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FORCED CONVECTION BOUNDARY LAYER SLIP FLOW OVER A PLATE IN DARCY-FORCHHEIMER POROUS MEDIUM

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By

SHAHIRAH BINTI ABU BAKAR

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirements for the Degree of Master of Science

December 2014

DEDICATIONS

To my precious Mom and Dad;

and my one and only husband.

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Master of Science

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December 2014

Chair: Norihan Md Arifin, Ph.D.

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The boundary layer flows play a central role in many aspects of fluid dynamics since they describe the motion of a viscous fluid close to the surface. The forced convection can be enhanced passively by changing flow geometry, boundary conditions or by enhancing thermal conductivity of the fluid. A mathematical modeling of forced convection boundary layer flow over a plate in Darcy-Forchheimer porous medium is performed in this thesis. The aim of this thesis is to analyze the effects of velocity and thermal slips, chemical reaction with MHD (magnetohydrodynamics) and stagnation-point flow over shrinking sheet. The governing partial differential equations are transformed into ordinary differential equations by similarity transformation, which is then solved numerically using shooting method in MAPLE 15 software. The characteristics of the flow and heat transfer features for various of parameters are discussed and analyzed in detail. We found that the heat transfer increases with increasing permeability and velocity slip parameters but it decreases with increasing thermal slip parameter whereas the skin friction decreases with increasing velocity slip. Our analysis also reveals that the increase of the velocity slip parameter reduces the momentum boundary layer thickness and also enhances the heat transfer from the plate.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk Ijazah Sarjana Sains

ALIRAN OLAKAN PAKSA PADA SLIP LAPISAN SEMPADAN YANG MELINTASI PLAT DI MEDIUM BERLIANG DARCY-FORCHHEIMER

Oleh

SHAHIRAH BINTI ABU BAKAR

Disember 2014

Pengerusi: Norihan Md Arifin, Ph.D.

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Aliran lapisan sempadan memainkan peranan yang penting di dalam bidang dinamik bendalir kerana gerakan bendalir likat yang berdekatan dengan permukaan. Olakan paksa boleh dipertingkatkan dengan mengubah aliran geometri, syarat sempadan atau meningkatkan kekonduksian terma di dalam bendalir. Sebuah model matematik bagi aliran lapisan sempadan olakan paksa yang melintasi plat di dalam medium berliang Darcy-Forchheimer telah ditunjukkan di dalam tesis ini. Tujuan utama tesis ini adalah untuk menganalisa kesan-kesan gelincir halaju dan terma, tindak balas bahan kimia dengan MHD (Magnet-Hidrodinamik) dan aliran titik genangan pada lapisan mengecut. Persamaan menakluk pembezaan separa kemudian dijelmakan kepada persamaan pembezaan biasa dengan menggunakan penjelmaan keserupaan yang kemudiannya diselesaikan secara berangka dengan kaedah tembakan dengan perisian MAPLE 15. Ciri-ciri aliran dan sifat-sifat pemindahan haba untuk pelbagai parameter telah dianalisa dan dibincangkan secara terperinci. Kami mendapati bahawa pemindahan haba meningkat dengan peningkatan parameter kebolehtelapan dan gelinciran halaju tetapi ia berkurang apabila parameter gelinciran terma meningkat manakala geseran kulit berkurang apabila gelinciran halaju meningkat. Analisis kami juga menunjukkan bahawa peningkatan parameter gelinciran halaju akan mengurangkan ketebalan lapisan sempadan dan juga meningkatkan pemindahan haba daripada plat.

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In the Name of Allah, the most Gracious and the most Merciful.

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I certify that a Thesis Examination Committee has met on **19 December 2014** to conduct the final examination of **Shahirah binti Abu Bakar** on her thesis entitled "**Forced Convection Boundary Layer Slip Flow over a Plate in Darcy-Forchheimer Porous Medium**" in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the **Master of Science**.

