UNIVERSITI PUTRA MALAYSIA

ONE-STEP BLOCK METHODS FOR DIRECT SOLVING OF LINEAR BOUNDARY VALUE DIRICHLET AND NEUMANN TYPE PROBLEMS

MOHD MUGHTI BIN HASNI

IPM 2014 12

ONE-STEP BLOCK METHODS FOR DIRECT SOLVING OF LINEAR BOUNDARY VALUE DIRICHLET AND NEUMANN TYPE PROBLEMS

By

MOHD MUGHTI BIN HASNI

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirements for the Degree of Master of Science

February 201

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Master of Science

ONE-STEP BLOCK METHODS FOR DIRECT SOLVING OF LINEAR BOUNDARY VALUE DIRICHLET AND NEUMANN TYPE PROBLEMS

By

MOHD MUGHTI BIN HASNI

February 2014

Chairman: Associate Professor Zanariah Abdul Majid, PhD
Faculty: Institute for Mathematical Research

In this research, the block methods have been used to solve the second order linear boundary value problems of Dirichlet and Neumann type. Mathematical problems which involve higher order ordinary differential equations were likely to be reduced into the system of first order equations. However, these block methods will solve the problems directly without reducing it into the first order equations using constant step size.

There are three methods that have been used in this research which are two point one-step block method, three point one-step block method and four point one-step block method. Each of these methods will be implemented to solve the second order linear boundary value problems with two different types of boundary conditions i.e. Dirichlet and Neumann type.

Those three methods will be implemented together with the linear shooting technique to construct the numerical solution. The stability for each method will be presented. Numerical results of the methods are compared with the existing methods.

As a conclusion, the proposed block methods can give better and comparable accuracy with the advantage of less costly. Thus, the proposed block methods are suitable to solve the second order linear boundary value problems of Dirichlet and Neumann type directly.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia
Sebagai memenuhi keperluan untuk ijazah Master Sains

KAEDAH SECARA LANGSUNG BLOK SATU-LANGKAH BAGI PENYELESAIAN MASALAH NILAI SEMPADAN LINEAR JENIS DIRICHLET DAN NEUMANN

Oleh

MOHD MUGHTI BIN HASNI

Februari 2014

Pengerusi: Profesor Madya Zanariah Binti Abdul Majid, PhD
Fakulti: Institut Penyelidikan Matematik

Dalam kajian ini, kaedah blok telah digunakan untuk menyelesaikan masalah nilai sempadan linear peringkat kedua jenis Dirichlet dan Neumann. Secara kebiasaannya, masalah matematik yang melibatkan peringkat persamaan pembezaan biasa yang lebih tinggi akan diubah ke dalam bentuk sistem persamaan peringkat pertama. Walau bagaimanapun, kaedah blok ini akan menyelesaikan ini secara langsung tanpa perlu mengubah ke dalam bentuk sistem persamaan peringkat pertama dengan menggunakan saiz langkah yang malar.

Terdapat tiga kaedah yang telah digunakan dalam kajian ini iaitu kaedah blok dua titik satu-langkah, kaedah blok tiga titik satu-langkah dan kaedah blok empat titik satu-langkah. Setiap satu daripada kaedah ini akan menyelesaikan masalah nilai sempadan linear peringkat kedua dengan dua jenis keadaan sempadan iaitu jenis Dirichlet dan Neumann.

Ketiga-tiga kaedah ini akan dilaksanakan bersama-sama dengan teknik tembakan linear untuk membina penyelesaian berangka. Kestabilan bagi setiap kaedah juga akan dibentangkan. Keputusan penyelesaian berangka bagi kaedah tersebut akan dibandingkan dengan kaedah yang sedia ada.

Sebagai kesimpulan, kaedah blok yang dicadangkan ini mampu memberi ketepatan yang setanding dan lebih baik dengan kelebihannya yang kurang mahal. Lantas, kaedah blok yang dicadangkan adalah sesuai untuk menyelesaikan masalah nilai sempadan linear peringkat kedua jenis Dirichlet dan Neumann secara langsung.

