



**UNIVERSITI PUTRA MALAYSIA**

***EFFECT OF CUBIC TEMPERATURE GRADIENT ON ONSET OF  
THERMAL CONVECTION IN A MICROPOLAR FLUID***

**NURUL AFIQAH BINTI MOHD ISA**

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**EFFECT OF CUBIC TEMPERATURE  
GRADIENT ON ONSET OF THERMAL  
CONVECTION IN A MICROPOLAR FLUID**

**NURUL AFIQAH BINTI MOHD ISA**

**MASTER OF SCIENCE  
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2018**



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**EFFECT OF CUBIC TEMPERATURE GRADIENT ON ONSET OF  
THERMAL CONVECTION IN A MICROPOLAR FLUID**

By

**NURUL AFIQAH BINTI MOHD ISA**

**Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in  
Fulfilment of the Requirements for the Degree of Master of Science**

**April 2018**



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## DEDICATIONS

**To**

*My Beloved Parent:*

*Dad: Mohd Isa Bin Othman,*

*Mum: Zainun Binti Ali,*

...

*My Amazing Siblings:*

*Nurul Maisarah Binti Mohd Isa,*

*Muhammad Hafeez Bin Mohd Isa,*

*Mohamad Akmal Haneef Bin Mohd Isa,*



Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Master of Science

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THERMAL CONVECTION IN A MICROPOLAR FLUID**

By

**NURUL AFIQAH BINTI MOHD ISA**

**April 2018**

**Chair: Prof Norihan Md Arifin, PhD**

**Faculty: Science Mathematic**

Thermal convection with shear flow has received widespread attention due to its importance in geophysical flows as well as several technological applications, such as heat exchangers and chemical vapour deposition. The thesis deals with two types of thermal convection in a fluid layer that is Rayleigh Benard convection (driven by buoyancy) and Marangoni convection (driven by surface tension). The Rayleigh-Benard and Marangoni stability problem for a fluid bound by bottom and top wall which are heated and cooled, respectively are studied numerically. The fluid layer with various boundary conditions at the different lower and upper boundary are investigated theoretically based on linear stability theory. The various boundary conditions are assumed for lower and upper boundaries to be free isothermal and free isothermal (FIFI), free isothermal and free adiabatic (FIFA), free isothermal and rigid adiabatic (FIRA), rigid isothermal and free isothermal (RIFI), rigid isothermal and free adiabatic (RIFA), and rigid isothermal and rigid adiabatic (RIRA). The effect of cubic temperature gradient, internal heat generation, feedback control, and electric field on the onset of Rayleigh-Bénard and Marangoni convection in an Eringen's micropolar fluid has been examined. Three types of non-uniform basic temperature gradients which are linear, cubic 1 and cubic 2 are considered. The single-term Galerkin method is applied to obtain the eigenvalue for FIFI, FIFA, FIRA, RIFI, RIFA, and RIRA boundary combination. Closed form analytical solutions, of the full governing equations, are derived and the governing parameters of the problem are the thermal critical Rayleigh number, the critical Marangoni number, couple stress,  $N_1$  coupling,  $N$  and micropolar heat conduction,  $N_5$  on the onset of convection has been analysed. It is found that cubic 1 is the most stabilizing temperature gradient and linear temperature

gradient is the most destabilizing temperature gradient. The results also indicate that the internal heat generation and electric field can acts as destabilizing on the system while feedback control can acts as controller of the stability of the system.



Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Master Sains

**KESAN KECERUNAN SUHU KUBIK KE ATAS OLAKAN TERMA DI  
DALAM BENDALIR MIKROPOLAR**

Oleh

**NURUL AFIQAH BINTI MOHD ISA**

**April 2018**

**Pengerusi: Prof Norihan Md Arifin, PhD**

**Fakulti : Sains Matematik**

Olakan terma dengan aliran ricih telah mendapat perhatian yang meluas kerana kepentingannya dalam aliran geofizik serta beberapa aplikasi teknologi, seperti penukar haba dan pemendapan wap kimia. Tesis ini adalah berkaitan dengan dua jenis olakan terma dalam lapisan bendalir iaitu olakan Rayleigh Benard (didorong oleh keapungan) dan olakan Marangoni (didorong oleh ketegangan permukaan). Masalah kestabilan Rayleigh-Benard dan Marangoni untuk bendalir dibatasi oleh dinding bawah dan atas yang dipanaskan dan disejukkan, masing-masing dikaji secara berangka. Lapisan bendalir dengan pelbagai syarat sempadan di sempadan bawah dan atas yang berbeza dikaji secara teori berdasarkan teori kestabilan linear. Pelbagai syarat sempadan diandaikan untuk lapisan bawah dan lapisan atas iaitu bebas isoterma dan bebas isoterma (FIFI), bebas isoterma dan adiabatik bebas (FIFA), isoterma bebas dan adiabatik tegar (FIRA), isothermal tegar dan isoterma bebas (RIFI), tegar isoterma dan adiabatik bebas (RIFA), dan adiabatik dan isoterma tegar (RIRA). Kesan kecerunan suhu kubik, penjana haba dalaman, kawalan suap balik, dan medan elektrik pada permulaan olakan Rayleigh-Bénard dan Marangoni dalam bendalir mikropolar Eringen telah diselidiki. Tiga jenis kecerunan suhu asas tidak seragam yang dipertimbangkan adalah linear, kubik 1 dan kubik 2. Kaedah Galerkin satu-sebutan digunakan untuk mendapatkan nilai eigen bagi gabungan sempadan FIFI, FIFA, FIRA, RIFI, RIFA, dan RIRA. Penyelesaian beranalitik dalam bentuk tertutup bagi persamaan yang diterbitkan sepenuhnya, diperolehi dan parameter terbitan bagi masalah ini adalah nombor Rayleigh kritikal, nombor Marangoni kritikal, tekanan gandingan,  $N_1$ , gandingan  $N_3$  dan konduksi haba mikropolar,  $N_5$  pada permulaan olakan telah dianalisis. Telah didapati bahawa sistem dengan kubik 1 adalah lebih stabil dan sistem dengan kecerunan suhu

linear adalah yang paling tidak stabil. Keputusan juga menunjukkan bahawa penjanaan haba dalaman dan medan elektrik boleh menyebabkan ketidakstabilan pada sistem manakala kawalan suap balik boleh bertindak sebagai pengawal untuk menstabilkan sistem.



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The writing of this thesis involves many valuable contributions from a number of people. Hence, I would like to take this golden opportunity to express my sincere appreciation to those who have been involved behind the development of this thesis for their help, love, guidance, insight and encouragement throughout this task.

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My deepest appreciation goes to my friends and research colleagues who kindly provided valuable and helpful comments in the preparation of this thesis. Finally, thank you very much to my family who had given their maximum support and great motivation without fail, for me to keep going to reach the peak, especially during the hard times. Thank you very much.

I certify that a Thesis Examination Committee has met on 30 April 2018 to conduct the final examination of Nurul Afiqah bt Mohd Isa on his thesis entitled "Effect of Cubic Temperature Gradient on Onset of Thermal Convection in a Micropolar Fluid" in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Master of Science.

Members of the Thesis Examination Committee were as follows:

**Siti Hasana bt Sapar, PhD**

Associate Professor  
Faculty of Science  
Universiti Putra Malaysia  
(Chairman)

**Fudziah binti Ismail, PhD**

Professor  
Faculty of Science  
Universiti Putra Malaysia  
(Internal Examiner)

**Seripah Awang Kechil, PhD**

Associate Professor  
Universiti Teknologi MARA  
Malaysia  
(External Examiner)



---

**RUSLI HAJI ABDULLAH, PhD**

Professor and Deputy Dean  
School of Graduate Studies  
Universiti Putra Malaysia

Date: 30 July 2018

This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfilment of the requirement for the degree of Master of Science. The members of the Supervisory Committee were as follows:

**Norihan Md Arifin, PhD**

Professor  
Faculty of Science  
Universiti Putra Malaysia  
(Chairman)

**Norfifah Bachok, PhD**

Associate Professor  
Faculty of Science  
Universiti Putra Malaysia  
(Member)

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**ROBIAH BINTI YUNUS, PhD**

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Supervisory Committee : Prof Madya Dr Norfifah Bt Bachok @ Lati