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LIST OF ABBREVIATIONS

BVP	Boundary Value Problem
IVP	Initial Value Problem
MHD	Magnetohydrodynamics
ODE	Ordinary Differential Equations
PDE	Partial Differential Equations
a	Straining rate parameter
B	Reaction rate parameter
B_0	Uniform transverse magnetic field
c	Stretching/shrinking parameter
C	Concentration
C_p	Specific heat at constant pressure
C_w	Plate concentration
C_∞	Concentration value in the free stream
Da_x	Darcy number
D_f	Diffusion coefficient
D_1	Thermal slip factor
f	Dimensionless stream function
k	Darcy permeability of the porous medium
k'	Forchheimer resistance factor
k_1	Porous medium parameter
k_2	Inertial parameter
L	Velocity slip factors
L_1	Velocity slip parameter
M	Magnetic parameter
Nu	Nusselt number

Pr	Prandtl number
q_m	Mass transfer
q_w	heat transfer from the sheet
R	Variable reaction rate
Re	Reynolds number
S	Suction/injection parameter
Sc	Schmidt number
Sh	Sherwood number
T	Temperature of the fluid
T_w	Temperature of the wall of the surface
T_∞	Free stream temperature
U_e	Straining velocity of the stagnation-point flow
U_w	Stretching surface velocity
U_∞	Free stream velocity
u, v	Velocity components along $x-$, $y-$ directions

Greek symbols

α	Velocity ratio parameter
β	Thermal slip parameter
δ	Velocity slip parameter
ϵ	Dimensionless stretching/shrinking parameter
η	Similarity variable
κ	Coefficient of thermal conductivity
μ	Dynamic viscosity
ν	Kinematic viscosity
ϕ	Non-dimensional concentration
ψ	Stream function
ρ	Density of fluid
σ	Constant electrical conductivity of the fluid
τ_w	Shear stress at the plate surface
θ	Non-dimensional temperature

CHAPTER 1

INTRODUCTION

1.1 Convection

Convective is the movement caused within a fluid by the tendency of hotter and therefore less dense material to rise, and colder, and denser material to sink under the gravity influence which results in heat transfer. In regular concept, convection is the concerted movement of collections of molecules within fluids, gases or rheids. Convection is usually known as the heat transfer form of dominant in liquids and gases. A good model of convection is when a Bunsen burner produces a heat source and placed it at any side of a glass full of a liquid, thus it will resulting a different levels of heat in the glass.

Two types of convections that are most commonly known are convective mass transfer and convective heat transfer. Convection of mass transfer cannot take place in solid directly since neither bulk current flows nor significant diffusion can take place in solids; however, diffusion of heat can take place in solids but is referred to separately in certain case as heat transfer. The dominant contribution for convection is due to the bulk, or gross, motion of fluid particles.

Convective heat transfer may be distinguished into two types: i) free or natural convection; and ii) forced convection. The combination of natural convection and forced convection known as mixed convection. The general concept of natural convection is a mechanism in which the fluid motion is not generated by any external sources but only by density differences in the fluid occurring due to temperature gradients. The heated fluid (at the bottom boundary) that received heat becomes less dense and rises. Forced convection occurred when a fluid is forced to flow over a surface by an external source and creating an artificially induced convection current. External sources may be classified as by using pump, suction device, fan, etc. These uses of external sources will provide high-velocity fluid (gas or liquid). The high-velocity fluid results in a decreased thermal resistance across the boundary layer from the fluid to the heated surface.

Forced convection is generally used to increase the rate of exchange heat. Many types of mixing utilized forced convection to distribute one substance within another. Forced convection also occurs as a by-product to other processes such as the action of propeller in a fluid or in aerodynamic heating. Other familiar examples of forced convection are fluid radiator systems and blood circulation in the body system which took an effect in heating and cooling of certain parts.

Forced convection also can exist in natural meaning that when the fire heat causes bulk air flow and air expansion. In microgravity, the heat rate along with diffusions is able to generate and draw in fresh oxygens to maintain them. However, some types of forced convection are more efficiency than natural convection as they are not limited by natural mechanisms. For example, a convection oven generates by forced convection will rapidly circulate forces heat of hot air into food. This process is faster than natural convection due to simple heating without the fan.

In addition, internal and external flow can also be classified as convection. Internal flow occurs when a fluid is close to a solid boundary layer such as when flowing through a pipe while external flow occurs when a fluid extends indefinitely without encountering a solid surface. Both of forced and natural convections can be external and internal because they are generally independent of each other.

