UNIVERSITI PUTRA MALAYSIA

CLASSIFICATION OF SECOND-CLASS 10-DIMENSIONAL COMPLEX FILIFORM LEIBNIZ ALGEBRAS

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By

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Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Master of Science

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April 2014

Chair: Professor Isamiddin S. Rakhimov, PhD
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This thesis is concerned on studying the classification problem of a subclass of $(n+1)$-dimensional complex filiform Leibniz algebras. Leibniz algebras that are non-commutative generalizations of Lie algebras are considered. Leibniz identity and Jacobi identity are equivalent when the multiplication is skew-symmetric. When studying a certain class of algebras, it is important to describe at least the algebras of lower dimensions up to an isomorphism. For Leibniz algebras, difficulties arise even when considering nilpotent algebras of dimension greater than four. Thus, a special class of nilpotent Leibniz algebras is introduced namely filiform Leibniz algebras. Filiform Leibniz algebras arise from two sources. The first source is a naturally graded non-Lie filiform Leibniz algebras and another one is a naturally graded filiform Lie algebras.

Naturally graded non-Lie filiform Leibniz algebras contains subclasses FLb_{n+1} and SLb_{n+1}. While there is only one subclass obtained from naturally graded filiform Lie algebras which is TLb_{n+1}. These three subclasses FLb_{n+1}, SLb_{n+1} and TLb_{n+1} are over a field of complex number, $\mathbb{C}$ where $n+1$ denotes the dimension of these subclasses starting with $n \geq 4$.

In particular, a method of simplification of the basis transformations of the arbitrary filiform Leibniz algebras which were obtained from naturally graded non-Lie filiform Leibniz algebras, that allows for the problem of classification of algebras is reduced to the problem of a description of the structural constants. The investigation of filiform Leibniz algebras which were obtained from naturally graded non-Lie filiform Leibniz algebras only for subclass SLb_{n+1} is the subject of this thesis.

This research is the continuation of the works on SLb_{n+1} which have been treated for the cases of $n < 9$. The main purpose of this thesis is to apply the Rakhimov-Bekbaev approach to classify SLb_{10}. These approach will give a complete classification of SLb_{10} in terms of algebraic invariants. Isomorphism criterion of SLb_{10} is used to split the set of algebras SLb_{10} into several disjoint subsets. For each of these subsets, the classification problem is solved separately. As a result, some of them are represented as a union of infinitely many orbit (parametric families) and others as single orbits (isolated orbits). Finally, the list of isomorphism classes of complex filiform Leibniz algebras with the table of multiplications are given.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Sarjana Sains

PENGKELASAN KELAS-KEDUA 10-DIMENSI ALJABAR LEIBNIZ FILIFORM KOMPLEKS

Oleh

SUZILA BINTI MOHD KASIM

April 2014

Pengerusi: Profesor Isamiddin S. Rakhimov, PhD
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Tesis ini adalah berkenaan kajian terhadap masalah pengkelasan daripada subkelas $(n+1)$-dimensional aljabar Leibniz filiform kompleks. Aljabar Leibniz yang umum bukan kalis tukar tertib aljabar Lie akan dipertimbangkan. Identiti Leibniz dan identiti Jacobi adalah setara apabila pendaraban adalah condong-simetri. Ketika mengkaji kelas tertentu aljabar, adalah penting untuk menerangkan aljabar yang sekurang-kurangnya dimensi yang lebih rendah terhadap satu isomorfisma. Bagi aljabar Leibniz, timbul kesukaran apabila mempertimbangkan aljabar nilpotent dimensi yang lebih besar daripada empat. Oleh itu, kelas khas aljabar Leibniz nilpotent diperkenalkan iaitu aljabar Leibniz filiform. Aljabar Leibniz filiform diterbitkan daripada dua sumber. Sumber pertama adalah aljabar Leibniz bukan-Lie filiform tergred secara semula jadi dan satu lagi adalah aljabar Lie filiform tergred secara semula jadi.

Aljabar Leibniz bukan-Lie filiform tergred secara semula jadi mengandungi subkelas-subkelas FLb_{n+1} dan SLb_{n+1}. Namun hanya ada satu subkelas yang diperoleh daripada aljabar Lie filiform tergred secara semula jadi iaitu TLb_{n+1}. Ketiga-tiga subkelas FLb_{n+1}, SLb_{n+1} dan TLb_{n+1} berada dalam nombor kompleks , $\mathbb{C}$ di mana $n+1$ menandakan dimensi subkelas ini bermula dengan $n \geq 4$.

Khususnya, kaedah memudahkan perubahan-perubahan dasar terhadap aljabar Leibniz filiform yang diperolehi daripada aljabar Leibniz bukan-Lie filiform tergred secara semula jadi membolehkan masalah pengkelasan aljabar dikurangkan kepada masalah penerangan mengenai struktur pemalar. Penyiasatan aljabar Leibniz filiform yang diperolehi daripada aljabar Leibniz bukan-Lie filiform tergred secara semula jadi adalah subjek tesis ini bagi subkelas SLb_{n+1} sahaja.

Kajian ini adalah kesinambungan kerja-kerja pada SLb_{n+1} yang dikaji bagi kes-kes $n < 9$. Tujuan utama tesis ini adalah untuk mengaplikasi pendekatan Rakhimov-Bekbaev untuk mengkelaskan SLb_{10}. Pendekatan ini akan memberikan pengkelasan lengkap SLb_{10} dalam sebutan aljabar tak varian. Kriteria isomorfisma SLb_{10} digunakan untuk memisahkan set aljabar SLb_{10} kepada beberapa subset terpisah. Bagi setiap subset ini, masalah pengkelasan ini diselesaikan secara berasingan. Hasilnya, sebahagian daripada subset diwakili sebagai kesatuan orbit tak terhingga banyaknya (keluarga parametrik) dan selebihnya sebagai orbit tunggal (orbit diasingkan). Akhirnya, senarai kelas isomorfisma aljabar Leibniz filiform kompleks dengan jadual pendaraban diberikan.

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I certify that a Thesis Examination Committee has met on 22 April 2014 to conduct the final examination of Suzila Mohd Kasim on her thesis entitled "Classification of Second-Class 10-Dimensional Complex Filiform Leibniz Algebras" in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Master of Science.

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This is to confirm that:

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LIST OF ABBREVIATIONS

$\mathbb{C}$	The field of complex numbers
$\mathbb{C}^*$	The field of nonzero complex numbers
Lb_{n+1}	The class of $(n+1)$-dimensional filiform Leibniz algebras
LB_n	The class of n-dimensional Leibniz algebras
SLb_{n+1}	Second class of $(n+1)$-dimensional filiform Leibniz algebras
SLb_{10}^a	Second class of 10-dimensional filiform Leibniz algebras ($\beta_3 \neq 0$)
SLb_{10}^b	Second class of 10-dimensional filiform Leibniz algebras ($\beta_3 = 0$)

CHAPTER 1

INTRODUCTION

The aim of this thesis is to classify a subclass of complex filiform Leibniz algebras. The complex filiform Leibniz algebras are arising from two sources, naturally graded non-Lie filiform Leibniz algebras and naturally graded filiform Lie algebras. The class of filiform Leibniz algebras derived from naturally graded non-Lie filiform Leibniz algebras is separated into two subclasses. There are first class filiform Leibniz algebras, FLb_{n+1} and second class filiform Leibniz algebras, SLb_{n+1} in dimension $(n+1)$, respectively.

An algebraic approach is focused to solve isomorphism problem in terms of algebraic invariants (invariant functions) in the classification of second class 10-dimensional complex filiform Leibniz algebras, SLb_{10}. By using this approach, a complete classification of SLb_{10} is obtained and the isomorphism criterion are given.

This chapter is organized by reviewing basic concepts of algebras and Leibniz algebras briefly and next some literature reviews are described and followed by a few research objectives, scope of research and research methodology. Outline of contents of the thesis is also provided.

1.0 Basic Concepts

Let V be a vector space of dimension n over an algebraically closed field K ($\mathrm{char}\, K = 0$). The set of bilinear maps $V \times V \to V$ form a vector space $Hom\,(V \otimes V, V)$ of dimension n^3, which can be considered together with its natural structure of an affine algebraic variety over K and be denoted by $Alg_n\,(K) \cong K^{n^3}$. An n-dimensional algebra L over K can be considered as an element L of $Alg_n\,(K)$ via bilinear mapping $L \otimes L \to L$ defining a binary algebraic operation on L: let $\{e_0, e_1, e_2, ..., e_n\}$ be a basis of the algebra L. The table of multiplication of L is represented by point $\left(\gamma_{ij}^k\right)$ as follows:

$$[e_i, e_j] = \sum_{k=1}^{n} \gamma_{ij}^k e_k, \quad i, j = 1, 2, \ldots, n.$$

Here γ_{ij}^k are called structural constants of L. The linear reductive group $GL_n\,(K)$ acts on $Alg_n\,(K)$ is said to be "transport of structure" with

$$(g * \lambda)\,(x, y) = g\left(\lambda\left(g^{-1}\,(x)\,, g^{-1}\,(y)\right)\right).$$

Two algebras L_1 and L_2 on V are isomorphic if and only if they belong to the same orbit under action of transport of structure.

Recall that an algebra L over a field K is called a *Leibniz algebra* if it satisfies the following Leibniz identity:

$$[x,[y,z]] = [[x,y],z] - [[x,z],y], \tag{1.1}$$

where $[\cdot,\cdot]$ denotes the multiplication in L.