ACKNOWLEDGEMENTS

In the name of God, the Most Gracious, the Most merciful.I would like to express my gratitude towards all people that help me during this journey whether direct or indirectly. Many thanks go to my supervisor, Associate Profesor Zanariah Abdul Majid for her guidance, ideas, encouragements and many more for helping finish this study. My gratitude also goes to my co-supervisor in which Dr. Norazak Senu for his advice all this time. Special thanks go to my senior and colleagues for their contribution in making this thesis. There are no such words to describe their kindness and contribution in making this thesis. I would like to thank to my family for the encouragements and moral support throughout my studies.

This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfilment of the requirement for the degree of Master of Science. The members of the Supervisory Committee were as follows:

Zanariah Abdul Majid, PhD
Associate Professor
Institute for Mathematical Research
Universiti Putra Malaysia
(Chairman)

Norazak Senu, PhD
Senior Lecturer
Faculty of Science
Universiti Putra Malaysia
(Member)

____

BUJANG BIN KIM HUAT, PhD
Professor and Dean
School of Graduate Studies
Universiti Putra Malaysia

Date:

Declaration by graduate student

I hereby confirm that:

- this thesis is my original work;
- quotations, illustrations and citations have been duly referenced;
- this thesis has not been submitted previously or concurrently for any other degree at any other institutions;
- intellectual property from the thesis and copyright of thesis are fully-owned by Universiti Putra Malaysia, as according to the Universiti Putra Malaysia (Research) Rules 2012;
- written permission must be obtained from supervisor and the office of Deputy Vice-Chancellor (Research and Innovation) before thesis is published (in the form of written, printed or in electronic form) including books, journals, modules, proceedings, popular writings, seminar papers, manuscripts, posters, reports, lecture notes, learning modules or any other materials as stated in the Universiti Putra Malaysia (Research) Rules 2012;
- there is no plagiarism or data falsification/fabrication in the thesis, and scholarly integrity is upheld as according to the Universiti Putra Malaysia (Graduate Studies) Rules 2003 (Revision 2012-2013) and the Universiti Putra Malaysia (Research) Rules 2012. The thesis has undergone plagiarism detection software.

Signature: ______________________________ Date: ________________________

Name and Matric No: ____

Declaration by Members of Supervisory Committee

This is to confirm that:

- the research conducted and the writing of this thesis was under our supervision;
- supervision responsibilities as stated in the Universiti Putra Malaysia (Graduate Studies) Rules 2003 (Revision 2012-2013) are adhered to.

Signature: ______________________
Name of Chairman of Supervisory Committee: ______________________

Signature: ______________________
Name of Member of Supervisory Committee: ______________________

Signature: ______________________
Name of Member of Supervisory Committee: ______________________

Signature: ______________________
Name of Member of Supervisory Committee: ______________________

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF ABBREVIATIONS

ODEs	:	Ordinary Differential Equations
PDEs	:	Partial Differential Equations
BVPs	:	Boundary Value Problems
IVPs	:	Initial Value Problems
ADM	:	Adomian Decomposition Method
EADM	:	Extended Adomian Decomposition Method
FDM	:	Finite Difference Method
FEM	:	Finite Element Method
FVM	:	Finite Volume Method
CBIM	:	Cubic B-Spline Interpolation
ECBIM	:	Extended Cubic B-Spline Interpolation
SPP	:	Singular Perturbation Problems
2LBVP	:	Implementation of the Direct Two Point One-Step Block Method For Solving The Linear Dirichlet and Neumann BVPs
3LBVP	:	Implementation of the Direct Three Point One-Step Block Method For Solving The Linear Dirichlet and Neumann BVPs
4LBVP	:	Implementation of the Direct Four Point One-Step Block Method For Solving The Linear Dirichlet and Neumann BVPs
RK4	:	Implementation of the Runge Kutta Order Four with Linear Shooting Technique
2PSN(4)	:	Implementation of the multistep block method order four using nonlinear shooting technique in Phang et al. (2011)
2PSN(5)	:	Implementation of the multistep block method order five using nonlinear shooting technique in Phang et al. (2011)
ECBIM(N)	:	Extended Cubic B-Spline Method Minimizing Using Newton's Method in Hamid et al. (2011)
ECBIM(B)	:	Extended Cubic B-Spline Method Minimizing Using Built-In Function in Hamid et al. (2011)
EADM	:	Extended Adomian Decomposition Method in Bongsoo (2008)
COLHW	:	Collocation method with the Haar Wavelets in Siraj et al. (2010)
SPLINE	:	Polynomial spline method in Li-Bin et al. (2011)