1.2 Boundary Layer

The boundary layer concept was introduced by Prandtl (1904) to describe the shallow fluid domain that adjoins the solid wall bathed by the flow, as mentioned by Anderson Jr (2005). His study simplifies the fluid flow equations by dividing the flow field into two areas: one dominated by viscosity and creating the majority of drag experienced by the boundary body inside the boundary layer; and another one where viscosity can be neglected without significant effects on the solution at outside the boundary layer. In physics and fluid dynamics study, a boundary layer is the layer of fluid in the immediate vicinity of a bounding surface where the effects of viscosity are significant. The existence of the heat transfer from and to a body also takes place within the boundary layer, again allowing the equations to be simplified in the flow field outside the boundary layer.

The analysis of convection heat transfer consists of recognizing the boundary layer regime which is identified to three types: i) laminar; ii) transition; or iii) turbulent. Generally starting as a laminar flow, the boundary layer will thick, and undergoes transition to turbulent and then continues to develop along the body surface.

The velocity of the boundary layer thickness can be explained as the distance from the solid body at which the viscous flow velocity is closed to the inviscid flow of the surface velocity. In addition, displacement thickness is an alternative definition stating that the boundary layer occurs a deficit in mass flow compared to inviscid flow with slip at the wall. It is the distance and gap where the wall should have to be displaced in the case of inviscid to give the same total mass flow as the viscous case. However, boundary condition without slip requires the velocity flow at the solid object surface to be total zero and the fluid temperature can be equalize to the boundary surface temperature. The flow velocity will then enhance rapidly towards the boundary layer.

The thickness of thermal boundary layer can be identified as the distance and gap from the body where the temperature is near to the temperature found from an inviscid solution. For a simple explanation, the ratio of the two thicknesses is governed by the Prandtl number, Pr, which is explained in detail in Chapter 3. If the Prandtl number is 1, thus the two boundary layers are having the same thickness. If the Prandtl number is less than 1, which is the case for air at standard conditions, the thermal boundary layer is thicker than the velocity boundary layer and if the Prandtl number is greater than 1, thus the thermal boundary layer is thinner than the velocity boundary layer.

1.3 Darcy-Forchheimer Porous Media

A porous medium, or also known as porous material, is a metal containing pores or voids, and basically the pores are filled with fluid. The skeletal material is usually a solid, but structures like foams are often also usefully analyzed using concept of porous media. Extremely small voids are called molecular interstices and large voids are called vugs or caverns. Pores, e.g. intergranular and intercrystalline, are intermediate sizes between caverns and molecular interstices. Nowadays, fluid flow in porous media is of considerable interest in many areas such as petroleum, environmental and groundwater hydrology, reservoir engineering in connection with thermal recovery process, etc. Accurate description of fluid flow behavior in the porous media is essential to the successful operation and design of projects in these areas.

Generally, Darcy's law is used to depicts the fluid flow behavior in a porous media due to pressure gradients as mentioned by Rajagopal (2007). According to Darcy's law, the pressure gradient is linearly proportional to the fluid velocity in a porous media as stated by Zeng and Grigg (2006). In most porous medium study, the pores are typically very small and the changes of velocity across the pores throat are mostly negligible, thus the term non-Darcy is used in place of inertial effects parameter.

Inherent in the development of the Darcy flow model can be made as following assumptions:

1. Darcy's law assumes laminar or viscous flow, where it does not involve in inertia term. This implies that the acceleration forces or inertia in the fluid are being neglected when compared to the Navier-Stokes equations.

2. Darcy's law assumes a large surface area is exposed to fluid flow in porous medium, hence the viscous resistance will greatly exceed acceleration forces in the fluid, unless if turbulence is set in.

The Darcy equation is an empirical relationship based on experimental observations of one-dimensional water flow through packed sands at low velocity. By this, the limitation of Darcy's law have longer been recognized and an accepted approach has been to use Forchheimer's equation, and it's inertial flow parameter had been used as an extension of Darcy's law beyond the region of linear flow. It is well known that when the velocity increases, the flow enters a non-linear laminar regime and the porous inertial effects are no longer negligible as explained by Shenoy (1993). Due to many researchers attempt have been made to correct the Darcy equation, Forchheimer (1901) introduced a square velocity term in addition to the Darcian velocity term for this effect which then Muskat (1946) called the theory as Forchheimer term. Forchheimer term can be explained as inertial term that added to Darcy's equation. This term is able to account for the pressure difference non-linear behavior and velocity data. Hence, in this preceding study, the focus is on the important and relevant Darcy-Forchheimer flow model.

