UNIVERSITI PUTRA MALAYSIA

CONVERGENCE AND ERROR STUDY OF DIFFERENT BASIS AND TESTING FUNCTIONS IN THE METHOD OF MOMENTS APPLIED TO ELECTROMAGNETIC WAVE SCATTERING FROM DIELECTRIC OBJECTS

NG TZE WEI

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By

NG TZE WEI

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirements for the Degree of Master of Science

JULY 2014

DEDICATION

To my parents and teachers,

Thanks for everything

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Master of Science

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Chairman: Associate Professor Zulkifly Abbas, PhD
Institute: Mathematical Research
The convergence and variation of error of numerical methods depends on the implementation of different types of basis and testing functions. This thesis describes a comparative analysis of different basis and testing functions used in the MoM for two dimensional dielectric objects. The basis and testing functions namely the sinusoid/pulse (SP), sinusoid/sinusoid (SS), sinusoid/triangle (ST), triangle/pulse (TP), triangle/sinusoid (TS) and triangle/triangle (TT) methods are considered in this work. These basis and testing functions used in conjunction with MoM integral equations which include the electric field integral equation (EFIE), magnetic field integral equation (MFIE), Poggio-Muller-Chu-Harrington-Wu (PMCHW) integral equation and the Muller integral equation. All the computations in this study are carried out using MATLAB on dielectric objects using personal computer with 2GB DDR3 RAM. The variation of mean relative error with samples per wavelength is calculated for different dielectric objects with outer and inner radii of 0.0521 m and 0.0313 m respectively. Using Gauss quadrature technique, the SP and TP methods give faster convergence than the SS, ST, TS and TT methods for a higher number of integral equations at 915 MHz. When the EFIE and MFIE are used in both TE and TM cases of the hollow dielectric cylinder with relative permittivity of 77.3-j37.2, the SS,ST,TS and TT methods require at least 1.5 and 1.75 times the samples per wavelength required by the SP and TP methods to achieve magnetic current error less than 0.01 respectively. For the dielectric coated conducting cylinder with relative permittivity of 33.2-j124.17, the SS, ST, TS and TT methods require 2 times the samples per wavelength required by the SP and TP methods for the surface magnetic current calculated using Gauss quadrature technique to be more accurate than the staircase approximation technique. The difference in the convergence due to different basis and testing function under the impedance boundary condition (IBC) is not as significant as under the exact boundary condition (EBC) for the dielectric coated impedance cylinder. The difference in the number of matrix elements between the SS, ST and SP methods and also between the TS, TT and TP methods to achieve magnetic current error less than 0.01 for the Muller integral equation is higher than the EFIE and MFIE when the EBC is utilised. The SP and TP methods provide faster convergence than the SS, ST, TS and TT methods with a higher

difference in the number of matrix elements between the SS, ST and SP methods and also between the TS, TT and TP methods to achieve an error less than 0.01 for the high permittivity hollow dielectric cylinder with large size compared to the one with small size.

Abstraktesis yang dikemukakan kepada SenatUniversiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Master Sains

KAJIAN RALAT AND PENUMPUAN FUNGSI ASAS DAN PEMBERAT BERBEZA DALAM APPLIKASI KAEDAH MOMEN UNTUK PENYERAKAN GELOMBANG ELEKTROMAGNET OLEH OBJEK DIELEKTRIK