If a Leibniz algebra has the property of antisymmetricity

$$[x,y] = -[y,x],$$

then the Leibniz identity is simplified into the Jacobi identity:

$$[[x,y],z] + [[y,z],x] + [[z,x],y] = 0.$$

Therefore Leibniz algebras are generalization of Lie algebras.

Let $LB_n(K)$ be a subvariety of $Alg_n(K)$ consisting of all n-dimensional Leibniz algebras over K. It is stable under the above mentioned action of $GL_n(K)$. As a subset of $Alg_n(K)$, the set $LB_n(K)$ is specified by the following polynomial equalities with respect to the structural constants γ_{ij}^k:

$$\gamma_{jk}^l \gamma_{il}^m - \gamma_{ij}^l \gamma_{lk}^m + \gamma_{ik}^l \gamma_{lj}^m = 0, \qquad i,j,k,l,m = 1,2,\ldots,n. \tag{1.2}$$

The solution by this system of equations in the set $LB_n(K)$ with respect to γ_{ij}^k has been done for low-dimensional cases of some classes of algebra ($n \leq 3$). The complexity of the computations increases much with increasing of dimension to classify $LB_n(K)$ for any fixed n. Hence, one considers some subclasses of $LB_n(K)$ to be classified.

1.1 Literature Review

Many papers concern with the study of Lie algebras case. According to the structural theory of Lie algebras a finite-dimensional Lie algebra is written as a semidirect sum of its semisimple subalgebra and the solvable radical (Levi's theorem). The semisimple part is a direct sum of simple Lie algebras which are completely classified in fifties of the last century. At the same period, the essential progress has been made in the solvable part by Malcev [20]. Malcev [20] reduced the problem of classification of solvable Lie algebras to the classification of nilpotent Lie algebras. Later all the classification results have been related to the nilpotent part.

The theory of Lie algebras is one of the most developed branches of modern algebra. It has been deeply investigated for many years. The investigation of the properties of Lie algebras leads to the generalization of Leibniz algebras.

Leibniz algebra was introduced by Loday [17] as a non-commutative generalization of Lie algebra. It is well-known that the Leibniz identity and the classical Jacobi identity are equivalent, when the multiplication is skew-symmetric. Leibniz algebras inherit on important property of Lie algebras which is the right multiplication operator of a Leibniz algebra is a derivation. Many results on Lie algebras have been extended to Leibniz algebras.

1.1.1 Relations on Leibniz Algebras

1. (Co)homological problems

   The (co)homology theory, representations and related problems of Leibniz algebras were studied by Loday and Pirashvili [19] and Frabetti [13]. Problems which related to the group theoretical realizations of Leibniz algebras were studied by Kinyon et al. [16]. A good survey about classification is given by Loday et al. [18]. In 1998, Mikhalev et al. [21] discussed the solution of the noncommutative analogue of the Jacobian conjecture in the affirmative for free Leibniz algebras, in the spirit of the corresponding results of Reutenauer [28], Shpilrain [29] and Umirbaev [31].

2. Deformations and contractions

   Deformation theory of Leibniz algebras and its related physical applications, was initiated by Fialowski et al. [12]. The algebraic variety of four-dimensional complex nilpotent Leibniz algebras was suggested by Albeverio et al. [2].

3. Structural problems

   In Leibniz algebras, the analogue of Levi's theorem was proved by Barnes [5]. He showed that any finite-dimensional complex Leibniz algebra is decomposed into a semidirect sum of the solvable radical and a semisimple Lie algebra. The semisimple part is composed by simple Lie algebras and the main issue in the classification problem of finite-dimensional complex Leibniz algebras is to study the solvable part. Thus solvable Leibniz algebras still play a central role and have been recently studied by Cañete et al. [6] and Casas et al. [7] which shows the importance of the consideration of their nilradicals in Leibniz algebras in dimensions less than 5. Casas et al. [8, 9] also devoted to the study of solvable Leibniz algebras by considering their nilradicals. The notion of simple Leibniz algebra was suggested by Dzhumadil'daev et al. [11], which obtained some results concerning special cases of simple Leibniz algebras.The problems investigating Cartan subalgebras were studied by Omirov [22]. Albeverio et al. [1] studied on Cartan subalgebras and solvability of Leibniz algebras.

   The classification, up to isomorphism, of any class of algebras is a fundamental and very difficult problem. It is one of the first problem that one encounters when trying to understand the structure of a member of this class of algebras. Due to result by Ayupov et al. [4], there are some structural results concerning nilpotency of Leibniz algebras and its multiplication in the year 2001. Ayupov et al. [4] described the so called complex non Lie filiform Leibniz algebras. In particular case, some equivalent conditions for Leibniz algebra to be filiform and classification of the set of nilpotent Leibniz algebras containing an algebra of maximal nilindex are given as well. Naturally graded complex Leibniz algebras also is characterized. In the year 2006, the classification of 4-dimensional nilpotent complex Leibniz algebras was studied by Albeverio et al. [3].

Gómez et al. [15] devised a method of simplication of the basis transformation of the arbitrary filiform Leibniz algebras obtained from naturally graded non Lie filiform Leibniz algebras. It was considered those transformations that change the structural constants in the description of such filiform Leibniz algebras. In 2011, an approach classifying a class of filiform Leibniz algebras in terms of algebraic invariants was developed by Rakhimov et al. [24]. The method is applicable to any fixed dimensional cases of filiform Leibniz algebras.

The results of algebraic invariants method can be performed in low dimensional cases of complex filiform Leibniz algebras were devoted by Rakhimov et al. [26, 27]. Subclasses of filiform Leibniz algebras appearing from naturally graded non Lie filiform Leibniz algebras in dimensions five until eight are classified named as first class and second class. Classification problem for dimension nine is presented by Sozan et al. [30] for first class case while, Deraman et al. [10] treated second class case.

Especially useful for this thesis is the work by Rakhimov et al. [24] which classified filiform Leibniz algebras in more invariant method. This approach is extended for the classification of second class 10-dimensional filiform Leibniz algebra over field $K = \mathbb{C}$.

1.2 Research Objectives

In this thesis, the classification problem of a class of complex filiform Leibniz algebras arising from naturally graded non-Lie filiform Leibniz algebras is considered. It is known that this class of filiform Leibniz algebras are split into two subclasses. Each of them are denoted as FLb_{n+1} and SLb_{n+1}, in dimension $(n+1)$, respectively. The investigation of classification problem of SLb_{10} for dimension 10 is followed by some research objectives:

(1) To study the algebraic and geometric properties of second class complex filiform Leibniz algebras.

(2) To create isomorphism criterion for second class 10-dimensional complex filiform Leibniz algebras in terms of invariant functions.

(3) To find the list of isomorphism classes of second class 10-dimensional complex filiform Leibniz algebras.

1.3 Scope of Research

The classification of a class of algebras corresponds to a fiber of this class, that being the isomorphism classes. Classification or algebraic classification means the determination of the types of isomorphic algebras.

This research can be beneficially applied in quantum mechanics and geometrical constructions as well as may triggers in fundamental physic and applied engineering.

1.4 Research Methodology

There are two sources to get classification of complex filiform Leibniz algebras. The first of them is naturally graded non-Lie complex filiform Leibniz algebras and the second one is naturally graded filiform Lie algebras. $(n+1)$-dimensional complex filiform Leibniz algebras appearing from naturally graded non Lie filiform Leibniz algebras is split into two subclasses denoted by FLb_{n+1} and SLb_{n+1} respectively. Classification of SLb_{n+1} is only considered in this thesis particularly for $n = 9$. Here some methods on classifying SLb_{10} are presented.

(1) Isomorphism criteria for SLb_{10}

An isomorphism criterion for subclass SLb_{10} is provided where it is expressed by system of equalities. The polynomial equalities are simplified into the simplest form.

(2) Specification of a union of disjoint subsets

The isomorphism criterion of SLb_{10} is split into several disjoint subsets. For each subset, the classification problem is considered separately. Some of these subsets will represented as union parametric families of orbits and others are stated as single orbits.

(3) Algebraic invariant functions and basis change

System of algebraic invariant functions are presented in parametric families of orbits. While in single orbits case, the basis change are leading to the representatives of these orbits.

(4) Adapted linear transformations and elementary basis changing

According to the table of multiplication of SLb_{10}, the action of the reductive linear group, GL_{10} on SLb_{10} is reduced to the action of a subgroup, consist of only transformations sending adapted basis to adapted basis. Then, one defines the so called elementary basis changing and it is proved that an adapted transformation can be represented as a product of the elementary basis changing.

(5) List of non-isomorphic classes

The list of non-isomorphic classes of SLb_{10} are given according to the table of multiplications.

1.5 Outline of Contents

The thesis consists of five chapters. Now, we briefly mention the layout of the thesis.

Chapter 1 introduces the basic concepts of Leibniz algebras.

Chapter 2 describes a method of classification of filiform Leibniz algebras. Here we introduce definition of nilpotent and filiform Leibniz algebras and followed by naturally graded non-Lie filiform Leibniz algebras. Basic concepts about adapted base change, adapted transformations and isomorphism criteria for filiform Leibniz algebras arising from naturally graded non-Lie filiform Leibniz algebras are discussed.

Chapter 3 gives a complete classification of SLb_{10} which is including all possible list of isomorphism classes and invariants of SLb_{10} containing parametric families of orbits and single orbits.

