

UNIVERSITI PUTRA MALAYSIA

ONE-STEP BLOCK METHODS FOR DIRECT SOLVING OF LINEAR BOUNDARY VALUE DIRICHLET AND NEUMANN TYPE PROBLEMS

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ONE-STEP BLOCK METHODS FOR DIRECT SOLVING OF LINEAR BOUNDARY VALUE DIRICHLET AND NEUMANN TYPE PROBLEMS

By

MOHD MUGHTI BIN HASNI

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirements for the Degree of Master of Science

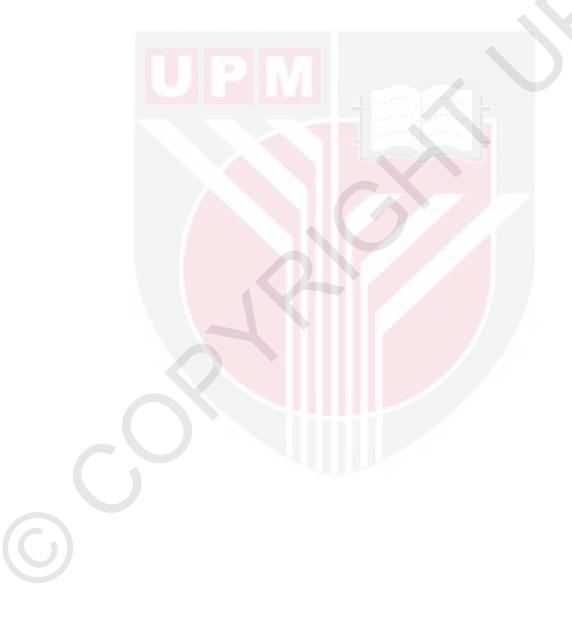
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Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Master of Science

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By

MOHD MUGHTI BIN HASNI

February 2014

Chairman: Associate Professor Zanariah Abdul Majid, PhD Faculty: Institute for Mathematical Research

In this research, the block methods have been used to solve the second order linear boundary value problems of Dirichlet and Neumann type. Mathematical problems which involve higher order ordinary differential equations were likely to be reduced into the system of first order equations. However, these block methods will solve the problems directly without reducing it into the first order equations using constant step size.

There are three methods that have been used in this research which are two point onestep block method, three point one-step block method and four point one-step block method. Each of these methods will be implemented to solve the second order linear boundary value problems with two different types of boundary conditions i.e. Dirichlet and Neumann type.

Those three methods will be implemented together with the linear shooting technique to construct the numerical solution. The stability for each method will be presented. Numerical results of the methods are compared with the existing methods.

As a conclusion, the proposed block methods can give better and comparable accuracy with the advantage of less costly. Thus, the proposed block methods are suitable to solve the second order linear boundary value problems of Dirichlet and Neumann type directly. Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia Sebagai memenuhi keperluan untuk ijazah Master Sains

KAEDAH SECARA LANGSUNG BLOK SATU-LANGKAH BAGI PENYELESAIAN MASALAH NILAI SEMPADAN LINEAR JENIS DIRICHLET DAN NEUMANN

Oleh

MOHD MUGHTI BIN HASNI

Februari 2014

Pengerusi: Profesor Madya Zanariah Binti Abdul Majid, PhD Fakulti: Institut Penyelidikan Matematik

Dalam kajian ini, kaedah blok telah digunakan untuk menyelesaikan masalah nilai sempadan linear peringkat kedua jenis Dirichlet dan Neumann. Secara kebiasaannya, masalah matematik yang melibatkan peringkat persamaan pembezaan biasa yang lebih tinggi akan diubah ke dalam bentuk sistem persamaan peringkat pertama. Walau bagaimanapun, kaedah blok ini akan menyelesaikan ini secara langsung tanpa perlu mengubah ke dalam bentuk sistem persamaan peringkat pertama dengan menggunakan saiz langkah yang malar.

