



UNIVERSITI PUTRA MALAYSIA

***CLASSIFICATION OF SECOND-CLASS
10-DIMENSIONAL COMPLEX FILIFORM LEIBNIZ
ALGEBRAS***

SUZILA BINTI MOHD KASIM

IPM 2014 10



**CLASSIFICATION OF SECOND-CLASS
10-DIMENSIONAL COMPLEX FILIFORM LEIBNIZ
ALGEBRAS**

By

SUZILA BINTI MOHD KASIM

**Thesis Submitted to the School of Graduate Studies,
Universiti Putra Malaysia, in Fulfilment of the
Requirements for the Degree of Master of Science**

April 2014

COPYRIGHT

All material contained within the thesis, including without limitation text, logos, icons, photographs and all other artwork, is copyright material of Universiti Putra Malaysia unless otherwise stated. Use may be made of any material contained within the thesis for non-commercial purposes from the copyright holder. Commercial use of material may only be made with the express, prior, written permission of Universiti Putra Malaysia.

Copyright ©Universiti Putra Malaysia



Abstract of thesis presented to the Senate of Universiti Putra Malaysia in
fulfilment of the requirement for the degree of Master of Science

CLASSIFICATION OF SECOND-CLASS 10-DIMENSIONAL COMPLEX FILIFORM LEIBNIZ ALGEBRAS

By

SUZILA BINTI MOHD KASIM

April 2014

Chair: Professor Isamiddin S. Rakhimov, PhD

Faculty: Institute for Mathematical Research

This thesis is concerned on studying the classification problem of a subclass of $(n + 1)$ -dimensional complex filiform Leibniz algebras. Leibniz algebras that are non-commutative generalizations of Lie algebras are considered. Leibniz identity and Jacobi identity are equivalent when the multiplication is skew-symmetric. When studying a certain class of algebras, it is important to describe at least the algebras of lower dimensions up to an isomorphism. For Leibniz algebras, difficulties arise even when considering nilpotent algebras of dimension greater than four. Thus, a special class of nilpotent Leibniz algebras is introduced namely filiform Leibniz algebras. Filiform Leibniz algebras arise from two sources. The first source is a naturally graded non-Lie filiform Leibniz algebras and another one is a naturally graded filiform Lie algebras.

Naturally graded non-Lie filiform Leibniz algebras contains subclasses FLb_{n+1} and SLb_{n+1} . While there is only one subclass obtained from naturally graded filiform Lie algebras which is TLb_{n+1} . These three subclasses FLb_{n+1} , SLb_{n+1} and TLb_{n+1} are over a field of complex number, \mathbb{C} where $n + 1$ denotes the dimension of these subclasses starting with $n \geq 4$.

In particular, a method of simplification of the basis transformations of the arbitrary filiform Leibniz algebras which were obtained from naturally graded non-Lie filiform Leibniz algebras, that allows for the problem of classification of algebras is reduced to the problem of a description of the structural constants. The investigation of filiform Leibniz algebras which were obtained from naturally graded non-Lie filiform Leibniz algebras only for subclass SLb_{n+1} is the subject of this thesis.

This research is the continuation of the works on SLb_{n+1} which have been treated for the cases of $n < 9$. The main purpose of this thesis is to apply the Rakhimov-Bekbaev approach to classify SLb_{10} . These approach will give a complete classification of SLb_{10} in terms of algebraic invariants. Isomorphism criterion of SLb_{10} is used to split the set of algebras SLb_{10} into several disjoint subsets. For each of these subsets, the classification problem is solved separately. As a result, some of them are represented as a union of infinitely many orbit (parametric families) and others as single orbits (isolated orbits). Finally, the list of isomorphism classes of complex filiform Leibniz algebras with the table of multiplications are given.



Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia
sebagai memenuhi keperluan untuk ijazah Sarjana Sains

**PENGGELASAN KELAS-KEDUA 10-DIMENSI ALJABAR
LEIBNIZ FILIFORM KOMPLEKS**

Oleh

SUZILA BINTI MOHD KASIM

April 2014

Pengerusi: Profesor Isamiddin S. Rakhimov, PhD
Fakulti: Institut Penyelidikan Matematik

Tesis ini adalah berkenaan kajian terhadap masalah pengkelasan daripada subkelas $(n + 1)$ -dimensional aljabar Leibniz filiform kompleks. Aljabar Leibniz yang umum bukan kalis tukar tertib aljabar Lie akan dipertimbangkan. Identiti Leibniz dan identiti Jacobi adalah setara apabila pendaraban adalah condong-simetri. Ketika mengkaji kelas tertentu aljabar, adalah penting untuk menerangkan aljabar yang sekurang-kurangnya dimensi yang lebih rendah terhadap satu isomorfisma. Bagi aljabar Leibniz, timbul kesukaran apabila mempertimbangkan aljabar nilpotent dimensi yang lebih besar daripada empat. Oleh itu, kelas khas aljabar Leibniz nilpotent diperkenalkan iaitu aljabar Leibniz filiform. Aljabar Leibniz filiform diterbitkan daripada dua sumber. Sumber pertama adalah aljabar Leibniz bukan-Lie filiform terged secara semula jadi dan satu lagi adalah aljabar Lie filiform terged secara semula jadi.

Aljabar Leibniz bukan-Lie filiform terged secara semula jadi mengandungi subkelas-subkelas FLb_{n+1} dan SLb_{n+1} . Namun hanya ada satu subkelas yang diperolehi daripada aljabar Lie filiform terged secara semula jadi iaitu TLe_{n+1} . Ketiga-tiga subkelas FLb_{n+1} , SLb_{n+1} dan TLe_{n+1} berada dalam nombor kompleks, \mathbb{C} di mana $n + 1$ menandakan dimensi subkelas ini bermula dengan $n \geq 4$.

Khususnya, kaedah memudahkan perubahan-perubahan dasar terhadap aljabar Leibniz filiform yang diperolehi daripada aljabar Leibniz bukan-Lie filiform terged secara semula jadi membolehkan masalah pengkelasan aljabar dikurangkan kepada masalah penerangan mengenai struktur pemalar. Penyiasatan aljabar Leibniz filiform yang diperolehi daripada aljabar Leibniz bukan-Lie filiform terged secara semula jadi adalah subjek tesis ini bagi subkelas SLb_{n+1} sahaja.

Kajian ini adalah kesinambungan kerja-kerja pada SLb_{n+1} yang dikaji bagi kes-kes $n < 9$. Tujuan utama tesis ini adalah untuk mengaplikasi pendekatan Rakhimov-Bekbaev untuk mengelaskan SLb_{10} . Pendekatan ini akan memberikan pengkelasan lengkap SLb_{10} dalam sebutan aljabar tak varian. Kriteria isomorfisma SLb_{10} digunakan untuk memisahkan set aljabar SLb_{10} kepada beberapa subset terpisah. Bagi setiap subset ini, masalah pengkelasan ini diselesaikan secara berasingan. Hasilnya, sebahagian daripada subset diwakili sebagai kesatuan orbit tak terhingga banyaknya (keluarga parametrik) dan selebihnya sebagai orbit tunggal (orbit diasingkan). Akhirnya, senarai kelas isomorfisma aljabar Leibniz filiform kompleks dengan jadual pendaraban diberikan.



ACKNOWLEDGEMENTS

First and foremost, all praises are to the almighty ALLAH for His blessings and mercy that enable me to learn and complete this thesis.

I am sincerely grateful to Professor Dr. Isamiddin S. Rakhimov, my supervisor, for giving me the opportunity to work under his supervision. He has been very generous and patient to contribute his valuable time to the numerous checking and rechecking sessions. I highly appreciate his assistance and commitment in preparation and completion of this thesis.

I want to thank my supervisory committee Dr Sharifah Kartini Said Husain for her helpful guidance. This thesis will not be possible without the contribution from her. I would also like to thank group of peoples in Algebraist Group UPM for their collaborations.

Many thanks to Professor Dato' Dr. Kamel Ariffin Mohd Atan, the Director of Institute for Mathematical Research (INSPEM) for his encouragements.

My master study was supported by UPM scholarship (Graduate Research Fellowship) and also a scholarship from Ministry of Higher Education (MyMaster).

Last but not least, I would like to express my gratitude to my beloved mother (Nekmah Ngah Derani), my late father (Mohd Kasim bin Alang Mahat), my sisters and brothers for their love, support, understanding and encouragements.

This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfilment of the requirement for the degree of Master of Science.

The members of the Supervisory Committee were as follows:

Isamiddin S. Rakhimov, PhD

Professor

Institute for Mathematical Research

Universiti Putra Malaysia

(Chairman)

Sharifah Kartini Said Husain, PhD

Lecturer

Institute for Mathematical Research

Universiti Putra Malaysia

(Member)



BUJANG KIM HUAT, PhD

Professor and Dean

School of Graduate Studies

Universiti Putra Malaysia

Date:

DECLARATION

Declaration by graduate student

I hereby confirm that:

- this thesis is my original work;
- quotations, illustrations and citations have been duly referenced;
- this thesis has not been submitted previously or concurrently for any other degree at any other institutions;
- intellectual property from the thesis and copyright of thesis are fully-owned by Universiti Putra Malaysia, as according to the Universiti Putra Malaysia (Research) Rules 2012;
- written permission must be obtained from supervisor and the office of Deputy Vice-Chancellor (Research and Innovation) before thesis is published (in the form of written, printed or in electronic form) including books, journals, modules, proceedings, popular writings, seminar papers, manuscripts, posters, reports, lecture notes, learning modules or any other materials as stated in the Universiti Putra Malaysia (Research) Rules 2012;
- there is no plagiarism or data falsification/fabrication in the thesis, and scholarly integrity is upheld as according to the Universiti Putra Malaysia (Graduate Studies) Rules 2003 (Revision 2012-2013) and the Universiti Putra Malaysia (Research) Rules 2012. The thesis has undergone plagiarism detection software.

Signature: _____ Date: _____

Name and Matric No.: Suzila binti Mohd Kasim (GS31371)

Declaration by Members of Supervisory Committee

This is to confirm that:

- the research conducted and the writing of this thesis was under our supervision;
- supervision responsibilities as stated in the Universiti Putra Malaysia (Graduate Studies) Rules 2003 (Revision 2012-2013) are adhered to.

Signature: _____ Date: _____

Name of

Chairman of

Supervisory

Committee: **Isamiddin S. Rakhimov, PhD**

Signature: _____ Date: _____

Name of

Member of

Supervisory

Committee: **Sharifah Kartini Said Husain, PhD**

TABLE OF CONTENTS

	Page
ABSTRACT	i
ABSTRAK	iii
ACKNOWLEDGEMENTS	v
APPROVAL	vi
DECLARATION	viii
LIST OF FIGURES	xii
LIST OF ABBREVIATIONS	xiii
 CHAPTER	
1 INTRODUCTION	1
1.0 Basic Concepts	1
1.1 Literature Review	2
1.1.1 Relations on Leibniz Algebras	3
1.2 Research Objectives	4
1.3 Scope of Research	5
1.4 Research Methodology	5
1.5 Outline of Contents	6
 2 CONCEPT OF FILIFORM LEIBNIZ ALGEBRAS	7
2.0 Introduction	7
2.1 Naturally Graded Filiform Leibniz Algebras	7
2.2 Adapted Base Change and Isomorphism Criteria for SLb_{n+1}	9
2.3 Conclusion	15
 3 CLASSIFICATION METHOD OF SLb_{10}	16
3.0 Introduction	16
3.1 Isomorphism Criterion for SLb_{10}	16
3.2 Specification of Disjoint Subsets	18
3.2.1 Notations in SLb_{10}^a	18
3.2.2 Notations in SLb_{10}^b	18
3.3 Description of Orbits in Parametric Families	51
3.4 Description of Isolated Orbits	81
3.5 Conclusion	86
 4 RESULTS ON CLASSIFICATION OF SLb_{10}	87
4.0 Introduction	87
4.1 The Representatives of SLb_{10}	87
4.2 Conclusion	93
4.3	94

5 CONCLUSION	95
5.1 Summary	95
5.2 Recommendations For Future Works	95
REFERENCES/BIBLIOGRAPHY	96
APPENDICES	98
A.1 The Program	99
A.1.1 Isomorphism Criteria	99
B.1 Classification of SLb_{10}^a	101
B.2 Classification of SLb_{10}^b	113
BIODATA OF STUDENT	133
LIST OF PUBLICATIONS	134



LIST OF FIGURES

Figure	Page
B.1 Graph tree of SLb_{10}^a	112
B.2 Graph tree of SLb_{10}^b	129
B.3 Graph tree of SLb_{10}^b (continued from Figure B.2)	130



LIST OF ABBREVIATIONS

\mathbb{C}	The field of complex numbers
\mathbb{C}^*	The field of nonzero complex numbers
Lb_{n+1}	The class of $(n + 1)$ -dimensional filiform Leibniz algebras
LB_n	The class of n -dimensional Leibniz algebras
SLb_{n+1}	Second class of $(n + 1)$ -dimensional filiform Leibniz algebras
SLb_{10}^a	Second class of 10-dimensional filiform Leibniz algebras ($\beta_3 \neq 0$)
SLb_{10}^b	Second class of 10-dimensional filiform Leibniz algebras ($\beta_3 = 0$)

CHAPTER 1

INTRODUCTION

The aim of this thesis is to classify a subclass of complex filiform Leibniz algebras. The complex filiform Leibniz algebras are arising from two sources, naturally graded non-Lie filiform Leibniz algebras and naturally graded filiform Lie algebras. The class of filiform Leibniz algebras derived from naturally graded non-Lie filiform Leibniz algebras is separated into two subclasses. There are first class filiform Leibniz algebras, FLb_{n+1} and second class filiform Leibniz algebras, SLb_{n+1} in dimension $(n + 1)$, respectively.

An algebraic approach is focused to solve isomorphism problem in terms of algebraic invariants (invariant functions) in the classification of second class 10-dimensional complex filiform Leibniz algebras, SLb_{10} . By using this approach, a complete classification of SLb_{10} is obtained and the isomorphism criterion are given.

This chapter is organized by reviewing basic concepts of algebras and Leibniz algebras briefly and next some literature reviews are described and followed by a few research objectives, scope of research and research methodology. Outline of contents of the thesis is also provided.

1.0 Basic Concepts

Let V be a vector space of dimension n over an algebraically closed field K ($\text{char } K = 0$). The set of bilinear maps $V \times V \rightarrow V$ form a vector space $\text{Hom}(V \otimes V, V)$ of dimension n^3 , which can be considered together with its natural structure of an affine algebraic variety over K and be denoted by $\text{Alg}_n(K) \cong K^{n^3}$. An n -dimensional algebra L over K can be considered as an element L of $\text{Alg}_n(K)$ via bilinear mapping $L \otimes L \rightarrow L$ defining a binary algebraic operation on L : let $\{e_0, e_1, e_2, \dots, e_n\}$ be a basis of the algebra L . The table of multiplication of L is represented by point (γ_{ij}^k) as follows:

$$[e_i, e_j] = \sum_{k=1}^n \gamma_{ij}^k e_k, \quad i, j = 1, 2, \dots, n.$$

Here γ_{ij}^k are called structural constants of L . The linear reductive group $GL_n(K)$ acts on $\text{Alg}_n(K)$ is said to be "transport of structure" with

$$(g * \lambda)(x, y) = g \left(\lambda \left(g^{-1}(x), g^{-1}(y) \right) \right).$$

Two algebras L_1 and L_2 on V are isomorphic if and only if they belong to the same orbit under action of transport of structure.

Recall that an algebra L over a field K is called a *Leibniz algebra* if it satisfies the following Leibniz identity:

$$[x, [y, z]] = [[x, y], z] - [[x, z], y], \quad (1.1)$$

where $[\cdot, \cdot]$ denotes the multiplication in L .

If a Leibniz algebra has the property of antisymmetry

$$[x, y] = -[y, x],$$

then the Leibniz identity is simplified into the Jacobi identity:

$$[[x, y], z] + [[y, z], x] + [[z, x], y] = 0.$$

Therefore Leibniz algebras are generalization of Lie algebras.

Let $LB_n(K)$ be a subvariety of $Alg_n(K)$ consisting of all n -dimensional Leibniz algebras over K . It is stable under the above mentioned action of $GL_n(K)$. As a subset of $Alg_n(K)$, the set $LB_n(K)$ is specified by the following polynomial equalities with respect to the structural constants γ_{ij}^k :

$$\gamma_{jk}^l \gamma_{il}^m - \gamma_{ij}^l \gamma_{lk}^m + \gamma_{ik}^l \gamma_{lj}^m = 0, \quad i, j, k, l, m = 1, 2, \dots, n. \quad (1.2)$$

The solution by this system of equations in the set $LB_n(K)$ with respect to γ_{ij}^k has been done for low-dimensional cases of some classes of algebra ($n \leq 3$). The complexity of the computations increases much with increasing of dimension to classify $LB_n(K)$ for any fixed n . Hence, one considers some subclasses of $LB_n(K)$ to be classified.

1.1 Literature Review

Many papers concern with the study of Lie algebras case. According to the structural theory of Lie algebras a finite-dimensional Lie algebra is written as a semidirect sum of its semisimple subalgebra and the solvable radical (Levi's theorem). The semisimple part is a direct sum of simple Lie algebras which are completely classified in fifties of the last century. At the same period, the essential progress has been made in the solvable part by Malcev [20]. Malcev [20] reduced the problem of classification of solvable Lie algebras to the classification of nilpotent Lie algebras. Later all the classification results have been related to the nilpotent part.

The theory of Lie algebras is one of the most developed branches of modern algebra. It has been deeply investigated for many years. The investigation of the properties of Lie algebras leads to the generalization of Leibniz algebras.

