point flow and heat transfer towards a shrinking sheet with thermal radiation. *International Journal of Heat and Mass Transfer*, 54(1):302–307.

Bhattacharyya, K. and Pop, I. (2011). MHD boundary layer flow due to an exponentially shrinking sheet. *Magnetohydrodynamics*, 47(4):337–44.

Bhattacharyya, K. and Vajravelu, K. (2012). Stagnation-point flow and heat transfer over an exponentially shrinking sheet. *Communications in Nonlinear Science and Numerical Simulation*, 17(7):2728–2734.

Brewster, M. Q. (1992). *Thermal radiative transfer and properties*. John Wiley & Sons.

Buongiorno, J. (2006). Convective transport in nanofluids. *Journal of Heat Transfer*, 128(3):240–250.

Cengel, Y. A. (2003). *Heat tranfer a practical approach.* McGraw-Hill.

Cengel, Y. A. and Ghajar, A. (2011). Heat and mass transfer (a practical approach, si version).

Chand, G., Jat, R., and Rajotia, D. (2015). Mhd slip flow over a stretching sheet with convective boundary condition. *Institute of Thermomechanics AS CR*, (2):60.

Chao, B. and Jeng, D. (1965). Unsteady stagnation point heat transfer. *J. Heat Transfer*, 87(2):221–230.

Chaudhary, M. and Merkin, J. (1995a). A simple isothermal model for homogeneous-heterogeneous reactions in boundary-layer flow. I Equal diffusivities. *Fluid dynamics research*, 16(6):311–333.

Chaudhary, M. and Merkin, J. (1995b). A simple isothermal model for homogeneous-heterogeneous reactions in boundary-layer flow. II Different diffusivities for reactant and autocatalyst. *Fluid dynamics research*, 16(6):335–359.

Chaudhary, M. and Merkin, J. (1996). Homogeneous-heterogeneous reactions in boundary-layer flow: Effects of loss of reactant. *Mathematical and computer modelling*, 24(3):21–28.

Chaudhary, S. and Kumar, P. (2015a). Magnetohydrodynamic boundary layer flow over an exponentially stretching sheet with radiation effects. *Applied Mathematical Sciences*, 9(23):1097–1106.

Chaudhary, S. and Kumar, P. (2015b). Unsteady magnetohydrodynamic boundary layer flow near the stagnation point towards a shrinking surface. *Journal of Applied Mathematics and Physics*, 3(07):921.

Chen, H., Liang, H., Wang, F., and Shen, M. (2016). Unsteady mhd stagnation-point flow toward a shrinking sheet with thermal radiation and slip effects. *Heat TransferAsian Research*, 45(8):730–745.

Choi, S. U. and Eastman, J. A. (1995). Enhancing thermal conductivity of fluids with nanoparticles. Technical report, Argonne National Lab., IL (United States).

Cohen, E. R. (2007). *Quantities, units and symbols in physical chemistry*. Royal Society of Chemistry.

Crane, L. J. (1970). Flow past a stretching plate. *Zeitschrift für angewandte Mathematik und Physik (ZAMP)*, 21(4):645–647.

Das, M., Mahatha, B., and Nandkeolyar, R. (2015). Mixed convection and nonlinear radiation in the stagnation point nanofluid flow towards a stretching sheet with homogenous-heterogeneous reactions effects. *Procedia Engineering*, 127:1018–1025.

Elbashbeshy, E. (2001). Heat transfer over an exponentially stretching continuous surface with suction. *Archives of Mechanics*, 53(6):643–651.

Fan, T., Xu, H., and Pop, I. (2010). Unsteady stagnation flow and heat transfer towards a shrinking sheet. *International Communications in Heat and Mass Transfer*, 37(10):1440–1446.

Hady, F., Eid, M. R., and Ahmed, M. A. (2014). Slip effects on unsteady mhd stagnation point flow of a nanofluid over stretching sheet in a porous medium with thermal radiation. *Journal of Pure and Applied Mathematics: Advances and Applications*, 12(2):181–206.

Hamid, R. A., Nazar, R., and Pop, I. (2015). Non-alignment stagnation-point flow of a nanofluid past a permeable stretching/shrinking sheet: Buongiornos model. *Scientific reports*, 5.