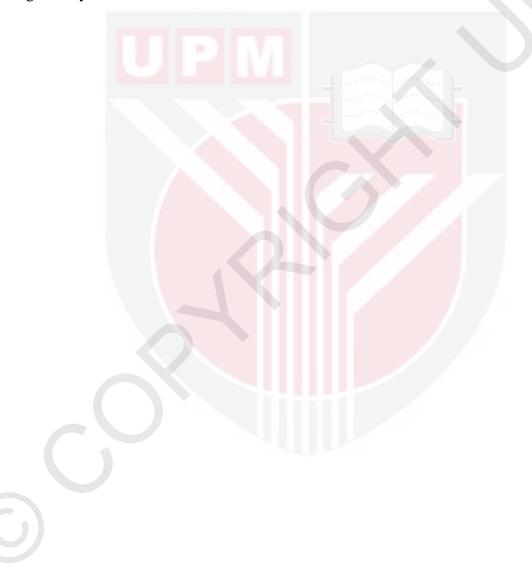
Terdapat tiga kaedah yang telah digunakan dalam kajian ini iaitu kaedah blok dua titik satu-langkah, kaedah blok tiga titik satu-langkah dan kaedah blok empat titik satu-langkah. Setiap satu daripada kaedah ini akan menyelesaikan masalah nilai sempadan linear peringkat kedua dengan dua jenis keadaan sempadan iaitu jenis Dirichlet dan Neumann.

Ketiga-tiga kaedah ini akan dilaksanakan bersama-sama dengan teknik tembakan linear untuk membina penyelesaian berangka. Kestabilan bagi setiap kaedah juga akan dibentangkan. Keputusan penyelesaian berangka bagi kaedah tersebut akan dibandingkan dengan kaedah yang sedia ada.

Sebagai kesimpulan, kaedah blok yang dicadangkan ini mampu memberi ketepatan yang setanding dan lebih baik dengan kelebihannya yang kurang mahal. Lantas, kaedah blok yang dicadangkan adalah sesuai untuk menyelesaikan masalah nilai sempadan linear peringkat kedua jenis Dirichlet dan Neumann secara langsung.

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I certify that a Thesis Examination Committee has met on 21 February 2014 to conduct the final examination of Mohd Mughti Bin Hasni on his thesis entitled "One-Step Block Methods for Direct Solving of Linear Boundary Value Dirichlet and Neumann Type Problems" in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Master of Science.

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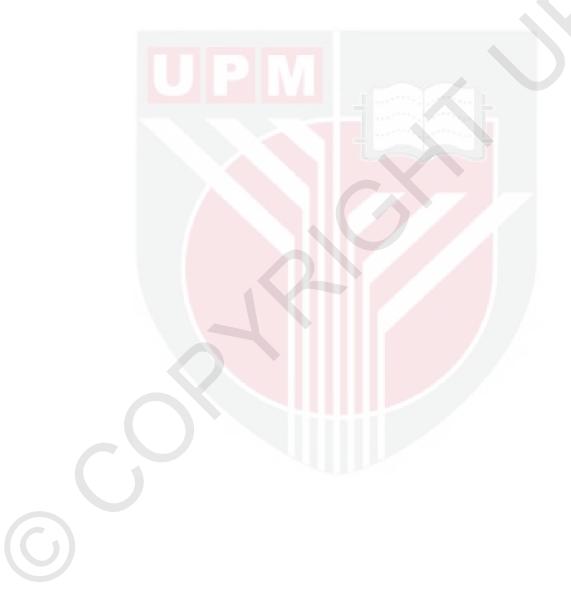
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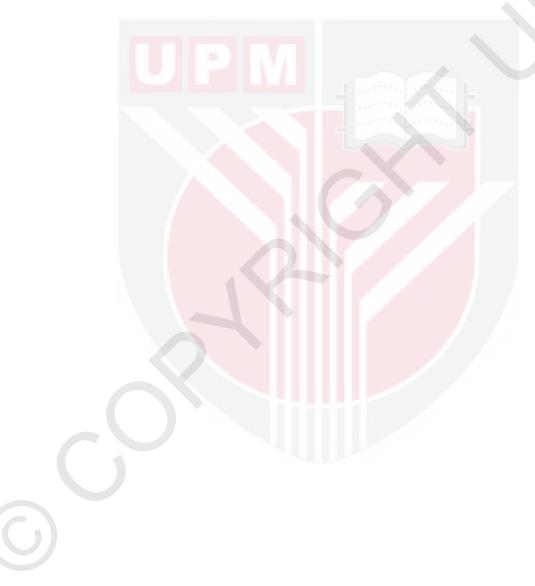
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LIST OF ABBREVIATIONS