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## LIST OF ABBREVIATIONS

$a$	Wave number
$a_c$	Critical wave number
$a_i$	Constants
$b$	Basic state
$C_v$	Specific heat
$d$	Depth
$E$	Electric field
$f(z)$	Basic non-uniform temperature gradient
$G(z)$	Spin
$g$	Acceleration due to gravity
$h_g$	Internal heat generation within micropolar fluid layer
$I$	Moment of inertia
$K$	Feedback control
$\hat{k}$	Unit vector z-direction
$L$	Electric number
$M_c$	Critical Marangoni number
$N_1$	Coupling parameter
$N_3$	Couple stress parameter
$N_5$	Micropolar heat conduction parameter

$p$	Pressure
$\mathbf{P}$	Dielectric polarization
$Q$	Internal Heat Generation
$\mathbf{q}$	Velocity
$R$	Rayleigh number
$R_c$	Critical Rayleigh number
$T$	Temperature
$T_b$	Basic-state temperature
$T_0$	Initial temperature
$\bar{T}_{os}$	Temperature at the upper free surface and $a_i$
$W(z)$	Amplitudes of the perturbations of vertical velocity
$\rho$	Density of the fluid

#### **Greek symbols**

$\rho_b$	Basic-state density of the fluid
$\lambda$	Bulk kinematic viscosity coefficients
$\eta$	Shear kinematic viscosity coefficients
$\lambda'$	Bulk spin velocity coefficients
$\eta'$	Shear spin velocity coefficients
$\zeta$	Coupling viscosity coefficient or vortex viscosity
$\beta$	Micropolar heat conduction coefficient

$\rho_0$	Density of the fluid at temperature $T = T_0$
$\omega$	Micro rotation
$\kappa$	Thermal conductivity
$\chi$	Thermal conductivity
$\chi_e$	Thermal susceptibility
$\phi$	Electrostatic potential
$\epsilon_0$	Electric permeability of free space
$\epsilon_r$	Dielectric constant
$\theta(z)$	Temperature
$\alpha$	Thermal expansion



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# CHAPTER 1

## INTRODUCTION

### 1.1 Heat Transfer

Heat is about all matter that made up from atoms and molecules. Basically, atoms are always in different types of motion or movement such as rotational, translation, and vibrational. Then, the motion of atoms and molecules will create the heat or thermal energy hence we assume that all matter has thermal energy. In fact, the more movement of the atoms or molecules, the more heat or thermal energy they will produce. Heat can travel from one place to another in three ways: conduction, convection and radiation. But, from these three ways, both conduction and convection require matter to transfer heat energy. If there is a temperature difference between two systems heat, it will always find a way to transfer from the higher system to the lower system.

Conduction is the transfer of heat between substances that are indirect contact with each other. The better the conductor, the more rapidly heat will be transferred. Metal is a one example of good conduction of heat. Conduction occurs when a substance is heated the particles will gain more energy, and vibrate more. These molecules then bump into nearby particles and transfer some of their energy to them. This then continues and passes the energy from the hot end down to the colder end of the substance.

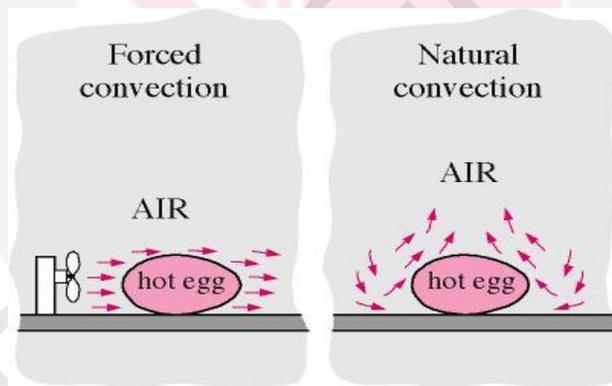
Thermal energy is transferred from hot places to cold places by convection. Convection occurs when warmer areas of a liquid or gas rise to cooler areas in the liquid or gas. Cooler liquid or gas then takes the place of the warmer areas which have risen higher. This results in a continuous circulation pattern. Water boiling in a pan is a good example of these convection currents. Another good example of convection is in the atmosphere. The earth's surface is warmed by the sun, the warm air rises and cool air moves in.