Oleh

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Penumpuan dan perubahan ralat kaedah berangka bergantung kepada implementasi fungsi asas dan pemberat yang berbeza. Tesis ini menerangkan analisis perbandingan fungsi asas dan pemberat berbeza dalam kaedah momen (MoM) untuk objek dua dimensi. Fungsi asas dan pemberat iaitu kaedah sinusoid/segiempat (SP), sinusoid/sinusoid (SS), sinusoid/segitiga (ST), segitiga/segiempat (TP), segitiga/sinusoid (TS) dan segitiga/segitiga (TT) telah dipertimbangkan. Fungsi asas dan pemberat ini digunakan sempena persamaan kamiran MoM iaitu persamaan kamiran medan elektrik (EFIE), persamaan kamiran medan magnet (MFIE), persamaan kamiran Poggio-Muller-Chu-Harrington-Wu (PMCHW) dan persamaan kamiran Muller. Komputasi dalam kajian ini telah dilaksanakan menggunakan MATLAB pada objek dielektrik menggunakan komputer peribadi 2GB DDR3 RAM. Perubahan purata ralat relatif terhadap sampel per panjang gelombang dihitung untuk objek dielektrik berbeza dengan jejari luaran and dalaman 0.0521 m dan 0.0313 m masing-masing. Dengan menggunakan kaedah Gauss kuadratur, kaedah SP dan TP memberikan penumpuan yang lebih cepat daripada kaedah SS, ST, TS dan TT untuk bilangan persamaan kamiran yang lebih tinggi pada 915 MHz. Apabila EFIE dan MFIE digunakan untuk silinder dielectric berongga dengan kebolehtelapan relatif 77.3-j37.2 dalam kes TE dan TM, kaedah SS, ST, TS dan TT memerlukan sekurang-kurangnya 1.5 dan 1.75 kali sample per panjang gelombang yang diperlukan oleh kaedah SP dan TP untuk mencapai ralat arus magnetik kurang daripada 0.01 masing-masing. Untuk silinder konduktor bersalut dielektrik dengan kebolehtelapan relatif 33.2-j124.17, kaedah SS, ST, TS and TT memerlukan 2 kali sampel per panjang gelombang yang diperlukan oleh kaedah SP and TP supaya arus magnetik yang dihitung menggunakan kaedah Gauss kuadratur lebih tepat berbanding kaedah anggaran tetangga. Perbezaan kadar penumpuan antara fungsi asas dan pemberat berbeza di bawah syarat sempadan impedans (IBC) adalah tidak seketara syarat sempadan tepat (EBC) untuk silinder impedans bersalut dielektrik. Perbezaan bilangan elemen matriks antara kaedah SS, ST dan SP dan juga antara kaedah TS, TT dan TP dalam mencapai ralat arus magnetik kurang daripada 0.01 untuk persamaan Muller adalah lebih tinggi daripada EFIE dan MFIE apabila EBC

digunakan. Kaedah SP dan TP juga memberikan penumpuan lebih cepat berbanding kaedah SS, ST, TS dan TT untuk bilangan persamaan kamiran lebih tinggi dengan perbezaan bilangan elemen matriks yang lebih tinggi didapati antara kaedah SS, ST dan SP dan juga antara kaedah TS, TT dan TP untuk mencapai ralat kurang daripada 0.01 apabila saiz silinder berongga dengan kebolehtelapan relatif yang tinggi adalah besar berbanding saiz silinder berongga yang kecil.

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I certify that an Examination Committee has met on _____ to conduct the final examination of Ng Tze Wei on his thesis entitled “Convergence And Error Study Of Different Basis And Testing Functions In Method Of Moments Applied To Electromagnetic Wave Scattering From Dielectric Objects” in accordance with Universities and University Collage Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A)106] 15 March 1998. The Committee recommends that the candidate be awarded the Master of Science.

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LIST OF ABBREVIATIONS

IEEE	-	Institute of Electrical and Electronics Engineers
MHz	-	Mega Hertz
FEM	-	Finite Element Method
MoM	-	Method of Moments
FDTD	-	Finite difference time domain
TM	-	Transverse Magnetic Mode
TE	-	Transverse Electric Mode
EFIE	-	Electric Field Integral Equation
MFIE	-	Magnetic Field Integral Equation
PMCHW	-	Poggio-Muller-Chu-Harrington-Wu
MATLAB	-	Matrix Laboratory
GB	-	Gigabyte
SP	-	Sinusoid Basis Pulse Testing
SS	-	Sinusoid Basis Sinusoid Testing
ST	-	Sinusoid Basis Triangle Testing
TP	-	Triangle Basis Pulse Testing
TS	-	Triangle Basis Sinusoid Testing
TT	-	Triangle Basis Triangle Testing
EBC	-	Exact boundary condition
IBC	-	Impedance boundary condition
PC	-	Personal computer
RCS	-	Radar cross section