ODE		
ODEs	:	Ordinary Differential Equations
PDEs	:	Partial Differential Equations
BVPs	:	Boundary Value Problems
IVPs	:	Initial Value Problems
ADM	:	Adomian Decomposition Method
EADM	:	Extended Adomian Decomposition Method
FDM	:	Finite Difference Method
FEM	:	Finite Element Method
FVM	:	Finite Volume Method
CBIM	:	Cubic B-Spline Interpolation
ECBIM	:	Extended Cubic B-Spline Interpolation
SPP	:	Singular Perturbation Problems
2LBVP	:	Implementation of the Direct Two Point One-Step Block Method For
		Solving The Linear Dirichlet and Neumann BVPs
3LBVP	:	Implementation of the Direct Three Point One-Step Block Method
		For Solving The Linear Dirichlet and Neumann BVPs
4LBVP	:	Implementation of the Direct Four Point One-Step Block Method For
		Solving The Linear Dirichlet and Neumann BVPs
RK4	:	Implementation of the Runge Kutta Order Four with Linear Shooting
		Technique
2PSN(4)	:	Implementation of the multistep block method order four using
		nonlinear shooting technique in Phang et al. (2011)
2PSN(5)	:	Implementation of the multistep block method order five using
		nonlinear shooting technique in Phang et al. (2011)
ECBIM(N)	:	Extended Cubic B-Spline Method Minimizing Using Newton's
× ,		Method in Hamid et al. (2011)
ECBIM(B)	:	Extended Cubic B-Spline Method Minimizing Using Built-In
- ()		Function in Hamid et al. (2011)
EADM	:	Extended Adomian Decomposition Method in Bongsoo (2008)
COLHW	•	Collocation method with the Haar Wavelets in Siraj et al. (2010)
SPLINE	:	Polynomial spline method in Li-Bin et al. (2011)
	·	

CHAPTER 1

INTRODUCTION

1.1 Introduction

In the fields of science, many physical phenomena have been modeled mathematically to provide a better understanding of the phenomena. These mathematical models often yield an equation that contains some derivatives of an unknown function. This kind of equation is called a differential equation. Differential equation plays an important part in wide variety of subjects (i.e. physical sciences, economics, medicine, psychology and operation research).

Differential equations can be divided into two categories which are ordinary differential equations (ODEs) and partial differential equations (PDEs). ODEs are differential equations which deal with function of single variables and its derivatives. In contrast to the ODEs, PDEs are differential equations which deal with multivariable functions and their partial derivatives.

There are many types of differential equation. One of them is a boundary value problems (BVPs). BVPs are subfields of differential equations. BVPs are common thing in diverse fields (i.e. science, engineering, technology, boundary layer theory in fluid mechanics, heat power transmission theory, space technology and optimization theory). Many researchers have carried out their research for solving this kind of problems. Some of them have developed powerful methods for solving the BVPs numerically. BVPs and initial value problems (IVPs) are almost similar but differ in terms of their boundary conditions. BVPs have a condition specified at their extreme boundaries, whereas IVPs have all the conditions specified by the same value which is the initial value. In terms of their solutions, BVPs can give whether a unique solution, no solution or many solutions compared to the IVPs which only give unique solution.

Usually, the well-known one-step method such as the Euler method computes only one approximation value at a time. The same thing goes to multistep method such as the Adam Moulton method that computes only one approximation value at a time. However, this is different with the block method that computes more than one approximation values simultaneously at a time. A block is a set of new approximation values which are evaluated during each application of the iteration formula. Block method is a numerical method that computes *n* approximation values at a time. A *n* points block will produce *n* new function values simultaneously at each computational steps. The block method can be categorized as one-step block method or multistep block method. One-step block methods only use one previous information from the last block to obtain *n* approximation values in the next block. The multistep block methods use several previous information from the last preceding block to obtain *n* approximation values in the next block. Let y_n denotes the approximation to the exact solution $y(x_n)$ at $x = x_n$. For n = ms, $m = 0,1, \dots, s$, a block of solution can be represented by the vector $Y_m = (y_{n+1}, y_{n+2}, \dots, y_{n+s})^T$ with y_{n+i} $(1 \le i \le s)$ is the approximate solution at

 $x_{n+i} = x_n + ih$, where x_n is the right-hand end point of the previous block. If the block method computes the values of y_{n+i} , where $(1 \le i \le s)$ from the value of y_n only, then it is called as one-step method. If that is not the case, the block method may also refer to several or all points in the preceding block which is called as multistep method.

1.2 Boundary Value Problem

Boundary value problems (BVPs) that would be solved in this research is an ODEs together with a set of additional restraints, called the boundary conditions. A solution to BVPs is a solution to the differential equations which also satisfies the boundary conditions. This is the type of problems where the unknown function or its derivatives are given at two different points (i.e. x = a and x = b). There are many types of BVPs. Some of them can be distinguished from their boundary conditions. Usually, there are three common types of boundary conditions which are the Dirichlet, Neumann and mixed boundary conditions.