Leibniz algebra was introduced by Loday [17] as a non-commutative generalization of Lie algebra. It is well-known that the Leibniz identity and the classical Jacobi identity are equivalent, when the multiplication is skew-symmetric. Leibniz algebras inherit an important property of Lie algebras which is the right multiplication operator of a Leibniz algebra is a derivation. Many results on Lie algebras have been extended to Leibniz algebras.

1.1.1 Relations on Leibniz Algebras

1. (Co)homological problems

The (co)homology theory, representations and related problems of Leibniz algebras were studied by Loday and Pirashvili [19] and Frabetti [13]. Problems which related to the group theoretical realizations of Leibniz algebras were studied by Kinyon et al. [16]. A good survey about classification is given by Loday et al. [18]. In 1998, Mikhalev et al. [21] discussed the solution of the noncommutative analogue of the Jacobian conjecture in the affirmative for free Leibniz algebras, in the spirit of the corresponding results of Reutenauer [28], Shpilrain [29] and Umirbaev [31].

2. Deformations and contractions

Deformation theory of Leibniz algebras and its related physical applications, was initiated by Fialowski et al. [12]. The algebraic variety of four-dimensional complex nilpotent Leibniz algebras was suggested by Albeverio et al. [2].

3. Structural problems

In Leibniz algebras, the analogue of Levi's theorem was proved by Barnes [5]. He showed that any finite-dimensional complex Leibniz algebra is decomposed into a semidirect sum of the solvable radical and a semisimple Lie algebra. The semisimple part is composed by simple Lie algebras and the main issue in the classification problem of finite-dimensional complex Leibniz algebras is to study the solvable part. Thus solvable Leibniz algebras still play a central role and have been recently studied by Cañete et al. [6] and Casas et al. [7] which shows the importance of the consideration of their nilradicals in Leibniz algebras in dimensions less than 5. Casas et al. [8, 9] also devoted to the study of solvable Leibniz algebras by considering their nilradicals. The notion of simple Leibniz algebra was suggested by Dzhumadil'daev et al. [11], which obtained some results concerning special cases of simple Leibniz algebras. The problems investigating Cartan subalgebras were studied by Omirov [22]. Albeverio et al. [1] studied on Cartan subalgebras and solvability of Leibniz algebras.

The classification, up to isomorphism, of any class of algebras is a fundamental and very difficult problem. It is one of the first problem that one encounters when trying to understand the structure of a member of this class of algebras. Due to result by Ayupov et al. [4], there are some structural results concerning nilpotency of Leibniz algebras and its multiplication

in the year 2001. Ayupov et al. [4] described the so called complex non Lie filiform Leibniz algebras. In particular case, some equivalent conditions for Leibniz algebra to be filiform and classification of the set of nilpotent Leibniz algebras containing an algebra of maximal nilindex are given as well. Naturally graded complex Leibniz algebras also is characterized. In the year 2006, the classification of 4-dimensional nilpotent complex Leibniz algebras was studied by Albeverio et al. [3].

Gómez et al. [15] devised a method of simplification of the basis transformation of the arbitrary filiform Leibniz algebras obtained from naturally graded non Lie filiform Leibniz algebras. It was considered those transformations that change the structural constants in the description of such filiform Leibniz algebras. In 2011, an approach classifying a class of filiform Leibniz algebras in terms of algebraic invariants was developed by Rakhimov et al. [24]. The method is applicable to any fixed dimensional cases of filiform Leibniz algebras.

The results of algebraic invariants method can be performed in low dimensional cases of complex filiform Leibniz algebras were devoted by Rakhimov et al. [26, 27]. Subclasses of filiform Leibniz algebras appearing from naturally graded non Lie filiform Leibniz algebras in dimensions five until eight are classified named as first class and second class. Classification problem for dimension nine is presented by Sozan et al. [30] for first class case while, Deraman et al. [10] treated second class case.

Especially useful for this thesis is the work by Rakhimov et al. [24] which classified filiform Leibniz algebras in more invariant method. This approach is extended for the classification of second class 10-dimensional filiform Leibniz algebra over field $K = \mathbb{C}$.

1.2 Research Objectives

In this thesis, the classification problem of a class of complex filiform Leibniz algebras arising from naturally graded non-Lie filiform Leibniz algebras is considered. It is known that this class of filiform Leibniz algebras are split into two subclasses. Each of them are denoted as FLb_{n+1} and SLb_{n+1} , in dimension $(n + 1)$, respectively. The investigation of classification problem of SLb_{10} for dimension 10 is followed by some research objectives:

- (1) To study the algebraic and geometric properties of second class complex filiform Leibniz algebras.
- (2) To create isomorphism criterion for second class 10-dimensional complex filiform Leibniz algebras in terms of invariant functions.
- (3) To find the list of isomorphism classes of second class 10-dimensional complex filiform Leibniz algebras.

1.3 Scope of Research

The classification of a class of algebras corresponds to a fiber of this class, that being the isomorphism classes. Classification or algebraic classification means the determination of the types of isomorphic algebras.

This research can be beneficially applied in quantum mechanics and geometrical constructions as well as may triggers in fundamental physic and applied engineering.

1.4 Research Methodology

There are two sources to get classification of complex filiform Leibniz algebras. The first of them is naturally graded non-Lie complex filiform Leibniz algebras and the second one is naturally graded filiform Lie algebras. $(n + 1)$ -dimensional complex filiform Leibniz algebras appearing from naturally graded non Lie filiform Leibniz algebras is split into two subclasses denoted by FLb_{n+1} and SLb_{n+1} respectively. Classification of SLb_{n+1} is only considered in this thesis particularly for $n = 9$. Here some methods on classifying SLb_{10} are presented.

(1) Isomorphism criteria for SLb_{10}

An isomorphism criterion for subclass SLb_{10} is provided where it is expressed by system of equalities. The polynomial equalities are simplified into the simplest form.

(2) Specification of a union of disjoint subsets

The isomorphism criterion of SLb_{10} is split into several disjoint subsets. For each subset, the classification problem is considered separately. Some of these subsets will represented as union parametric families of orbits and others are stated as single orbits.

(3) Algebraic invariant functions and basis change

System of algebraic invariant functions are presented in parametric families of orbits. While in single orbits case, the basis change are leading to the representatives of these orbits.

(4) Adapted linear transformations and elementary basis changing

According to the table of multiplication of SLb_{10} , the action of the reductive linear group, GL_{10} on SLb_{10} is reduced to the action of a subgroup, consist of only transformations sending adapted basis to adapted basis. Then, one defines the so called elementary basis changing and it is proved that an adapted transformation can be represented as a product of the elementary basis changing.

(5) List of non-isomorphic classes

The list of non-isomorphic classes of SLb_{10} are given according to the table of multiplications.

1.5 Outline of Contents

The thesis consists of five chapters. Now, we briefly mention the layout of the thesis.

Chapter 1 introduces the basic concepts of Leibniz algebras.

Chapter 2 describes a method of classification of filiform Leibniz algebras. Here we introduce definition of nilpotent and filiform Leibniz algebras and followed by naturally graded non-Lie filiform Leibniz algebras. Basic concepts about adapted base change, adapted transformations and isomorphism criteria for filiform Leibniz algebras arising from naturally graded non-Lie filiform Leibniz algebras are discussed.

Chapter 3 gives a complete classification of SLb_{10} which is including all possible list of isomorphism classes and invariants of SLb_{10} containing parametric families of orbits and single orbits.

Chapter 4 presents the main result of the classification. All of the representatives in SLb_{10} joined with the table of multiplication of each algebra are given in this chapter.

Chapter 5 contains some summaries on results of the thesis and suggests a few problem for future work in these areas.

BIBLIOGRAPHY

- [1] Albeverio, S., Ayupov, S. A. and Omirov, B. A. (2006). On Cartan subalgebras, weight spaces and criterion of solvability of finite dimensional Leibniz algebras. *Revista Mathematica Complutense*, 19 (1): 183–195.
- [2] Albeverio, S., Omirov, B. A. and Rakhimov, I. S. (2005). Varieties of nilpotent complex Leibniz algebras of dimension less than five. *Communication in Algebra*, 33: 1575–1585.
- [3] Albeverio, S., Omirov, B. A. and Rakhimov, I. S. (2006). Classification of 4-dimensional Complex Leibniz algebras. *Extracta Mathematicae*, 21 (3): 197–210.
- [4] Ayupov, S. A. and Omirov, B. A. (2001). On some classes of nilpotent Leibniz algebras. *Siberian Math. J.*, 42 (1): 18–29.
- [5] Barnes, D. W. (2012). On Levi’s theorem for Leibniz algebras. *Bull. Aust. Math. Soc.*, 86 (2): 184–185.
- [6] Cañete, E. M. and Khudoyberdiyev, A. K. (2013). The classification of 4-dimensional Leibniz algebras. *Linear Algebra and its Applications*, 439 (1): 273–288.
- [7] Casas, J. M., Insua, M. A., Ladra, M. and Ladra, S. (2012). An algorithm for the classification of 3-dimensional complex Leibniz algebras. *Linear Algebra and its Application*, 436: 3747–3756.
- [8] Casas, J. M., Ladra, M., Omirov, B. A. and Karimjanov, I. A. (2013). Classification of solvable Leibniz algebras with naturally graded filiform nilradical. *Linear Algebra and its Applications*, 438 (7): 2973–3000.
- [9] Casas, J. M., Ladra, M., Omirov, B. A. and Karimjanov, I. A. (2013). Classification of solvable Leibniz algebras with null-filiform nilradical. *Linear and Multilinear Algebra*, 61 (6): 758–774.
- [10] Deraman, F., Rakhimov, I. S. and Said Husain, S. K. (2012). Isomorphism classes and invariants for a subclass of nine-dimensional filiform Leibniz algebras. *AIP Conference Proceedings*, 1450: 326–331.
- [11] Dzhumadil’dav, A. and Abdykassymova, S. (2001). Leibniz algebras in characteristic p . *C.R. Acad. Sci., Paris*, 332: 1047–1052. Serie I.
- [12] Fialowski, A., Mandal, A. and Mukherjee, G. (2009). Versal deformations of Leibniz algebras. *Journal of K-theory: K-theory and its Applications to Algebra, Geometry, and Topology*, 3: 327–358.
- [13] Frabetti, A. (1998). Leibniz homology of dialgebras of matrices. *Journal Pure and Applied Algebra*, 129: 123–141.
- [14] Gómez, J. R. and Khakimjanov, Y. (1998). Low dimensional filiform Lie algebras. *Journal of Pure and Applied Algebra*, 130: 133–158.

- [15] Gómez, J. R. and Omirov, B. A. (2012). On classification of complex filiform Leibniz algebras. *Available at* arXiv:math/0612735v2 [math.RA] .
- [16] Kinyon, M. K. and Weinstein, A. (2001). Leibniz algebras, Courant algebroids, and multiplications on reductive homogeneous spaces. *American Journal of Mathematics*, 123 (3): 525–550.
- [17] Loday, J. L. (1993). Une version non commutative des algèbres de Lie: les algèbres de Leibniz. *Ens. Math.*, 39: 269–293.
- [18] Loday, J. L., Frabetti, A., Chapoton, F. and Goichot, F. (2001). Dialgebras and related operads. *Springer-Verlag*, IV.
- [19] Loday, J. L. and Pirashvili, T. (1993). Universal enveloping algebras of Leibniz algebras and (co)homology. *Math. Ann.*, 296: 139–158.
- [20] Malcev, A. (1945). On solvable Lie algebras. *Amer. Math. Soc. Transl.*, 9: 228–262.
- [21] Mikhalev, A. A. and Umirbaev, U. U. (1998). Subalgebras of free Leibniz algebras. *Communication Algebra*, 26: 435–446.
- [22] Omirov, B. A. (2006). Conjugacy of Cartan subalgebras of complex finite-dimensional Leibniz algebras. *Journal of Algebra*, 302: 887–896.
- [23] Omirov, B. A. and Rakhimov, I. S. (2009). On Lie-like complex filiform Leibniz algebras. *Bull. Aust. Math. Soc.*, 79: 391–404.
- [24] Rakhimov, I. S. and Bekbaev, U. D. (2010). On isomorphisms and invariants of finite dimensional complex filiform Leibniz algebras. *Communications in Algebra*, 38 (12): 4705–4738.
- [25] Rakhimov, I. S. and Hassan, M. A. (2011). On low-dimensional filiform Leibniz algebras and their invariants. *Bull. Malaysian Mathematical Science Society (2)*, 34 (3): 475–485.
- [26] Rakhimov, I. S. and Said Husain, S. K. (2011). Classification of a subclass of low-dimensional complex filiform Leibniz algebras. *Linear and Multilinear Algebra*, 59 (3): 339–354.
- [27] Rakhimov, I. S. and Said Husain, S. K. (2011). On isomorphism classes and invariants of a subclass of low-dimensional complex filiform Leibniz algebras. *Linear and Multilinear Algebra*, 59 (2): 205–220.
- [28] Reutenauer, C. (1992). Applications of a noncommutative Jacobian matrix. *Journal of Pure and Applied Algebra* , 77: 169–181.
- [29] Shpilran, V. (1993). On generators of L/R^2 Lie algebras. *Proceedings of the American Mathematical Society*, 119: 1039–1043.

- [30] Sozan, J., Rakhimov, I. S. and Mohd Atan, K. A. (2010). *Classification of first class of complex filiform Leibniz algebras*. Lambert Academic Publishing.
- [31] Umirbaev, U. U. (1993). Partial derivatives and endomorphisms of some relatively free Lie algebras. *Sibirskii Matematicheskii Zhurnal*, 34 (6): 179–188.



Procedure

```

F := proc (n) local  $\beta, t, s, \Gamma$ ;
if  $n \leq 3$  then return 0; end if;
for t from 3 to n do
if  $t = 3$  then q:=0; else  $q := 1$ ; end if;

```

$$\beta_t := \frac{BD\gamma}{A^t} + \frac{1}{A^{t-1}} \left(Db_t - \sum_{k=3}^{t-1} \left(\left(\prod_{i=1}^1 (k-i) A^{k-2} B b_{t+2-k} \right) + \left(\frac{\prod_{i=1}^2 (k-i)}{2} A^{k-3} B^2 \sum_{i[1]=k+2}^t b_{t+3-i_1} b_{i_1+1-k} \right) + \left(\frac{\prod_{i=1}^3 (k-i)}{6} A^{k-4} B^3 \sum_{i_3=k+4}^t \right. \right. \right. \\ \left. \left. \sum_{i_2=k+3}^t \left(\sum_{i_1=k+3}^{i_2} b_{t+3-i_2} b_{i_2+3-i_1} b_{i_1-k} \right) \right) + \left(\frac{\prod_{i=1}^4 (k-i)}{24} A^{k-5} B^4 \sum_{i_3=k+4}^t \left(\sum_{i_2=k+4}^{i_3} \left(\sum_{i_1=k+4}^{i_2} b_{t+3-i_3} b_{i_3+3-i_2} b_{i_2+3-i_1} b_{i_1-1-k} \right) \right) \right) \beta_k \right);$$

```
end do;
```

```
for s from 3 to n do
```

```
print (simplify( $\beta_s$ ));
```

```
end do;
```

$$\Gamma := \frac{1}{A^{n-2}} \left(\frac{D}{A} \right)^2 \gamma,$$

```
print (simplify( $\Gamma$ ));
```

```
end proc;
```

APPENDIX B

Classification Procedures

This part contains some procedures of classification of subclass SLb_{10} . It is known that SLb_{10} is divided into two cases: (i) SLb_{10}^a and (ii) SLb_{10}^b . Section B.1 presents classification procedures in SLb_{10}^a while Section B.2 gives a few methods of classification of subclass SLb_{10}^b .

B.1 Classification of SLb_{10}^a

The isomorphism criteria for SLb_{10}^a can be expressed as follows:

$$\begin{aligned} \Lambda'_3 &= \frac{1}{A} \left(\frac{D}{A} \right) \Lambda_3, & f_7(\Lambda') &= \frac{1}{A^8} \left(\frac{D}{A} \right)^4 f_7(\Lambda), \\ \Lambda'_4 &= 0, & f_8(\Lambda') &= \frac{1}{A^{10}} \left(\frac{D}{A} \right)^5 f_8(\Lambda), \\ \Lambda'_5 &= \frac{1}{A^4} \left(\frac{D}{A} \right)^2 \Lambda_5, & f_9(\Lambda') &= \frac{1}{A^{12}} \left(\frac{D}{A} \right)^6 f_9(\Lambda), \\ f_6(\Lambda') &= \frac{1}{A^6} \left(\frac{D}{A} \right)^3 f_6(\Lambda), & \Gamma' &= \frac{1}{A^7} \left(\frac{D}{A} \right)^2 \Gamma. \end{aligned} \quad (B.1)$$

Under condition $\Lambda_3 \neq 0$, the system (B.1) is isomorphic to the following system:

$$\begin{aligned} \Lambda'_3 &= 1, & \frac{f_7(\Lambda')}{\Lambda_3'^4} &= \frac{1}{A^4} \frac{f_7(\Lambda)}{\Lambda_3^4}, \\ \Lambda'_4 &= 0, & \frac{f_8(\Lambda')}{\Lambda_3'^5} &= \frac{1}{A^5} \frac{f_8(\Lambda)}{\Lambda_3^5}, \\ \frac{\Lambda'_5}{\Lambda_3'^2} &= \frac{1}{A^2} \frac{\Lambda_5}{\Lambda_3^2}, & \frac{f_9(\Lambda')}{\Lambda_3'^6} &= \frac{1}{A^6} \frac{f_9(\Lambda)}{\Lambda_3^6}, \\ \frac{f_6(\Lambda')}{\Lambda_3'^3} &= \frac{1}{A^3} \frac{f_6(\Lambda)}{\Lambda_3^3}, & \frac{\Gamma'}{\Lambda_3'^2} &= \frac{1}{A^5} \frac{\Gamma}{\Lambda_3^2}. \end{aligned} \quad (B.2)$$

It shows that $SLb_{10}^a = F_1 \cup F_2$, where $F_1 = \{L(\Lambda) \in SLb_{10}^a : \Lambda_3 \neq 0, \Lambda_5 \neq 0\}$ and $F_2 = \{L(\Lambda) \in SLb_{10}^a : \Lambda_3 \neq 0, \Lambda_5 = 0\}$.