Radiation is a method of heat transfer that does not rely upon any contact between the heat source and the heated object as is the case with conduction and convection. Heat can be transmitted though empty space by thermal radiation often called infrared radiation. This is a type of electromagnetic radiation. No mass is exchanged and no medium is required in the process of radiation. Example of radiation is the heat from the sun, or heat released from the filament of a light bulb.

## 1.2 Convection

Convection in deep meaning is the transfer of internal energy into or out of an object by the physical movement of a surrounding fluid that transfers the internal energy along with its mass. Although the heat is transferred early between an object and a fluid by conduction, a bulk transfer of energy comes from the motion of the fluid. Convection can arise spontaneously (or naturally or freely) through the creation of convection cells or can be forced by propelling the fluid across the object or by the object through the fluid.

There are two types of convection that was free convection and forced convection. Figure 1.1 shows the comparison the cooling of the boiled egg by forced and natural convection. For the natural process of convection, the block has to heat the air to a certain extent to make lighter (say up to 50 degree). When the fan is turned on the air is constantly replaced thus the temperature of the air adjacent to the hot plate is around 25 degrees. Hence the potential of heat transfer is much higher, thus the heat transfer coefficient is higher.



**Figure 1. 1: Difference between forced and natural convection.** (Adapted from: <http://www.mhhe.com/engcs/mech/cengel/notes/ConvectionHeatTransfer.html>)

### 1.2.1 Free or Natural Convection

Free or natural convection can be described when fluid motion is caused by buoyancy forces the results from the density variations due to variations of thermal temperature in the fluid. In the absence of an external source, when the fluid is in contact with a hot surface, its molecules separate and scatter, causing the fluid to be less dense. As a consequence, the fluid is displaced while the cooler fluid gets denser and the fluid sinks. Thus, the hotter volume transfers heat towards the cooler volume of that fluid.

Familiar examples are the upward flow of air due to a fire or hot object and the circulation of water in a pot that is heated from below.

## 1.2.2 Forced Convection

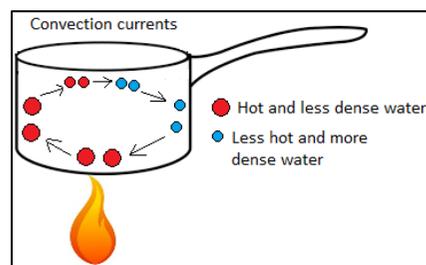
Mechanism when the fluid motion produced from an external source like a pump, fan, or other device called as forced convection. Term of forced convection also can be used to refer any fluid and it is commonly relatable forced air cooling.

The effect of high airspeeds involved with forced convection can cause the significant amounts of heat can be transported rapidly and effectively. The quantity of the surface area on the heat sink is an important factor in helping meet the desired thermal performance, but too much surface area will cause the heat sink to have a large pressure drop. The greater the drop in pressure, the greater the strain put on the fan, which results in decreased fan performance.

## 1.3 Application of convection

### 1.3.1 Boiling water in a cooking pot

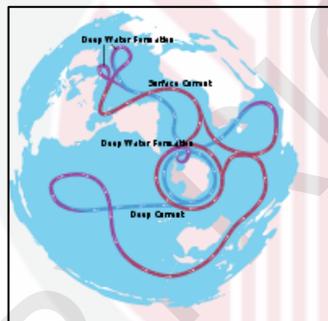
Boiling water in a cooking pot is one great example for convection. When water is boiled, the liquid moves around quickly. Water at the bottom of the pan receives more heat. This process will make the water at the bottom of the pan less dense and rise to the top. When it on top surface, it will receives less heat and become more dense. This will make the water go back to the bottom of the pot again. Once at the bottom, it will go right back up, and the process repeats. The back and forth movements are called convection currents. Figure 1.2 shows the illustration of boiling water in a cooking pot. The big red circles show when water is heated, it will expand and become less dense.