CHAPTER 1

INTRODUCTION

1.1 Introduction

In the fields of science, many physical phenomena have been modeled mathematically to provide a better understanding of the phenomena. These mathematical models often yield an equation that contains some derivatives of an unknown function. This kind of equation is called a differential equation. Differential equation plays an important part in wide variety of subjects (i.e. physical sciences, economics, medicine, psychology and operation research).

Differential equations can be divided into two categories which are ordinary differential equations (ODEs) and partial differential equations (PDEs). ODEs are differential equations which deal with function of single variables and its derivatives. In contrast to the ODEs, PDEs are differential equations which deal with multivariable functions and their partial derivatives.

There are many types of differential equation. One of them is a boundary value problems (BVPs). BVPs are subfields of differential equations. BVPs are common thing in diverse fields (i.e. science, engineering, technology, boundary layer theory in fluid mechanics, heat power transmission theory, space technology and optimization theory). Many researchers have carried out their research for solving this kind of problems. Some of them have developed powerful methods for solving the BVPs numerically. BVPs and initial value problems (IVPs) are almost similar but differ in terms of their boundary conditions. BVPs have a condition specified at their extreme boundaries, whereas IVPs have all the conditions specified by the same value which is the initial value. In terms of their solutions, BVPs can give whether a unique solution, no solution or many solutions compared to the IVPs which only give unique solution.

Usually, the well-known one-step method such as the Euler method computes only one approximation value at a time. The same thing goes to multistep method such as the Adam Moulton method that computes only one approximation value at a time. However, this is different with the block method that computes more than one approximation values simultaneously at a time. A block is a set of new approximation values which are evaluated during each application of the iteration formula. Block method is a numerical method that computes n approximation values at a time. A n points block will produce n new function values simultaneously at each computational steps. The block method can be categorized as one-step block method or multistep block method. One-step block methods only use one previous information from the last block to obtain n approximation values in the next block. The multistep block methods use several previous information from the last preceding block to obtain n approximation values in the next block. Let y_n denotes the approximation to the exact solution $y(x_n)$ at $x = x_n$. For $n = ms$, $m = 0,1, \ldots ,s$, a block of solution can be represented by the vector $Y_m = (y_{n+1}, y_{n+2}, \ldots , y_{n+s})^T$ with y_{n+i} $(1 \le i \le s)$ is the approximate solution at

$x_{n+i} = x_n + ih$, where x_n is the right-hand end point of the previous block. If the block method computes the values of y_{n+i}, where $(1 \le i \le s)$ from the value of y_n only, then it is called as one-step method. If that is not the case, the block method may also refer to several or all points in the preceding block which is called as multistep method.

1.2 Boundary Value Problem

Boundary value problems (BVPs) that would be solved in this research is an ODEs together with a set of additional restraints, called the boundary conditions. A solution to BVPs is a solution to the differential equations which also satisfies the boundary conditions. This is the type of problems where the unknown function or its derivatives are given at two different points (i.e. $x = a$ and $x = b$). There are many types of BVPs. Some of them can be distinguished from their boundary conditions. Usually, there are three common types of boundary conditions which are the Dirichlet, Neumann and mixed boundary conditions.

Suppose the linear second order BVPs with the different boundary conditions as follows:

$$y'' = p(x)y' + q(x)y + r(x), \; q(x) > 0, \; [a,b], \qquad (1.3.1)$$

First type of boundary conditions (Dirichlet boundary conditions):

$$y(a) = \alpha \text{ and } y(b) = \beta, \qquad (1.3.2)$$

where α and β are constants.
Second type of boundary conditions (Neumann boundary conditions):

$$y'(a) = \xi \text{ and } y'(b) = \psi, \qquad (1.3.3)$$

where ξ and ψ are constants.
Third type of boundary conditions (mixed boundary conditions):

$$y'(a) + c_1 y(a) = \sigma \text{ and } y'(b) + c_2 y(b) = \varsigma, \qquad (1.3.4)$$

where σ, ς, c_1 and c_2 are constants.
Refer Fausett (2003).