(i) $F_1 = \{L(\Lambda) \in SLb_{10}^a : \Lambda_3 \neq 0, \Lambda_5 \neq 0\}.$

(a) $F_1 = U_1 \cup F_3 :$

- $U_1 = \{L(\Lambda) \in SLb_{10}^a : \Lambda_3 \neq 0, \Lambda_5 \neq 0, f_6(\Lambda) \neq 0\}.$

By reducing $\Lambda'_3 = 1$ and $\Lambda'_4 = 0$, then we obtain $A = \frac{f_6(\Lambda)}{\Lambda_3\Lambda_5}$ through conditions $\Lambda_5 \neq 0$ and $f_6(\Lambda) \neq 0$ in (B.2). Automatically, the values of $B = \frac{\Lambda_4 A}{2\Lambda_3^3\Lambda_5}$ and $D = \frac{A^2}{\Lambda_3}$ are obtained. Subset U_1 brings the following system of equations:

$$\begin{aligned} \Lambda'_3 &= 1, \\ \Lambda'_4 &= 0, \\ \frac{\Lambda'_5}{\Lambda_3'^2} &= \left(\frac{\Lambda_3\Lambda_5}{f_6(\Lambda)} \right)^2 \frac{\Lambda_5}{\Lambda_3^2}, \\ \frac{f_6(\Lambda')}{\Lambda_3'^3} &= \left(\frac{\Lambda_3\Lambda_5}{f_6(\Lambda)} \right)^3 \frac{f_6(\Lambda)}{\Lambda_3^3}, \\ \frac{f_7(\Lambda')}{\Lambda_3'^4} &= \left(\frac{\Lambda_3\Lambda_5}{f_6(\Lambda)} \right)^4 \frac{f_7(\Lambda)}{\Lambda_3^4}, \\ \frac{f_8(\Lambda')}{\Lambda_3'^5} &= \left(\frac{\Lambda_3\Lambda_5}{f_6(\Lambda)} \right)^5 \frac{f_8(\Lambda)}{\Lambda_3^5}, \\ \frac{f_9(\Lambda')}{\Lambda_3'^6} &= \left(\frac{\Lambda_3\Lambda_5}{f_6(\Lambda)} \right)^6 \frac{f_9(\Lambda)}{\Lambda_3^6}, \\ \frac{\Gamma'}{\Lambda_3'^2} &= \left(\frac{\Lambda_3\Lambda_5}{f_6(\Lambda)} \right)^5 \frac{\Gamma}{\Lambda_3^2}. \end{aligned} \tag{B.3}$$

New parameters are represented as $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ and λ_5 to imply that $\Lambda'_5 = \frac{\Lambda_5^3}{f_6^2(\Lambda)} = \lambda_1$, $f_6(\Lambda') = \frac{\Lambda_5^3}{f_6^2(\Lambda)} = \lambda_1$, $f_7(\Lambda') = \frac{f_7(\Lambda)\Lambda_5^4}{f_6^4(\Lambda)} = \lambda_2$, $f_8(\Lambda') = \frac{f_8(\Lambda)\Lambda_5^5}{f_6^5(\Lambda)} = \lambda_3$, $f_9(\Lambda') = \frac{f_9(\Lambda)\Lambda_5^6}{f_6^6(\Lambda)} = \lambda_4$ and $\Gamma' = \frac{\Lambda_3^3\Lambda_5^5\Gamma}{f_6^5(\Lambda)} = \lambda_5$. Thus all algebras from U_1 are isomorphic to $L(1, 0, \lambda_1, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5)$ for $\lambda_1 \in \mathbb{C}^*$ and $\lambda_2, \lambda_3, \lambda_4, \lambda_5 \in \mathbb{C}$.

- $F_3 = \{L(\Lambda) \in SLb_{10}^a : \Lambda_3 \neq 0, \Lambda_5 \neq 0, f_6(\Lambda) = 0\}$.
Through the conditions in F_3 , the system of equations in (B.2) becomes as follows:

$$\begin{aligned}
\Lambda_3' &= 1, & \frac{f_7(\Lambda')}{\Lambda_3'^4} &= \frac{1}{A^4} \frac{f_7(\Lambda)}{\Lambda_3^4}, \\
\Lambda_4' &= 0, & \frac{f_8(\Lambda')}{\Lambda_3'^5} &= \frac{1}{A^5} \frac{f_8(\Lambda)}{\Lambda_3^5}, \\
\frac{\Lambda_5'}{\Lambda_3'^2} &= \frac{1}{A^2} \frac{\Lambda_5}{\Lambda_3^2}, & \frac{f_9(\Lambda')}{\Lambda_3'^6} &= \frac{1}{A^6} \frac{f_9(\Lambda)}{\Lambda_3^6}, \\
\frac{f_6(\Lambda')}{\Lambda_3'^3} &= 0, & \frac{\Gamma'}{\Lambda_3'^2} &= \frac{1}{A^5} \frac{\Gamma}{\Lambda_3^2}.
\end{aligned} \tag{B.4}$$

We can rewrite F_3 as $F_3 = U_2 \cup F_4$, where

$$\begin{aligned}
U_2 &= \{L(\Lambda) \in SLb_{10}^a : \Lambda_3 \neq 0, \Lambda_5 \neq 0, f_6(\Lambda) = 0, f_8(\Lambda) \neq 0\} \text{ and} \\
F_4 &= \{L(\Lambda) \in SLb_{10}^a : \Lambda_3 \neq 0, \Lambda_5 \neq 0, f_6(\Lambda) = 0, f_8(\Lambda) = 0\}.
\end{aligned}$$

(b) $F_3 = U_2 \cup F_4$:

- $U_2 = \{L(\Lambda) \in SLb_{10}^a : \Lambda_3 \neq 0, \Lambda_5 \neq 0, f_6(\Lambda) = 0, f_8(\Lambda) \neq 0\}$.

By reducing $\frac{\Lambda_5'}{\Lambda_3'^2} = \frac{1}{A^2} \frac{\Lambda_5}{\Lambda_3^2}$ and $\frac{f_8(\Lambda')}{\Lambda_3'^5} = \frac{1}{A^5} \frac{f_8(\Lambda)}{\Lambda_3^5}$ in (B.4), then we obtain

$$A = \frac{f_8(\Lambda)}{\Lambda_3 \Lambda_5^2}. \text{ The values of } B \text{ and } D \text{ appear as } B = \frac{\Lambda_4 A}{2 \Lambda_3 \Lambda_5} \text{ and } D = \frac{A^2}{\Lambda_3}.$$

Then by using value $\frac{1}{A} = \frac{\Lambda_3 \Lambda_5^2}{f_8(\Lambda)}$, the system of equalities for U_2 is as follows:

$$\begin{aligned}
\Lambda_3' &= 1, \\
\Lambda_4' &= 0, \\
\frac{\Lambda_5'}{\Lambda_3'^2} &= \left(\frac{\Lambda_3 \Lambda_5^2}{f_8(\Lambda)} \right)^2 \frac{\Lambda_5}{\Lambda_3^2}, \\
\frac{f_6(\Lambda')}{\Lambda_3'^3} &= 0, \\
\frac{f_7(\Lambda')}{\Lambda_3'^4} &= \left(\frac{\Lambda_3 \Lambda_5^2}{f_8(\Lambda)} \right)^4 \frac{f_7(\Lambda)}{\Lambda_3^4}, \\
\frac{f_8(\Lambda')}{\Lambda_3'^5} &= \left(\frac{\Lambda_3 \Lambda_5^2}{f_8(\Lambda)} \right)^5 \frac{f_8(\Lambda)}{\Lambda_3^5}, \\
\frac{f_9(\Lambda')}{\Lambda_3'^6} &= \left(\frac{\Lambda_3 \Lambda_5^2}{f_8(\Lambda)} \right)^6 \frac{f_9(\Lambda)}{\Lambda_3^6}, \\
\frac{\Gamma'}{\Lambda_3'^2} &= \left(\frac{\Lambda_3 \Lambda_5^2}{f_8(\Lambda)} \right)^5 \frac{\Gamma}{\Lambda_3^2}.
\end{aligned} \tag{B.5}$$

New parameters are represented as $\lambda_1, \lambda_2, \lambda_3$ and λ_4 to imply that $\Lambda'_5 = \frac{\Lambda_5^5}{f_8^2(\Lambda)} = \lambda_1$, $f_7(\Lambda') = \frac{\Lambda_5^8 f_7(\Lambda)}{f_8^4(\Lambda)} = \lambda_2$, $f_8(\Lambda') = \frac{\Lambda_5^{10}}{f_8^4(\Lambda)} = \lambda_1^2$, $f_9(\Lambda') = \frac{f_9(\Lambda) \Lambda_5^{12}}{f_8^6(\Lambda)} = \lambda_3$ and $\Gamma' = \frac{\Lambda_3^3 \Lambda_5^{10} \Gamma}{f_8^5(\Lambda)} = \lambda_4$. Thus all algebras from U_2 are isomorphic to $L(1, 0, \lambda_1, 0, \lambda_2, \lambda_1^2, \lambda_3, \lambda_4)$ for $\lambda_1 \in \mathbb{C}^*$ and $\lambda_2, \lambda_3, \lambda_4 \in \mathbb{C}$.

- $F_4 = \{L(\Lambda) \in SLb_{10}^a : \Lambda_3 \neq 0, \Lambda_5 \neq 0, f_6(\Lambda) = 0, f_8(\Lambda) = 0\}$.
The system of equalities in F_4 is as below:

$$\begin{aligned} \Lambda'_3 &= 1, \\ \Lambda'_4 &= 0, \\ \frac{\Lambda'_5}{\Lambda_3'^2} &= \frac{1}{A^2} \frac{\Lambda_5}{\Lambda_3^2}, \\ \frac{f_6(\Lambda')}{\Lambda_3'^3} &= 0, \\ \frac{f_7(\Lambda')}{\Lambda_3'^4} &= \frac{1}{A^4} \frac{f_7(\Lambda)}{\Lambda_3^4}, \\ \frac{f_8(\Lambda')}{\Lambda_3'^5} &= 0, \\ \frac{f_9(\Lambda')}{\Lambda_3'^6} &= \frac{1}{A^6} \frac{f_9(\Lambda)}{\Lambda_3^6}, \\ \frac{\Gamma'}{\Lambda_3'^2} &= \frac{1}{A^5} \frac{\Gamma}{\Lambda_3^2}. \end{aligned} \tag{B.6}$$

The subset of F_4 can be written as $F_4 = U_3 \cup U_4$ where $U_3 = \{L(\Lambda) \in SLb_{10}^a : \Lambda_3 \neq 0, \Lambda_5 \neq 0, f_6(\Lambda) = 0, f_8(\Lambda) = 0, \Gamma \neq 0\}$ and $U_4 = \{L(\Lambda) \in SLb_{10}^a : \Lambda_3 \neq 0, \Lambda_5 \neq 0, f_6(\Lambda) = 0, f_8(\Lambda) = 0, \Gamma = 0\}$.

(c) For $F_4 = U_3 \cup U_4$:

- $U_3 = \{L(\Lambda) \in SLb_{10}^a : \Lambda_3 \neq 0, \Lambda_5 \neq 0, f_6(\Lambda) = 0, f_8(\Lambda) = 0, \Gamma \neq 0\}$

By reducing $\frac{\Lambda'_5}{\Lambda_3'^2} = \frac{1}{A^2} \frac{\Lambda_5}{\Lambda_3^2}$ and $\frac{\Gamma'}{\Lambda_3'^2} = \frac{1}{A^5} \frac{\Gamma}{\Lambda_3^2}$ in (B.6), then we obtain

$A = \frac{\Lambda_3^2 \Gamma}{\Lambda_5^2}$. The values of B and D appear as $B = \frac{\Lambda_4 A}{2 \Lambda_3^3 \Lambda_5}$ and $D = \frac{A^2}{\Lambda_3}$.

Then by using value $\frac{1}{A} = \frac{\Lambda_5^2}{\Lambda_3^2 \Gamma}$, the system of equalities for U_3 is as follows:

$$\begin{aligned}
 \Lambda_3' &= 1, \\
 \Lambda_4' &= 0, \\
 \frac{\Lambda_5'}{\Lambda_3'^2} &= \left(\frac{\Lambda_5^2}{\Lambda_3^2 \Gamma} \right)^2 \frac{\Lambda_5}{\Lambda_3^2}, \\
 \frac{f_6(\Lambda')}{\Lambda_3'^3} &= 0, \\
 \frac{f_7(\Lambda')}{\Lambda_3'^4} &= \left(\frac{\Lambda_5^2}{\Lambda_3^2 \Gamma} \right)^4 \frac{f_7(\Lambda)}{\Lambda_3^4}, \\
 \frac{f_8(\Lambda')}{\Lambda_3'^5} &= 0, \\
 \frac{f_9(\Lambda')}{\Lambda_3'^6} &= \left(\frac{\Lambda_5^2}{\Lambda_3^2 \Gamma} \right)^6 \frac{f_9(\Lambda)}{\Lambda_3^6}, \\
 \frac{\Gamma'}{\Lambda_3'^2} &= \left(\frac{\Lambda_5^2}{\Lambda_3^2 \Gamma} \right)^5 \frac{\Gamma}{\Lambda_3^2}.
 \end{aligned} \tag{B.7}$$

New parameters are represented as λ_1, λ_2 and λ_3 to imply that $\Lambda_5' = \frac{\Lambda_5^5}{\Lambda_3^6 \Gamma^2} = \lambda_1$, $f_7(\Lambda') = \frac{\Lambda_5^8 f_7(\Lambda)}{\Lambda_3^{12} \Gamma^4} = \lambda_2$, $f_9(\Lambda') = \frac{f_9(\Lambda) \Lambda_5^{12}}{\Lambda_3^{18} \Gamma^6} = \lambda_3$ and $\Gamma^6 = \frac{\Lambda_5^{10}}{\Lambda_3^{12} \Gamma^4} = \lambda_1$. Thus all algebras from U_3 are isomorphic to $L(1, 0, \lambda_1, 0, \lambda_2, 0, \lambda_3, \lambda_1^2)$ for $\lambda_1 \in \mathbb{C}^*$ and $\lambda_2, \lambda_3 \in \mathbb{C}$.

- $U_4 = \{L(\Lambda) \in SLb_{10}^a : \Lambda_3 \neq 0, \Lambda_5 \neq 0, f_6(\Lambda) = 0, f_8(\Lambda) = 0, \Gamma = 0\}$.

By reducing $\frac{\Lambda_5'}{\Lambda_3'^2} = \frac{1}{A^2} \frac{\Lambda_5}{\Lambda_3^2}$ in (B.6), then we obtain $A^2 = \frac{\Lambda_5}{\Lambda_3^2}$. The values of

B and D appear as $B = \frac{\Lambda_4 A}{2 \Lambda_3^3 \Lambda_5}$ and $D = \frac{A^2}{\Lambda_3}$.

Then by using value $\frac{1}{A^2} = \frac{\Lambda_3^2}{\Lambda_5}$, the system of equalities for U_4 is as follows:

$$\begin{aligned}
 \Lambda_3' &= 1, \\
 \Lambda_4' &= 0, \\
 \frac{\Lambda_5'}{\Lambda_3'^2} &= \left(\frac{\Lambda_3^2}{\Lambda_5} \right) \frac{\Lambda_5}{\Lambda_3^2}, \\
 \frac{f_6(\Lambda')}{\Lambda_3'^3} &= 0, \\
 \frac{f_7(\Lambda')}{\Lambda_3'^4} &= \left(\frac{\Lambda_3^2}{\Lambda_5} \right)^2 \frac{f_7(\Lambda)}{\Lambda_3^4}, \\
 \frac{f_8(\Lambda')}{\Lambda_3'^5} &= 0, \\
 \frac{f_9(\Lambda')}{\Lambda_3'^6} &= \left(\frac{\Lambda_3^2}{\Lambda_5} \right)^3 \frac{f_9(\Lambda)}{\Lambda_3^6}, \\
 \frac{\Gamma'}{\Lambda_3'^2} &= 0.
 \end{aligned} \tag{B.8}$$

New parameters are represented as λ_1 and λ_2 to imply that $\Lambda_5' = 1$, $f_7(\Lambda') = \frac{f_7(\Lambda)}{\Lambda_5^2} = \lambda_1$, $f_9(\Lambda') = \frac{f_9(\Lambda)}{\Lambda_5^3} = \lambda_2$. Thus all algebras from U_4 are isomorphic to $L(1, 0, 1, 0, \lambda_1, 0, \lambda_2, 0)$ for $\lambda_1, \lambda_2 \in \mathbb{C}$.