**Figure 1. 2: Example of boiling water in a cooking pot.** (Adapt from: <https://www.introduction-to-physics.com/examples-of-convection.html>)

### 1.3.2 Oceanic circulation

Ocean circulation is the large scale movement of waters in the ocean basins. The mechanism occurs when warm water from the equator move to circulate toward the poles and cold water heads towards to the equator. The surface currents are initially dictated by surface wind conditions while the trade winds blow westward in the tropics, and the westerlies blow eastward at mid-latitudes. This wind pattern applies a stress to the subtropical ocean surface with negative curl across the Northern Hemisphere and the reverse across the Southern Hemisphere. The ocean current pattern produced by the wind-induced Ekman transport called Sverdrup transport is equator ward because of conservation of potential vorticity caused by the poleward-moving winds on the subtropical ridge's western periphery and the increased relative vorticity of poleward moving water, transport is balanced by a narrow, accelerating poleward current, which flows along the western boundary of the ocean basin, outweighing the effects of friction with the cold western boundary current which originates from high latitudes. The overall process is known as western intensification, causes currents on the western boundary of an ocean basin to be stronger than those on the eastern boundary. Figure 1.3 below show the illustration of the ocean circulation.

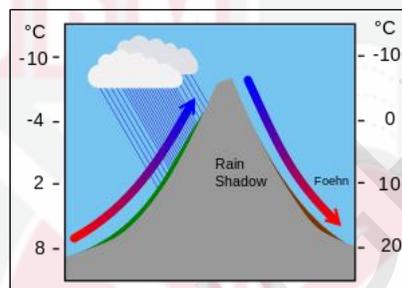


**Figure 1. 3: Ocean currents.** (Adapt from: <https://pediaview.com/openpedia/Convection>)

As it travels poleward, warm water transported by strong warm water current undergoes evaporative cooling. The cooling is wind driven: wind moving over water cools the water and also causes evaporation, leaving saltier brine. In this process, the water becomes saltier and denser, and decreases in temperature. Once sea ice forms, salts are left out of the ice, a process known as brine exclusion. These two processes produce water that is denser and colder or more precisely, water that is still liquid at a lower temperature. The water across the northern Atlantic Ocean becomes so dense that it begins to sink down through less salty and less dense water. The convective action is not unlike that of a lava lamp. This downdraft of heavy, cold and dense water becomes a part of the North Atlantic Deep Water, a south going stream.

### 1.3.3 Weather

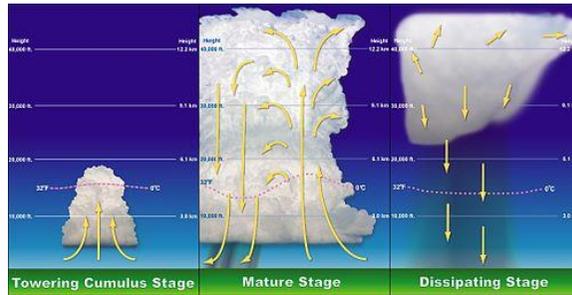
Some more localized phenomena than global atmospheric movement are also due to convection, including wind and some of the hydrologic cycle. For example, a foehn wind is a down-slope wind which occurs on the downwind side of a mountain range. A föhn or foehn is a type of dry, warm, down-slope wind that occurs in the lee (downwind side) of a mountain range. Figure 1.4 is the illustration of how the foehn wind is produced. It results from the adiabatic warming of air which has dropped most of its moisture on windward slopes. Because of the different adiabatic lapse rates of moist and dry air, the air on the leeward slopes becomes warmer than at the same height on the windward slopes.



**Figure 1.4: How foehn is produced.** (Adapt from: <https://commons.wikimedia.org/wiki/File:Föhn1.svg>)

A thermal column (or thermal) is a vertical section of rising air in the lower altitudes of the Earth's atmosphere. Thermals are generated by the irregular heating of Earth's surface from solar radiation. The warm ground affected by the sun will turn the warm air which is directly below it. Then, the warmer air will rise and become less dense around the surrounding of air mass, and creating a thermal low. The mass of lighter air rises, and as it does, it cools by expansion at lower air pressures. It stops rising when it had the same temperature as encircle the air. Associated with a thermal is a downward flow around the thermal area. The downward moving exterior is affected by colder air being replaced at the upper surface of the thermal. Sea breeze is one example of convection driven weather.