LIST OF SYMBOLS

∇^{tan}	-	Tangential component of gradient
$X^{e}_{i'j'}$	-	Coefficient of the edge connected by node i' and j' of the *e*-th element
$\vec{N}^{e}_{ij}$	-	Vector testing function along node *i* to node *j* of the *e*-th element
$\vec{N}^{e}_{i'j'}$	-	Vector basis function along node i' to node j' of the *e*-th element
l_{ij}	-	Length of element from node *i* to node *j*
L^{e}_{j}	-	Linear pyramid function of *j*-th node of element *e*
E_x	-	x-directed electric field
E_y	-	y-directed electric field
E_z	-	Axially or z-directed electric field
H_x	-	x-directed magnetic field
H_y	-	y-directed magnetic field
H_z	-	Axially or z-directed magnetic field
$\sigma^{(m)}$	-	Magnetic conductivity (Ω/m)
Δt	-	Time step
G	-	Green function
G_m	-	Green function of region *m*
$\vec{r}$	-	3 dimensional observation vector
$\vec{r}'$	-	3 dimensional source vector
$\vec{\rho}$	-	2 dimensional observation vector
$\vec{\rho}'$	-	2 dimensional source vector
ε_r	-	Relative dielectric constant
ε_m	-	Permittivity of region *m*

μ_0	-	Permeability of free space
μ_m	-	Permeability of region *m*
ε'	-	Dielectric Constant
ε''	-	Dielectric Loss Factor
φ	-	Angle with respect to x-axis in the xy plane
μ_r	-	Relative permeability
J_n, Y_n	-	Bessel function of first and second kind
J_n', Y_n'	-	derivative of Bessel function of first and second kind
$H_0^{(2)}$	-	Hankel function of second kind
$\vec{E}$	-	Total electric field intensity (V/m)
$\vec{H}$	-	Total magnetic field intensity (A/m)
$\vec{E}^{inc}$	-	Incident electric field (V/m)
$\vec{H}^{inc}$	-	Incident magnetic field (A/m)
$\vec{E}_m^s$	-	Scattered electric field in region *m* (V/m)
$\vec{H}_m^s$	-	Scattered magnetic field in region *m* (A/m)
S	-	Object contour
$\hat{t}$	-	Unit tangent vector
$\hat{n}$	-	Outward unit normal vector
$\vec{J}_{de}$	-	Outer layer electric current density of dielectric coated cylinder (A/m) on S_{de}
$\vec{M}_{de}$	-	Outer layer magnetic current density of dielectric coated cylinder (V/m) on S_{de}
$\vec{J}_{cd}$	-	Inner layer electric current density of dielectric coated cylinder (A/m) on S_{cd}
$\vec{J}_{de1}$	-	Outer layer electric current density of hollow dielectric cylinder (A/m) on S_{de1}
$\vec{M}_{de1}$	-	Outer layer magnetic current density of hollow dielectric cylinder (V/m) on S_{de1}

$\vec{J}_{de2}$	-	Inner layer electric current density of hollow dielectric cylinder (A/m) on S_{de2}
$\vec{M}_{de2}$	-	Inner layer electric current density of hollow dielectric cylinder (V/m) on S_{de2}
η_0	-	Free space intrinsic impedance
η	-	Surface impedance
η_1	-	Medium intrinsic impedance
β	-	Inverse of surface impedance
J_z	-	Axially directed surface electric current density (A/m)
J_t	-	Transverse directed surface electric current density (A/m)
M_z	-	Axially directed surface magnetic current density (V/m)
M_t	-	Transverse directed surface magnetic current density (V/m)
K_z	-	Axially directed surface current coefficients
K_t	-	Transverse directed surface current coefficients
$[Z]$	-	Impedance matrix
$[V]$	-	Excitation matrix
$[I]$	-	Current coefficient matrix
$Z_{m,n}$	-	Impedance matrix elements
$V_{m,n}$	-	Excitation matrix elements
σ_{TM}	-	Radar cross section for TM case (m)
σ_{TE}	-	Radar radar cross section for TE case (m)
$E_{z,m}$	-	Axially directed electric field in the *m*-th region
$H_{\varphi,m}$	-	Transverse directed magnetic field in the *m*-th region
K_n	-	Normalisation constant
C_n	-	Hankel function coefficient