- (ii) $F_2 = \{L(\Lambda) \in SLb_{10}^a : \Lambda_3 \neq 0, \Lambda_5 = 0\}$

The subset F_2 brings the following system of equations:

$$\begin{aligned}
 \Lambda_3' &= 1, & \frac{f_7(\Lambda')}{\Lambda_3'^4} &= \frac{1}{A^4} \frac{f_7(\Lambda)}{\Lambda_3^4}, \\
 \Lambda_4' &= 0, & \frac{f_8(\Lambda')}{\Lambda_3'^5} &= \frac{1}{A^5} \frac{f_8(\Lambda)}{\Lambda_3^5}, \\
 \frac{\Lambda_5'}{\Lambda_3'^2} &= 0, & \frac{f_9(\Lambda')}{\Lambda_3'^6} &= \frac{1}{A^6} \frac{f_9(\Lambda)}{\Lambda_3^6}, \\
 \frac{f_6(\Lambda')}{\Lambda_3'^3} &= \frac{1}{A^3} \frac{f_6(\Lambda)}{\Lambda_3^3}, & \frac{\Gamma'}{\Lambda_3'^2} &= \frac{1}{A^5} \frac{\Gamma}{\Lambda_3^2}.
 \end{aligned} \tag{B.9}$$

(a) $F_2 = F_5 \cup F_6 :$

- $F_5 = \{L(\Lambda) \in SLb_{10}^a : \Lambda_3 \neq 0, \Lambda_5 = 0, f_6(\Lambda) \neq 0\}$

The subset F_5 brings the following system of equations:

$$\begin{aligned} \Lambda_3' &= 1, & \frac{f_7(\Lambda')}{\Lambda_3'^4} &= \frac{1}{A^4} \frac{f_7(\Lambda)}{\Lambda_3^4}, \\ \Lambda_4' &= 0, & \frac{f_8(\Lambda')}{\Lambda_3'^5} &= \frac{1}{A^5} \frac{f_8(\Lambda)}{\Lambda_3^5}, \\ \frac{\Lambda_5'}{\Lambda_3'^2} &= 0, & \frac{f_9(\Lambda')}{\Lambda_3'^6} &= \frac{1}{A^6} \frac{f_9(\Lambda)}{\Lambda_3^6}, \\ \frac{f_6(\Lambda')}{\Lambda_3'^3} &= \frac{1}{A^3} \frac{f_6(\Lambda)}{\Lambda_3^3}, & \frac{\Gamma'}{\Lambda_3'^2} &= \frac{1}{A^5} \frac{\Gamma}{\Lambda_3^2}. \end{aligned} \quad (\text{B.10})$$

- $F_6 = \{L(\Lambda) \in SLb_{10}^a : \Lambda_3 \neq 0, \Lambda_5 = 0, f_6(\Lambda) = 0\}$

The system of equations of subset F_6 is as following:

$$\begin{aligned} \Lambda_3' &= 1, & \frac{f_7(\Lambda')}{\Lambda_3'^4} &= \frac{1}{A^4} \frac{f_7(\Lambda)}{\Lambda_3^4}, \\ \Lambda_4' &= 0, & \frac{f_8(\Lambda')}{\Lambda_3'^5} &= \frac{1}{A^5} \frac{f_8(\Lambda)}{\Lambda_3^5}, \\ \frac{\Lambda_5'}{\Lambda_3'^2} &= 0, & \frac{f_9(\Lambda')}{\Lambda_3'^6} &= \frac{1}{A^6} \frac{f_9(\Lambda)}{\Lambda_3^6}, \\ \frac{f_6(\Lambda')}{\Lambda_3'^3} &= 0, & \frac{\Gamma'}{\Lambda_3'^2} &= \frac{1}{A^5} \frac{\Gamma}{\Lambda_3^2}. \end{aligned} \quad (\text{B.11})$$

(b) $F_5 = U_5 \cup F_7 :$

- $U_5 = \{L(\Lambda) \in SLb_{10}^a : \Lambda_3 \neq 0, \Lambda_5 = 0, f_6(\Lambda) \neq 0, f_7(\Lambda) \neq 0\}$

By reducing $\frac{f_6(\Lambda')}{\Lambda_3'^3} = \frac{1}{A^3} \frac{f_6(\Lambda)}{\Lambda_3^3}$ and $\frac{f_7(\Lambda')}{\Lambda_3'^4} = \frac{1}{A^4} \frac{f_7(\Lambda)}{\Lambda_3^4}$ in (B.10), then we obtain $A = \frac{f_7(\Lambda)}{\Lambda_3 f_6(\Lambda)}$. The values of B and D appear as $B = \frac{\Lambda_4 A}{2\Lambda_3^3 \Lambda_5}$ and $D = \frac{A^2}{\Lambda_3}$. Then by using value $\frac{1}{A} = \frac{\Lambda_3 f_6(\Lambda)}{f_7(\Lambda)}$, the system of equalities for U_5 is as follows:

$$\begin{aligned}
\Lambda_3' &= 1, \\
\Lambda_4' &= 0, \\
\frac{\Lambda_5'}{\Lambda_3'^2} &= 0, \\
\frac{f_6(\Lambda')}{\Lambda_3'^3} &= \left(\frac{\Lambda_3 f_6(\Lambda)}{f_7(\Lambda)} \right)^3 \frac{f_6(\Lambda)}{\Lambda_3^3}, \\
\frac{f_7(\Lambda')}{\Lambda_3'^4} &= \left(\frac{\Lambda_3 f_6(\Lambda)}{f_7(\Lambda)} \right)^4 \frac{f_7(\Lambda)}{\Lambda_3^4}, \\
\frac{f_8(\Lambda')}{\Lambda_3'^5} &= \left(\frac{\Lambda_3 f_6(\Lambda)}{f_7(\Lambda)} \right)^5 \frac{f_8(\Lambda)}{\Lambda_3^5}, \\
\frac{f_9(\Lambda')}{\Lambda_3'^6} &= \left(\frac{\Lambda_3 f_6(\Lambda)}{f_7(\Lambda)} \right)^6 \frac{f_9(\Lambda)}{\Lambda_3^6}, \\
\frac{\Gamma'}{\Lambda_3'^2} &= \left(\frac{\Lambda_3 f_6(\Lambda)}{f_7(\Lambda)} \right)^5 \frac{\Gamma}{\Lambda_3^2}.
\end{aligned} \tag{B.12}$$

New parameters are represented as $\lambda_1, \lambda_2, \lambda_3$ and λ_4 to imply that $f_6(\Lambda') = \frac{f_6^4(\Lambda)}{f_7^3(\Lambda)} = \lambda_1$, $f_7(\Lambda') = \frac{f_6^4(\Lambda)}{f_7^3(\Lambda)} = \lambda_1$, $f_8(\Lambda') = \frac{f_6^5(\Lambda)f_8(\Lambda)}{f_7^5(\Lambda)} = \lambda_2$, $f_9(\Lambda') = \frac{f_6^6(\Lambda)f_9(\Lambda)}{f_7^6(\Lambda)} = \lambda_3$ and $\Gamma' = \frac{\Lambda_3^3 f_6^5(\Lambda)\Gamma}{f_7^5(\Lambda)} = \lambda_4$. Thus all algebras from U_5 are isomorphic to $L(1, 0, 0, \lambda_1, \lambda_1, \lambda_2, \lambda_3, \lambda_4)$ for $\lambda_1 \in \mathbb{C}^*$ and $\lambda_2, \lambda_3, \lambda_4 \in \mathbb{C}$.

- $F_7 = \{L(\Lambda) \in SLb_{10}^a : \Lambda_3 \neq 0, \Lambda_5 = 0, f_6(\Lambda) \neq 0, f_7(\Lambda) = 0\}$
Under the conditions of subset F_7 , the system of equations appear as below:

$$\begin{aligned}
\Lambda_3' &= 1, & \frac{f_7(\Lambda')}{\Lambda_3'^4} &= 0, \\
\Lambda_4' &= 0, & \frac{f_8(\Lambda')}{\Lambda_3'^5} &= \frac{1}{A^5} \frac{f_8(\Lambda)}{\Lambda_3^5}, \\
\frac{\Lambda_5'}{\Lambda_3'^2} &= 0, & \frac{f_9(\Lambda')}{\Lambda_3'^6} &= \frac{1}{A^6} \frac{f_9(\Lambda)}{\Lambda_3^6}, \\
\frac{f_6(\Lambda')}{\Lambda_3'^3} &= \frac{1}{A^3} \frac{f_6(\Lambda)}{\Lambda_3^3}, & \frac{\Gamma'}{\Lambda_3'^2} &= \frac{1}{A^5} \frac{\Gamma}{\Lambda_3^2}.
\end{aligned} \tag{B.13}$$

(c) $F_7 = U_6 \cup F_8$:

- $U_6 = \{L(\Lambda) \in SLb_{10}^a : \Lambda_3 \neq 0, \Lambda_5 = 0, f_6(\Lambda) \neq 0, f_7(\Lambda) = 0, f_8(\Lambda) \neq 0\}$.

By reducing $\frac{f_6(\Lambda')}{\Lambda_3'^3} = \frac{1}{A^3} \frac{f_6(\Lambda)}{\Lambda_3^3}$ and $\frac{f_8(\Lambda')}{\Lambda_3'^5} = \frac{1}{A^5} \frac{f_8(\Lambda)}{\Lambda_3^5}$ in (B.13), then we obtain $A = \frac{f_6^2(\Lambda)}{\Lambda_3 f_8(\Lambda)}$. The values of B and D appear as $B = \frac{\Lambda_4 A}{2\Lambda_3^3 \Lambda_5}$ and $D = \frac{A^2}{\Lambda_3}$. Then by using value $\frac{1}{A} = \frac{\Lambda_3 f_8(\Lambda)}{f_6^2(\Lambda)}$, the system of equalities for U_6 is as the following page:

$$\begin{aligned}
 \Lambda_3' &= 1 \\
 \Lambda_4' &= 0 \\
 \Lambda_5' &= 0 \\
 \Lambda_3'^2 &= 0 \\
 \frac{f_6(\Lambda')}{\Lambda_3'^3} &= \left(\frac{\Lambda_3 f_8(\Lambda)}{f_6^2(\Lambda)} \right)^3 \frac{f_6(\Lambda)}{\Lambda_3^3} \\
 \frac{f_7(\Lambda')}{\Lambda_3'^4} &= 0 \\
 \frac{f_8(\Lambda')}{\Lambda_3'^5} &= \left(\frac{\Lambda_3 f_8(\Lambda)}{f_6^2(\Lambda)} \right)^5 \frac{f_8(\Lambda)}{\Lambda_3^5} \\
 \frac{f_9(\Lambda')}{\Lambda_3'^6} &= \left(\frac{\Lambda_3 f_8(\Lambda)}{f_6^2(\Lambda)} \right)^6 \frac{f_9(\Lambda)}{\Lambda_3^6} \\
 \frac{\Gamma'}{\Lambda_3'^2} &= \left(\frac{\Lambda_3 f_8(\Lambda)}{f_6^2(\Lambda)} \right)^5 \frac{\Gamma}{\Lambda_3^2}
 \end{aligned} \tag{B.14}$$

New parameters are represented as λ_1, λ_2 and λ_3 to imply that $f_6(\Lambda') = \frac{f_8^3(\Lambda)}{f_6^5(\Lambda)} = \lambda_1$, $f_8(\Lambda') = \frac{f_8^6(\Lambda)}{f_6^{10}(\Lambda)} = \lambda_1^2$, $f_9(\Lambda') = \frac{f_8^6(\Lambda)f_9(\Lambda)}{f_6^{12}(\Lambda)} = \lambda_2$ and $\Gamma' = \frac{\Lambda_3^3 f_8^5(\Lambda)\Gamma}{f_6^{10}(\Lambda)} = \lambda_3$. Thus all algebras from U_6 are isomorphic to $L(1, 0, 0, \lambda_1, 0, \lambda_1^2, \lambda_2, \lambda_3)$ for $\lambda_1 \in \mathbb{C}^*$ and $\lambda_2, \lambda_3 \in \mathbb{C}$.

- $F_8 = \{L(\Lambda) \in SLb_{10}^a : \Lambda_3 \neq 0, \Lambda_5 = 0, f_6(\Lambda) \neq 0, f_7(\Lambda) = 0, f_8(\Lambda) = 0\}$
Subset F_8 shows the following system of equations:

$$\begin{aligned}
\Lambda_3' &= 1, & \frac{f_7(\Lambda')}{\Lambda_3'^4} &= 0, \\
\Lambda_4' &= 0, & \frac{f_8(\Lambda')}{\Lambda_3'^5} &= 0, \\
\frac{\Lambda_5'}{\Lambda_3'^2} &= 0, & \frac{f_9(\Lambda')}{\Lambda_3'^6} &= \frac{1}{A^6} \frac{f_9(\Lambda)}{\Lambda_3^6}, \\
\frac{f_6(\Lambda')}{\Lambda_3'^3} &= \frac{1}{A^3} \frac{f_6(\Lambda)}{\Lambda_3^3}, & \frac{\Gamma'}{\Lambda_3'^2} &= \frac{1}{A^5} \frac{\Gamma}{\Lambda_3^2}.
\end{aligned} \tag{B.15}$$

(d) $F_8 = U_7 \cup U_8$:

- $U_7 = \{L(\Lambda) \in SLb_{10}^a :$

$$\Lambda_3 \neq 0, \Lambda_5 = 0, f_6(\Lambda) \neq 0, f_7(\Lambda) = 0, f_8(\Lambda) = 0, \Gamma \neq 0\}$$

By reducing $\frac{f_6(\Lambda')}{\Lambda_3'^3} = \frac{1}{A^3} \frac{f_6(\Lambda)}{\Lambda_3^3}$ and $\frac{\Gamma'}{\Lambda_3'^2} = \frac{1}{A^5} \frac{\Gamma}{\Lambda_3^2}$ in (B.15), then we obtain

$$A = \frac{f_6^2(\Lambda)}{\Lambda_3^4 \Gamma}. \text{ The values of } B \text{ and } D \text{ appear as } B = \frac{\Lambda_4 A}{2\Lambda_3^3 \Lambda_5} \text{ and } D = \frac{A^2}{\Lambda_3}.$$

Then by using value $\frac{1}{A} = \frac{\Lambda_3^4 \Gamma}{f_6^2(\Lambda)}$, the system of equalities for U_7 is as the following page:

$$\begin{aligned}
\Lambda_3' &= 1, \\
\Lambda_4' &= 0, \\
\frac{\Lambda_5'}{\Lambda_3'^2} &= 0, \\
\frac{f_6(\Lambda')}{\Lambda_3'^3} &= \left(\frac{\Lambda_3^4 \Gamma}{f_6^2(\Lambda)} \right)^3 \frac{f_6(\Lambda)}{\Lambda_3^3}, \\
\frac{f_7(\Lambda')}{\Lambda_3'^4} &= 0, \\
\frac{f_8(\Lambda')}{\Lambda_3'^5} &= 0, \\
\frac{f_9(\Lambda')}{\Lambda_3'^6} &= \left(\frac{\Lambda_3^4 \Gamma}{f_6^2(\Lambda)} \right)^6 \frac{f_9(\Lambda)}{\Lambda_3^6}, \\
\frac{\Gamma'}{\Lambda_3'^2} &= \left(\frac{\Lambda_3^4 \Gamma}{f_6^2(\Lambda)} \right)^5 \frac{\Gamma}{\Lambda_3^2}.
\end{aligned} \tag{B.16}$$

New parameters are represented as λ_1 and λ_2 to imply that

$$f_6(\Lambda') = \frac{\Lambda_3^9 \Gamma^3}{f_6^5(\Lambda)} = \lambda_1, \quad f_9(\Lambda') = \frac{\Lambda_3^{18} \Gamma^6 f_9(\Lambda)}{f_6^{12}(\Lambda)} = \lambda_2 \quad \text{and} \quad \Gamma' = \frac{\Lambda_3^{18} \Gamma^6}{f_6^{10}(\Lambda)} = \lambda_1^2.$$

Thus all algebras from U_7 are isomorphic to $L(1, 0, 0, \lambda_1, 0, 0, \lambda_2, \lambda_1^2)$ for $\lambda_1 \in \mathbb{C}^*$ and $\lambda_2 \in \mathbb{C}$.

- $U_8 = \{L(\Lambda) \in SLb_{10}^a : \Lambda_3 \neq 0, \Lambda_5 = 0, f_6(\Lambda) \neq 0, f_7(\Lambda) = 0, f_8(\Lambda) = 0, \Gamma = 0\}$.