In this investigation, only two types of boundary conditions will be considered which are the Dirichlet and Neumann boundary conditions. BVPs also can be distinguished whether it is linear or nonlinear. When the function of $f(x, y, y')$ has in the form of $f(x, y, y') = p(x)y' + q(x)y + r(x)$, then the $y' = f(x, y, y')$ is called linear differential equation.

Theorem 1.1 (Burden and Faires (1993))

If the linear BVPs in (1.3.1) is continuous on the set
$D = \{(x, y, y') | a \le x \le b, -\infty < y < \infty, -\infty < y' < \infty\}$,
and the partial derivatives f_y and $f_{y'}$ are also continuous on D. If

i. $f_y(x, y, y') > 0$ for all $(x, y, y') \in D$, and
ii. a constant M exists, with

$$\left|f_{y'}(x,y,y')\right| \le M, \text{ for all } (x,y,y') \in D.$$

Then, the BVPs has a unique solution. Readers can refer to Keller (1968) for the proof of this theorem.

Corollary 1.1 (Burden and Faires (1993))

If the linear BVP in (1.3.1) satisfies

i. *p(x)*, *q(x)*, and *r(x)* are continuous on $[a,b]$,
ii. *q(x)* > 0 on $[a,b]$.

Then, the BVPs has a unique solution.

1.3 Problem statement

The problems that will be considered in this research were equation (1.3.1) with two types of boundary conditions which are (1.3.2) and (1.3.3). Boundary value problems have become one of the main interests among the researchers nowadays. The difficulties when solving the BVPs arise from the existence of the boundary conditions at their extreme points, unlike the IVPs which have all the information specified in their initial conditions. To overcome this difficulty, the already well-known method such as shooting and finite difference method are often employed by several researchers such as Ha (2001) and Tirmizi and Twizell (2002). Ha (2001) proposes a new nonlinear shooting method for solving the nonlinear two-point boundary value problems. Ha (2001) has used the fourth order Runge-Kutta method and the Newton method implemented with the new nonlinear shooting method. Another researcher who has implemented the nonlinear shooting method was Phang et al. (2011). Phang et al. (2011) use the multistep block method and the three-step iterative method implemented with the nonlinear shooting technique for solving the second order nonlinear BVPs. However, there is another type of shooting technique which is linear shooting technique use mainly for solving linear two-point BVPs. Burden and Faires (1993) distinguish the two types of shooting technique which is the linear shooting method for solving linear second order BVPs and the nonlinear shooting method for solving nonlinear second order BVPs. Basically, the linear shooting technique is normally used for solving linear BVPs although the nonlinear shooting technique also could be used. Majid et al. (2012b), Mukhtar et al. (2011) and Mukhtar et al. (2012) have proposed the two-point, three-point and four point one-step block method for solving second order ODEs. In this research, we would like to extend the two point, three point and four point one-step block method implements with the linear shooting technique for solving linear two point second order BVPs. The numerical comparison will be provided to show the advantages of using the linear shooting technique over the nonlinear shooting technique for solving linear second order BVPs.

1.4 Objective of the thesis

The purpose of this research is to propose three types of block method for solving BVPs of Dirichlet and Neumann type. The objectives of this research are:

i. To apply the one-step block method for solving second order linear BVPs directly.
ii. To develop algorithm for one-step block method implement with linear shooting technique for solving second order two point linear BVPs of Dirichlet type directly using constant step size.
iii. To develop algorithm for one-step block method implement with linear shooting technique for solving second order two point linear BVPs of Neumann type directly using constant step size.
iv. To determine the stability analysis, order, error constant and the stability region for each method in this research.
v. To discuss and analyze the numerical solution obtain by linear shooting technique.