Warm air has a lower density than cool air, so warm air rises within cooler air, similar to hot air balloons. Clouds form as relatively warmer air carrying moisture rises within cooler air. As the moist air rises, it cools, causing some of the water vapour in the rising packet of air to condense. When the moisture condenses, it releases energy known as latent heat of condensation which permits the rising packet of air to cool less than its surrounding air, continuing the cloud's ascension. If enough instability is present in the atmosphere, this mechanism will continue until it forms the cumulonimbus clouds, where it will support lightning and thunder. Generally, thunderstorm requires three conditions to form: moisture, an unstable air mass, and a lifting force (heat). Figure 1.5 below show three stages of a thunderstorm life.

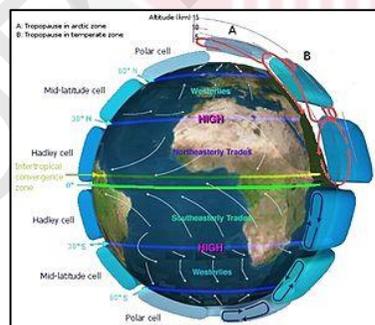


**Figure 1. 5: Stage of a thunderstorm's life.** (Adapt from: [https://en.wikipedia.org/wiki/Cumulonimbus\\_cloud](https://en.wikipedia.org/wiki/Cumulonimbus_cloud))

Thunderstorms will go through three stages that was the developing stage, the mature stage, and the dissipation stage. The average thunderstorm has a 24 km (15 mi) diameter. Depending on the conditions present in the atmosphere, these three stages take an average of 30 minutes to go through.

### 1.3.4 Atmospheric circulation

Atmospheric circulation is the major movement of air where the thermal energy is dispersed on the surface of the earth, together with much slower (lagged) ocean circulation system. The wide structure of the atmospheric circulation varies from time to time, but the basic climatological structure remains fairly constant.



**Figure 1. 6: Idealised depiction of the global circulation on Earth.** (Adapt from: [https://en.wikipedia.org/wiki/Atmospheric\\_circulation](https://en.wikipedia.org/wiki/Atmospheric_circulation))

Latitudinal circulation occurs because of incident solar radiation per unit area is highest at the heat equator and decreases while the latitude increases also it reached minima at

the poles. It consists of two primary convection cells, the Hadley cell and the polar vortex, with the Hadley cell experiencing stronger convection due to the release of latent heat energy by condensation of water vapour at higher altitudes during cloud formation.

The longitudinal circulation occurs because the ocean greater specific heat capacity than the ground and also thermal conductivity allowing the heat to penetrate further beneath the surface, and consequently absorbs and releases more heat, but the temperature changes less than the ground. Longitudinal circulation consists of two cells, the Walker circulation and El Niño / Southern Oscillation.

#### 1.4 Rayleigh-Benard convection

Rayleigh-Bénard convection is a fluid flow (thermal convection) due to a non-uniform temperature distribution in a plane horizontal fluid layer where it is heated from below. Such flows result from development of convective instability, if the static vertical temperature gradient (the gradient that would be present in a motionless fluid under the same conditions) is large enough. Figure 1.7 shows straight convection roll on Rayleigh-Bénard convection. The fluid at the bottom will rise, cool and return in an overturning flow. The arrows indicate the flow pattern, with bright and dark regions corresponding to warm and cool fluid, respectively.

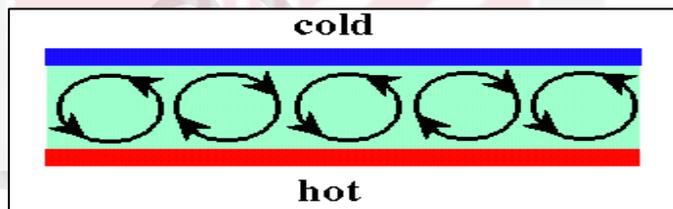


Figure 1. 7: Rayleigh-Bénard convection showing straight convection rolls.

A horizontal layer of the convection fluid is the most comprehensively studied example of nonlinear systems exhibiting self-organization (pattern-forming systems). Rayleigh-Bénard convection, which shares a number of important properties with many other pattern-formation mechanisms is considered the as a "granddaddy of canonical examples used to study pattern formation and a behaviour in spatially extended systems" (Newell et al. 1993).

