UNIVERSITI PUTRA MALAYSIA

BLOCK ONE-STEP METHODS FOR SOLVING STIFF DIFFERENTIAL EQUATIONS

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By

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September 2014

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Faculty: Institute for Mathematical Research

In this research, both stiff ordinary differential equations (ODEs) and parabolic partial differential equation (PDEs) are solved using the *A*-stable one-step block method with Newton's iteration with constant step size.

Two-point block one-step method and three-point block one-step method had been proposed in this research. These two methods are used to approximate the solutions for stiff ODEs and parabolic PDEs at two and three points simultaneously. The implementation of these methods will be in predictor and corrector mode. The predictor formulae is formulated from the modified block method itself. Newton's iteration is adapted in implementation of the block methods. The order, error constant, convergence and stability of each method are also discussed.

This study also focused on solving parabolic PDEs. In order to solve parabolic PDEs using the proposed methods, we reduced the form of parabolic PDEs into ODEs by discretizing the parabolic equation using method of line. To illustrate the applicability of the proposed method, several numerical results are shown and compared with the results obtained by the existing methods

In conclusion, the proposed methods are suitable for solving stiff ordinary differential equations at varies stepsizes especially when the stepsizes are larger. Other than that, the proposed method also appropriate for solving stiff parabolic partial differential equations due to acceptable results that had been produced.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Master Sains

KAEDAH BLOK SATU-LANGKAH UNTUK PENYELESAIAN PERSAMAAN PEMBEZAAN KAKU

Oleh

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September 2014

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Dalam kajian ini, kedua-dua persamaan pembezaan biasa kaku dan persamaan pembezaan separa parabolik telah diselesaikan dengan menggunakan kaedah blok *A*-stabil satu-langkah dengan lelaran Newton menggunakan saiz langkah yang malar.

Terdapat dua kaedah yang telah dicadangkan dalam kajian ini iaitu kaedah blok dua-titik satu-langkah dan kaedah blok tiga-titik satu-langkah. Kedua-dua kaedah telah digunakan untuk mengira penyelesaian bagi persamaan pembezaan biasa kaku dan persamaan pembezaan separan parabolik, pada dua dan tiga titik serentak. Dalam pelaksanaan kedua-dua keadah, kaedah peramal dan pembetul akan digunakan. Formula peramal dirumuskan daripada kaedah blok yang diubah suai sendiri. Lelaran Newton telah disesuaikan ke dalam setiap kaedah blok sebagai pembetul. Peringkat, pemalar ralat, penumpuan dan kestabilan setiap kaedah juga telah dibincangkan.

Persamaan pembezaan separa dalam kajian ini akan lebih tertumpu kepada penyelesaian persamaan pembezaan separa parabolik. Untuk menyelesaikan persamaan pembezaan separa parabolik dengan menggunakan kaedah yang dicadangkan, kami akan menurunkan bentuk persamaan pembezaan separa parabolik kepada persamaan pembezaan biasa dengan mendiskretkan persamaan parabolik menggunakan kaedah garis. Beberapa keputusan berangka ditunjukkan untuk dibandingkan dengan keputusan yang diperolehi melalui kaedah yang sedia ada untuk menggambarkan kesesuaian kaedah yang dicadangkan.

Secara keseluruhan, kita dapat membuat kesimpulan bahawa kaedah yang dicadangkan adalah sesuai untuk menyelesaikan persamaan pembezaan biasa kaku pada saiz langkah yang bervariasi terutama ketika saiz langkah adalah lebih besar. Selain daripada itu, kaedah yang dicadangkan juga sesuai untuk menyelesaikan persamaan pembezaan separa dengan memberi keputusan yang boleh diterima pakai.

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I certify that a Thesis Examination Committee has met on 25 September 2014 to conduct the final examination of Muhammad Izzat Zakwan Bin Mohd Zabidi on his thesis entitled "Block One-step Methods for Solving Stiff Differential Equations" in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Master of Science.

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LIST OF ABBREVIATIONS

ODEs	:	Ordinary Differential Equations
PDEs	:	Partial Differential Equations
IVPs	:	Initial Value Problems
LMM	:	Linear Multistep Method
2PBOSM	:	Implementation of Two-point One-step block Method with Newton's iteration for Solving Stiff Ordinary Differential Equations and Parabolic Partial Differential Equations
2PBOSM*	:	Implementation of Two-point One-step block Method with Fix Point Iteration (*PECE*) for Solving Stiff Ordinary Differential Equations
2PPDE	:	Implementation of Two-point one-step block method with Newton's iteration for solving parabolic PDEs
3PBOSM	:	Implementation of Three-point One-step block Method with Newton's iteration (*PECE*) for Solving Stiff Ordinary Differential Equations and Parabolic Partial Differential Equations
3PBOSM*	:	Implementation of Three-point One-step block Method with Fix Point Iteration for Solving Stiff Ordinary Differential Equations
3PPDE	:	Implementation of Three-point one-step block method with Newton's iteration for solving parabolic PDEs
3BEBDF	:	Implementation of Fourth Order 3-point Block Extended Backward Differential Formula for Solving Stiff Ordinary Differential Equations
BBDF(5)	:	Implementation of Fifth Order Block Backward Differential Formula for Solving Stiff Ordinary Differential Equations
RK3	:	Implementation of Third Order Runge-Kutta for Solving Parabolic Partial Differential Equations
MCHW-RK3	:	Implementation of Modified C_0M Weights Runge-Kutta Three Method for Parabolic Partial Differential Equations
RK4	:	Implementation of Fourth Order Runge-Kutta for Solving Parabolic Partial Differential Equations
CoM	:	Implementation of New Fourth Order Runge-Kutta Formula Based On The Contra-Harmonic (C_0M) Mean for Solving Parabolic Partial Differential Equations

CHAPTER 1

INTRODUCTION

1.1 Introduction

Differential equations occur frequently in many branches of science, including pure and applied mathematics. Those branches of science are chemical process, mechanical processes and mathematical models of electrical.