1.4 Permeable Surface (Suction and Injection)

Suction or injection (blowing) of a fluid through the bounding surfaces can significantly change the flow field and hence, affects the rate of heat transfer from the surface. For a better concept, suction tends to enhance the heat transfer and skin friction coefficients while injection acts to decrease both coefficients (Al-Sanea, 2004). The flows with the presence of suction and injection have some interesting features; for example, when suction is large, the breaking of the secondary flow into loops is obliterated. While when suction is small, every streamline will start at the inner sphere and ends up at the outer sphere.

Suction also can cause a fluid or solid to be drawn into an interior space or to adhere a surface because of the difference between the internal and external pressures. A force acting on a fluid caused by the difference in the pressure of internal and external regions and tends to make the fluid flow from the higher pressure region to lower pressure region. The pressure gradient between this region and the ambient pressure will propel towards the lower pressure area, and produce a partial vacuum by the removal of air in order to force fluid into a vacant space of procure adhesion. One application of suction in technology nowadays is the usage of suction in pump or fan where the act of reducing pressure is to create such a force.

Injection can be explained when the pressure energy of a fluid is converted to velocity energy which creates a low pressure zone that draws in and entrains a suction fluid. When an injection passed by the injector, the mixed fluid expands and reduced the velocity which then resulting in recompressing the mixed fluids by converting back the velocity energy to pressure energy. Instead, fluid under higher pressure is converted into a high velocity gradient which then creates a low pressure at that point. In other words, the pressure energy is converted to kinetic energy in the form of velocity head which then the kinetic energy is converted back to pressure energy at the diffuser outlet when the mixed fluid expands in the divergent diffuser. The use of injections in various industrial applications has become quite importance due to the their relative simplicity and adaptability such as in thermal power stations, steam jet cooling systems, enhanced oil recovery, boiling water nuclear reactors and water eductors.

1.5 Slip Surface

Generally, the no-slip boundary condition, or can be assumed as the liquid adheres to a solid boundary, is one of the central tenets in most study. However, there are situations where the condition does not hold perfectly. In this case, partial velocity slip may occur on the stretching boundary when the fluid is particulate. The first experiment about slip conditions was analyzed by Beavers and Joseph (1967), in connection with the investigations of viscous flow past permeable surfaces and that is when viscous fluid flows past the permeable surface body and the viscous shear stress effects can penetrate into porous media to form a boundary layer region adjacent to the interface. However, the problem arising when Darcy's law is incompatible with the existence of such boundary layer region and yet no shear stress tensor associated with it due to the nature of Darcy's equation. To incorporate this, they considered slip boundary condition for plane boundaries since the usage of no-slip condition at the permeable surface was not satisfactory.

Velocity slip is commonly known as the non-adherence of the fluid to a solid boundary. This phenomenon has been observed under certain circumstances because of its importance in radial impellers and is useful in determining the accurate estimation of work input or energy transfer between the impeller and the fluid. The advantage of slip factor is it can accommodate for a slip loss which affects the developed net power and increases the flow-rate. Instead, slip velocity can be described as the relative motion between large particles and a turbulent flow. This definition is essential because the slip velocity used in the standard drag model fails when particle size falls within the ambient turbulence inertial subrange.

1.6 Application in Industry

Major developments have been made in modeling transport phenomena in porous media, including several important conceptual breakthroughs in the past couple decades. The physics of fluid flow in different media and conduits is a well researched area in engineering field. Equations describing flows in media such as cylindrical pipes, rectangular conduits, chemical industry and other forms and shapes of conduits have been developed analytically over the years.

Instead, the boundary layer flow and heat transfer phenomenon in porous media is of considerable interest due to its ever increasing industrial applications and a myriad of technological processes since they describe the motion of a viscous fluid close to the surface. Processes involving heat and mass transfer in porous media are frequently encountered in geophysics, petroleum engineering and chemical industry.