Convective motion enhances dramatically the heat transfer through the layer compared to the molecular heat conduction. The moving fluid parcels, which are agents of heat exchange, normally have velocities and effective free paths much greater than the corresponding figures for molecules. Therefore, the heat flux via the layer of convection fluid may be several orders of magnitude higher than the heat flux due to molecular thermal conductivity.

## 1.5 Marangoni convection

Consider a horizontal layer bounded by free upper surface and rigid lower surface boundary. We assumed a free upper surface with the presence of surface tension force as a function of temperature. The lower rigid boundary has high temperature value compare to upper free boundary because the heat is placed at the bottom of boundary. Then, at the free surface, we assume that a warm spot (infinitesimal disturbance of the temperature) occurs to lead to a local decrease in the surface tension when temperature derivative of surface tension is negative. Presence of gradients in the surface tension cause surface traction forces move from the warmer region (lower surface tension) to cooler region (higher surface tension). The fluid beneath warm spot move consequently to upper surface, and then it will be cooled there, finally flowing down to the bottom layer. The viscosity will stabilize the fluid movement at the surface and the fluid will spread nearby.

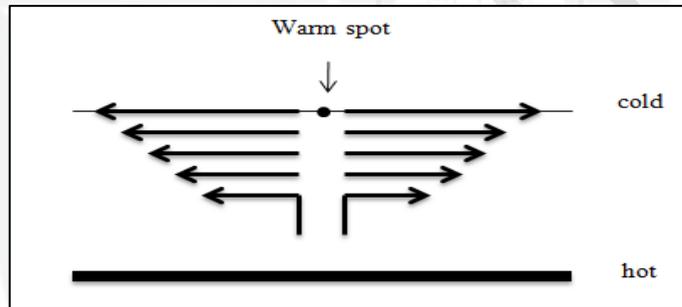


Figure 1.8: Convection mechanism caused by surface tension

Figure 1.8 above shows the convection mechanism caused by the surface tension. The movement of the fluid can be maintained if the effect of viscosity of the fluid is less than the effect of surface tension. This condition can be occurred if the layer is heated from below and the energy can be provided by a vertical temperature difference across the fluid layer. If sufficient energy is supplied, then a critical temperature gradient is reached and finally the convection will commence and will be sustained.

## 1.6 Micropolar fluid

Micropolar fluids had special characteristic microstructure. Micropolar fluid corresponds to the group of fluids with non-symmetric stress tensor that the other name with polar fluids. They also include as a special case of the well-established Navier-Stokes model of classical fluids that we can called as ordinary fluids. Basically, micropolar fluids may illustrate as fluids consist of rigid and it is randomly oriented (or spherical) particles suspended in a viscous medium, where the deformation of fluid

particles is neglected. The model of micropolar fluids introduced in Eringen (1965) is valuation of studying because it is very well balanced structure. Based on Eringen (1965), micropolar fluids is a subclass of microfluids. The theory of microfluids presented by Eringen (1964) meets with a group of fluid where it shows definite microscopic effects increasing from original structure and micro-motions of fluid elements. Micropolar fluids are also the fluids that can help stress moments and body moments that were influenced by spin inertia.

Micropolar fluids show the effects and inertia of micro-rotational. This group of fluids owned certain simplicity and sophistication in their mathematical formulation which should appeal to mathematicians. The micropolar fluids can support couple stress and body couples only. Basically they may show adequately the fluids including of dipole elements. Certain anisotropic fluids, e.g. liquid crystals which are made up of dumbbell molecules, are of this type. In fact, animal blood also include into this category.

In this research, we are interested with micropolar fluids for the reasons of stability of micropolar fluids and we want to investigate is there any parameters that can stabilize the system and delays the convection.

### **1.7 Problem Statement**

The problem of the onset of thermal instability in horizontal layer of micropolar fluids heated from below is illustrated using mathematical model based on the set of the hydrodynamic equations in Oberbeck-Boussinesq approximations. For the present study, the term stability on the system over three different problems with the effect of cubic temperature gradient is studied. Some issues about this study are:

1. What are the effects of cubic temperature profile on Rayleigh Benard convection with six different combination boundary conditions?
2. What are the effects of heat generation on the critical Rayleigh number?
3. How does the presence of internal heat generation and electric field give impact on Rayleigh Benard convection over six different boundary conditions?
4. What is the effect of heat generation and electric field on Marangoni convection?