By reducing $\frac{f_6(\Lambda')}{\Lambda_3'^3} = \frac{1}{A^3} \frac{f_6(\Lambda)}{\Lambda_3^3}$ in (B.15), then we obtain $A^3 = \frac{f_6(\Lambda)}{\Lambda_3^3}$. The

values of B and D appear as $B = \frac{\Lambda_4 A}{2\Lambda_3^3 \Lambda_5}$ and $D = \frac{A^2}{\Lambda_3}$. Then by using value

$\frac{1}{A^3} = \frac{\Lambda_3^3}{f_6(\Lambda)}$, the system of equalities for U_8 is as the following page:

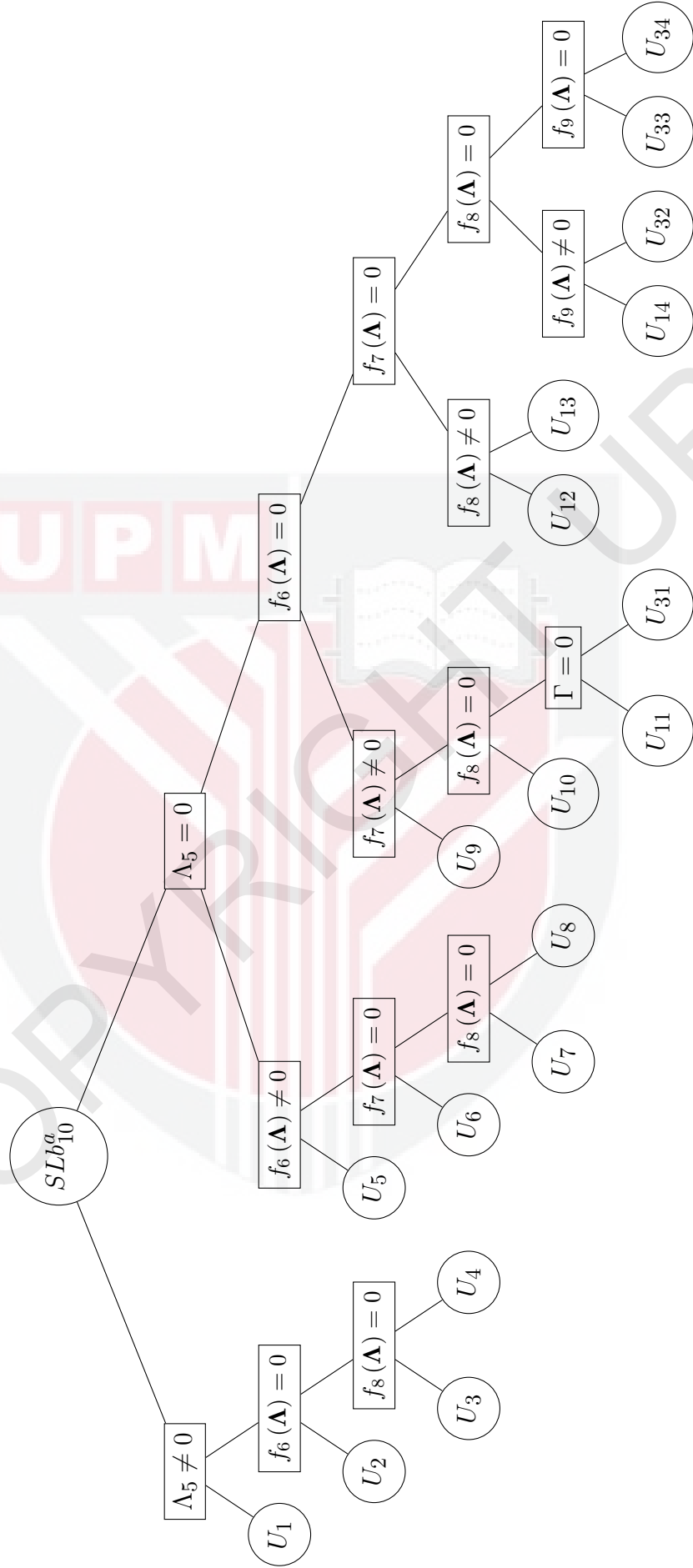
$$\begin{aligned} \frac{\Lambda_3'}{\Lambda_3^3} &= 1, \\ \frac{\Lambda_4'}{\Lambda_3^4} &= 0, \\ \frac{\Lambda_5'}{\Lambda_3'^2} &= 0, \\ \frac{f_6(\Lambda')}{\Lambda_3'^3} &= \left(\frac{\Lambda_3^3}{f_6(\Lambda)} \right) \frac{f_6(\Lambda)}{\Lambda_3^3}, \\ \frac{f_7(\Lambda')}{\Lambda_3'^4} &= 0, \\ \frac{f_8(\Lambda')}{\Lambda_3'^5} &= 0, \\ \frac{f_9(\Lambda')}{\Lambda_3'^6} &= \left(\frac{\Lambda_3^3}{f_6(\Lambda)} \right)^2 \frac{f_9(\Lambda)}{\Lambda_3^6}, \\ \frac{\Gamma'}{\Lambda_3'^2} &= 0. \end{aligned} \tag{B.17}$$

New parameter is represented as λ to imply that $f_9(\Lambda') = \frac{f_9(\Lambda)}{f_6^2(\Lambda)} = \lambda$.

Thus all algebras from U_8 are isomorphic to $L(1, 0, 0, 1, 0, 0, \lambda, 0)$ for $\lambda \in \mathbb{C}$.

The subsets in SLb_{10}^a are represented by the following graph tree.

Figure B.1: Graph tree of SLb_{10}^a



B.2 Classification of SLb_{10}^b

The isomorphism criteria for SLb_{10}^b can be expressed as follows:

$$\begin{aligned} \Lambda'_3 &= 0, & t_7(\Lambda') &= \frac{1}{A^{11}} \left(\frac{D}{A}\right)^4 t_7(\Lambda), \\ \Lambda'_4 &= \frac{1}{A^2} \left(\frac{D}{A}\right) \Lambda_4, & t_8(\Lambda') &= \frac{1}{A^{14}} \left(\frac{D}{A}\right)^5 t_8(\Lambda), \\ \Lambda'_5 &= \frac{1}{A^3} \left(\frac{D}{A}\right) \Lambda_5, & t_9(\Lambda') &= \frac{1}{A^{17}} \left(\frac{D}{A}\right)^6 t_9(\Lambda), \\ \Lambda'_6 &= 0, & \Gamma' &= \frac{1}{A^7} \left(\frac{D}{A}\right)^2 \Gamma. \end{aligned} \quad (\text{B.18})$$

Under condition $\Lambda_3 = 0$ and $\Lambda_4 \neq 0$, the system (B.18) is isomorphic to the following system of equations:

$$\begin{aligned} \Lambda'_3 &= 0, & \frac{t_7(\Lambda')}{\Lambda_4'^4} &= \frac{1}{A^3} \frac{t_7(\Lambda)}{\Lambda_4^4}, \\ \Lambda'_4 &= 1, & \frac{t_8(\Lambda')}{\Lambda_4'^5} &= \frac{1}{A^4} \frac{t_8(\Lambda)}{\Lambda_4^5}, \\ \frac{\Lambda'_5}{\Lambda_4'} &= \frac{1}{A} \frac{\Lambda_5}{\Lambda_4}, & \frac{t_9(\Lambda')}{\Lambda_4'^6} &= \frac{1}{A^5} \frac{t_9(\Lambda)}{\Lambda_4^6}, \\ \Lambda'_6 &= 0, & \frac{\Gamma'}{\Lambda_4'^2} &= \frac{1}{A^3} \frac{\Gamma}{\Lambda_4^2}. \end{aligned} \quad (\text{B.19})$$

It shows that $SLb_{10}^b = T_1 \cup T_2$, where $T_1 = \{L(\Lambda) \in SLb_{10}^b : \Lambda_3 = 0, \Lambda_4 \neq 0\}$ and $T_2 = \{L(\Lambda) \in SLb_{10}^b : \beta_3 = 0, \beta_4 = 0\}$.

(i) $T_1 = \{L(\Lambda) \in SLb_{10}^b : \Lambda_3 = 0, \Lambda_4 \neq 0\}$

(a) $T_1 = U_{15} \cup T_3 :$

- $U_{15} = \{L(\Lambda) \in SLb_{10}^b : \Lambda_3 = 0, \Lambda_4 \neq 0, \Lambda_5 \neq 0\}$

By reducing $\frac{\Lambda'_5}{\Lambda_4'} = \frac{1}{A} \frac{\Lambda_5}{\Lambda_4}$ in (B.19), then we obtain $A = \frac{\Lambda_5}{\Lambda_4}$. The values of

B and D appear as $B = \frac{\Lambda_6 A}{3\Lambda_4^2}$ and $D = \frac{A^3}{\Lambda_4}$. Then by using value $\frac{1}{A} = \frac{\Lambda_4}{\Lambda_5}$, the system of equalities for U_{15} is as follows:

$$\begin{aligned}
\Lambda_3' &= 0, & \frac{t_7(\Lambda')}{\Lambda_4'^4} &= \left(\frac{\Lambda_4}{\Lambda_5}\right)^3 \frac{t_7(\Lambda)}{\Lambda_4^4}, \\
\Lambda_4' &= 1, & \frac{t_8(\Lambda')}{\Lambda_4'^5} &= \left(\frac{\Lambda_4}{\Lambda_5}\right)^4 \frac{t_8(\Lambda)}{\Lambda_4^5}, \\
\frac{\Lambda_5'}{\Lambda_4'} &= \left(\frac{\Lambda_4}{\Lambda_5}\right) \frac{\Lambda_5}{\Lambda_4}, & \frac{t_9(\Lambda')}{\Lambda_4'^6} &= \left(\frac{\Lambda_4}{\Lambda_5}\right)^5 \frac{t_9(\Lambda)}{\Lambda_4^6}, \\
\Lambda_6' &= 0, & \frac{\Gamma'}{\Lambda_4'^2} &= \left(\frac{\Lambda_4}{\Lambda_5}\right)^3 \frac{\Gamma}{\Lambda_4^2}.
\end{aligned} \tag{B.20}$$

New parameters are represented as $\lambda_1, \lambda_2, \lambda_3$ and λ_4 to imply that $t_7(\Lambda') = \frac{t_7(\Lambda)}{\Lambda_4 \Lambda_5^3} = \lambda_1, t_8(\Lambda') = \frac{t_8(\Lambda)}{\Lambda_4 \Lambda_5^4} = \lambda_2, t_9(\Lambda') = \frac{t_9(\Lambda)}{\Lambda_4 \Lambda_5^5} = \lambda_3$ and $\Gamma' = \frac{\Lambda_4 \Gamma}{\Lambda_5^3} = \lambda_4$. Thus all algebras from U_{15} are isomorphic to $L(0, 1, 1, 0, \lambda_1, \lambda_2, \lambda_3, \lambda_4)$ for $\lambda_1, \lambda_2, \lambda_3, \lambda_4 \in \mathbb{C}$.

- $T_3 = \left\{ L(\Lambda) \in SLb_{10}^b : \Lambda_3 = 0, \Lambda_4 \neq 0, \Lambda_5 = 0 \right\}$

The system of equations in subset T_3 becomes:

$$\begin{aligned}
\Lambda_3' &= 0, & \frac{t_7(\Lambda')}{\Lambda_4'^4} &= \frac{1}{A^3} \frac{t_7(\Lambda)}{\Lambda_4^4}, \\
\Lambda_4' &= 1, & \frac{t_8(\Lambda')}{\Lambda_4'^5} &= \frac{1}{A^4} \frac{t_8(\Lambda)}{\Lambda_4^5}, \\
\frac{\Lambda_5'}{\Lambda_4'} &= 0, & \frac{t_9(\Lambda')}{\Lambda_4'^6} &= \frac{1}{A^5} \frac{t_9(\Lambda)}{\Lambda_4^6}, \\
\Lambda_6' &= 0, & \frac{\Gamma'}{\Lambda_4'^2} &= \frac{1}{A^3} \frac{\Gamma}{\Lambda_4^2}.
\end{aligned} \tag{B.21}$$

(b) $T_3 = T_4 \cup T_5 :$

- $T_4 = \left\{ L(\Lambda) \in SLb_{10}^b : \Lambda_3 = 0, \Lambda_4 \neq 0, \Lambda_5 = 0, t_7(\Lambda) \neq 0 \right\}$

The system of equations in subset T_4 becomes:

$$\begin{aligned} \Lambda_3' &= 0, & \frac{t_7(\Lambda')}{\Lambda_4'^4} &= \frac{1}{A^3} \frac{t_7(\Lambda)}{\Lambda_4^4}, \\ \Lambda_4' &= 1, & \frac{t_8(\Lambda')}{\Lambda_4'^5} &= \frac{1}{A^4} \frac{t_8(\Lambda)}{\Lambda_4^5}, \\ \frac{\Lambda_5'}{\Lambda_4'} &= 0, & \frac{t_9(\Lambda')}{\Lambda_4'^6} &= \frac{1}{A^5} \frac{t_9(\Lambda)}{\Lambda_4^6}, \\ \Lambda_6' &= 0, & \frac{\Gamma'}{\Lambda_4'^2} &= \frac{1}{A^3} \frac{\Gamma}{\Lambda_4^2}. \end{aligned} \quad (\text{B.22})$$

- $T_5 = \left\{ L(\Lambda) \in SLb_{10}^b : \Lambda_3 = 0, \Lambda_4 \neq 0, \Lambda_5 = 0, t_7(\Lambda) = 0 \right\}$

The system of equations in subset T_5 becomes:

$$\begin{aligned} \Lambda_3' &= 0, & \frac{t_7(\Lambda')}{\Lambda_4'^4} &= 0, \\ \Lambda_4' &= 1, & \frac{t_8(\Lambda')}{\Lambda_4'^5} &= \frac{1}{A^4} \frac{t_8(\Lambda)}{\Lambda_4^5}, \\ \frac{\Lambda_5'}{\Lambda_4'} &= 0, & \frac{t_9(\Lambda')}{\Lambda_4'^6} &= \frac{1}{A^5} \frac{t_9(\Lambda)}{\Lambda_4^6}, \\ \Lambda_6' &= 0, & \frac{\Gamma'}{\Lambda_4'^2} &= \frac{1}{A^3} \frac{\Gamma}{\Lambda_4^2}. \end{aligned} \quad (\text{B.23})$$

(c) $T_4 = U_{16} \cup T_6 :$

- $U_{16} = \left\{ L(\Lambda) \in SLb_{10}^b : \Lambda_3 = 0, \Lambda_4 \neq 0, \Lambda_5 = 0, t_7(\Lambda) \neq 0, t_8(\Lambda) \neq 0 \right\}$

By reducing $\frac{t_7(\Lambda')}{\Lambda_4'^4} = \frac{1}{A^3} \frac{t_7(\Lambda)}{\Lambda_4^4}$ and $\frac{t_8(\Lambda')}{\Lambda_4'^5} = \frac{1}{A^4} \frac{t_8(\Lambda)}{\Lambda_4^5}$ in (B.22), then we obtain $A = \frac{t_8(\Lambda)}{\Lambda_4 t_7(\Lambda)}$. The values of B and D appear as $B = \frac{\Lambda_6 A}{3\Lambda_4^2}$ and $D = \frac{A^3}{\Lambda_4}$. Then by using value $\frac{1}{A} = \frac{\Lambda_4 t_7(\Lambda)}{t_8(\Lambda)}$, the system of equalities for U_{16} is as follows:

$$\begin{aligned}
\Lambda_3' &= 0, & \frac{t_7(\Lambda')}{\Lambda_4'^4} &= \left(\frac{\Lambda_4 t_7(\Lambda)}{t_8(\Lambda)} \right)^3 \frac{t_7(\Lambda)}{\Lambda_4^4}, \\
\Lambda_4' &= 1, & \frac{t_8(\Lambda')}{\Lambda_4'^5} &= \left(\frac{\Lambda_4 t_7(\Lambda)}{t_8(\Lambda)} \right)^4 \frac{t_8(\Lambda)}{\Lambda_4^5}, \\
\frac{\Lambda_5'}{\Lambda_4'} &= 0, & \frac{t_9(\Lambda')}{\Lambda_4'^6} &= \left(\frac{\Lambda_4 t_7(\Lambda)}{t_8(\Lambda)} \right)^5 \frac{t_9(\Lambda)}{\Lambda_4^6}, \\
\Lambda_6' &= 0, & \frac{\Gamma'}{\Lambda_4'^2} &= \left(\frac{\Lambda_4 t_7(\Lambda)}{t_8(\Lambda)} \right)^3 \frac{\Gamma}{\Lambda_4^2}.
\end{aligned} \tag{B.24}$$

New parameters are represented as λ_1, λ_2 and λ_3 to imply that $t_7(\Lambda') = \frac{t_7^4(\Lambda)}{\Lambda_4 t_8^3(\Lambda)} = \lambda_1, t_8(\Lambda') = \frac{t_8^4(\Lambda)}{\Lambda_4 t_8^3(\Lambda)} = \lambda_1, t_9(\Lambda') = \frac{t_9^5(\Lambda) t_9(\Lambda)}{\Lambda_4 t_8^5(\Lambda)} = \lambda_2$ and $\Gamma' = \frac{\Lambda_4 t_7^3(\Lambda) \Gamma}{t_8^3(\Lambda)} = \lambda_3$. Thus all algebras from U_{16} are isomorphic to $L(0, 1, 0, 0, \lambda_1, \lambda_1, \lambda_2, \lambda_3)$ for $\lambda_1 \in \mathbb{C}^*$ and $\lambda_2, \lambda_3 \in \mathbb{C}$.

- $T_6 = \left\{ L(\Lambda) \in SLb_{10}^b : \Lambda_3 = 0, \Lambda_4 \neq 0, \Lambda_5 = 0, t_7(\Lambda) \neq 0, t_8(\Lambda) = 0 \right\}$
The system of equations in subset T_6 becomes:

$$\begin{aligned}
\Lambda_3' &= 0, & \frac{t_7(\Lambda')}{\Lambda_4'^4} &= \frac{1}{A^3} \frac{t_7(\Lambda)}{\Lambda_4^4}, \\
\Lambda_4' &= 1, & \frac{t_8(\Lambda')}{\Lambda_4'^5} &= 0, \\
\frac{\Lambda_5'}{\Lambda_4'} &= 0, & \frac{t_9(\Lambda')}{\Lambda_4'^6} &= \frac{1}{A^5} \frac{t_9(\Lambda)}{\Lambda_4^6}, \\
\Lambda_6' &= 0, & \frac{\Gamma'}{\Lambda_4'^2} &= \frac{1}{A^3} \frac{\Gamma}{\Lambda_4^2}.
\end{aligned} \tag{B.25}$$

(d) $T_6 = U_{17} \cup U_{18}$:

- $U_{17} = \left\{ L(\Lambda) \in SLb_{10}^b : \right.$
 $\Lambda_3 = 0, \Lambda_4 \neq 0, \Lambda_5 = 0, t_7(\Lambda) \neq 0, t_8(\Lambda) = 0, t_9(\Lambda) \neq 0 \}$
By reducing $\frac{t_7(\Lambda')}{\Lambda_4'^4} = \frac{1}{A^3} \frac{t_7(\Lambda)}{\Lambda_4^4}$ and $\frac{t_9(\Lambda')}{\Lambda_4'^6} = \frac{1}{A^5} \frac{t_9(\Lambda)}{\Lambda_4^6}$ in (B.25), then we obtain $A = \frac{t_7^2(\Lambda)}{\Lambda_4^2 t_9(\Lambda)}$. The values of B and D appear as $B = \frac{\Lambda_6 A}{3 \Lambda_4^2}$ and $D = \frac{A^3}{\Lambda_4}$. Then by using value $\frac{1}{A} = \frac{\Lambda_4^2 t_9(\Lambda)}{t_7^2(\Lambda)}$, the system of equalities for

U_{17} is as follows:

$$\begin{aligned}
\Lambda'_3 = 0, & \quad \frac{t_7(\Lambda')}{\Lambda_4'^4} = \left(\frac{\Lambda_4^2 t_9(\Lambda)}{t_7^2(\Lambda)} \right)^3 \frac{t_7(\Lambda)}{\Lambda_4^4}, \\
\Lambda'_4 = 1, & \quad \frac{t_8(\Lambda')}{\Lambda_4'^5} = 0, \\
\frac{\Lambda'_5}{\Lambda_4'} = 0, & \quad \frac{t_9(\Lambda')}{\Lambda_4'^6} = \left(\frac{\Lambda_4^2 t_9(\Lambda)}{t_7^2(\Lambda)} \right)^5 \frac{t_9(\Lambda)}{\Lambda_4^6}, \\
\Lambda'_6 = 0, & \quad \frac{\Gamma'}{\Lambda_4'^2} = \left(\frac{\Lambda_4^2 t_9(\Lambda)}{t_7^2(\Lambda)} \right)^3 \frac{\Gamma}{\Lambda_4^2}.
\end{aligned} \tag{B.26}$$

New parameters are represented as λ_1 and λ_2 to imply that $t_7(\Lambda') = \frac{\Lambda_4^2 t_9^3(\Lambda)}{t_7^3(\Lambda)} = \lambda_1$, $t_9(\Lambda') = \frac{\Lambda_4^4 t_9^6(\Lambda)}{t_7^{10}(\Lambda)} = \lambda_1^2$ and $\Gamma' = \frac{\Lambda_4^4 t_9^3(\Lambda) \Gamma}{t_7^6(\Lambda)} = \lambda_2$. Thus all algebras from U_{17} are isomorphic to $L(0, 1, 0, 0, \lambda_1, 0, \lambda_1^2, \lambda_2)$ for $\lambda_1 \in \mathbb{C}^*$ and $\lambda_2 \in \mathbb{C}$.