Harris, S., Ingham, D., and Pop, I. (2009). Mixed convection boundary-layer flow near the stagnation point on a vertical surface in a porous medium: Brinkman model with slip. *Transport in Porous Media*, 77(2):267–285.

Hayat, T., Abbas, F., Awais, M., and Alsaedi, A. (2014). Magnetohydrodynamic stretched flow of maxwell fluid in presence of homogeneous–heterogeneous chemical reactions by three different approaches. *Journal of Computational and Theoretical Nanoscience*, 11(4):953–957.

Heldman, D. R. (2003). *Encyclopedia of Agricultural, Food, and Biological Engineering (Print)*. Crc Press.

Hiemenz, K. (1911). Die grenzschicht an einem in den gleichformigen flussigkeitsstrom eingetauchten geraden kreiszlynder. *Dinglers J.*, 326:321–324.

Ibrahim, W., Shankar, B., and Nandeppanavar, M. M. (2013). Mhd stagnation point flow and heat transfer due to nanofluid towards a stretching sheet. *International Journal of Heat and Mass Transfer*, 56(1):1–9.

Ishak, A. (2014). Flow and heat transfer over a shrinking sheet: A stability analysis. *International Journal of Mechanical, Aerospace, Industrial and Mechatronics Engineering*, 8(5):905–9.

Jat, R., Chand, G., and Rajotia, D. (2014). Mhd heat and mass transfer for viscous flow over nonlinearly stretching sheet in a porous medium. *Therm Energy Power Eng*, 3:191–197.

Jat, R. and Chaudhary, S. (2010). Radiation effects on the mhd flow near the stagnation point of a stretching sheet. *Zeitschrift für Angewandte Mathematik und Physik (ZAMP)*, 61(6):1151–1154.

Jat, R. and Neemawat, A. (2012). Similarity solution for mhd stagnation point flow and heat transfer over a non-linear stretching sheet. *International Journal of Recent Research and Review*, 3:32–51.

Kakaç, S. and Pramuanjaroenkij, A. (2009). Review of convective heat transfer enhancement with nanofluids. *International Journal of Heat and Mass Transfer*, 52(13-14):3187–3196.

Kameswaran, P., Shaw, S., Sibanda, P., and Murthy, P. (2013). Homogeneous–heterogeneous reactions in a nanofluid flow due to a porous stretching sheet. *International journal of heat and mass transfer*, 57(2):465–472.

Khalili, S., Dinarvand, S., Hosseini, R., Tamim, H., and Pop, I. (2014). Unsteady mhd flow and heat transfer near stagnation point over a stretching/shrinking sheet in porous medium filled with a nanofluid. *Chinese Physics B*, 23(4):048203.

Khan, S. K. and Sanjayanand, E. (2005). Viscoelastic boundary layer flow and heat transfer over an exponential stretching sheet. *International Journal of Heat and Mass Transfer*, 48(8):1534–1542.

Khan, W. and Pop, I. (2010a). Boundary-layer flow of a nanofluid past a stretching sheet. *International journal of heat and mass transfer*, 53(11):2477–2483.

Khan, W. and Pop, I. (2010b). Flow near the two-dimensional stagnation-point on an infinite permeable wall with a homogeneous–heterogeneous reaction. *Communications in Nonlinear Science and Numerical Simulation*, 15(11):3435–3443.

Khan, W. and Pop, I. (2012). Effects of homogeneous–heterogeneous reactions on the viscoelastic fluid toward a stretching sheet. *Journal of Heat Transfer*, 134(6):064506.

Khanafer, K., Vafai, K., and Lightstone, M. (2003). Buoyancy-driven heat transfer enhancement in a two-dimensional enclosure utilizing nanofluids. *International Journal of Heat and Mass Transfer*, 46(19):3639–3653.

Kumar Ch, K. and Bandari, S. (2014). Melting heat transfer in boundary layer stagnation-point flow of a nanofluid towards a stretching–shrinking sheet. *Canadian Journal of Physics*, 92(12):1703–1708.