## 1.8 Objective

The objectives of the present study is to analyse the onset of thermal convection on the horizontal micropolar fluid layer, heated from below and cooled from above with effect of cubic temperature gradient. The study is divided into three problems:

- a. The onset of Rayleigh-Benard convection, with the lower boundary layer and the upper boundary layer is assumed to be
  - i. Free isothermal and free isothermal (FIFI)
  - ii. Free isothermal and free adiabatic (FIFA)
  - iii. Free isothermal and rigid adiabatic (FIRA)
  - iv. Rigid isothermal and free isothermal (RIFI)
  - v. Rigid isothermal and free adiabatic (RIFA)
  - vi. Rigid isothermal and rigid adiabatic (RIRA)
- b. The onset of Rayleigh-Benard convection, in the presence of internal heat generation and electric field with the lower boundary layer and the upper boundary layer as FIFI, FIFA, FIRA, RIFI, RIFA and RIRA.
- c. The onset of Marangoni convection in the presence of internal heat generation, electric field and feedback control with upper boundary at which the surface tension act is free adiabatic and the lower boundary is assumed to be rigid isothermal.

## 1.9 Scope

In this thesis, the effects of various hydrodynamic boundary combination from free and rigid surface such as free isothermal and free isothermal (FIFI), free isothermal and free adiabatic (FIFA), free isothermal and rigid adiabatic (FIRA), rigid isothermal and free isothermal (RIFI), rigid isothermal and free adiabatic (RIFA), rigid isothermal and rigid adiabatic (RIRA) have been used. Effect of cubic temperature gradient upon Rayleigh Benard and Marangoni convection are considered. We apply the single Galerkin method to solve the problems.

## 1.10 Thesis Outline

In Chapter 1, we discuss on the definition of heat energy and explanation on the different between convection, radiation and conduction. Then, a brief explanation on types of convection is given and followed by five examples to demonstrate the application of convection. Then, the different mechanisms of generating convection is discussed which are induced by buoyancy (Benard convection) and driven by surface tension variation (Marangoni convection). We also explain on the characteristic of

micropolar fluid. Finally, we stated the objectives of our present works and the scope of this thesis.

Chapter 2 discussed the pioneering studies on the onset of Rayleigh-Benard, and Marangoni convection theoretically and experimentally. After that, we review the research on the onset of thermal convection focused on the non-uniform basic temperature gradient with the presence of magnetic field, internal heat generation and feedback control on horizontal fluid. Then, we analysed thermal convection on the electric field in different boundary condition. Lastly, we discussed the paper related to the internal heat generation and the presence of feedback control towards the thermal convection.

Methodology for Chapter 4, 5, and 6 are discussed in Chapter 3. We have explained in details the step that should be taken to solve all the problems in this research. We started from the governing equation and include the effect of cubic basic temperature gradient, internal heat, electric field, and feedback control.

Chapter 4 indicates the first problem that we present in this thesis is the effect of cubic temperature gradient on Rayleigh benard convection in a horizontal micropolar fluid. The problem is solved and discussion of all results and conclusion is given at the end of chapter.

In Chapter 5, we present the effect of cubic temperature gradient on Rayleigh benard electro convection in a presence of internal heat in a horizontal micropolar fluid. We analysed new governing equation that should be used in this problem for the effect of electric field and internal heat generation. We have showed how to solve the problem and the linear stability theory will be discussed in this chapter.

In Chapter 6, we focus on Marangoni convection with the boundary surface condition at the upper free adiabatic layer and lower rigid isothermal layer. We also presented the effect of internal heat generation on thermal convection with the presence of feedback control and electric field. The comparison between all parameters that have been used in this thesis will be given.

In Chapter 7, we discussed briefly what are the functions of all the parameters and the aim of this thesis. We also suggest some future works that can be extended from this research. MAPLE programs for this research are given in appendices.



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