$A_{m,n}, B_{m,n}$	-	Field coefficients
k_m	-	Wave number in the *m*-th region
k_0	-	Free space wave number
r_m	-	Radius of m-th layer
j	-	Square root of (-1)
h	-	Segment length
$\vec{T}$	-	Weighing function of tangential vector finite elements
d	-	Coating thickness (m)
σ	-	Ionic Conductivity
ω	-	Angular Frequency
v	-	Velocity (m/s)
&	-	And
λ_0	-	Free space wavelength
h	-	Width (m)
L	-	Integral operator
$\vec{K}$	-	Unknown function
$\vec{g}$	-	Known excitation
n_λ	-	Samples per wavelength
r	-	Radius
f	-	Frequency
V	-	Excitation
E_t	-	Transverse directed electric field
H_t	-	Transverse directed magnetic field
%	-	Percentage
N	-	Number of linear segmentations

$J_{exact,n}$	-	n-th point electric current density using exact eigenfunction (A/m)
$M_{exact,n}$	-	n-th point magnetic current density using exact eigenfunction (V/m)
$RCS_{exct,n}$	-	n-th point radar cross section using exact eigenfunction (m)
$J_{Gauss,n}$	-	n-th point electric current density using Gauss quadrature (A/m)
$M_{Gauss,n}$	-	n-th point magnetic current density using Gauss quadrature (V/m)
$RCS_{Gauss,n}$	-	n-th point radar cross section using Gauss quadrature (m)
$J_{stair,n}$	-	n-th point electric current density using staircase approximation (A/m)
$M_{stair,n}$	-	n-th point magnetic current density using staircase approximation (V/m)
$RCS_{stair,n}$	-	n-th point radar cross section using staircase approximation (m)
ΔJ_{Gauss}	-	Electric current density mean relative error using Gauss quadrature
ΔM_{Gauss}	-	Magnetic current density mean relative error using Gauss quadrature
ΔRCS_{Gauss}	-	Radar cross section mean relative error using Gauss quadrature
ΔJ_{stair}	-	Electric current density mean relative error using staircase approximation
ΔM_{stair}	-	Magnetic current density mean relative error using staircase approximation
ΔRCS_{stair}	-	Radar cross section mean relative error using staircase approximation
ΔJ_{diff}	-	Difference of electric current density mean relative error between Gauss quadrature and staircase approximation
ΔM_{diff}	-	Difference of magnetic current density mean relative error between Gauss quadrature and staircase approximation
ΔRCS_{dff}	-	Difference of radar cross section mean relative error between Gauss quadrature and staircase approximation

CHAPTER 1

INTRODUCTION

1.1 Background of the study

Electromagnetic problems can be solved using analytical and numerical methods. Among the analytical techniques available are separation of variables, series expansion and conformal mapping which allows solution of simple geometry to be constructed however, not many practical problems could be solved using analytical techniques due to the complex structure which defines the problem (Sadiku, 2000; Ishimaru, 1991). Numerical methods such as the method of moments (MoM), finite difference time domain (FDTD), finite element method (FEM) allows solution of complex electromagnetic problem where analytical solution does not exist, however numerical methods are not as accurate as the analytical techniques (Sadiku, 2000).

Numerical results computed are verified using exact solutions and solutions obtained from other techniques such as physical optics (PO) and geometrical theory of diffraction (GTD) providing the user confidence regarding the accuracy of the numerical solution (Davis and Warnick, 2005). Numerical methods are required to solve electromagnetic problems that do not have exact solutions (Sadiku, 2000). Hence, the efficiency of numerical EM solutions has to be investigated. In this thesis, a particular method known as the method of moments (MoM) is utilised to solve electromagnetic scattering problems.