Chapter 4 presents the main result of the classification. All of the representatives in SLb_{10} joined with the table of multiplication of each algebra are given in this chapter.

Chapter 5 contains some summaries on results of the thesis and suggests a few problem for future work in these areas.

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Procedure

$F :=$ `proc` (n) `local` $\beta, t, s, \Gamma;$

`if` $n \leq 3$ `then return` $0;$ `end if;`

`for` t `from` 3 `to` n `do`

`if` $t = 3$ `then` `q:=0;` `else` $q := 1;$ `end if;`

$$\beta_t := \frac{BD\gamma}{A^t} + \frac{1}{A^{t-1}} \left(Db_t - \sum_{k=3}^{t-1} \left(\left(\prod_{i=1}^{1} (k-i) A^{k-2} B b_{t+2-k} \right) + \left(\frac{\prod_{i=1}^{2}(k-i)}{2} A^{k-3} B^2 \sum_{i[1]=k+2}^{t} b_{t+3-i_1} b_{i_1+1-k} \right) \right) + \left(\frac{\prod_{i=1}^{3}(k-i)}{6} A^{k-4} B^3 \sum_{i_2=k+3}^{t} \left(\sum_{i_1=k+3}^{i_2} b_{t+3-i_2} b_{i_2+3-i_1} b_{i_1-k} \right) \right) + \left(\frac{\prod_{i=1}^{4}(k-i)}{24} A^{k-5} B^4 \sum_{i_3=k+4}^{t} \left(\sum_{i_2=k+4}^{i_3} \left(\sum_{i_1=k+4}^{i_2} b_{t+3-i_3} b_{i_3+3-i_2} b_{i_2+3-i_1} b_{i_1-1-k} \right) \right) \right) \beta_k \right);$$

`end do;`

`for` s `from` 3 `to` n `do`

$print\,(simplify\,(\beta_s));$

`end do;`

$$\Gamma := \frac{1}{A^{n-2}} \left(\frac{D}{A} \right)^2 \gamma,$$

$print\,(simplify\,(\Gamma));$

`end proc;`

APPENDIX B

Classification Procedures

This part contains some procedures of classification of subclass SLb_{10}. It is known that SLb_{10} is divided into two cases: (i) SLb_{10}^{a} and (ii) SLb_{10}^{b}. Section B.1 presents classification procedures in SLb_{10}^{a} while Section B.2 gives a few methods of classification of subclass SLb_{10}^{b}.

B.1 Classification of SLb_{10}^{a}

The isomorphism criteria for SLb_{10}^{a} can be expressed as follows:

$$\begin{aligned}
\Lambda_3^{'} &= \frac{1}{A}\left(\frac{D}{A}\right)\Lambda_3, & f_7\left(\mathbf{\Lambda}^{'}\right) &= \frac{1}{A^8}\left(\frac{D}{A}\right)^4 f_7\left(\mathbf{\Lambda}\right), \\
\Lambda_4^{'} &= 0, & f_8\left(\mathbf{\Lambda}^{'}\right) &= \frac{1}{A^{10}}\left(\frac{D}{A}\right)^5 f_8\left(\mathbf{\Lambda}\right), \\
\Lambda_5^{'} &= \frac{1}{A^4}\left(\frac{D}{A}\right)^2\Lambda_5, & f_9\left(\mathbf{\Lambda}^{'}\right) &= \frac{1}{A^{12}}\left(\frac{D}{A}\right)^6 f_9\left(\mathbf{\Lambda}\right), \\
f_6\left(\mathbf{\Lambda}^{'}\right) &= \frac{1}{A^6}\left(\frac{D}{A}\right)^3 f_6\left(\mathbf{\Lambda}\right), & \Gamma^{'} &= \frac{1}{A^7}\left(\frac{D}{A}\right)^2\Gamma.
\end{aligned} \tag{B.1}$$

Under condition $\Lambda_3 \neq 0$, the system (B.1) is isomorphic to the following system:

$$\begin{aligned}
\Lambda_3^{'} &= 1, & \frac{f_7\left(\mathbf{\Lambda}'\right)}{\Lambda_3'^4} &= \frac{1}{A^4}\frac{f_7\left(\mathbf{\Lambda}\right)}{\Lambda_3^4}, \\
\Lambda_4^{'} &= 0, & \frac{f_8\left(\mathbf{\Lambda}'\right)}{\Lambda_3'^5} &= \frac{1}{A^5}\frac{f_8\left(\mathbf{\Lambda}\right)}{\Lambda_3^5}, \\
\frac{\Lambda_5^{'}}{\Lambda_3'^2} &= \frac{1}{A^2}\frac{\Lambda_5}{\Lambda_3^2}, & \frac{f_9\left(\mathbf{\Lambda}'\right)}{\Lambda_3'^6} &= \frac{1}{A^6}\frac{f_9\left(\mathbf{\Lambda}\right)}{\Lambda_3^6}, \\
\frac{f_6\left(\mathbf{\Lambda}'\right)}{\Lambda_3'^3} &= \frac{1}{A^3}\frac{f_6\left(\mathbf{\Lambda}\right)}{\Lambda_3^3}, & \frac{\Gamma'}{\Lambda_3'^2} &= \frac{1}{A^5}\frac{\Gamma}{\Lambda_3^2}.
\end{aligned} \tag{B.2}$$

It shows that $SLb_{10}^{a} = F_1 \bigcup F_2$, where $F_1 = \left\{L\left(\mathbf{\Lambda}\right) \in SLb_{10}^{a} : \Lambda_3 \neq 0, \Lambda_5 \neq 0\right\}$ and $F_2 = \left\{L\left(\mathbf{\Lambda}\right) \in SLb_{10}^{a} : \Lambda_3 \neq 0, \Lambda_5 = 0\right\}$.

(i) $F_1 = \left\{L\left(\mathbf{\Lambda}\right) \in SLb_{10}^{a} : \Lambda_3 \neq 0, \Lambda_5 \neq 0\right\}$.

(a) $F_1 = U_1 \bigcup F_3 :$

- $U_1 = \left\{L\left(\mathbf{\Lambda}\right) \in SLb_{10}^{a} : \Lambda_3 \neq 0, \Lambda_5 \neq 0, f_6\left(\mathbf{\Lambda}\right) \neq 0\right\}$.

By reducing $\Lambda_3^{'}=1$ and $\Lambda_4^{'}=0$, then we obtain $A=\frac{f_6(\mathbf{\Lambda})}{\Lambda_3\Lambda_5}$ through conditions $\Lambda_5\neq 0$ and $f_6(\mathbf{\Lambda})\neq 0$ in (B.2). Automatically, the values of $B=\frac{\Lambda_4 A}{2\Lambda_3^3\Lambda_5}$ and $D=\frac{A^2}{\Lambda_3}$ are obtained. Subset U_1 brings the following system of equations:

$$\begin{aligned}
\Lambda_3^{'} &= 1, \\
\Lambda_4^{'} &= 0, \\
\frac{\Lambda_5^{'}}{\Lambda_3^{'2}} &= \left(\frac{\Lambda_3\Lambda_5}{f_6(\mathbf{\Lambda})}\right)^2\frac{\Lambda_5}{\Lambda_3^2}, \\
\frac{f_6(\mathbf{\Lambda}')}{\Lambda_3^{'3}} &= \left(\frac{\Lambda_3\Lambda_5}{f_6(\mathbf{\Lambda})}\right)^3\frac{f_6(\mathbf{\Lambda})}{\Lambda_3^3}, \\
\frac{f_7(\mathbf{\Lambda}')}{\Lambda_3^{'4}} &= \left(\frac{\Lambda_3\Lambda_5}{f_6(\mathbf{\Lambda})}\right)^4\frac{f_7(\mathbf{\Lambda})}{\Lambda_3^4}, \\
\frac{f_8(\mathbf{\Lambda}')}{\Lambda_3^{'5}} &= \left(\frac{\Lambda_3\Lambda_5}{f_6(\mathbf{\Lambda})}\right)^5\frac{f_8(\mathbf{\Lambda})}{\Lambda_3^5}, \\
\frac{f_9(\mathbf{\Lambda}')}{\Lambda_3^{'6}} &= \left(\frac{\Lambda_3\Lambda_5}{f_6(\mathbf{\Lambda})}\right)^6\frac{f_9(\mathbf{\Lambda})}{\Lambda_3^6}, \\
\frac{\Gamma'}{\Lambda_3^{'2}} &= \left(\frac{\Lambda_3\Lambda_5}{f_6(\mathbf{\Lambda})}\right)^5\frac{\Gamma}{\Lambda_3^2}.
\end{aligned} \tag{B.3}$$

New parameters are represented as $\lambda_1,\lambda_2,\lambda_3,\lambda_4$ and λ_5 to imply that $\Lambda_5^{'}=\frac{\Lambda_5^3}{f_6^2(\mathbf{\Lambda})}=\lambda_1$, $f_6(\mathbf{\Lambda}')=\frac{\Lambda_5^3}{f_6^2(\mathbf{\Lambda})}=\lambda_1$, $f_7(\mathbf{\Lambda}')=\frac{f_7(\mathbf{\Lambda})\Lambda_5^4}{f_6^4(\mathbf{\Lambda})}=\lambda_2$, $f_8(\mathbf{\Lambda}')=\frac{f_8(\mathbf{\Lambda})\Lambda_5^5}{f_6^5(\mathbf{\Lambda})}=\lambda_3$, $f_9(\mathbf{\Lambda}')=\frac{f_9(\mathbf{\Lambda})\Lambda_5^6}{f_6^6(\mathbf{\Lambda})}=\lambda_4$ and $\Gamma'=\frac{\Lambda_3^3\Lambda_5^5\Gamma}{f_6^5(\mathbf{\Lambda})}=\lambda_5$. Thus all algebras from U_1 are isomorphic to $L(1,0,\lambda_1,\lambda_1,\lambda_2,\lambda_3,\lambda_4,\lambda_5)$ for $\lambda_1\in\mathbb{C}^*$ and $\lambda_2,\lambda_3,\lambda_4,\lambda_5\in\mathbb{C}$.