- $U_{18} = \left\{ L(\Lambda) \in SLb_{10}^b : \Lambda_3 = 0, \Lambda_4 \neq 0, \Lambda_5 = 0, t_7(\Lambda) \neq 0, t_8(\Lambda) = 0, t_9(\Lambda) = 0 \right\}$
 By reducing $\frac{t_7(\Lambda')}{\Lambda_4'^4} = \frac{1}{A^3} \frac{t_7(\Lambda)}{\Lambda_4^4}$ in (B.25), then we obtain $A^3 = \frac{t_7(\Lambda)}{\Lambda_4^4}$. The values of B and D appear as $B = \frac{\Lambda_6 A}{3\Lambda_4^2}$ and $D = \frac{A^3}{\Lambda_4}$. Then by using value $\frac{1}{A^3} = \frac{\Lambda_4^4}{t_7(\Lambda)}$, the system of equalities for U_{18} is as follows:

$$\begin{aligned}
\Lambda'_3 = 0, & \quad \frac{t_7(\Lambda')}{\Lambda_4'^4} = \left(\frac{\Lambda_4^4}{t_7(\Lambda)} \right) \frac{t_7(\Lambda)}{\Lambda_4^4}, \\
\Lambda'_4 = 1, & \quad \frac{t_8(\Lambda')}{\Lambda_4'^5} = 0, \\
\frac{\Lambda'_5}{\Lambda_4'} = 0, & \quad \frac{t_9(\Lambda')}{\Lambda_4'^6} = 0, \\
\Lambda'_6 = 0, & \quad \frac{\Gamma'}{\Lambda_4'^2} = \left(\frac{\Lambda_4^4}{t_7(\Lambda)} \right) \frac{\Gamma}{\Lambda_4^2}.
\end{aligned} \tag{B.27}$$

New parameters are represented as λ to imply that $\Gamma' = \frac{\Lambda_4^2 \Gamma}{t_7(\Lambda)} = \lambda$. Thus all algebras from U_{18} are isomorphic to $L(0, 1, 0, 0, 1, 0, 0, \lambda)$ for $\lambda \in \mathbb{C}$.

(e) $T_5 = T_8 \cup T_9$:

- $T_8 = \left\{ L(\Lambda) \in SLb_{10}^b : \Lambda_3 = 0, \Lambda_4 \neq 0, \Lambda_5 = 0, t_7(\Lambda) = 0, t_8(\Lambda) \neq 0 \right\}$

The system of equations in subset T_8 becomes:

$$\begin{aligned} \Lambda'_3 &= 0, & \frac{t_7(\Lambda')}{\Lambda_4'^4} &= 0, \\ \Lambda'_4 &= 1, & \frac{t_8(\Lambda')}{\Lambda_4'^5} &= \frac{1}{A^4} \frac{t_8(\Lambda)}{\Lambda_4^5}, \\ \frac{\Lambda'_5}{\Lambda_4'} &= 0, & \frac{t_9(\Lambda')}{\Lambda_4'^6} &= \frac{1}{A^5} \frac{t_9(\Lambda)}{\Lambda_4^6}, \\ \Lambda'_6 &= 0, & \frac{\Gamma'}{\Lambda_4'^2} &= \frac{1}{A^3} \frac{\Gamma}{\Lambda_4^2}. \end{aligned} \tag{B.28}$$

- $T_9 = \left\{ L(\Lambda) \in SLb_{10}^b : \Lambda_3 = 0, \Lambda_4 \neq 0, \Lambda_5 = 0, t_7(\Lambda) = 0, t_8(\Lambda) = 0 \right\}$

The system of equations in subset T_9 becomes:

$$\begin{aligned} \Lambda'_3 &= 0, & \frac{t_7(\Lambda')}{\Lambda_4'^4} &= 0, \\ \Lambda'_4 &= 1, & \frac{t_8(\Lambda')}{\Lambda_4'^5} &= 0, \\ \frac{\Lambda'_5}{\Lambda_4'} &= 0, & \frac{t_9(\Lambda')}{\Lambda_4'^6} &= \frac{1}{A^5} \frac{t_9(\Lambda)}{\Lambda_4^6}, \\ \Lambda'_6 &= 0, & \frac{\Gamma'}{\Lambda_4'^2} &= \frac{1}{A^3} \frac{\Gamma}{\Lambda_4^2}. \end{aligned} \tag{B.29}$$

(f) $T_8 = U_{19} \cup T_{10}$:

- $U_{19} = \left\{ L(\Lambda) \in SLb_{10}^b : \Lambda_3 = 0, \Lambda_4 \neq 0, \Lambda_5 = 0, t_7(\Lambda) = 0, t_8(\Lambda) \neq 0, t_9(\Lambda) \neq 0 \right\}$

By reducing $\frac{t_8(\Lambda')}{\Lambda_4'^5} = \frac{1}{A^4} \frac{t_8(\Lambda)}{\Lambda_4^5}$ and $\frac{t_9(\Lambda')}{\Lambda_4'^6} = \frac{1}{A^5} \frac{t_9(\Lambda)}{\Lambda_4^6}$ in (B.28), then we obtain $A = \frac{t_9(\Lambda)}{\Lambda_4 t_8(\Lambda)}$. The values of B and D appear as $B = \frac{\Lambda_6 A}{3\Lambda_4^2}$ and $D = \frac{A^3}{\Lambda_4}$.

Then by using value $\frac{1}{A} = \frac{\Lambda_4 t_8(\Lambda)}{t_9(\Lambda)}$, the system of equalities for U_{19} is as follows:

$$\begin{aligned}
\Lambda_3' &= 0, & \frac{t_7(\Lambda')}{\Lambda_4'^4} &= 0, \\
\Lambda_4' &= 1, & \frac{t_8(\Lambda')}{\Lambda_4'^5} &= \left(\frac{\Lambda_4 t_8(\Lambda)}{t_9(\Lambda)} \right)^4 \frac{t_8(\Lambda)}{\Lambda_4^5}, \\
\frac{\Lambda_5'}{\Lambda_4'} &= 0, & \frac{t_9(\Lambda')}{\Lambda_4'^6} &= \left(\frac{\Lambda_4 t_8(\Lambda)}{t_9(\Lambda)} \right)^5 \frac{t_9(\Lambda)}{\Lambda_4^6}, \\
\Lambda_6' &= 0, & \frac{\Gamma'}{\Lambda_4'^2} &= \left(\frac{\Lambda_4 t_8(\Lambda)}{t_9(\Lambda)} \right)^3 \frac{\Gamma}{\Lambda_4^2}.
\end{aligned} \tag{B.30}$$

New parameters are represented as λ_1 and λ_2 to imply that $t_8(\Lambda') = \frac{t_8^5(\Lambda)}{\Lambda_4 t_9^4(\Lambda)} = \lambda_1$, $t_9(\Lambda') = \frac{t_8^5(\Lambda)}{\Lambda_4 t_9^4(\Lambda)} = \lambda_1$ and $\Gamma' = \frac{\Lambda_4 t_8^3(\Lambda) \Gamma}{t_9^3(\Lambda)} = \lambda_2$. Thus all algebras from U_{19} are isomorphic to $L(0, 1, 0, 0, 0, \lambda_1, \lambda_1, \lambda_2)$ for $\lambda_1 \in \mathbb{C}^*$ and $\lambda_2 \in \mathbb{C}$.

- $T_{10} = \left\{ L(\Lambda) \in SLb_{10}^b : \Lambda_3 = 0, \Lambda_4 \neq 0, \Lambda_5 = 0, t_7(\Lambda) = 0, t_8(\Lambda) \neq 0, t_9(\Lambda) = 0 \right\}$
The system of equations in subset T_{10} becomes:

$$\begin{aligned}
\Lambda_3' &= 0, & \frac{t_7(\Lambda')}{\Lambda_4'^4} &= 0, \\
\Lambda_4' &= 1, & \frac{t_8(\Lambda')}{\Lambda_4'^5} &= \frac{1}{A^4} \frac{t_8(\Lambda)}{\Lambda_4^5}, \\
\frac{\Lambda_5'}{\Lambda_4'} &= 0, & \frac{t_9(\Lambda')}{\Lambda_4'^6} &= 0, \\
\Lambda_6' &= 0, & \frac{\Gamma'}{\Lambda_4'^2} &= \frac{1}{A^3} \frac{\Gamma}{\Lambda_4^2}.
\end{aligned} \tag{B.31}$$

(g) $T_{10} = U_{20} \cup U_{37}$:

- $U_{20} = \left\{ L(\Lambda) \in SLb_{10}^b : \Lambda_3 = 0, \Lambda_4 \neq 0, \Lambda_5 = 0, t_7(\Lambda) = 0, t_8(\Lambda) \neq 0, t_9(\Lambda) = 0, \Gamma \neq 0 \right\}$

By reducing $\frac{t_8(\Lambda')}{\Lambda_4'^5} = \frac{1}{A^4} \frac{t_8(\Lambda)}{\Lambda_4^5}$ and $\frac{\Gamma'}{\Lambda_4'^2} = \frac{1}{A^3} \frac{\Gamma}{\Lambda_4^2}$ in (B.31), then we obtain

$A = \frac{t_8(\Lambda)}{\Lambda_4^3 \Gamma}$. The values of B and D appear as $B = \frac{\Lambda_6 A}{3 \Lambda_4^2}$ and $D = \frac{A^3}{\Lambda_4}$. Then

by using value $\frac{1}{A} = \frac{\Lambda_4^3 \Gamma}{t_8(\Lambda)}$, the system of equalities for U_{20} is as follows:

$$\begin{aligned}
\Lambda_3' &= 0, & \frac{t_7(\Lambda')}{\Lambda_4'^4} &= 0, \\
\Lambda_4' &= 1, & \frac{t_8(\Lambda')}{\Lambda_4'^5} &= \left(\frac{\Lambda_4^3 \Gamma}{t_8(\Lambda)} \right)^4 \frac{t_8(\Lambda)}{\Lambda_4^5}, \\
\frac{\Lambda_5'}{\Lambda_4'} &= 0, & \frac{t_9(\Lambda')}{\Lambda_4'^6} &= 0, \\
\Lambda_6' &= 0, & \frac{\Gamma'}{\Lambda_4'^2} &= \left(\frac{\Lambda_4^3 \Gamma}{t_8(\Lambda)} \right)^3 \frac{\Gamma}{\Lambda_4^2}.
\end{aligned} \tag{B.32}$$

New parameter is represented as λ to imply that $t_8(\Lambda') = \frac{\Lambda_4^7 \Gamma^4}{t_8^3(\Lambda)} = \lambda$ and $\Gamma' = \frac{\Lambda_4^7 \Gamma^4}{t_8^3(\Lambda)} = \lambda$. Thus all algebras from U_{20} are isomorphic to $L(0, 1, 0, 0, 0, \lambda, 0, \lambda)$ for $\lambda \in \mathbb{C}^*$.

- $U_{37} = \left\{ L(\Lambda) \in SLb_{10}^b : \Lambda_3 = 0, \Lambda_4 \neq 0, \Lambda_5 = 0, t_7(\Lambda) = 0, t_8(\Lambda) \neq 0, \right.$
 $\left. t_9(\Lambda) = 0, \Gamma = 0 \right\}$

By reducing $\frac{t_8(\Lambda')}{\Lambda_4'^5} = \frac{1}{A^4} \frac{t_8(\Lambda)}{\Lambda_4^5}$ in (B.31), then we obtain $A^4 = \frac{t_8(\Lambda)}{\Lambda_4^5}$. The values of B and D appear as $B = \frac{\Lambda_6 A}{3\Lambda_4^2}$ and $D = \frac{A^3}{\Lambda_4}$. Then by using value $\frac{1}{A^4} = \frac{\Lambda_4^5}{t_8(\Lambda)}$, the system of equalities for U_{37} is as follows:

$$\begin{aligned}
\Lambda_3' &= 0, & \frac{t_7(\Lambda')}{\Lambda_4'^4} &= 0, \\
\Lambda_4' &= 1, & \frac{t_8(\Lambda')}{\Lambda_4'^5} &= \left(\frac{\Lambda_4^5}{t_8(\Lambda)} \right) \frac{t_8(\Lambda)}{\Lambda_4^5}, \\
\frac{\Lambda_5'}{\Lambda_4'} &= 0, & \frac{t_9(\Lambda')}{\Lambda_4'^6} &= 0, \\
\Lambda_6' &= 0, & \frac{\Gamma'}{\Lambda_4'^2} &= 0.
\end{aligned} \tag{B.33}$$

Thus all algebras from U_{37} are isomorphic to $L(0, 1, 0, 0, 0, 1, 0, 0)$.

(h) $T_9 = T_{11} \cup T_{12}$:

- $T_{11} = \left\{ L(\Lambda) \in SLb_{10}^b : \Lambda_3 = 0, \Lambda_4 \neq 0, \Lambda_5 = 0, t_7(\Lambda) = 0, t_8(\Lambda) = 0, \right.$
 $\left. t_9(\Lambda) \neq 0 \right\}$

The system of equations in subset T_{11} becomes:

$$\begin{aligned} \Lambda_3' &= 0, & \frac{t_7(\Lambda')}{\Lambda_4'^4} &= 0, \\ \Lambda_4' &= 1, & \frac{t_8(\Lambda')}{\Lambda_4'^5} &= 0, \\ \frac{\Lambda_5'}{\Lambda_4'} &= 0, & \frac{t_9(\Lambda')}{\Lambda_4'^6} &= \frac{1}{A^5} \frac{t_9(\Lambda)}{\Lambda_4^6}, \\ \Lambda_6' &= 0, & \frac{\Gamma'}{\Lambda_4'^2} &= \frac{1}{A^3} \frac{\Gamma}{\Lambda_4^2}. \end{aligned} \tag{B.34}$$

- $T_{12} = \left\{ L(\Lambda) \in SLb_{10}^b : \Lambda_3 = 0, \Lambda_4 \neq 0, \Lambda_5 = 0, t_7(\Lambda) = 0, t_8(\Lambda) = 0, \right.$
 $\left. t_9(\Lambda) = 0 \right\}$

The system of equations in subset T_{12} becomes:

$$\begin{aligned} \Lambda_3' &= 0, & \frac{t_7(\Lambda')}{\Lambda_4'^4} &= 0, \\ \Lambda_4' &= 1, & \frac{t_8(\Lambda')}{\Lambda_4'^5} &= 0, \\ \frac{\Lambda_5'}{\Lambda_4'} &= 0, & \frac{t_9(\Lambda')}{\Lambda_4'^6} &= 0, \\ \Lambda_6' &= 0, & \frac{\Gamma'}{\Lambda_4'^2} &= \frac{1}{A^3} \frac{\Gamma}{\Lambda_4^2}. \end{aligned} \tag{B.35}$$

(i) $T_{11} = U_{21} \cup U_{38}$:

- $U_{21} = \left\{ L(\Lambda) \in SLb_{10}^b : \Lambda_3 = 0, \Lambda_4 \neq 0, \Lambda_5 = 0, t_7(\Lambda) = 0, t_8(\Lambda) = 0, \right.$
 $\left. t_9(\Lambda) \neq 0, \Gamma \neq 0 \right\}$

By reducing $\frac{t_9(\Lambda')}{\Lambda_4'^6} = \frac{1}{A^5} \frac{t_9(\Lambda)}{\Lambda_4^6}$ and $\frac{\Gamma'}{\Lambda_4'^2} = \frac{1}{A^3} \frac{\Gamma}{\Lambda_4^2}$ in (B.34), then we obtain

$A = \frac{\Lambda_4^2 \Gamma^2}{t_9(\Lambda)}$. The values of B and D appear as $B = \frac{\Lambda_6 A}{3\Lambda_4^2}$ and $D = \frac{A^3}{\Lambda_4}$. Then

by using value $\frac{1}{A} = \frac{t_9(\Lambda)}{\Lambda_4^2 \Gamma^2}$, the system of equalities for U_{21} is as follows:

$$\begin{aligned}
\Lambda_3' &= 0, & \frac{t_7(\Lambda')}{\Lambda_4'^4} &= 0, \\
\Lambda_4' &= 1, & \frac{t_8(\Lambda')}{\Lambda_4'^5} &= 0, \\
\frac{\Lambda_5'}{\Lambda_4'} &= 0, & \frac{t_9(\Lambda')}{\Lambda_4'^6} &= \left(\frac{t_9(\Lambda)}{\Lambda_4^2 \Gamma^2} \right)^5 \frac{t_9(\Lambda)}{\Lambda_4^6}, \\
\Lambda_6' &= 0, & \frac{\Gamma'}{\Lambda_4'^2} &= \left(\frac{t_9(\Lambda)}{\Lambda_4^2 \Gamma^2} \right)^3 \frac{\Gamma}{\Lambda_4^2}.
\end{aligned} \tag{B.36}$$

New parameter is represented as λ to imply that

$t_9(\Lambda') = \frac{t_9^6(\Lambda)}{\Lambda_4^{16} \Gamma^{10}} = \lambda^2$ and $\Gamma' = \frac{t_9^3(\Lambda)}{\Lambda_4^8 \Gamma^5} = \lambda$. Thus all algebras from U_{21} are isomorphic to $L(0, 1, 0, 0, 0, 0, \lambda^2, \lambda)$ for $\lambda \in \mathbb{C}^*$.