The purpose of this work is to perform investigation on the convergence and variation of error using the method of moments due to different basis and testing functions on dielectric objects. The basis function is the function that represents the source current in the MoM whilst the testing function is the function used to match the observation points on the scatterer using an integral equation (Balanis, 1989). The analysis is valuable because it provides a quantitative understanding of how different basis and testing functions can affect the convergence and variation of error of the numerical solution. Therefore, with the appropriate selection of basis and testing functions, the uncertainties in the numerical solution can be minimised, and therefore this allows faster convergence of numerical solutions using a smaller impedance matrix size.

1.2 Dielectric properties of materials

To understand the physical processes associated with various radio frequency and microwave devices, it is necessary to know the dielectric properties of media that interact with EM waves (Ishimaru, 1991). The dielectric properties are intrinsic properties that are expressed in a complex form as the relative complex permittivity $\varepsilon_r = \varepsilon_r' - j\varepsilon_r''$ (Balanis, 1989). The real part (the dielectric constant) is associated with the capability of a material to store energy whereas the imaginary part (the loss factor) is associated with loss of electric field energy in the material, which is usually dissipated as heat (Nyfors and Vainikainen, 1989). One of the most important parameters that is used to describe dielectrics is the ratio ε_r' to ε_r'', the loss tangent, $\tan\left(\varepsilon_r''/\varepsilon_r'\right)$.

Lossy materials have high loss factor or loss tangent values where high loss factor is due to material absorption (Balanis, 1989). Good examples of lossy material are ferrites due to its ability to absorb microwave signals and microwave signals are strongly absorbed by the water molecules (Nyfors and Vainikainen, 1989). The dielectric properties in liquid are described by the single-relaxation Debye model and are useful for non-destructive determination of important characteristics such as scattering cross section and variations of complex dielectric permittivity over a wide frequency range are important for communication and radar devices (Nyfors and Vainikainen, 1989).

1.3 Computational Electromagnetics

Computational electromagnetics deals with the art and science of solving Maxwell's equations using electronic computers and numerical methods (Umashankar and Taflove, 1991). The MoM technique is based on the systematic functional space description and the FDTD solves Maxwell's time dependent curl equations whilst the FEM approximates the solution of partial differential equations by converting them into a set of linear equations (Umashankar and Taflove, 1991). Geometrical optics is based on electromagnetic waves that propagate as optical rays (Jin, 2010). In the geometrical theory of diffraction, the solution obtained is added to the geometrical optics solution. The geometrical optic solution is found through an approximate asymptotic solution for the diffracted field using a straight edge at normal incidence that is then extended to oblique incidences and curved edges (Jin, 2010).

Using the uniform theory of diffraction (UTD), in the geometrical theory of diffraction, the discontinuous field is compensated by a transition function that is obtained using a more accurate evaluation of the diffracted field (Jin, 2010). In the physical optics, induced current is approximated to be equal to twice the magnetic field on the illuminated side of the surface and is approximated to be equal to zero on the dark side of the surface that is improved by the physical theory of diffraction by taking the effect of edges into account (Jin, 2010).

Maxwell's theory can predict the experimental outcomes if Maxwell's equations are solved correctly (Jin, 2010). Therefore, there is always a quest to solve Maxwell's equations accurately using numerical methods for increasingly complex problems. Early analyses were carried out for simple shapes such as spheres and cylinders after the establishment of Maxwell's theory (Umashankar and Taflove, 1991). Solutions to more complex geometries were needed and as a result, approximate techniques were developed to solve Maxwell's equations (Jin, 2010).

1.4 Method of moments as applied to electromagnetic problems

One of the numerical techniques to solve electromagnetic scattering problem, namely the method of moments (MoM) is reviewed in this section. The MoM is used for the analysis of electromagnetic radiation and scattering in antenna design, remote sensing, and other applications (Davis and Warnick, 2005). With increasing complexity of electromagnetic problems, computer simulation is needed to predict the behaviour of such systems by using sophisticated techniques developed to solve electromagnetic problems (Jin, 2010).