A better understanding of forced convection through porous medium with the presence of slip condition can benefit several areas such as insulation design, heat exchangers, aerodynamic extrusion, filtering devices, underground nuclear waste storage sites, metal processing and others. In a general concept, forced convection is typically used to increase the exchange heat rate. Many types of mixing utilized forced convection to distribute one substance within another. Forced convection also occurs as a by-product to other processes such as the action of propeller in a fluid or in aerodynamics heating. Other familiar examples of forced convection are fluid radiator systems or blood circulation in the body system which took an effect in heating and cooling of certain parts.

The importance of Darcy's law through a porous media is one of considerable interest in variety of different industrial applications, such as packed-bed chemical reactors, petroleum reservoirs, geothermal operations and building thermal insulation. Recently, the boundary layer flow due to a shrinking sheet has attracted considerable interest where there are plenty of applications in shrinking sheet problems in industries and engineering fields. The shrinking sheet occurs, for example, on a rising shrinking balloon. At the same time, stagnation-point flow is one of importance topic in fluid mechanics since stagnation-points appear virtually in all flow fields of science and engineering.

1.7 Objective

The objectives of the present study are to construct and analyze mathematical model for the following three problems:

1. Forced convection boundary layer slip flow over a permeable porous plate in Darcy-Forchheimer porous medium.

2. MHD forced convection boundary layer slip flow in Darcy-Forchheimer porous medium with chemical reaction.

3. Forced convection boundary layer stagnation-point slip flow in Darcy-Forchheimer porous medium towards a shrinking sheet.

1.8 Scope of Study

The aim of this study was to determine and investigate the problem of forced convection boundary layer slip flow over a plate in Darcy-Forchheimer porous medium. This study was extended to three different types of problems:

i. permeable porous plate;

ii. chemical reaction with MHD;

iii. stagnation-point flow over shrinking sheet.

The mathematical model was extended from Bhattacharrya et al. (2011b) which is it was the best choice and one that fits the onset of this problem. The sources of this study are from well-known journals and published papers.

This study identified a number of key findings including the effects of velocity and thermal slip on the Darcy-Forchheimer model, and the effects of existing parameters on the model. Before continuing on examining the mathematical model, a comparison has to be made with the other authors to identify the accuracy of the results. Each of these problems has undergone the comparisons and the results should be in excellent agreement. This study also highlighted the effect of several parameters on skin friction coefficient, thermal gradient, concentration gradient, velocity profile, temperature profile and concentration profile.

The scope of our study is for the research purposes. In addition, mathematical model and analysis of Darcy-Forchheimer porous media flow model is becoming a key to solving many challenging problems in engineering and applied sciences, and instead, the knowledge and results gained from this thesis will be useful in adequately evaluating production performance in several areas.

1.9 Thesis Outline

This thesis is divided into seven chapters where Chapter 1 is the preliminary chapter consisting of general introduction of convection, mechanism of convection heat transfer, introduction of Darcy-Forchheimer porous medium, boundary layer with partial slip, objective of the study, scope of study and thesis outline.

Chapter 2 reviews the pioneering studies on the onset of boundary layer flow in Darcy porous medium by experimentally and numerically. We also highlighted the investigators who studied the onset of convection in Darcy-Forchheimer porous medium and the boundary layer with the presence of slip flow in Darcy-Forchheimer porous media.

Chapter 3 reviews the forced convection boundary layer slip flow over a permeable porous plate in Darcy-Forchheimer porous medium is investigated. In this chapter, the figure for this problem is included and we will derived the problem equations and the last equation obtained will be the subject to continue for evaluation in Chapter 4 and Chapter 5. Several investigations about this problem are also referred and discussed in this chapter. This problem analyzed the effect of suction and injection in both velocity and temperature profiles. The analytical solutions are discussed. To verify the accuracy of this analysis, the results are compared with the previous studies.

Later, MHD forced convection boundary layer slip flow in Darcy-Forchheimer porous medium with chemical reaction effects is discussed in Chapter 4. The review of this study is listed in this chapter. This problem discussed the dual effects of chemical reaction and magnetic field in porous medium.

In Chapter 5, we discussed forced convection boundary layer stagnation-point slip flow in Darcy-Forchheimer porous medium towards a shrinking sheet. We included several researchers and authors who had studied within this problem in this chapter. This problem discussed the effects of velocity ratio parameters and slip parameters in both velocity and temperature profiles. We compared our present work with previous studies to verify the accuracy of our analysis.

The final chapter, namely Chapter 6, contains the summary of present research and also some possible further research.

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