- $F_3=\left\{L(\mathbf{\Lambda})\in SLb_{10}^a : \Lambda_3\neq 0,\Lambda_5\neq 0, f_6(\mathbf{\Lambda})=0\right\}$.
  Through the conditions in F_3, the system of equations in (B.2) becomes as follows:

$$\Lambda_3^{'} = 1, \qquad \frac{f_7(\mathbf{\Lambda}')}{\Lambda_3^{'4}} = \frac{1}{A^4}\frac{f_7(\mathbf{\Lambda})}{\Lambda_3^4}, \tag{B.4}$$
$$\Lambda_4^{'} = 0, \qquad \frac{f_8(\mathbf{\Lambda}')}{\Lambda_3^{'5}} = \frac{1}{A^5}\frac{f_8(\mathbf{\Lambda})}{\Lambda_3^5},$$
$$\frac{\Lambda_5^{'}}{\Lambda_3^{'2}} = \frac{1}{A^2}\frac{\Lambda_5}{\Lambda_3^2}, \qquad \frac{f_9(\mathbf{\Lambda}')}{\Lambda_3^{'6}} = \frac{1}{A^6}\frac{f_9(\mathbf{\Lambda})}{\Lambda_3^6},$$
$$\frac{f_6(\mathbf{\Lambda}')}{\Lambda_3^{'3}} = 0, \qquad \frac{\Gamma'}{\Lambda_3^{'2}} = \frac{1}{A^5}\frac{\Gamma}{\Lambda_3^2}.$$

We can rewrite F_3 as $F_3 = U_2 \bigcup F_4$, where
$U_2 = \left\{L(\Lambda) \in SLb_{10}^a \,:\, \Lambda_3 \neq 0, \Lambda_5 \neq 0, f_6(\mathbf{\Lambda}) = 0, f_8(\mathbf{\Lambda}) \neq 0\right\}$ and
$F_4 = \left\{L(\Lambda) \in SLb_{10}^a \,:\, \Lambda_3 \neq 0, \Lambda_5 \neq 0, f_6(\mathbf{\Lambda}) = 0, f_8(\mathbf{\Lambda}) = 0\right\}$.

(b) $F_3 = U_2 \bigcup F_4$:

- $U_2 = \left\{L(\mathbf{\Lambda}) \in SLb_{10}^a \,:\, \Lambda_3 \neq 0, \Lambda_5 \neq 0, f_6(\mathbf{\Lambda}) = 0, f_8(\mathbf{\Lambda}) \neq 0\right\}$.
  By reducing $\frac{\Lambda_5^{'}}{\Lambda_3^{'2}} = \frac{1}{A^2}\frac{\Lambda_5}{\Lambda_3^2}$ and $\frac{f_8(\mathbf{\Lambda}')}{\Lambda_3^{'5}} = \frac{1}{A^5}\frac{f_8(\mathbf{\Lambda})}{\Lambda_3^5}$ in (B.4), then we obtain $A = \frac{f_8(\mathbf{\Lambda})}{\Lambda_3\Lambda_5^2}$. The values of B and D appear as $B = \frac{\Lambda_4 A}{2\Lambda_3^3\Lambda_5}$ and $D = \frac{A^2}{\Lambda_3}$. Then by using value $\frac{1}{A} = \frac{\Lambda_3\Lambda_5^2}{f_8(\mathbf{\Lambda})}$, the system of equalities for U_2 is as follows:

$$\begin{aligned}
\Lambda_3^{'} &= 1, \\
\Lambda_4^{'} &= 0, \\
\frac{\Lambda_5^{'}}{\Lambda_3^{'2}} &= \left(\frac{\Lambda_3\Lambda_5^2}{f_8(\mathbf{\Lambda})}\right)^2 \frac{\Lambda_5}{\Lambda_3^2}, \\
\frac{f_6(\mathbf{\Lambda}')}{\Lambda_3^{'3}} &= 0, \\
\frac{f_7(\mathbf{\Lambda}')}{\Lambda_3^{'4}} &= \left(\frac{\Lambda_3\Lambda_5^2}{f_8(\mathbf{\Lambda})}\right)^4 \frac{f_7(\mathbf{\Lambda})}{\Lambda_3^4}, \\
\frac{f_8(\mathbf{\Lambda}')}{\Lambda_3^{'5}} &= \left(\frac{\Lambda_3\Lambda_5^2}{f_8(\mathbf{\Lambda})}\right)^5 \frac{f_8(\mathbf{\Lambda})}{\Lambda_3^5}, \\
\frac{f_9(\mathbf{\Lambda}')}{\Lambda_3^{'6}} &= \left(\frac{\Lambda_3\Lambda_5^2}{f_8(\mathbf{\Lambda})}\right)^6 \frac{f_9(\mathbf{\Lambda})}{\Lambda_3^6}, \\
\frac{\Gamma'}{\Lambda_3^{'2}} &= \left(\frac{\Lambda_3\Lambda_5^2}{f_8(\mathbf{\Lambda})}\right)^5 \frac{\Gamma}{\Lambda_3^2}.
\end{aligned} \tag{B.5}$$

New parameters are represented as $\lambda_1, \lambda_2, \lambda_3$ and λ_4 to imply that $\Lambda_5^{'} = \frac{\Lambda_5^5}{f_8^2(\mathbf{\Lambda})} = \lambda_1$, $f_7\left(\mathbf{\Lambda}'\right) = \frac{\Lambda_5^8 f_7(\mathbf{\Lambda})}{f_8^4(\mathbf{\Lambda})} = \lambda_2$, $f_8\left(\mathbf{\Lambda}'\right) = \frac{\Lambda_5^{10}}{f_8^4(\mathbf{\Lambda})} = \lambda_1^2$, $f_9\left(\mathbf{\Lambda}'\right) = \frac{f_9(\mathbf{\Lambda})\Lambda_5^{12}}{f_8^6(\mathbf{\Lambda})} = \lambda_3$ and $\Gamma' = \frac{\Lambda_3^3\Lambda_5^{10}\Gamma}{f_8^5(\mathbf{\Lambda})} = \lambda_4$. Thus all algebras from U_2 are isomorphic to $L\left(1, 0, \lambda_1, 0, \lambda_2, \lambda_1^2, \lambda_3, \lambda_4\right)$ for $\lambda_1 \in \mathbb{C}^*$ and $\lambda_2, \lambda_3, \lambda_4 \in \mathbb{C}$.

- $F_4 = \left\{L\left(\mathbf{\Lambda}\right) \in SLb_{10}^a : \Lambda_3 \neq 0, \Lambda_5 \neq 0, f_6\left(\mathbf{\Lambda}\right) = 0, f_8\left(\mathbf{\Lambda}\right) = 0\right\}$.
The system of equalities in F_4 is as below:

$$\begin{aligned}
\Lambda_3^{'} &= 1, \\
\Lambda_4^{'} &= 0, \\
\frac{\Lambda_5^{'}}{\Lambda_3^{'2}} &= \frac{1}{A^2}\frac{\Lambda_5}{\Lambda_3^2}, \\
\frac{f_6\left(\mathbf{\Lambda}'\right)}{\Lambda_3^{'3}} &= 0, \\
\frac{f_7\left(\mathbf{\Lambda}'\right)}{\Lambda_3^{'4}} &= \frac{1}{A^4}\frac{f_7\left(\mathbf{\Lambda}\right)}{\Lambda_3^4}, \\
\frac{f_8\left(\mathbf{\Lambda}'\right)}{\Lambda_3^{'5}} &= 0, \\
\frac{f_9\left(\mathbf{\Lambda}'\right)}{\Lambda_3^{'6}} &= \frac{1}{A^6}\frac{f_9\left(\mathbf{\Lambda}\right)}{\Lambda_3^6}, \\
\frac{\Gamma'}{\Lambda_3^{'2}} &= \frac{1}{A^5}\frac{\Gamma}{\Lambda_3^2}.
\end{aligned} \tag{B.6}$$

The subset of F_4 can be written as $F_4 = U_3 \bigcup U_4$ where
$U_3 = \left\{L\left(\Lambda\right) \in SLb_{10}^a : \Lambda_3 \neq 0, \Lambda_5 \neq 0, f_6\left(\mathbf{\Lambda}\right) = 0, f_8\left(\mathbf{\Lambda}\right) = 0, \Gamma \neq 0\right\}$ and
$U_4 = \left\{L\left(\Lambda\right) \in SLb_{10}^a : \Lambda_3 \neq 0, \Lambda_5 \neq 0, f_6\left(\mathbf{\Lambda}\right) = 0, f_8\left(\mathbf{\Lambda}\right) = 0, \Gamma = 0\right\}$.