Differential equations can be classified into two parts which are ordinary differential equations (ODEs) and partial differential equations (PDEs). ODE is equation involving functions and derivatives while the PDEs involving functions and their partial derivatives.

An ODE is a differential equation in which the unknown function is a function of a single independent variable. In the simplest form, the unknown function is a real or complex valued function, but more generally, it may be vector-valued or matrix-valued: this corresponds to a system of ODEs for a single function. When a different part of ODEs system has different time dependencies, this ODES equation become stiff ODEs. A small step size is needed in order to get an accurate result when we use existing numerical method except that the method is with A-stability properties.

An unknown function will be called as PDEs when the unknown function is a function of various independent variables and involving partial derivatives. PDEs can be classified into three major categories which are elliptic, parabolic and hyperbolic.

Elliptic PDEs can be considered as Poisson equation:

$$\frac{\partial^2 u}{\partial x^2}(x,y)+\frac{\partial^2 u}{\partial y^2}(x,y)= f(x,y),$$

The function $f(x,y)$ describes the input to the problem on a plane R whose boundary denote by S. Typically, this type of equation arises in the study of various time-dependent physical problems such as the energy potential of a point in a plane acted on by the force of gravity plane, two-dimensional steady-state problems involving in compressible fluids and steady-state heat distribution in a plane region. In the study of steady-state distribution of heat in a plane region requires that $f(x,y) = 0$, in the result will simplify the Poisson equation into:

$$\frac{\partial^2 u}{\partial x^2}(x,y)+\frac{\partial^2 u}{\partial y^2}(x,y)=0,$$

which is called Laplace’s equation.

PDEs can be described as the physical problems of heat flow along the rod length *l* (see Figure 1.1), which is considered to have a uniform temperature in each element of the cross-section. In this condition, a perfectly insulated on a rod lateral surface is required. The form of parabolic PDEs:

$$\frac{\partial u}{\partial t}(x,t) = \alpha^2 \frac{\partial^2 u}{\partial x^2}(x,t),$$

where the constant α is determined by the heat-conductive properties of the material of which rod is composed and is assumed to be independent of the position in the rod.

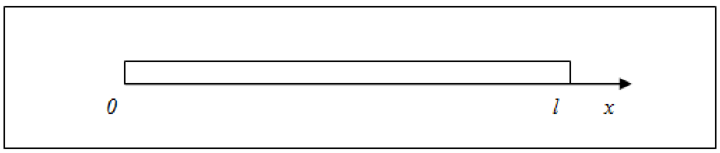

Figure 1.1 The flow of heat on a insulation rod.
(Source Burden *et al.* 2005)

Wave equation is an example of hyperbolic PDEs. For example, an elastic string of length *l* is stretched between two supports at the same horizontal level. When the string is in a motion state and its vibration is on a vertical plane, the vertical displacement $u(x,t)$ of point x at time t satisfies the PDEs

$$\frac{\partial^2 u}{\partial t^2}(x,t) = \alpha^2 \frac{\partial^2 u}{\partial x^2}(x,t), \qquad 0 < x < l, \qquad 0 < l,$$

provided that damping effects are neglected and the amplitude is too large.
In this research, we will only focus on solving parabolic PDEs using the proposed method.
Source: Burden *et al.* (2005)

1.2 Objective of the thesis

The goal of this study is to propose two types of block method in solving stiff ODEs and parabolic PDEs. The objectives of the research are:

a) To establish the order, convergent and stability region for two-point and three-point block one-step methods.
b) To formulate the two-point and three-point block one-step methods for solving stiff ODEs using constant step size.
c) To formulate the two-point and three-point block one-step methods for solving parabolic PDEs using constant step size.

1.3 Scope of the study

The scope of this study will focus on solving two types of problems, i.e. first order ODEs equations and one dimensional of PDEs. Two-point and three-point block one-step methods will be implemented to compute both problems using constant step size. In addition, C program will be used to run the develop code of the methods. MATHEMATICA software will be used to plot the stability region of both block methods.

1.4 Outline of the thesis

There are six chapters in this thesis. In chapter one, the introduction of this thesis will be discussed. It covers the general introduction for differential equations, the main objectives of this thesis and the scope of study for this thesis.

Chapter two consists of preliminary mathematical concepts and literature review of the previous work will be discussed at the end of this chapter.

Chapter three presents the two-point block one-step method for solving stiff ODEs using constant step size. The derivation of this method, stability region, order, algorithm of this method, problem tested and the implementation of this method also have been discussed in this chapter. Discussion will be at the end of this chapter.

Chapter four deals with three-point block one-step method for solving stiff ODEs using constant step size. The derivation of this method, stability region, order, algorithm of this method, problem tested and the implementation of this method have also been discussed in this chapter. This chapter will finish with the discussion of the results.

Chapter five discusses the PDEs that will solve using block method. This chapter also includes the introduction of PDEs, method of lines, implementation of the block method, algorithm of the method and the test problem. The numerical results and discussion will be at the end this chapter.

Chapter six consists of the summary and the findings of this study. It also provides recommendations for future work.

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