1.5 Scope of Study

The scope of this research will be mainly focused on solving the single second order linear two point BVPs of the Dirichlet and Neumann types by one-step block methods. Only three types of method with different orders will be considered in this research which are two, three and four point one-step block method with the order of three, four and five respectively. In this research, the step size used is constant.

1.6 Methodology

The direct two, three and four point one-step block method will be used in this research for solving second order linear BVP. The stability for each method will be tested by determining its zero stability, consistency and convergence. Each one-step block method will be implemented with the linear shooting technique. For n points one-step block method where n = 2, 3, and 4, the interval of [a, b] will be evenly separated for each block contains n points. The n points one-step block method will compute n approximation values by using only one point (right-hand end point) from the preceding block. These block methods will be used to solve the problem directly without reducing it into the system of first order ODEs. The C language is used to develop the codes for these three types of method which are two, three and four points one-step block method. With these codes, we will obtain the accuracy, total number of function call, total number of steps and the computational time. Then, comparison will be made with the Runge-Kutta order four and several other existing methods.

1.7 Outline of the thesis

This thesis basically can be divided into six chapters. Chapter 1 covers for the introductory part. Problem statements are introduced in this chapter. The main objectives of this thesis and the scope of this ressearch are also given in this chapter.

Chapter 2 will cover the literature review and the basic mathematical concepts. Chapter 3 and 4 present the two and three point one-step block method respectively, for solving linear BVPs of Dirichlet and Neumann type directly using constant step size. The derivation, stability analysis, order, error constant and the stability region of the two and three point one-step block method is included in Chapter 3 and 4 respectively. The implementation and the algorithm for the two point one-step block method are given in Chapter 3. The numerical results and the discussion will also be presented. Chapter 5 shows the derivation of the four point one-step block method. In this chapter, we will show the stability analysis, order, error constant, stability region and the numerical results. Finally, the discussion of the method will end this chapter.

Chapter 6 is the last chapter in this thesis and all the finding obtained throughout this research will be concluded and the suggestion for the future work is also provided.

REFERENCES

Aziz, A. K. (1975). *Numerical Solution of Boundary Value Problems for Ordinary Differential Equations*. Academic Press, New York.

Agarwal, R. P. (1985). On Numerov's method for solving two point boundary value problems. *Utilitas Mathematica*. 28: 159-174.

Agarwal, R. P., & Wang, Y. –M. (2001). Some recent developments of the Numerov's method. *Computer and Mathematics with Applications*.42: 561-592.

Burden, R. L. & Faires, J. D. (1993). *Numerical Analysis*. Fifth Edition. Boston: PWS-KENT Publishing Company.

Bongsoo, J. (2008). Two-point boundary value problems by the extended Adomian decomposition method. *Journal of Computational and Applied Mathematics*. 219(1): 253-262.

Chawla, M. M., & Subramanian, R. (1987). A new fourth order cubic spline method for nonlinear two-point boundary value problems. *International Journal of Computer Mathematics.* 22: 321-341.

Chawla, M. M., & ShivKumar, P. N. (1996). An efficient finite difference methods for two point boundary value problems. *Neural Parallel and Scientific Computation.* 4: 387C95.

Caglar, H., Caglar, N., & Elfaituri, K. (2006). B-spline interpolation compared with finite difference, finite element and finite volume methods which applied to two-point boundary value problems. *Applied Mathematics and Computation.* 175: 72-79.

Caglar, H., Caglar, N., & Ozer, M. (2009). B-spline solution of non–linear singular boundary value problems arising in physiology. *Chaos, Solitons and Fractals* 39: 1232-1237.

Chew, K. T., Majid, Z. A., Suleiman, M., & Senu, N. (2012). Solving Linear Two-Point Boundary Value Problems by Direct Adams Moulton Method. *Applied Mathematics Sciences*. 99(6): 4921-4929.

Emad, H. A., Abdelhalim, E., & Randolph, R. (2012). Advances in the Adomian decomposition method for solving two-point nonlinear boundary value problems with Neumann boundary conditions. *Computer and Mathematics with Applications* 63: 1056-1065.