Suppose the linear second order BVPs with the different boundary conditions as follows: y'' = p(x)y' + q(x)y + r(x), q(x) > 0, [a, b], (1.3.1)

First type of boundary conditions (Dirichlet boundary conditions):

$$y(a) = \alpha$$
 and $y(b) = \beta$, (1.3.2)

where α and β are constants.

Second type of boundary conditions (Neumann boundary conditions):

$$f(a) = \xi \text{ and } y'(b) = \psi,$$
 (1.3.3)

where ξ and ψ are constants.

Third type of boundary conditions (mixed boundary conditions):

v'

$$y'(a) + c_1 y(a) = \sigma \text{ and } y'(b) + c_2 y(b) = \varsigma,$$
 (1.3.4)

where σ , ς , c_1 and c_2 are constants. Refer Fausett (2003).

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In this investigation, only two types of boundary conditions will be considered which are the Dirichlet and Neumann boundary conditions. BVPs also can be distinguished whether it is linear or nonlinear. When the function of f(x, y, y') has in the form of f(x, y, y') = p(x)y' + q(x)y + r(x), then the y' = f(x, y, y') is called linear differential equation.

Theorem 1.1 (Burden and Faires (1993))

If the linear BVPs in (1.3.1) is continuous on the set $D = \{(x, y, y') | a \le x \le b, -\infty < y < \infty, -\infty < y' < \infty\},$

and the partial derivatives f_{v} and f_{v} are also continuous on D. If

i. $f_y(x, y, y') > 0$ for all $(x, y, y') \in D$, and

ii. a constant M exists, with

$$f_{y'}(x, y, y') \leq M$$
, for all $(x, y, y') \in D$.

Then, the BVPs has a unique solution. Readers can refer to Keller (1968) for the proof of this theorem.

Corollary 1.1 (Burden and Faires (1993))

If the linear BVP in (1.3.1) satisfies

- i. p(x), q(x), and r(x) are continuous on [a,b],
- ii. q(x) > 0 on [a, b].

Then, the BVPs has a unique solution.

1.3 Problem statement

The problems that will be considered in this research were equation (1.3.1) with two types of boundary conditions which are (1.3.2) and (1.3.3). Boundary value problems have become one of the main interests among the researchers nowadays. The difficulties when solving the BVPs arise from the existence of the boundary conditions at their extreme points, unlike the IVPs which have all the information specified in their initial conditions. To overcome this difficulty, the already well-known method such as shooting and finite difference method are often employed by several researchers such as Ha (2001) and Tirmizi and Twizell (2002). Ha (2001) proposes a new nonlinear shooting method for solving the nonlinear two-point boundary value problems. Ha (2001) has used the fourth order Runge-Kutta method and the Newton method implemented with the new nonlinear shooting method. Another researcher who has implemented the nonlinear shooting method was Phang et al. (2011). Phang et al. (2011) use the multistep block method and the three-step iterative method implemented with the nonlinear shooting technique for solving the second order nonlinear BVPs. However, there is another type of shooting technique which is linear shooting technique use mainly for solving linear two-point BVPs. Burden and Faires (1993) distinguish the two types of shooting technique which is the linear shooting method for solving linear second order BVPs and the nonlinear shooting method for solving nonlinear second order BVPs. Basically, the linear shooting technique is normally used for solving linear BVPs although the nonlinear shooting technique also could be used. Majid et al. (2012b), Mukhtar et al. (2011) and Mukhtar et al. (2012) have proposed the two-point, three-point and four point one-step block method for solving second order ODEs. In this research, we would like to extend the two point, three point and four point one-step block method implements with the linear shooting technique for solving linear two point second order BVPs. The numerical comparison will be provided to show the advantages of using the linear shooting technique over the nonlinear shooting technique for solving linear second order BVPs.

1.4 Objective of the thesis

The purpose of this research is to propose three types of block method for solving BVPs of Dirichlet and Neumann type. The objectives of this research are:

- i. To apply the one-step block method for solving second order linear BVPs directly.
- ii. To develop algorithm for one-step block method implement with linear shooting technique for solving second order two point linear BVPs of Dirichlet type directly using constant step size.
- iii. To develop algorithm for one-step block method implement with linear shooting technique for solving second order two point linear BVPs of Neumann type directly using constant step size.
- iv. To determine the stability analysis, order, error constant and the stability region for each method in this research.
- v. To discuss and analyze the numerical solution obtain by linear shooting technique.