(c) For $F_4 = U_3 \bigcup U_4$:

- $U_3 = \left\{L\left(\mathbf{\Lambda}\right) \in SLb_{10}^a : \Lambda_3 \neq 0, \Lambda_5 \neq 0, f_6\left(\mathbf{\Lambda}\right) = 0, f_8\left(\mathbf{\Lambda}\right) = 0, \Gamma \neq 0\right\}$
By reducing $\frac{\Lambda_5^{'}}{\Lambda_3^{'2}} = \frac{1}{A^2}\frac{\Lambda_5}{\Lambda_3^2}$ and $\frac{\Gamma'}{\Lambda_3^{'2}} = \frac{1}{A^5}\frac{\Gamma}{\Lambda_3^2}$ in (B.6), then we obtain $A = \frac{\Lambda_3^2\Gamma}{\Lambda_5^2}$. The values of B and D appear as $B = \frac{\Lambda_4 A}{2\Lambda_3^3\Lambda_5}$ and $D = \frac{A^2}{\Lambda_3}$.

Then by using value $\frac{1}{A} = \frac{\Lambda_5^2}{\Lambda_3^2 \Gamma}$, the system of equalities for U_3 is as follows:

$$
\begin{aligned}
\Lambda_3^{'} &= 1, \\
\Lambda_4^{'} &= 0, \\
\frac{\Lambda_5^{'}}{\Lambda_3^{'2}} &= \left(\frac{\Lambda_5^2}{\Lambda_3^2 \Gamma}\right)^2 \frac{\Lambda_5}{\Lambda_3^2}, \\
\frac{f_6\left(\mathbf{\Lambda}'\right)}{\Lambda_3^{'3}} &= 0, \\
\frac{f_7\left(\mathbf{\Lambda}'\right)}{\Lambda_3^{'4}} &= \left(\frac{\Lambda_5^2}{\Lambda_3^2 \Gamma}\right)^4 \frac{f_7\left(\mathbf{\Lambda}\right)}{\Lambda_3^4}, \\
\frac{f_8\left(\mathbf{\Lambda}'\right)}{\Lambda_3^{'5}} &= 0, \\
\frac{f_9\left(\mathbf{\Lambda}'\right)}{\Lambda_3^{'6}} &= \left(\frac{\Lambda_5^2}{\Lambda_3^2 \Gamma}\right)^6 \frac{f_9\left(\mathbf{\Lambda}\right)}{\Lambda_3^6}, \\
\frac{\Gamma'}{\Lambda_3^{'2}} &= \left(\frac{\Lambda_5^2}{\Lambda_3^2 \Gamma}\right)^5 \frac{\Gamma}{\Lambda_3^2}.
\end{aligned}
\tag{B.7}
$$

New parameters are represented as λ_1, λ_2 and λ_3 to imply that $\Lambda_5^{'} = \frac{\Lambda_5^5}{\Lambda_3^6 \Gamma^2} = \lambda_1$, $f_7\left(\mathbf{\Lambda}'\right) = \frac{\Lambda_5^8 f_7(\mathbf{\Lambda})}{\Lambda_3^{12} \Gamma^4} = \lambda_2$, $f_9\left(\mathbf{\Lambda}'\right) = \frac{f_9(\mathbf{\Lambda}) \Lambda_5^{12}}{\Lambda_3^{18} \Gamma^6} = \lambda_3$ and $\Gamma^6 = \frac{\Lambda_5^{10}}{\Lambda_3^{12} \Gamma^4} = \lambda_1$. Thus all algebras from U_3 are isomorphic to $L\left(1, 0, \lambda_1, 0, \lambda_2, 0, \lambda_3, \lambda_1^2\right)$ for $\lambda_1 \in \mathbb{C}^*$ and $\lambda_2, \lambda_3 \in \mathbb{C}$.

- $U_4 = \{L\left(\mathbf{\Lambda}\right) \in SLb_{10}^a :$ $\Lambda_3 \neq 0, \Lambda_5 \neq 0, f_6\left(\mathbf{\Lambda}\right) = 0, f_8\left(\mathbf{\Lambda}\right) = 0, \Gamma = 0\}$.
  By reducing $\frac{\Lambda_5^{'}}{\Lambda_3^{'2}} = \frac{1}{A^2} \frac{\Lambda_5}{\Lambda_3^2}$ in (B.6), then we obtain $A^2 = \frac{\Lambda_5}{\Lambda_3^2}$. The values of B and D appear as $B = \frac{\Lambda_4 A}{2 \Lambda_3^3 \Lambda_5}$ and $D = \frac{A^2}{\Lambda_3}$.

Then by using value $\frac{1}{A^2}=\frac{\Lambda_3^2}{\Lambda_5}$, the system of equalities for U_4 is as follows:

$$
\begin{aligned}
\Lambda_3^{'} &= 1, \\
\Lambda_4^{'} &= 0, \\
\frac{\Lambda_5^{'}}{\Lambda_3^{'2}} &= \left(\frac{\Lambda_3^2}{\Lambda_5}\right)\frac{\Lambda_5}{\Lambda_3^2}, \\
\frac{f_6(\mathbf{\Lambda}')}{\Lambda_3^{'3}} &= 0, \\
\frac{f_7(\mathbf{\Lambda}')}{\Lambda_3^{'4}} &= \left(\frac{\Lambda_3^2}{\Lambda_5}\right)^2\frac{f_7(\mathbf{\Lambda})}{\Lambda_3^4}, \\
\frac{f_8(\mathbf{\Lambda}')}{\Lambda_3^{'5}} &= 0, \\
\frac{f_9(\mathbf{\Lambda}')}{\Lambda_3^{'6}} &= \left(\frac{\Lambda_3^2}{\Lambda_5}\right)^3\frac{f_9(\mathbf{\Lambda})}{\Lambda_3^6}, \\
\frac{\Gamma'}{\Lambda_3^{'2}} &= 0.
\end{aligned}
\tag{B.8}
$$

New parameters are represented as λ_1 and λ_2 to imply that $\Lambda_5^{'}=1$, $f_7(\mathbf{\Lambda}')=\frac{f_7(\mathbf{\Lambda})}{\Lambda_5^2}=\lambda_1$, $f_9(\mathbf{\Lambda}')=\frac{f_9(\mathbf{\Lambda})}{\Lambda_5^3}=\lambda_2$. Thus all algebras from U_4 are isomorphic to $L(1,0,1,0,\lambda_1,0,\lambda_2,0)$ for $\lambda_1,\lambda_2\in\mathbb{C}$.

(ii) $F_2=\left\{L(\mathbf{\Lambda})\in SLb_{10}^a : \Lambda_3\neq 0, \Lambda_5=0\right\}$
The subset F_2 brings the following system of equations:

$$
\begin{aligned}
\Lambda_3^{'} &= 1, & \frac{f_7(\mathbf{\Lambda}')}{\Lambda_3^{'4}} &= \frac{1}{A^4}\frac{f_7(\mathbf{\Lambda})}{\Lambda_3^4}, \\
\Lambda_4^{'} &= 0, & \frac{f_8(\mathbf{\Lambda}')}{\Lambda_3^{'5}} &= \frac{1}{A^5}\frac{f_8(\mathbf{\Lambda})}{\Lambda_3^5}, \\
\frac{\Lambda_5^{'}}{\Lambda_3^{'2}} &= 0, & \frac{f_9(\mathbf{\Lambda}')}{\Lambda_3^{'6}} &= \frac{1}{A^6}\frac{f_9(\mathbf{\Lambda})}{\Lambda_3^6}, \\
\frac{f_6(\mathbf{\Lambda}')}{\Lambda_3^{'3}} &= \frac{1}{A^3}\frac{f_6(\mathbf{\Lambda})}{\Lambda_3^3}, & \frac{\Gamma'}{\Lambda_3^{'2}} &= \frac{1}{A^5}\frac{\Gamma}{\Lambda_3^2}.
\end{aligned}
\tag{B.9}
$$

(a) $F_2 = F_5 \bigcup F_6$:

- $F_5 = \left\{L(\mathbf{\Lambda}) \in SLb_{10}^a : \Lambda_3 \neq 0, \Lambda_5 = 0, f_6(\mathbf{\Lambda}) \neq 0\right\}$
The subset F_5 brings the following system of equations:

$$\begin{aligned} \Lambda_3^{'} &= 1, & \frac{f_7(\mathbf{\Lambda}')}{\Lambda_3'^4} &= \frac{1}{A^4}\frac{f_7(\mathbf{\Lambda})}{\Lambda_3^4}, \\ \Lambda_4^{'} &= 0, & \frac{f_8(\mathbf{\Lambda}')}{\Lambda_3'^5} &= \frac{1}{A^5}\frac{f_8(\mathbf{\Lambda})}{\Lambda_3^5}, \\ \frac{\Lambda_5^{'}}{\Lambda_3'^2} &= 0, & \frac{f_9(\mathbf{\Lambda}')}{\Lambda_3'^6} &= \frac{1}{A^6}\frac{f_9(\mathbf{\Lambda})}{\Lambda_3^6}, \\ \frac{f_6(\mathbf{\Lambda}')}{\Lambda_3'^3} &= \frac{1}{A^3}\frac{f_6(\mathbf{\Lambda})}{\Lambda_3^3}, & \frac{\Gamma'}{\Lambda_3'^2} &= \frac{1}{A^5}\frac{\Gamma}{\Lambda_3^2}. \end{aligned} \tag{B.10}$$