Fatunla, S. O. (1991). Block methods for second order ODEs. *International Journal of Computer Mathematics*. 41: 55-63.

Fang, Q., Tsuchiya, T., & Yamamoto, T. (2002). Finite difference, finite element and finite volume methods applied to two-point boundary value problems. *Journal of Computational and Applied Mathematics*. 139(1): 9-19.

Fausett, L. V. (2003). *Numerical Methods: Algorithms and application*. Pearson Education. Inc.

Gasparo, M. G. & Macconi, M. (1990). Initial-value methods for second-order singularly perturbed boundary value problems. *Journal of Optimization Theory and Applications.* 66(2): 197-210.

Ha, S. N. (2001). A nonlinear shooting method for two-point boundary value problems. *Computer and Mathematics with Applications.* 42: 1441-1420.

Hamid, N. N. A., Majid, A. A., & Ismail, A. I. M. (2011). Extended cubic B-spline Method for Linear Two-Point Boundary Value Problems. *Sains Malaysiana* 40(11): 1285-1290.

Keller, H. B. (1968). *Numerical Methods for Two Point Boundary Value Problems*. London: Blaisdel.

Keller, H. B. (1976). Numerical Solution of Two-Point BVPs. *CBMS Regional Conference Series in Applied Mathematics (24) SIAM, Philadelphia.*

Kramer, M. E., & Mattheij, R. M. M. (1993). Application Of Global Methods In Parallel Shooting. *SIAM J. Numer. Anal.* 30(6): 1723-1739.

Khan, A. (2004). Parametric cubic spline solution of two-point boundary value problems. *Applied Mathematical Computation.* 154: 175-182.

Kadalbajoo, M. K. & Gupta, V. (2009). Numerical solution of singularly perturbed convection-diffusion problem using parameter uniform B-spline collocation method. *Journal of Mathematical Analysis and Applications* . 355: 439-452.

Kumar, M., Mishra, H. K. & Singh, P. (2009). A boundary value approach for a class of linear singular perturbed boundary value problems. *Advance in Engineering Software.* 40: 298-304.

Lambert, J. D. (1993). *Numerical Methods For Ordinary Differential Systems. The Initial Value Problem.* New York: John Wiley & Sons, Inc.

Li-Bin, L., Huan-Wen, L., & Yanping, C. (2011). Polynomial spline approach for solving second-order boundary-value problems with Neumann conditions. *Applied Mathematics and Computation* 217: 6872-6882.

Milne, W. E. (1953). *Numerical Solution of Differential Equations*. John Wiley, New York.

Majid, Z. A., Suleiman, M., Ismail, F., & Othman, M. (2003). 2-Point Implicit Block One-Step Method half Gauss-Seidel for Solving First Order Ordinary Differential Equations. *Matematika*. 19: 91-100.

Majid, Z. A., Azmi, N. A., & Suleiman, M. (2009). Solving Second Order Ordinary Differential Equations using Two Point Four Step Implicit Block Method. *European Journal of Scientific Research.* 31(1): 29-36.

Majid, Z. A., Mukhtar, N. Z. & Suleiman, M. (2012b). Direct Two-Point Block One-Step Method for Solving General Second-Order Ordinary Differential Equations. *Mathematical Problems in Engineering* vol. 2012, Article ID 184253, 16 pages, 2012. doi:10.1155/2012/184253.

Mukhtar, N. Z., Majid, Z. A., & Ismail, F. (2011). Solutions of general second order ODEs using direct block method of runge-kutta type. *Journal of Quality Measurement and Analysis (JQMA)* 7(2): 145-154. ISSN 1823-5670.

Mukhtar, N. Z., Majid, Z. A., Ismail, F. & Suleiman, M. (2012). Numerical Solutions for second order Ordinary Differential Equations Using Block Method from *Int. J. Mod. Phys. Conf. Ser 09* 560-565.

Majid, Z. A., Phang, P. S., & Suleiman, M. (2012a). Application of block method for solving nonlinear two point boundary value problem. *Advance Science Letter* 13: 754-757.