Kuznetsov, A. (2010). The onset of nanofluid bioconvection in a suspension containing both nanoparticles and gyrotactic microorganisms. *International Communications in Heat and Mass Transfer*, 37(10):1421–1425.

Liu, I.-C. (2005). A note on heat and mass transfer for a hydromagnetic flow over a stretching sheet. *International Communications in Heat and Mass Transfer*, 32(8):1075–1084.

Magyari, E. and Keller, B. (1999). Heat and mass transfer in the boundary layers on an exponentially stretching continuous surface. *Journal of Physics D: Applied Physics*, 32(5):577.

Mahapatra, T. R. and Nandy, S. (2011). Stability analysis of dual solutions in stagnation-point flow and heat transfer over a power-law shrinking surface. *International Journal of Nonlinear Science*, 12(1):86–94.

Mahapatra, T. R. and Nandy, S. K. (2013). Stability of dual solutions in stagnation-point flow and heat transfer over a porous shrinking sheet with thermal radiation. *Meccanica*, 48(1):23–32.

Mahapatra, T. R., Nandy, S. K., and Gupta, A. S. (2010). Dual solution of MHD stagnation-point flow towards a stretching surface. *Engineering*, 2(04):299.

Mahapatra, T. R., Nandy, S. K., and Pop, I. (2014). Dual solutions in magnetohydrodynamic stagnation-point flow and heat transfer over a shrinking surface with partial slip. *Journal of Heat Transfer*, 136(10):104501.

Majeed, A., Javed, T., Ghaffari, A., and Pop, I. (2016). Numerical study of unsteady mixed convection stagnation point flow over a stretching cylinder with sinusoidal surface temperature. *Revista Mexicana de Física*, 62(4).

Malik, M., Salahuddin, T., Hussain, A., Bilal, S., and Awais, M. (2015). Homogeneous-heterogeneous reactions in williamson fluid model over a stretching cylinder by using keller box method. *AIP Advances*, 5(10):107227.

Malvandi, A., Hedayati, F., and Ganji, D. (2014). Slip effects on unsteady stagnation point flow of a nanofluid over a stretching sheet. *Powder Technology*, 253:377–384.

Mc0rosky, W. (1977). Some current research in unsteady fluid dynamics. *ASME J. Fluids Eng*, 99(1):8–39.

Meade, D. B., Haran, B. S., and White, R. E. (1996). The shooting technique for the solution of two-point boundary value problems. *Maple Technical Newsletter*, 3(1):1–8.

Merkin, J. (1986). On dual solutions occurring in mixed convection in a porous medium. *Journal of engineering Mathematics*, 20(2):171–179.

Merkin, J. (1996). A model for isothermal homogeneous-heterogeneous reactions in boundary-layer flow. *Mathematical and Computer Modelling*, 24(8):125–136.

Merkin, J. and Kumaran, V. (2010). The unsteady mhd boundary-layer flow on a shrinking sheet. *European Journal of Mechanics-B/Fluids*, 29(5):357–363.

Merrill, K., Beauchesne, M., Previte, J., Paullet, J., and Weidman, P. (2006). Final steady flow near a stagnation point on a vertical surface in a porous medium. *International journal of heat and mass transfer*, 49(23):4681–4686.

Miklavcic, M. and Wang, C. (2006). Viscous flow due to a shrinking sheet. *Quarterly of Applied Mathematics*, 64(2):283–290.

Misra, J., Shit, G., and Rath, H. J. (2008). Flow and heat transfer of a mhd viscoelastic fluid in a channel with stretching walls: Some applications to haemodynamics. *Computers & Fluids*, 37(1):1–11.

Moore, F. K. (1951). Unsteady laminar boundary-layer flow. Technical report, National aeronautics and space administration Washington dc.

Mustafa, M., Farooq, M. A., Hayat, T., and Alsaedi, A. (2013). Numerical and series solutions for stagnation-point flow of nanofluid over an exponentially stretching sheet. *Plos One*, 8(5):e61859.

Mustafa, M., Hayat, T., Pop, I., Asghar, S., and Obaidat, S. (2011). Stagnation-point flow of a nanofluid towards a stretching sheet. *International Journal of Heat and Mass Transfer*, 54(25):5588–5594.