1.5 Scope of Study

The scope of this research will be mainly focused on solving the single second order linear two point BVPs of the Dirichlet and Neumann types by one-step block methods. Only three types of method with different orders will be considered in this research which are two, three and four point one-step block method with the order of three, four and five respectively. In this research, the step size used is constant.

1.6 Methodology

The direct two, three and four point one-step block method will be used in this research for solving second order linear BVP. The stability for each method will be tested by determining its zero stability, consistency and convergence. Each one-step block method will be implemented with the linear shooting technique. For n points one-step block method where n = 2, 3, and 4, the interval of [a, b] will be evenly separated for each block contains n points. The n points one-step block method will compute n approximation values by using only one point (right-hand end point) from the preceding block. These block methods will be used to solve the problem directly without reducing it into the system of first order ODEs. The C language is used to develop the codes for these three types of method which are two, three and four points one-step block method. With these codes, we will obtain the accuracy, total number of function call, total number of steps and the computational time. Then, comparison will be made with the Runge-Kutta order four and several other existing methods.

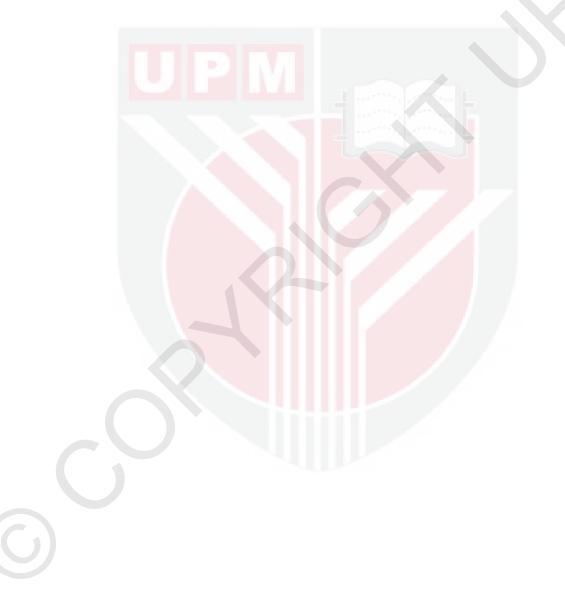
1.7 Outline of the thesis

This thesis basically can be divided into six chapters. Chapter 1 covers for the introductory part. Problem statements are introduced in this chapter. The main objectives of this thesis and the scope of this ressearch are also given in this chapter.

Chapter 2 will cover the literature review and the basic mathematical concepts. Chapter 3 and 4 present the two and three point one-step block method respectively, for solving linear BVPs of Dirichlet and Neumann type directly using constant step size. The derivation, stability analysis, order, error constant and the stability region of the two and

three point one-step block method is included in Chapter 3 and 4 respectively. The implementation and the algorithm for the two point one-step block method are given in Chapter 3. The numerical results and the discussion will also be presented. Chapter 5 shows the derivation of the four point one-step block method. In this chapter, we will show the stability analysis, order, error constant, stability region and the numerical results. Finally, the discussion of the method will end this chapter.

Chapter 6 is the last chapter in this thesis and all the finding obtained throughout this research will be concluded and the suggestion for the future work is also provided.



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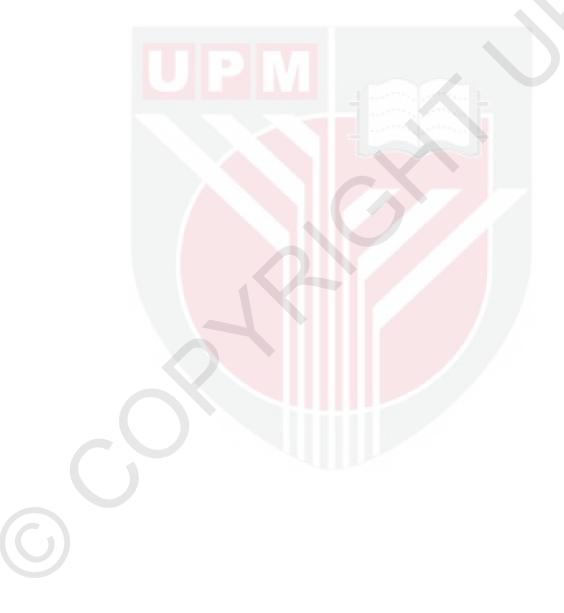
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LIST OF PUBLICATIONS

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