- $F_6 = \left\{L(\mathbf{\Lambda}) \in SLb_{10}^a : \Lambda_3 \neq 0, \Lambda_5 = 0, f_6(\mathbf{\Lambda}) = 0\right\}$
The system of equations of subset F_6 is as following:

$$\begin{aligned} \Lambda_3^{'} &= 1, & \frac{f_7(\mathbf{\Lambda}')}{\Lambda_3'^4} &= \frac{1}{A^4}\frac{f_7(\mathbf{\Lambda})}{\Lambda_3^4}, \\ \Lambda_4^{'} &= 0, & \frac{f_8(\mathbf{\Lambda}')}{\Lambda_3'^5} &= \frac{1}{A^5}\frac{f_8(\mathbf{\Lambda})}{\Lambda_3^5}, \\ \frac{\Lambda_5^{'}}{\Lambda_3'^2} &= 0, & \frac{f_9(\mathbf{\Lambda}')}{\Lambda_3'^6} &= \frac{1}{A^6}\frac{f_9(\mathbf{\Lambda})}{\Lambda_3^6}, \\ \frac{f_6(\mathbf{\Lambda}')}{\Lambda_3'^3} &= 0, & \frac{\Gamma'}{\Lambda_3'^2} &= \frac{1}{A^5}\frac{\Gamma}{\Lambda_3^2}. \end{aligned} \tag{B.11}$$

(b) $F_5 = U_5 \bigcup F_7$:

- $U_5 = \left\{L(\mathbf{\Lambda}) \in SLb_{10}^a : \Lambda_3 \neq 0, \Lambda_5 = 0, f_6(\mathbf{\Lambda}) \neq 0, f_7(\mathbf{\Lambda}) \neq 0\right\}$
By reducing $\frac{f_6(\mathbf{\Lambda}')}{\Lambda_3'^3} = \frac{1}{A^3}\frac{f_6(\mathbf{\Lambda})}{\Lambda_3^3}$ and $\frac{f_7(\mathbf{\Lambda}')}{\Lambda_3'^4} = \frac{1}{A^4}\frac{f_7(\mathbf{\Lambda})}{\Lambda_3^4}$ in (B.10), then we obtain $A = \frac{f_7(\mathbf{\Lambda})}{\Lambda_3 f_6(\mathbf{\Lambda})}$. The values of B and D appear as $B = \frac{\Lambda_4 A}{2\Lambda_3^3\Lambda_5}$ and $D = \frac{A^2}{\Lambda_3}$. Then by using value $\frac{1}{A} = \frac{\Lambda_3 f_6(\mathbf{\Lambda})}{f_7(\mathbf{\Lambda})}$, the system of equalities for U_5 is as follows:

$$\begin{aligned}
\Lambda_3^{'} &= 1, \qquad\qquad (B.12)\\
\Lambda_4^{'} &= 0,\\
\frac{\Lambda_5^{'}}{\Lambda_3^{'2}} &= 0,\\
\frac{f_6\left(\mathbf{\Lambda}'\right)}{\Lambda_3^{'3}} &= \left(\frac{\Lambda_3 f_6\left(\mathbf{\Lambda}\right)}{f_7\left(\mathbf{\Lambda}\right)}\right)^3 \frac{f_6\left(\mathbf{\Lambda}\right)}{\Lambda_3^3},\\
\frac{f_7\left(\mathbf{\Lambda}'\right)}{\Lambda_3^{'4}} &= \left(\frac{\Lambda_3 f_6\left(\mathbf{\Lambda}\right)}{f_7\left(\mathbf{\Lambda}\right)}\right)^4 \frac{f_7\left(\mathbf{\Lambda}\right)}{\Lambda_3^4},\\
\frac{f_8\left(\mathbf{\Lambda}'\right)}{\Lambda_3^{'5}} &= \left(\frac{\Lambda_3 f_6\left(\mathbf{\Lambda}\right)}{f_7\left(\mathbf{\Lambda}\right)}\right)^5 \frac{f_8\left(\mathbf{\Lambda}\right)}{\Lambda_3^5},\\
\frac{f_9\left(\mathbf{\Lambda}'\right)}{\Lambda_3^{'6}} &= \left(\frac{\Lambda_3 f_6\left(\mathbf{\Lambda}\right)}{f_7\left(\mathbf{\Lambda}\right)}\right)^6 \frac{f_9\left(\mathbf{\Lambda}\right)}{\Lambda_3^6},\\
\frac{\Gamma'}{\Lambda_3^{'2}} &= \left(\frac{\Lambda_3 f_6\left(\mathbf{\Lambda}\right)}{f_7\left(\mathbf{\Lambda}\right)}\right)^5 \frac{\Gamma}{\Lambda_3^2}.
\end{aligned}$$

New parameters are represented as $\lambda_1, \lambda_2, \lambda_3$ and λ_4 to imply that $f_6\left(\mathbf{\Lambda}'\right) = \frac{f_6^4(\mathbf{\Lambda})}{f_7^3(\mathbf{\Lambda})} = \lambda_1,\ f_7\left(\mathbf{\Lambda}'\right) = \frac{f_6^4(\mathbf{\Lambda})}{f_7^3(\mathbf{\Lambda})} = \lambda_1, f_8\left(\mathbf{\Lambda}'\right) = \frac{f_6^5(\mathbf{\Lambda})f_8(\mathbf{\Lambda})}{f_7^5(\mathbf{\Lambda})} = \lambda_2$, $f_9\left(\mathbf{\Lambda}'\right) = \frac{f_6^6(\mathbf{\Lambda})f_9(\mathbf{\Lambda})}{f_7^6(\mathbf{\Lambda})} = \lambda_3$ and $\Gamma' = \frac{\Lambda_3^3 f_6^5(\mathbf{\Lambda})\Gamma}{f_7^5(\mathbf{\Lambda})} = \lambda_4$. Thus all algebras from U_5 are isomorphic to $L\left(1,0,0,\lambda_1,\lambda_1,\lambda_2,\lambda_3,\lambda_4\right)$ for $\lambda_1 \in \mathbb{C}^*$ and $\lambda_2, \lambda_3, \lambda_4 \in \mathbb{C}$.

- $F_7 = \left\{L\left(\mathbf{\Lambda}\right) \in SLb_{10}^a : \Lambda_3 \neq 0, \Lambda_5 = 0, f_6\left(\mathbf{\Lambda}\right) \neq 0, f_7\left(\mathbf{\Lambda}\right) = 0\right\}$
  Under the conditions of subset F_7, the system of equations appear as below:

$$\begin{aligned}
\Lambda_3^{'} &= 1, & \frac{f_7\left(\mathbf{\Lambda}'\right)}{\Lambda_3^{'4}} &= 0, \qquad (B.13)\\
\Lambda_4^{'} &= 0, & \frac{f_8\left(\mathbf{\Lambda}'\right)}{\Lambda_3^{'5}} &= \frac{1}{A^5}\frac{f_8\left(\mathbf{\Lambda}\right)}{\Lambda_3^5},\\
\frac{\Lambda_5^{'}}{\Lambda_3^{'2}} &= 0, & \frac{f_9\left(\mathbf{\Lambda}'\right)}{\Lambda_3^{'6}} &= \frac{1}{A^6}\frac{f_9\left(\mathbf{\Lambda}\right)}{\Lambda_3^6},\\
\frac{f_6\left(\mathbf{\Lambda}'\right)}{\Lambda_3^{'3}} &= \frac{1}{A^3}\frac{f_6\left(\mathbf{\Lambda}\right)}{\Lambda_3^3}, & \frac{\Gamma'}{\Lambda_3^{'2}} &= \frac{1}{A^5}\frac{\Gamma}{\Lambda_3^2}.
\end{aligned}$$

(c) $F_7 = U_6 \bigcup F_8$:

- $U_6 = \{L\left(\mathbf{\Lambda}\right) \in SLb_{10}^a$:
  $\Lambda_3 \neq 0, \Lambda_5 = 0, f_6\left(\mathbf{\Lambda}\right) \neq 0, f_7\left(\mathbf{\Lambda}\right) = 0, f_8\left(\mathbf{\Lambda}\right) \neq 0\}$.