Majid, Z. A., & Mukhtar, N. Z. (2012). Five step block method for solving general second order ODEs directly. *The 8th East Asia SIAM Conference (EASIAM 2012), NTU.*

Phang, P. S., Majid, Z. A., & Suleiman, M. (2011). Solving Nonlinear Two-Point Boundary Value Problem Using Two Step Direct Method. *Journal of Quality Measurement and Analysis.* 7(1): 129-140.

Rosser, J. B., (1967). A Runge-Kutta for all seasons. *SIAM Review* 9; 417-452.

Reddien, G. W. (1980). Projection Methods For Two-Point Boundary Value Problems. *SIAM J. Review*. 22(2): 156-171.

Ramadan, M. A., Lashien, I. F., & Zahra, W. K. (2007). Polynomial and nonpolynomial spline approaches to the numerical solution of second order boundary value problems. *Applied Mathematics and Computation* 184: 476-484.

Ravi Kanth, A. S. V. (2007). Cubic spline polynomial for non-linear singular two-point boundary value problems. *Applied Mathematical Computation.* 189: 2017-2022.

Shampine, L. F. & Watts, H. A. (1969). Block implicit one-step methods. *Mathematics of Computation.* 23: 731-740.

Sen, R. N., & Bellal-Hussain, M. D. (1996). Finite difference methods for certain two point boundary value problems. *Journal Computational and Applied Mathematics.* 70: 33-50.

Siraj, U. I., Imran, A., & Bozidar, S. (2010). The numerical solution of second-order boundary-value problems by collocation method with the Haar wavelets. *Mathematical and Computer Modelling.* 52: 1577-1590.

Tamme, E. (1968). Solutions of a nonlinear boundary value problem for a second-order ordinary differential equation by the method of finite differences. *USSR Computational Mathematics and Mathematical Physics.* 8(5): 69-86.

Tirmizi, I. A., & Twizell, E. H. (2002). Higher order finite difference methods for nonlinear second order two point boundary value problems. *Applied Mathematics Letter*. 15: 897-902.

Wang, Y. –M., & Guo, B. –Y. (1998). On Numerov scheme for nonlinear two-points boundary value problem. *Journal of Computational Mathematics.* 16: 345-366.

Wang, Y. –M. (2007). The extrapolation of Numerov's scheme for nonlinear two-point boundary value problems. *Applied Numerical Mathematics.* 57: 253-269.

Wang, Y. G., Song, H. F. & Li, D. (2009). Solving two-point boundary value problems using combined homotophy perturbation method and Green's function method. *Applied Mathematics and Computation.* 212(2): 366-376.

BIODATA OF STUDENT

Mohd Mughti Bin Hasni was born in Hospital Besar Alor Setar. In 1996, he attends the primary school in Sekolah Rendah Kebangsaan Suka Menanti. Later, he moves into the secondary school in Sekolah Menengah Sains Pokok Sena. Then, he pursues his study in Penang Matriculation College. After that, he begins his journey in Universiti Putra Malaysia and obtained his first degree in Bachelor of Science (Honours) Mathematics. Then, he continues his study as a Master student in the field of computational mathematics.

LIST OF PUBLICATIONS

Hasni, M. M., Majid, Z. A., & Senu, N. (2012). Direct Two-Point One-Step Method for Solving Linear Dirichlet Boundary Value Problems. *Extended Abstracts of Fundamental Science Congress. Mathematics Symposium.*47-48.

Hasni, M. M., Majid, Z. A., & Senu, N. (2013). Numerical Solution of Linear Dirichlet Two-Point Boundary Value Problems Using Block Method. *International Journal of Pure and Applied Mathematics* 85(3): 495-506.

Hasni, M. M., Majid, Z. A., & Senu, N. (2013). Solving Linear Neumann Boundary Value Problems Using Block Methods. *International AIP Conference Proceedings.* 1552(1): 347-353.

Majid, Z. A., Hasni, M. M., & Senu, N. (2013). Solving Second Order Linear Dirichlet and Neumann Boundary Value Problems by Block Method. *IAENG International Journal of Applied Mathematics* 43(2): 71-76.