Nandkeolyar, R., Motsa, S., and Sibanda, P. (2013). Viscous and joule heating in the stagnation point nanofluid flow through a stretching sheet with homogenous–heterogeneous reactions and nonlinear convection. *Journal of Nanotechnology in Engineering and Medicine*, 4(4):041002.

Nandy, S. K. (2015). Unsteady flow of maxwell fluid in the presence of nanoparticles toward a permeable shrinking surface with navier slip. *Journal of the Taiwan Institute of Chemical Engineers*, 52:22–30.

Nandy, S. K. and Pop, I. (2014). Effects of magnetic field and thermal radiation on stagnation flow and heat transfer of nanofluid over a shrinking surface. *International Communications in Heat and Mass Transfer*, 53:50–55.

Nazar, R., Jaradat, M., Arifin, N., and Pop, I. (2011). Stagnation-point flow past a shrinking sheet in a nanofluid. *Open Physics*, 9(5):1195–1202.

Nield, D. and Kuznetsov, A. (2009). The cheng–minkowycz problem for natural convective boundary-layer flow in a porous medium saturated by a nanofluid. *International Journal of Heat and Mass Transfer*, 52(25):5792–5795.

Partha, M., Murthy, P., and Rajasekhar, G. (2005). Effect of viscous dissipation on the mixed convection heat transfer from an exponentially stretching surface. *Heat and Mass Transfer*, 41(4):360–366.

Paullet, J. and Weidman, P. (2007). Analysis of stagnation point flow toward a stretching sheet. *International Journal of Non-Linear Mechanics*, 42(9):1084–1091.

Pavlov, K. (1974). Magnetohydrodynamic flow of an incompressible viscous fluid caused by deformation of a plane surface. *Magnitnaya Gidrodinamika*, 4(1):146–147.

Pop, I., Ishak, A., and Aman, F. (2011). Radiation effects on the mhd flow near the stagnation point of a stretching sheet: revisited. *Zeitschrift für Angewandte Mathematik und Physik (ZAMP)*, 62(5):953–956.

Pop, I. and Na, T.-Y. (1998). A note on mhd flow over a stretching permeable surface. *Mechanics Research Communications*, 25(3):263–269.

Pop, S., Grosan, T., and Pop, I. (2004). Radiation effects on the flow near the stagnation point of a stretching sheet. *Technische Mechanik*, 25(2):100–106.

Postelnicu, A. and Pop, I. (2011). Falkner–Skan boundary layer flow of a power-law fluid past a stretching wedge. *Applied Mathematics and Computation*, 217(9):4359–4368.

Rohni, A. M., Ahmad, S., Ismail, A. I. M., and Pop, I. (2013). Boundary layer flow and heat transfer over an exponentially shrinking vertical sheet with suction. *International Journal of Thermal Sciences*, 64:264–272.

Rohni, A. M., Ahmad, S., and Pop, I. (2012). Flow and heat transfer over an unsteady shrinking sheet with suction in nanofluids. *International Journal of Heat and Mass Transfer*, 55(7):1888–1895.

Rohni, A. M., Ahmad, S., and Pop, I. (2014). Flow and heat transfer at a stagnation-point over an exponentially shrinking vertical sheet with suction. *International Journal of Thermal Sciences*, 75:164–170.

Roşca, A. V. and Pop, I. (2013a). Flow and heat transfer over a vertical permeable stretching/shrinking sheet with a second order slip. *International Journal of Heat and Mass Transfer*, 60:355–364.

Roşca, N. C. and Pop, I. (2013b). Mixed convection stagnation point flow past a vertical flat plate with a second order slip: heat flux case. *International Journal of Heat and Mass Transfer*, 65:102–109.

Sajid, M. and Hayat, T. (2008). Influence of thermal radiation on the boundary layer flow due to an exponentially stretching sheet. *International Communications in Heat and Mass Transfer*, 35(3):347–356.

Sakiadis, B. (1961). Boundary-layer behavior on continuous solid surfaces: I. boundary-layer equations for two-dimensional and axisymmetric flow. *AIChE Journal*, 7(1):26–28.