By reducing $\frac{f_6(\mathbf{\Lambda'})}{\Lambda_3'^3} = \frac{1}{A^3}\frac{f_6(\mathbf{\Lambda})}{\Lambda_3^3}$ and $\frac{f_8(\mathbf{\Lambda'})}{\Lambda_3'^5} = \frac{1}{A^5}\frac{f_8(\mathbf{\Lambda})}{\Lambda_3^5}$ in (B.13), then we obtain $A = \frac{f_6^2(\mathbf{\Lambda})}{\Lambda_3 f_8(\mathbf{\Lambda})}$. The values of B and D appear as $B = \frac{\Lambda_4 A}{2\Lambda_3^3\Lambda_5}$ and $D = \frac{A^2}{\Lambda_3}$. Then by using value $\frac{1}{A} = \frac{\Lambda_3 f_8(\mathbf{\Lambda})}{f_6^2(\mathbf{\Lambda})}$, the system of equalities for U_6 is as the following page:

$$\begin{aligned}
\Lambda_3^{'} &= 1 \\
\Lambda_4^{'} &= 0 \\
\frac{\Lambda_5^{'}}{\Lambda_3'^2} &= 0 \\
\frac{f_6\left(\mathbf{\Lambda'}\right)}{\Lambda_3'^3} &= \left(\frac{\Lambda_3 f_8\left(\mathbf{\Lambda}\right)}{f_6^2\left(\mathbf{\Lambda}\right)}\right)^3 \frac{f_6\left(\mathbf{\Lambda}\right)}{\Lambda_3^3} \\
\frac{f_7\left(\mathbf{\Lambda'}\right)}{\Lambda_3'^4} &= 0 \\
\frac{f_8\left(\mathbf{\Lambda'}\right)}{\Lambda_3'^5} &= \left(\frac{\Lambda_3 f_8\left(\mathbf{\Lambda}\right)}{f_6^2\left(\mathbf{\Lambda}\right)}\right)^5 \frac{f_8\left(\mathbf{\Lambda}\right)}{\Lambda_3^5} \\
\frac{f_9\left(\mathbf{\Lambda'}\right)}{\Lambda_3'^6} &= \left(\frac{\Lambda_3 f_8\left(\mathbf{\Lambda}\right)}{f_6^2\left(\mathbf{\Lambda}\right)}\right)^6 \frac{f_9\left(\mathbf{\Lambda}\right)}{\Lambda_3^6} \\
\frac{\Gamma'}{\Lambda_3'^2} &= \left(\frac{\Lambda_3 f_8\left(\mathbf{\Lambda}\right)}{f_6^2\left(\mathbf{\Lambda}\right)}\right)^5 \frac{\Gamma}{\Lambda_3^2}
\end{aligned} \tag{B.14}$$

New parameters are represented as λ_1, λ_2 and λ_3 to imply that $f_6\left(\mathbf{\Lambda'}\right) = \frac{f_8^3(\mathbf{\Lambda})}{f_6^5(\mathbf{\Lambda})} = \lambda_1$, $f_8\left(\mathbf{\Lambda'}\right) = \frac{f_8^6(\mathbf{\Lambda})}{f_6^{10}(\mathbf{\Lambda})} = \lambda_1^2$, $f_9\left(\mathbf{\Lambda'}\right) = \frac{f_8^6(\mathbf{\Lambda}) f_9(\mathbf{\Lambda})}{f_6^{12}(\mathbf{\Lambda})} = \lambda_2$ and $\Gamma' = \frac{\Lambda_3^3 f_8^5(\mathbf{\Lambda})\Gamma}{f_6^{10}(\mathbf{\Lambda})} = \lambda_3$. Thus all algebras from U_6 are isomorphic to $L\left(1,0,0,\lambda_1,0,\lambda_1^2,\lambda_2,\lambda_3\right)$ for $\lambda_1 \in \mathbb{C}^*$ and $\lambda_2, \lambda_3 \in \mathbb{C}$.

- $F_8 = \{L\left(\mathbf{\Lambda}\right) \in SLb_{10}^a :$
  $\Lambda_3 \neq 0, \Lambda_5 = 0, f_6\left(\mathbf{\Lambda}\right) \neq 0, f_7\left(\mathbf{\Lambda}\right) = 0, f_8\left(\mathbf{\Lambda}\right) = 0\}$
  Subset F_8 shows the following system of equations:

$$\Lambda_3^{'} = 1, \qquad \frac{f_7(\mathbf{\Lambda}')}{\Lambda_3'^4} = 0, \tag{B.15}$$

$$\Lambda_4^{'} = 0, \qquad \frac{f_8(\mathbf{\Lambda}')}{\Lambda_3'^5} = 0,$$

$$\frac{\Lambda_5^{'}}{\Lambda_3'^2} = 0, \qquad \frac{f_9(\mathbf{\Lambda}')}{\Lambda_3'^6} = \frac{1}{A^6}\frac{f_9(\mathbf{\Lambda})}{\Lambda_3^6},$$

$$\frac{f_6(\mathbf{\Lambda}')}{\Lambda_3'^3} = \frac{1}{A^3}\frac{f_6(\mathbf{\Lambda})}{\Lambda_3^3}, \qquad \frac{\Gamma'}{\Lambda_3'^2} = \frac{1}{A^5}\frac{\Gamma}{\Lambda_3^2}.$$

(d) $F_8 = U_7 \bigcup U_8$:

- $U_7 = \{L(\mathbf{\Lambda}) \in SLb_{10}^a :$ $\Lambda_3 \neq 0, \Lambda_5 = 0, f_6(\mathbf{\Lambda}) \neq 0, f_7(\mathbf{\Lambda}) = 0, f_8(\mathbf{\Lambda}) = 0, \Gamma \neq 0\}$
  By reducing $\frac{f_6(\mathbf{\Lambda}')}{\Lambda_3'^3} = \frac{1}{A^3}\frac{f_6(\mathbf{\Lambda})}{\Lambda_3^3}$ and $\frac{\Gamma'}{\Lambda_3'^2} = \frac{1}{A^5}\frac{\Gamma}{\Lambda_3^2}$ in (B.15), then we obtain $A = \frac{f_6^2(\mathbf{\Lambda})}{\Lambda_3^4\Gamma}$. The values of B and D appear as $B = \frac{\Lambda_4 A}{2\Lambda_3^3\Lambda_5}$ and $D = \frac{A^2}{\Lambda_3}$. Then by using value $\frac{1}{A} = \frac{\Lambda_3^4\Gamma}{f_6^2(\mathbf{\Lambda})}$, the system of equalities for U_7 is as the following page:

$$\begin{aligned}
\Lambda_3^{'} &= 1, \\
\Lambda_4^{'} &= 0, \\
\frac{\Lambda_5^{'}}{\Lambda_3'^2} &= 0, \\
\frac{f_6(\mathbf{\Lambda}')}{\Lambda_3'^3} &= \left(\frac{\Lambda_3^4\Gamma}{f_6^2(\mathbf{\Lambda})}\right)^3 \frac{f_6(\mathbf{\Lambda})}{\Lambda_3^3}, \\
\frac{f_7(\mathbf{\Lambda}')}{\Lambda_3'^4} &= 0, \\
\frac{f_8(\mathbf{\Lambda}')}{\Lambda_3'^5} &= 0, \\
\frac{f_9(\mathbf{\Lambda}')}{\Lambda_3'^6} &= \left(\frac{\Lambda_3^4\Gamma}{f_6^2(\mathbf{\Lambda})}\right)^6 \frac{f_9(\mathbf{\Lambda})}{\Lambda_3^6}, \\
\frac{\Gamma'}{\Lambda_3'^2} &= \left(\frac{\Lambda_3^4\Gamma}{f_6^2(\mathbf{\Lambda})}\right)^5 \frac{\Gamma}{\Lambda_3^2}.
\end{aligned} \tag{B.16}$$

New parameters are represented as λ_1 and λ_2 to imply that $f_6\left(\mathbf{\Lambda}'\right) = \frac{\Lambda_3^9\Gamma^3}{f_6^5(\mathbf{\Lambda})} = \lambda_1,\ f_9\left(\mathbf{\Lambda}'\right) = \frac{\Lambda_3^{18}\Gamma^6 f_9(\mathbf{\Lambda})}{f_6^{12}(\mathbf{\Lambda})} = \lambda_2$ and $\Gamma' = \frac{\Lambda_3^{18}\Gamma^6}{f_6^{10}(\mathbf{\Lambda})} = \lambda_1^2$. Thus all algebras from U_7 are isomorphic to $L\left(1,0,0,\lambda_1,0,0,\lambda_2,\lambda_1^2\right)$ for $\lambda_1 \in \mathbb{C}^*$ and $\lambda_2 \in \mathbb{C}$.