- $U_{38} = \left\{ L(\Lambda) \in SLb_{10}^b : \Lambda_3 = 0, \Lambda_4 \neq 0, \Lambda_5 = 0, t_7(\Lambda) = 0, t_8(\Lambda) = 0, \right.$
 $\left. t_9(\Lambda) \neq 0, \Gamma = 0 \right\}$

By reducing $\frac{t_9(\Lambda')}{\Lambda_4'^6} = \frac{1}{A^5} \frac{t_9(\Lambda)}{\Lambda_4^6}$ in (B.34), then we obtain $A^5 = \frac{t_9(\Lambda)}{\Lambda_4^6}$. The values of B and D appear as $B = \frac{\Lambda_6 A}{3\Lambda_4^2}$ and $D = \frac{A^3}{\Lambda_4}$. Then by using value $\frac{1}{A^5} = \frac{\Lambda_4^6}{t_9(\Lambda)}$, the system of equalities for U_{38} is as follows:

$$\begin{aligned}
\Lambda_3' &= 0, & \frac{t_7(\Lambda')}{\Lambda_4'^4} &= 0, \\
\Lambda_4' &= 1, & \frac{t_8(\Lambda')}{\Lambda_4'^5} &= 0, \\
\frac{\Lambda_5'}{\Lambda_4'} &= 0, & \frac{t_9(\Lambda')}{\Lambda_4'^6} &= \left(\frac{\Lambda_4^6}{t_9(\Lambda)} \right) \frac{t_9(\Lambda)}{\Lambda_4^6}, \\
\Lambda_6' &= 0, & \frac{\Gamma'}{\Lambda_4'^2} &= 0.
\end{aligned} \tag{B.37}$$

Thus all algebras from U_{38} are isomorphic to $L(0, 1, 0, 0, 0, 0, 1, 0)$.

(j) $T_{12} = U_{39} \cup U_{40}$:

- $U_{39} = \left\{ L(\Lambda) \in SLb_{10}^b : \Lambda_3 = 0, \Lambda_4 \neq 0, \Lambda_5 = 0, t_7(\Lambda) = 0, t_8(\Lambda) = 0, \right.$
 $\left. t_9(\Lambda) = 0, \Gamma \neq 0 \right\}$

By reducing $\frac{\Gamma'}{\Lambda_4'^2} = \frac{1}{A^3} \frac{\Gamma}{\Lambda_4^2}$ in (B.35), then we obtain $A^3 = \frac{\Gamma}{\Lambda_4^2}$. The values of

B and D appear as $B = \frac{\Lambda_6 A}{3\Lambda_4^2}$ and $D = \frac{A^3}{\Lambda_4}$.

Then by using value $\frac{1}{A^3} = \frac{\Lambda_4^2}{\Gamma}$, the system of equalities for U_{39} is as follows:

$$\begin{aligned} \Lambda_3' &= 0, & \frac{t_7(\Lambda')}{\Lambda_4'^4} &= 0, \\ \Lambda_4' &= 1, & \frac{t_8(\Lambda')}{\Lambda_4'^5} &= 0, \\ \frac{\Lambda_5'}{\Lambda_4'} &= 0, & \frac{t_9(\Lambda')}{\Lambda_4'^6} &= 0, \\ \Lambda_6' &= 0, & \frac{\Gamma'}{\Lambda_4'^2} &= \left(\frac{\Lambda_4^2}{\Gamma}\right) \frac{\Gamma}{\Lambda_4^2}. \end{aligned} \quad (\text{B.38})$$

Thus all algebras from U_{39} are isomorphic to $L(0, 1, 0, 0, 0, 0, 0, 1)$.

- $U_{40} = \left\{ L(\Lambda) \in SLb_{10}^b : \Lambda_3 = 0, \Lambda_4 \neq 0, \Lambda_5 = 0, t_7(\Lambda) = 0, t_8(\Lambda) = 0, \right.$
 $\left. t_9(\Lambda) = 0, \Gamma = 0 \right\}$

All algebras from U_{40} are isomorphic to $L(0, 1, 0, 0, 0, 0, 0, 0)$.

- (ii) $T_2 = \left\{ L(\beta) \in SLb_{10}^b : \beta_3 = 0, \beta_4 = 0 \right\}$

The subset of T_2 brings the following system of equations:

$$\begin{aligned} \beta_3' &= 0, & \beta_7' &= \frac{1}{A^5} \left(\frac{D}{A}\right) \beta_7, \\ \beta_4' &= 0, & \beta_8' &= \frac{1}{A^6} \left(\frac{D}{A}\right) \left[\beta_8 - 4 \left(\frac{B}{A}\right) \beta_5^2 \right], \\ \beta_5' &= \frac{1}{A^3} \left(\frac{D}{A}\right) \beta_5, & \beta_9' &= \frac{1}{A^7} \left(\frac{D}{A}\right) \left[\beta_9 + \left(\frac{B}{A}\right) \gamma - 9 \left(\frac{B}{A}\right) \beta_5 \beta_6 \right], \\ \beta_6' &= \frac{1}{A^4} \left(\frac{D}{A}\right) \beta_6, & \gamma' &= \frac{1}{A^7} \left(\frac{D}{A}\right)^2 \gamma. \end{aligned} \quad (\text{B.39})$$

- (a) $T_2 = T_{13} \cup T_{14} :$

- $T_{13} = \left\{ L(\beta) \in SLb_{10}^b : \beta_3 = 0, \beta_4 = 0, \beta_5 \neq 0 \right\}$

The system of equations of subset T_{13} is as following:

$$\begin{aligned}
\beta'_3 &= 0, & \frac{\beta'_7}{\beta'_5} &= \frac{1}{A^2} \frac{\beta_7}{\beta_5}, \\
\beta'_4 &= 0, & \frac{\beta'_8}{\beta'_5} &= \frac{1}{A^3} \left[\frac{\beta_8}{\beta_5} - 4 \left(\frac{B}{A} \right) \beta_5 \right], \\
\beta'_5 &= 1, & \frac{\beta'_9}{\beta'_5} &= \frac{1}{A^4} \left[\frac{\beta_9}{\beta_5} + \left(\frac{B}{A} \right) \frac{\gamma}{\beta_5} - 9 \left(\frac{B}{A} \right) \beta_6 \right], \\
\frac{\beta'_6}{\beta'_5} &= \frac{1}{A} \frac{\beta_6}{\beta_5}, & \frac{\gamma'}{\beta_5'^2} &= \frac{1}{A} \frac{\gamma}{\beta_5^2}.
\end{aligned} \tag{B.40}$$

- $T_{14} = \left\{ L(\beta) \in SLb_{10}^b : \beta_3 = 0, \beta_4 = 0, \beta_5 = 0 \right\}$

The subset of T_{14} brings the following system of equations:

$$\begin{aligned}
\beta'_3 &= 0, & \beta'_7 &= \frac{1}{A^5} \left(\frac{D}{A} \right) \beta_7, \\
\beta'_4 &= 0, & \beta'_8 &= \frac{1}{A^6} \left(\frac{D}{A} \right) \beta_8, \\
\beta'_5 &= 0, & \beta'_9 &= \frac{1}{A^7} \left(\frac{D}{A} \right) \left[\beta_9 + \left(\frac{B}{A} \right) \gamma \right], \\
\beta'_6 &= \frac{1}{A^4} \left(\frac{D}{A} \right) \beta_6, & \gamma' &= \frac{1}{A^7} \left(\frac{D}{A} \right)^2 \gamma.
\end{aligned} \tag{B.41}$$

(b) $T_{13} = U_{22} \cup T_{15}$:

- $U_{22} = \left\{ L(\beta) \in SLb_{10}^b : \beta_3 = 0, \beta_4 = 0, \beta_5 \neq 0, \beta_6 \neq 0 \right\}$

By reducing $\frac{\beta'_6}{\beta'_5} = \frac{1}{A} \frac{\beta_6}{\beta_5}$ in (B.40), then we obtain $A = \frac{\beta_6}{\beta_5}$. The values of B and D appear as $B = \frac{\beta_8 A}{4\beta_5^2}$ and $D = \frac{A^4}{\beta_5}$. Then by using value $\frac{1}{A} = \frac{\beta_5}{\beta_6}$ and $\frac{B}{A} = \frac{\beta_8 \beta_6}{4\beta_5^3}$, the system of equalities for U_{22} is as follows:

$$\begin{aligned}
\beta'_3 &= 0, \\
\beta'_4 &= 0, \\
\beta'_5 &= 1, \\
\frac{\beta'_6}{\beta'_5} &= \left(\frac{\beta_5}{\beta_6}\right) \frac{\beta_6}{\beta_5}, \\
\frac{\beta'_7}{\beta'_5} &= \left(\frac{\beta_5}{\beta_6}\right)^2 \frac{\beta_7}{\beta_5}, \\
\frac{\beta'_8}{\beta'_5} &= 0, \\
\frac{\beta'_9}{\beta'_5} &= \left(\frac{\beta_5}{\beta_6}\right)^4 \left[\frac{\beta_9}{\beta_5} + \left(\frac{\beta_8\beta_6}{4\beta_5^3}\right) \frac{\gamma}{\beta_5} - 9 \left(\frac{\beta_8\beta_6}{4\beta_5^3}\right) \beta_6 \right], \\
\frac{\gamma'}{\beta_5'^2} &= \left(\frac{\beta_5}{\beta_6}\right) \frac{\gamma}{\beta_5^2}.
\end{aligned} \tag{B.42}$$

New parameters are represented as λ_1, λ_2 and λ_3 to imply that $\beta'_7 = \frac{\beta_5\beta_7}{\beta_6^2} = \lambda_1$, $\beta'_9 = \frac{1}{4\beta_6^4} (4\beta_5^3\beta_9 + \beta_6\beta_8\gamma - 9\beta_5\beta_6^2\beta_8) = \lambda_2$ and $\gamma' = \frac{\gamma}{\beta_5\beta_6} = \lambda_3$. Thus all algebras from U_{22} are isomorphic to $L(0, 0, 1, 1, \lambda_1, 0, \lambda_2, \lambda_3)$ for $\lambda_1, \lambda_2, \lambda_3 \in \mathbb{C}$.

- $T_{15} = \left\{ L(\beta) \in SLb_{10}^b : \beta_3 = 0, \beta_4 = 0, \beta_5 \neq 0, \beta_6 = 0 \right\}$
The system of equations of subset T_{15} is as following:

$$\begin{aligned}
\beta'_3 &= 0, & \frac{\beta'_7}{\beta'_5} &= \frac{1}{A^2} \frac{\beta_7}{\beta_5}, \\
\beta'_4 &= 0, & \frac{\beta'_8}{\beta'_5} &= 0, \\
\beta'_5 &= 1, & \frac{\beta'_9}{\beta'_5} &= \frac{1}{A^4} \left[\frac{\beta_9}{\beta_5} + \left(\frac{B}{A}\right) \frac{\gamma}{\beta_5} \right], \\
\frac{\beta'_6}{\beta'_5} &= 0, & \frac{\gamma'}{\beta_5'^2} &= \frac{1}{A} \frac{\gamma}{\beta_5^2}.
\end{aligned} \tag{B.43}$$

(c) $T_{15} = T_{16} \cup T_{17}$:

- $T_{16} = \left\{ L(\beta) \in SLb_{10}^b : \beta_3 = 0, \beta_4 = 0, \beta_5 \neq 0, \beta_6 = 0, \beta_7 \neq 0 \right\}$

The system of equations of subset T_{16} is as following:

$$\begin{aligned}
\beta'_3 &= 0, & \frac{\beta'_7}{\beta'_5} &= \frac{1}{A^2} \frac{\beta_7}{\beta_5}, \\
\beta'_4 &= 0, & \frac{\beta'_8}{\beta'_5} &= 0, \\
\beta'_5 &= 1, & \frac{\beta'_9}{\beta'_5} &= \frac{1}{A^4} \left[\frac{\beta_9}{\beta_5} + \left(\frac{B}{A} \right) \frac{\gamma}{\beta_5} \right], \\
\frac{\beta'_6}{\beta'_5} &= 0, & \frac{\gamma'}{\beta'^2_5} &= \frac{1}{A} \frac{\gamma}{\beta^2_5}.
\end{aligned} \tag{B.44}$$

- $T_{17} = \left\{ L(\beta) \in SLb_{10}^b : \beta_3 = 0, \beta_4 = 0, \beta_5 \neq 0, \beta_6 = 0, \beta_7 = 0 \right\}$

The system of equations of subset T_{17} is as following:

$$\begin{aligned}
\beta'_3 &= 0, & \frac{\beta'_7}{\beta'_5} &= 0, \\
\beta'_4 &= 0, & \frac{\beta'_8}{\beta'_5} &= 0, \\
\beta'_5 &= 1, & \frac{\beta'_9}{\beta'_5} &= \frac{1}{A^4} \left[\frac{\beta_9}{\beta_5} + \left(\frac{B}{A} \right) \frac{\gamma}{\beta_5} \right], \\
\frac{\beta'_6}{\beta'_5} &= 0, & \frac{\gamma'}{\beta'^2_5} &= \frac{1}{A} \frac{\gamma}{\beta^2_5}.
\end{aligned} \tag{B.45}$$

(d) $T_{16} = U_{23} \cup T_{18}$:

- $U_{23} = \left\{ L(\beta) \in SLb_{10}^b : \beta_3 = 0, \beta_4 = 0, \beta_5 \neq 0, \beta_6 = 0, \beta_7 \neq 0, \gamma \neq 0 \right\}$

By reducing $\frac{\beta'_6}{\beta'_5} = \frac{1}{A} \frac{\beta_6}{\beta_5}$ in (B.44), then we obtain $A = \frac{\beta_5 \beta_7}{\gamma}$. The values of

B and D appear as $B = \frac{\beta_8 A}{4\beta_5^2}$ and $D = \frac{A^4}{\beta_5}$. Then by using value $\frac{1}{A} = \frac{\gamma}{\beta_5 \beta_7}$ and $\frac{B}{A} = \frac{\beta_7 \beta_8}{4\beta_5 \gamma}$, the system of equalities for U_{23} is as follows:

$$\begin{aligned}
\beta'_3 &= 0, \\
\beta'_4 &= 0, \\
\beta'_5 &= 1, \\
\frac{\beta'_6}{\beta'_5} &= 0, \\
\frac{\beta'_7}{\beta'_5} &= \left(\frac{\gamma}{\beta_5 \beta_7} \right)^2 \frac{\beta_7}{\beta_5}, \\
\frac{\beta'_8}{\beta'_5} &= 0, \\
\frac{\beta'_9}{\beta'_5} &= \left(\frac{\gamma}{\beta_5 \beta_7} \right)^4 \left[\frac{\beta_9}{\beta_5} + \left(\frac{\beta_7 \beta_8}{4 \beta_5 \gamma} \right) \frac{\gamma}{\beta_5} \right], \\
\frac{\gamma'}{\beta'^2_5} &= \left(\frac{\gamma}{\beta_5 \beta_7} \right) \frac{\gamma}{\beta_5^2}.
\end{aligned} \tag{B.46}$$

New parameters are represented as λ_1 and λ_2 to imply that $\beta'_7 = \frac{\gamma^2}{\beta_5^3 \beta_7} = \lambda_1$, $\beta'_9 = \frac{\gamma^4}{4 \beta_5^6 \beta_7^4} (4 \beta_5 \beta_9 + \beta_7 \beta_8) = \lambda_2$ and $\gamma' = \frac{\gamma^2}{\beta_5^3 \beta_7} = \lambda_1$. Thus all algebras from U_{23} are isomorphic to $L(0, 0, 1, 0, \lambda_1, 0, \lambda_2, \lambda_1)$ for $\lambda_1 \in \mathbb{C}^*$ and $\lambda_2 \in \mathbb{C}$.

- $T_{18} = \left\{ L(\beta) \in SLb_{10}^b : \beta_3 = 0, \beta_4 = 0, \beta_5 \neq 0, \beta_6 = 0, \beta_7 \neq 0, \gamma = 0 \right\}$

The system of equations of subset T_{18} is as following:

$$\begin{aligned}
\beta'_3 &= 0, & \frac{\beta'_7}{\beta'_5} &= \frac{1}{A^2} \frac{\beta_7}{\beta_5}, \\
\beta'_4 &= 0, & \frac{\beta'_8}{\beta'_5} &= 0, \\
\beta'_5 &= 1, & \frac{\beta'_9}{\beta'_5} &= \frac{1}{A^4} \frac{\beta_9}{\beta_5}, \\
\frac{\beta'_6}{\beta'_5} &= 0, & \frac{\gamma'}{\beta'^2_5} &= 0.
\end{aligned} \tag{B.47}$$

(e) $T_{18} = U_{24} \cup U_{41}$:

- $U_{24} = \left\{ L(\beta) \in SLb_{10}^b : \beta_3 = 0, \beta_4 = 0, \beta_5 \neq 0, \beta_6 = 0, \beta_7 \neq 0, \gamma = 0, \beta_9 \neq 0 \right\}$

By reducing $\frac{\beta'_7}{\beta'_5} = \frac{1}{A^2} \frac{\beta_7}{\beta_5}$ and $\frac{\beta'_9}{\beta'_5} = \frac{1}{A^4} \frac{\beta_9}{\beta_5}$ in (B.47), then we obtain $A^2 = \frac{\beta_9}{\beta_7}$.