Sanjayanand, E. and Khan, S. K. (2006). On heat and mass transfer in a viscoelastic boundary layer flow over an exponentially stretching sheet. *International Journal of Thermal Sciences*, 45(8):819–828.

Seth, G. S., Sharma, R., Kumbhakar, B., and Chamkha, A. J. (2016). Hydromagnetic flow of heat absorbing and radiating fluid over exponentially stretching sheet with partial slip and viscous and joule dissipation. *Engineering Computations*, 33(3):907–925.

Shampine, L. F., Gladwell, I., and Thompson, S. (2003). *Solving ODEs with matlab*. Cambridge University Press.

Shampine, L. F., Kierzenka, J., and Reichelt, M. W. (2000). Solving boundary value problems for ordinary differential equations in matlab with bvp4c. *Tutorial notes*, 2000:1–27.

Soid, S., Ishak, A., and Pop, I. (2015). Mhd stagnation point flow over a stretching/shrinking sheet. In *Mathematical Sciences and Computing Research (iSMSC), International Symposium on*, pages 355–360. IEEE.

Song, X., Williams, W., Schmidt, L., and Aris, R. (1991). Bifurcation behavior in homogeneous-heterogeneous combustion: II. Computations for stagnation-point flow. *Combustion and Flame*, 84(3-4):292–311.

Suali, M., Nik Long, N., and Ariffin, N. M. (2012). Unsteady stagnation point flow and heat transfer over a stretching/shrinking sheet with suction or injection. *Journal of Applied Mathematics*, 2012.

Tamim, H., Dinarvand, S., Hosseini, R., Khalili, S., and Pop, I. (2014). Unsteady mixed convection flow of a nanofluid near orthogonal stagnation point on a vertical permeable surface. *Proceedings of the Institution of Mechanical Engineers, Part E: Journal of Process Mechanical Engineering*, 228(3):226–237.

TC, C. (1994). Stagnation-point flow towards a stretching plate. *Journal of the physical society of Japan*, 63(6):2443–2444.

Tie-Gang, F., Ji, Z., and Shan-Shan, Y. (2009). Viscous flow over an unsteady shrinking sheet with mass transfer. *Chinese Physics Letters*, 26(1):014703.

Tiwari, R. K. and Das, M. K. (2007). Heat transfer augmentation in a two-sided lid-driven differentially heated square cavity utilizing nanofluids. *International Journal of Heat and Mass Transfer*, 50(9):2002–2018.

Uddin, M. J., Pop, I., and Ismail, A. (2012). Free convection boundary layer flow of a nanofluid from a convectively heated vertical plate with linear momentum slip boundary condition. *Sains Malaysiana*, 41(11):1475–1482.

Wang, C. (2008). Stagnation flow towards a shrinking sheet. *International Journal of Non-Linear Mechanics*, 43(5):377–382.

Weidman, P., Kubitschek, D., and Davis, A. (2006). The effect of transpiration on self-similar boundary layer flow over moving surfaces. *International journal of engineering science*, 44(11):730–737.

Welty, J. R., Wicks, C. E., Rorrer, G., and Wilson, R. E. (2009). *Fundamentals of momentum, heat, and mass transfer*. John Wiley & Sons.

Williams, W., Stenzel, M., Song, X., and Schmidt, L. (1991a). Bifurcation behavior in homogeneous-heterogeneous combustion: I. Experimental results over platinum. *Combustion and flame*, 84(3-4):277–291.

Williams, W. R., Zhao, J., and Schmidt, L. D. (1991b). Ignition and extinction of surface and homogeneous oxidation of nh3 and ch4. *AIChE journal*, 37(5):641–649.

Yunus, A. C. and Cimbala, J. M. (2006). Fluid mechanics fundamentals and applications. *International Edition, McGraw Hill Publication*, 185201.

Zaimi, K., Ishak, A., and Pop, I. (2014). Flow past a permeable stretching/shrinking sheet in a nanofluid using two-phase model. *Plos one*, 9(11):e111743.

Zargartalebi, H., Ghalambaz, M., Noghrehabadi, A., and Chamkha, A. (2015). Stagnation-point heat transfer of nanofluids toward stretching sheets with variable thermo-physical properties. *Advanced Powder Technology*, 26(3):819–829.