With the development of computers and programming languages, electromagnetic problems that cannot be solved analytically has received attention as solutions to more complex geometries were needed (Gibson, 2007; Davidson, 2005; Sadiku, 2000). The equations must be discretised using an appropriate numerical technique so that the equations can be programmed on a computer and in the MoM, the integral equations are linearised by converting the integral operators into matrix equation (Harrington, 1968) where the equations then solved for the basis function coefficients. These are then used to calculate important parameters such as the radar cross section (Peterson *et al.*, 1998).

1.5 Problem statement

The triangle basis pulse testing (TP), triangle basis triangle testing (TT), sinusoid basis pulse testing (SP), and sinusoid basis sinusoid testing (SS) have been used for scattering from conducting cylinder and wire antenna analysis (Davis and Warnick, 2004; Peterson *et al.*, 1998; Klein and Mittra, 1975). Commonly used basis and testing functions in two dimensional scattering from conducting cylinder are the triangle basis pulse testing and triangle basis triangle testing (Davis and Warnick, 2004; Peterson *et al.*, 1998). The sinusoid basis pulse testing and sinusoid basis sinusoid testing functions are typically used in wire antenna analysis (Peterson *et al.*, 1998; Klein and Mittra, 1975). However, to-date no intercomparison work has been done to test the actual performance of all the different basis and testing functions for dielectric object. Implementation of MoM demands detailed study on the variation of error, convergence and matrix size requirements when different basis and testing functions are utilised on dielectric object.

Selection of appropriate basis and testing functions may result in faster convergence by using a smaller impedance matrix size (Alad and Chakrabarty, 2012). Therefore, a comparative study on the effect of different basis and testing functions towards the variation of error with samples per wavelength for dielectric scatterers is worthwhile because the basis and testing functions that give faster convergence can be selected to save memory requirements and computation time. In addition, different computing techniques can give different convergence rate and memory usage even though the numerical technique used is the same (Jin, 2010). This may depend on the basis and testing functions utilised where the effect of different numerical implementations has yet to be investigated.

The efficiency in minimizing the error of the numerical solution affects the variation of error with samples per wavelength. Since different computing techniques can give different efficiencies in the minimizing the error even though the same basis and testing functions are utilised, the effect of different basis and testing functions on the difference in the variation of error between different computing techniques has yet to be examined. The effect of different boundary conditions may affect the convergence of numerical solutions and this has yet to be tested on different basis and testing functions. For high permittivity dielectric object, the total number of surface electric and magnetic current densities is high which will result in the use of large MoM impedance matrix size. The effect of object size on the convergence due to different basis and testing functions has yet to be investigated for high permittivity purely dielectric object.

1.6 Research Objectives

The comparative study of different basis and testing functions consists of several objectives. The main objectives are to evaluate the effect of different numerical implementations, different boundary conditions and size effect on the convergence and variation of error of the surface current densities and the radar cross section when different basis and testing functions are used on different integral equations in the transverse electric (TE) and transverse magnetic (TM) scattering from dielectric objects. The objectives of this work are as follows

1. To determine the effect of high permittivity dielectric objects on the convergence due to different basis and testing functions under different numerical implementations.
2. To examine the effect of different basis and testing functions on the difference in the variation of error between different computing techniques.
3. To evaluate the effect of different surface boundary conditions on the convergence due to different basis and testing functions for impedance coated objects.
4. To analyze the effect of object size on the convergence due to different basis and testing functions for high permittivity purely dielectric objects.

1.7 Thesis outline

This thesis consists of five main components that are described separately in the following five chapters: Chapter 2 critically reviews and discusses some previous research and summarises the application of MoM in electromagnetic scattering problems. Chapter 3 discusses the theoretical computation of surface current densities and radar cross sections of different dielectric objects using different integral equations; namely the electric field integral equation (EFIE), magnetic field integral equation (MFIE), Poggio-Muller-Chu-Harrington-Wu (PMCHW) and Muller integral equation. Chapter 4 describes the evaluation of generic integrals using different basis and testing functions using different numerical implementations and boundary conditions as well as the construction of program structure using MATLAB for the MoM and exact eigenfunction. Chapter 5 focuses on results and discussions of the objectives outlined in Section 1.6. Chapter 6 summarizes the contributions and the need for further studies. Appendices contain the details of the computer program for the understanding of Chapters 3 and 4.

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