The values of B and D appear as $B = \frac{\beta_8 A}{4\beta_5^2}$ and $D = \frac{A^4}{\beta_5}$. Then by using value $\frac{1}{A^2} = \frac{\beta_7}{\beta_9}$, the system of equalities for U_{24} is as follows:

$$\begin{aligned} \beta'_3 &= 0, & \frac{\beta'_7}{\beta'_5} &= \left(\frac{\beta_7}{\beta_9}\right) \frac{\beta_7}{\beta_5}, \\ \beta'_4 &= 0, & \frac{\beta'_8}{\beta'_5} &= 0, \\ \beta'_5 &= 1, & \frac{\beta'_9}{\beta'_5} &= \left(\frac{\beta_7}{\beta_9}\right)^2 \frac{\beta_9}{\beta_5}, \\ \frac{\beta'_6}{\beta'_5} &= 0, & \frac{\gamma'}{\beta'^2_5} &= 0. \end{aligned} \quad (\text{B.48})$$

New parameter is represented as λ to imply that $\beta'_7 = \frac{\beta_7^2}{\beta_5\beta_9} = \lambda$ and $\beta'_9 = \frac{\beta_7^2}{\beta_5\beta_9} = \lambda$. Thus all algebras from U_{24} are isomorphic to $L(0, 0, 1, 0, \lambda, 0, \lambda, 0)$ for $\lambda \in \mathbb{C}^*$.

- $U_{41} = \left\{ L(\beta) \in SLb_{10}^b : \beta_3 = 0, \beta_4 = 0, \beta_5 \neq 0, \beta_6 = 0, \beta_7 \neq 0, \gamma = 0, \beta_9 = 0 \right\}$

By reducing $\frac{\beta'_7}{\beta'_5} = \frac{1}{A^2} \frac{\beta_7}{\beta_5}$ in (B.47), then we obtain $A^2 = \frac{\beta_7}{\beta_5}$. The values of B and D appear as $B = \frac{\beta_8 A}{4\beta_5^2}$ and $D = \frac{A^4}{\beta_5}$. Then by using value $\frac{1}{A^2} = \frac{\beta_5}{\beta_7}$, the system of equalities for U_{41} is as follows:

$$\begin{aligned} \beta'_3 &= 0, & \frac{\beta'_7}{\beta'_5} &= \left(\frac{\beta_5}{\beta_7}\right) \frac{\beta_7}{\beta_5}, \\ \beta'_4 &= 0, & \frac{\beta'_8}{\beta'_5} &= 0, \\ \beta'_5 &= 1, & \frac{\beta'_9}{\beta'_5} &= 0, \\ \frac{\beta'_6}{\beta'_5} &= 0, & \frac{\gamma'}{\beta'^2_5} &= 0. \end{aligned} \quad (\text{B.49})$$

Thus all algebras from U_{41} are isomorphic to $L(0, 0, 1, 0, 1, 0, 0, 0)$.

The subsets in SLb_{10}^b are illustrated by the following graph trees.

Figure B.2: Graph tree of SLb_{10}^b

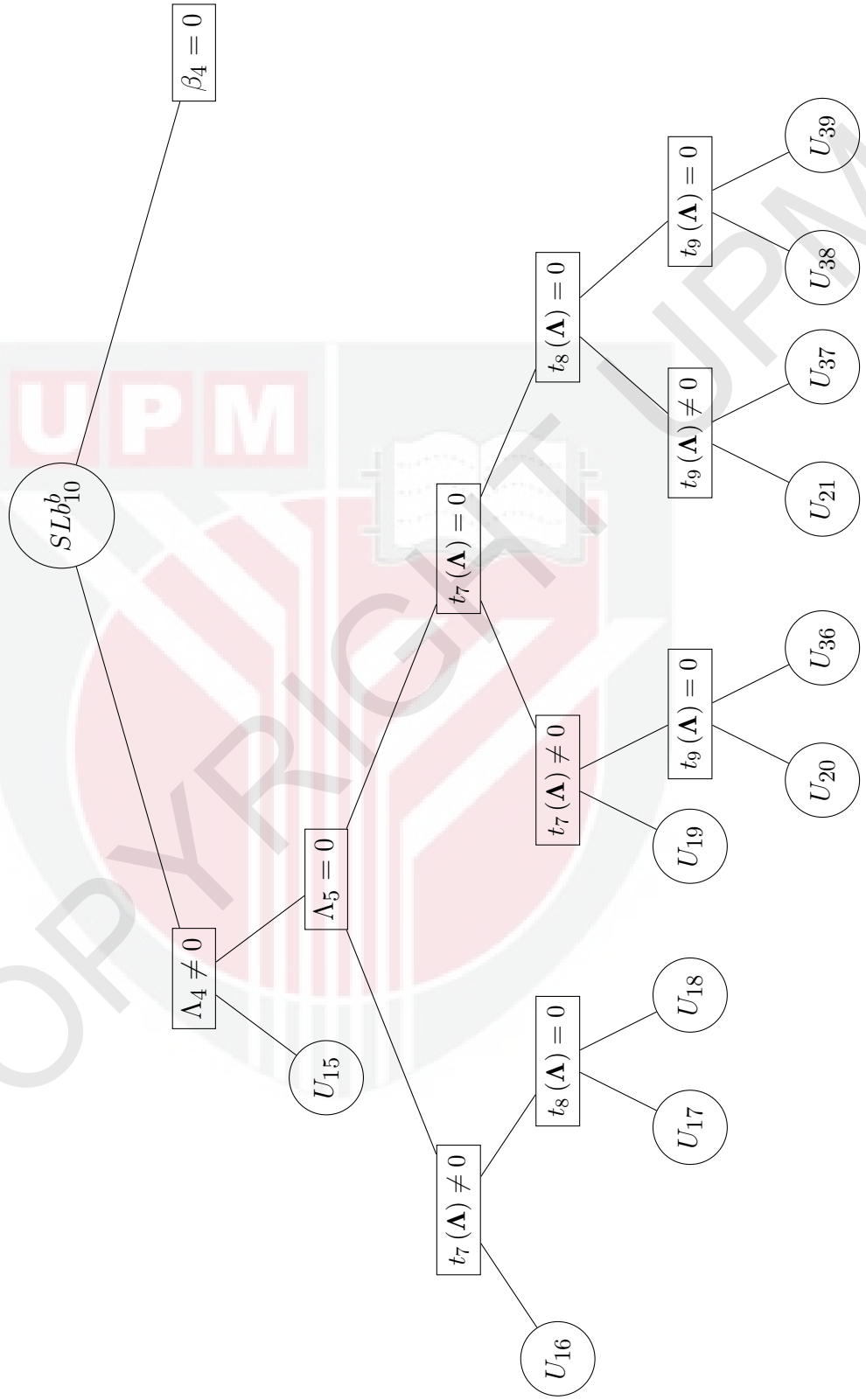
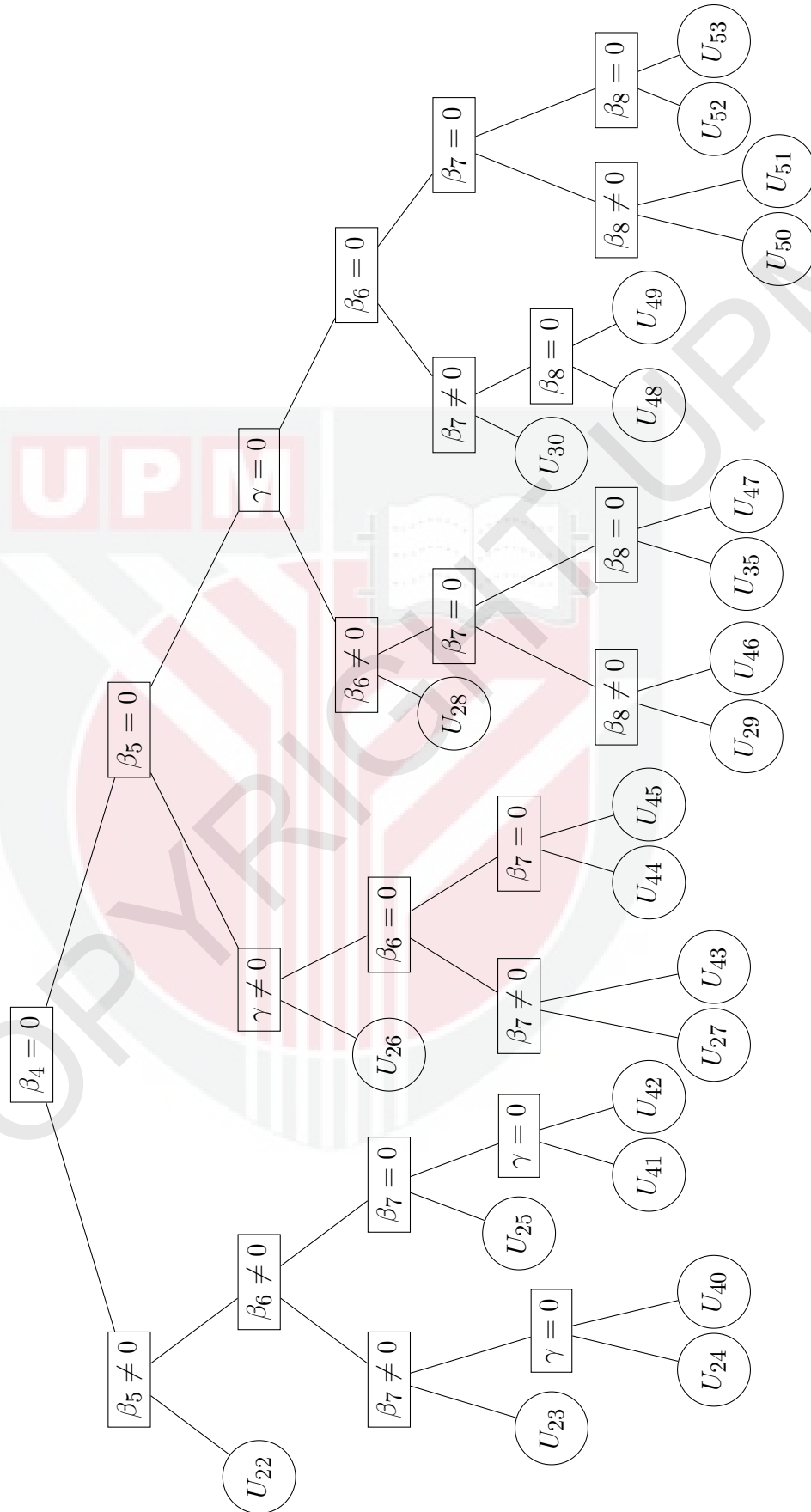


Figure B.3: Graph tree of SL_{10}^b (continued from Figure B.2)



APPENDIX C

This part includes two examples of computer program to represent the list of algebras of SLb_{10} into table of multiplications. Here orbits U_{30} and U_{52} are used to illustrate the examples.

Calling Sequence:

SC represents the list of algebras of SLb_{10} either in structural constants $\beta_3, \beta_4, \dots, \beta_9, \gamma$ or $\Lambda_3, \Lambda_4, f_5(\Lambda), \dots, f_9(\Lambda), \Gamma$ or $\Lambda_3, \Lambda_4, \Lambda_5, t_6(\Lambda), \dots, t_9(\Lambda), \Gamma$.

Parameters:

isom₁ — $e_0 e_1$
 isom₂ — $e_1 e_1$
 isom₃ — $e_2 e_1$
 isom₄ — $e_3 e_1$
 isom₅ — $e_4 e_1$
 isom₆ — $e_5 e_1$
 isom₇ — $e_6 e_1$
 isom₈ — $e_7 e_1$

Example of U_{52} :

Program

```
SC:=[ $\beta_3 = 0, \beta_4 = 0, \beta_5 = 0, \beta_6 = 0, \beta_7 = 0, \beta_8 = 0, \beta_9 = 1, \gamma = 0$ :
isom1:=eval( $\beta_3 e_3 + \beta_4 e_4 + \beta_5 e_5 + \beta_6 e_6 + \beta_7 e_7 + \beta_8 e_8 + \beta_9 e_9$ , SC);
isom2:=eval( $\gamma e_9$ , SC);
isom3:=eval( $\beta_3 e_4 + \beta_4 e_5 + \beta_5 e_6 + \beta_6 e_7 + \beta_7 e_8 + \beta_8 e_9$ , SC);
isom4:=eval( $\beta_3 e_5 + \beta_4 e_6 + \beta_5 e_7 + \beta_6 e_8 + \beta_7 e_9$ , SC);
isom5:=eval( $\beta_3 e_6 + \beta_4 e_7 + \beta_5 e_8 + \beta_6 e_9$ , SC);
isom6:=eval( $\beta_3 e_7 + \beta_4 e_8 + \beta_5 e_9$ , SC);
isom7:=eval( $\beta_3 e_8 + \beta_4 e_9$ , SC);
isom8:=eval( $\beta_3 e_9$ , SC);
```

Result

$$\begin{aligned} isom_1 &:= e_9 \\ isom_2 &:= 0 \\ isom_3 &:= 0 \\ isom_4 &:= 0 \\ isom_5 &:= 0 \\ isom_6 &:= 0 \\ isom_7 &:= 0 \\ isom_8 &:= 0 \end{aligned}$$

Example of U_{30} :

Program

```
SC:=[ $\beta_3 = 0, \beta_4 = 0, \beta_5 = 0, \beta_6 = 0, \beta_7 = 1, \beta_8 = 1, \beta_9 = \lambda, \gamma = 0$ :  
isom1:=eval( $\beta_3 e_3 + \beta_4 e_4 + \beta_5 e_5 + \beta_6 e_6 + \beta_7 e_7 + \beta_8 e_8 + \beta_9 e_9$ , SC);  
isom2:=eval( $\gamma e_9$ , SC);  
isom3:=eval( $\beta_3 e_4 + \beta_4 e_5 + \beta_5 e_6 + \beta_6 e_7 + \beta_7 e_8 + \beta_8 e_9$ , SC);  
isom4:=eval( $\beta_3 e_5 + \beta_4 e_6 + \beta_5 e_7 + \beta_6 e_8 + \beta_7 e_9$ , SC);  
isom5:=eval( $\beta_3 e_6 + \beta_4 e_7 + \beta_5 e_8 + \beta_6 e_9$ , SC);  
isom6:=eval( $\beta_3 e_7 + \beta_4 e_8 + \beta_5 e_9$ , SC);  
isom7:=eval( $\beta_3 e_8 + \beta_4 e_9$ , SC);  
isom8:=eval( $\beta_3 e_9$ , SC);
```

Result

$$\begin{aligned} isom_1 &:= e_7 + e_8 + \lambda e_9 \\ isom_2 &:= 0 \\ isom_3 &:= e_8 + e_9 \\ isom_4 &:= 0 \\ isom_5 &:= 0 \\ isom_6 &:= 0 \\ isom_7 &:= 0 \\ isom_8 &:= 0 \end{aligned}$$

BIODATA OF STUDENT

The student, **Suzila binti Mohd Kasim**, was born in 1989. The author started her tertiary education at matriculation level in 2007 at Kolej Matrikulasi Perak, Perak. She obtained her Bachelor of Science (Honours) Mathematics in 2011 from Universiti Putra Malaysia, Serdang, Selangor. In the same university, she pursued her Masters study in 2011.

The student can be contacted via her supervisor,
Professor Dr. Isamiddin S. Rakhimov, by address:

Institute for Mathematical Research,
Universiti Putra Malaysia,
43400 Serdang,
Selangor,
Malaysia.

Email: risamiddin@gmail.com



LIST OF PUBLICATIONS

Publications that arise from the study are:

1. S. Mohd Kasim, N. S. Mohamed, I. S. Rakhimov and S. K. Said Husain. *Isomorphism Criteria of Leibniz Algebras Arising from the Naturally Graded Non-Lie Filiform Leibniz Algebra*, Proceedings of 2nd Regional Conference on Applied and Engineering Mathematics 2012, p225-228, 2012. (Penang, Malaysia)
2. S. Mohd Kasim, N. S. Mohamed, I. S. Rakhimov and S. K. Said Husain. *Algorithm for Constructing the Isomorphism Criteria for 10-Dimensional Complex Filiform Leibniz Algebras*, Proceedings of the Fundamental Science Congress 2012, p51-52, 2012. (UPM Serdang, Selangor, Malaysia)
3. S. Mohd Kasim, I. S. Rakhimov and S. K. Said Husain. *Appearance of Odd Parts of Catalan Numbers in the Classification Problem of Complex Filiform Leibniz Algebras*, Proceedings in the International Science Postgraduate Conference 2012, p465-477, 2012. (UTM Skudai, Johor, Malaysia)
4. S. Mohd Kasim, I. S. Rakhimov and S. K. Said Husain. *On Isomorphism Classes and Invariants of A Subclass Filiform Leibniz Algebras Arising from Naturally Graded Non-Lie Filiform Leibniz Algebras in Dimension 10*, Proceedings of the Fundamental Science Congress 2013, p512-516, 2013. (UPM Serdang, Selangor, Malaysia)
5. S. Mohd Kasim, I. S. Rakhimov and S. K. Said Husain. *Isomorphism Classes of 10-Dimensional Filiform Leibniz Algebras*, AIP Conference Proceedings, 1602, p708-715, 2014.