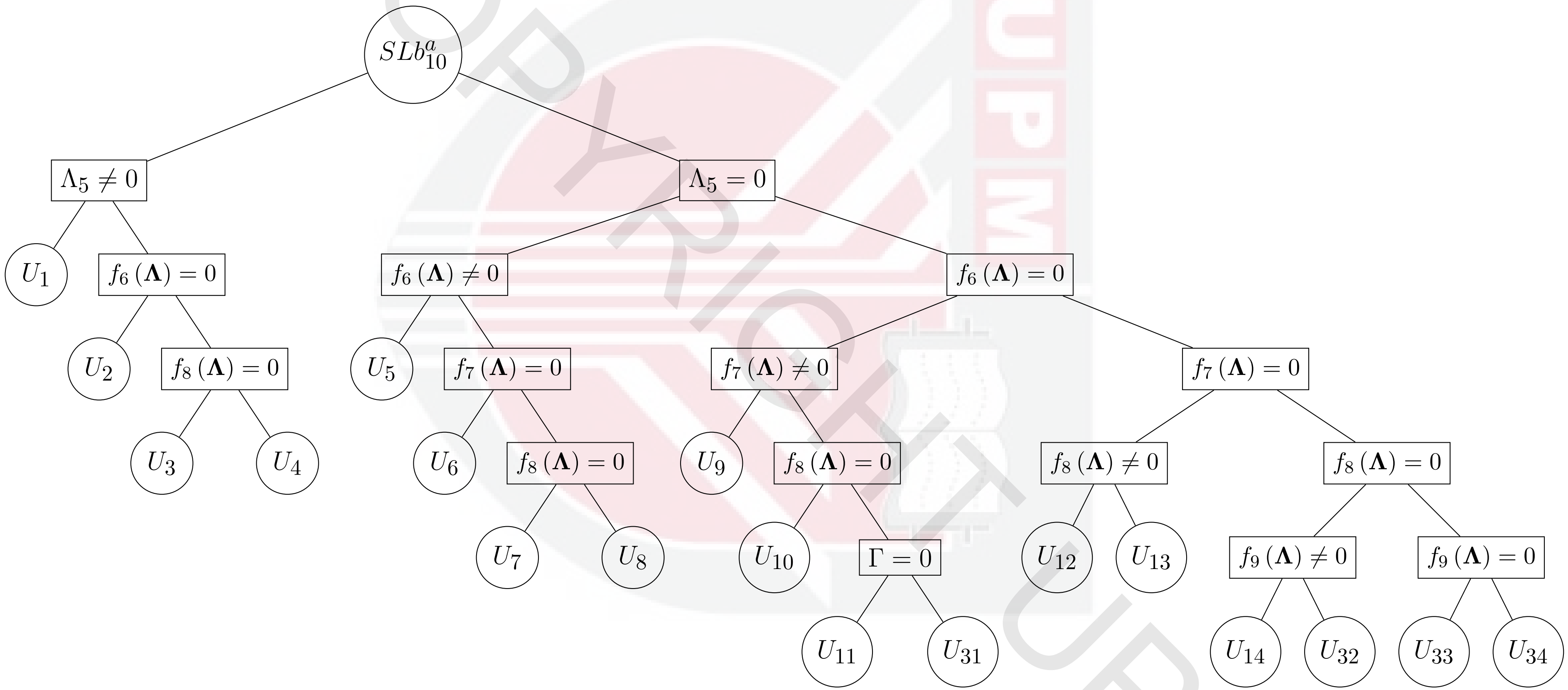
- $U_8 = \{L\left(\mathbf{\Lambda}\right) \in SLb_{10}^a :$
$\Lambda_3 \neq 0, \Lambda_5 = 0, f_6\left(\mathbf{\Lambda}\right) \neq 0, f_7\left(\mathbf{\Lambda}\right) = 0, f_8\left(\mathbf{\Lambda}\right) = 0, \Gamma = 0\}$.
By reducing $\frac{f_6(\mathbf{\Lambda}')}{\Lambda_3'^3} = \frac{1}{A^3}\frac{f_6(\mathbf{\Lambda})}{\Lambda_3^3}$ in (B.15), then we obtain $A^3 = \frac{f_6(\mathbf{\Lambda})}{\Lambda_3^3}$. The values of B and D appear as $B = \frac{\Lambda_4 A}{2\Lambda_3^3\Lambda_5}$ and $D = \frac{A^2}{\Lambda_3}$. Then by using value $\frac{1}{A^3} = \frac{\Lambda_3^3}{f_6(\mathbf{\Lambda})}$, the system of equalities for U_8 is as the following page:

$$
\begin{aligned}
\Lambda_3^{'} &= 1, \\
\Lambda_4^{'} &= 0, \\
\frac{\Lambda_5^{'}}{\Lambda_3^{'2}} &= 0, \\
\frac{f_6\left(\mathbf{\Lambda}'\right)}{\Lambda_3'^3} &= \left(\frac{\Lambda_3^3}{f_6\left(\mathbf{\Lambda}\right)}\right)\frac{f_6\left(\mathbf{\Lambda}\right)}{\Lambda_3^3}, \\
\frac{f_7\left(\mathbf{\Lambda}'\right)}{\Lambda_3'^4} &= 0, \\
\frac{f_8\left(\mathbf{\Lambda}'\right)}{\Lambda_3'^5} &= 0, \\
\frac{f_9\left(\mathbf{\Lambda}'\right)}{\Lambda_3'^6} &= \left(\frac{\Lambda_3^3}{f_6\left(\mathbf{\Lambda}\right)}\right)^2\frac{f_9\left(\mathbf{\Lambda}\right)}{\Lambda_3^6}, \\
\frac{\Gamma'}{\Lambda_3'^2} &= 0.
\end{aligned}
\tag{B.17}
$$

New parameter is represented as λ to imply that $f_9\left(\mathbf{\Lambda}'\right) = \frac{f_9(\mathbf{\Lambda})}{f_6^2(\mathbf{\Lambda})} = \lambda$. Thus all algebras from U_8 are isomorphic to $L\left(1,0,0,1,0,0,\lambda,0\right)$ for $\lambda \in \mathbb{C}$.

The subsets in SLb_{10}^a are represented by the following graph tree.

Figure B.1: **Graph tree of** SLb_{10}^{a}



B.2 Classification of SLb_{10}^{b}

The isomorphism criteria for SLb_{10}^{b} can be expressed as follows:

$$\begin{aligned}
\Lambda_3^{'} &= 0, & t_7\left(\mathbf{\Lambda}^{'}\right) &= \frac{1}{A^{11}}\left(\frac{D}{A}\right)^4 t_7\left(\mathbf{\Lambda}\right),\\
\Lambda_4^{'} &= \frac{1}{A^2}\left(\frac{D}{A}\right)\Lambda_4, & t_8\left(\mathbf{\Lambda}^{'}\right) &= \frac{1}{A^{14}}\left(\frac{D}{A}\right)^5 t_8\left(\mathbf{\Lambda}\right),\\
\Lambda_5^{'} &= \frac{1}{A^3}\left(\frac{D}{A}\right)\Lambda_5, & t_9\left(\mathbf{\Lambda}^{'}\right) &= \frac{1}{A^{17}}\left(\frac{D}{A}\right)^6 t_9\left(\mathbf{\Lambda}\right),\\
\Lambda_6^{'} &= 0, & \Gamma^{'} &= \frac{1}{A^7}\left(\frac{D}{A}\right)^2\Gamma.
\end{aligned} \tag{B.18}$$

Under condition $\Lambda_3 = 0$ and $\Lambda_4 \neq 0$, the system (B.18) is isomorphic to the following system of equations:

$$\begin{aligned}
\Lambda_3^{'} &= 0, & \frac{t_7\left(\mathbf{\Lambda}'\right)}{\Lambda_4'^{4}} &= \frac{1}{A^3}\frac{t_7\left(\mathbf{\Lambda}\right)}{\Lambda_4^4},\\
\Lambda_4^{'} &= 1, & \frac{t_8\left(\mathbf{\Lambda}'\right)}{\Lambda_4'^{5}} &= \frac{1}{A^4}\frac{t_8\left(\mathbf{\Lambda}\right)}{\Lambda_4^5},\\
\frac{\Lambda_5^{'}}{\Lambda_4^{'}} &= \frac{1}{A}\frac{\Lambda_5}{\Lambda_4}, & \frac{t_9\left(\mathbf{\Lambda}'\right)}{\Lambda_4'^{6}} &= \frac{1}{A^5}\frac{t_9\left(\mathbf{\Lambda}\right)}{\Lambda_4^6},\\
\Lambda_6^{'} &= 0, & \frac{\Gamma'}{\Lambda_4'^{2}} &= \frac{1}{A^3}\frac{\Gamma}{\Lambda_4^2}.
\end{aligned} \tag{B.19}$$

It shows that $SLb_{10}^{b} = T_1 \bigcup T_2$, where $T_1 = \left\{L\left(\mathbf{\Lambda}\right) \in SLb_{10}^{b} \,:\, \Lambda_3 = 0, \Lambda_4 \neq 0\right\}$ and $T_2 = \left\{L\left(\mathbf{\Lambda}\right) \in SLb_{10}^{b} \,:\, \beta_3 = 0, \beta_4 = 0\right\}$.

(i) $T_1 = \left\{L\left(\mathbf{\Lambda}\right) \in SLb_{10}^{b} \,:\, \Lambda_3 = 0, \Lambda_4 \neq 0\right\}$

(a) $T_1 = U_{15} \bigcup T_3$:

- $U_{15} = \left\{L\left(\mathbf{\Lambda}\right) \in SLb_{10}^{b} \,:\, \Lambda_3 = 0, \Lambda_4 \neq 0, \Lambda_5 \neq 0\right\}$
  By reducing $\frac{\Lambda_5^{'}}{\Lambda_4^{'}} = \frac{1}{A}\frac{\Lambda_5}{\Lambda_3}$ in (B.19), then we obtain $A = \frac{\Lambda_5}{\Lambda_4}$. The values of B and D appear as $B = \frac{\Lambda_6 A}{3\Lambda_4^2}$ and $D = \frac{A^3}{\Lambda_4}$. Then by using value $\frac{1}{A} = \frac{\Lambda_4}{\Lambda_5}$, the system of equalities for U_{15} is as follows:

$$\begin{aligned}
\Lambda_3^{'} &= 0, & \frac{t_7(\mathbf{\Lambda}')}{\Lambda_4^{'4}} &= \left(\frac{\Lambda_4}{\Lambda_5}\right)^3 \frac{t_7(\mathbf{\Lambda})}{\Lambda_4^4}, \\
\Lambda_4^{'} &= 1, & \frac{t_8(\mathbf{\Lambda}')}{\Lambda_4^{'5}} &= \left(\frac{\Lambda_4}{\Lambda_5}\right)^4 \frac{t_8(\mathbf{\Lambda})}{\Lambda_4^5}, \\
\frac{\Lambda_5^{'}}{\Lambda_4^{'}} &= \left(\frac{\Lambda_4}{\Lambda_5}\right)\frac{\Lambda_5}{\Lambda_4}, & \frac{t_9(\mathbf{\Lambda}')}{\Lambda_4^{'6}} &= \left(\frac{\Lambda_4}{\Lambda_5}\right)^5 \frac{t_9(\mathbf{\Lambda})}{\Lambda_4^6}, \\
\Lambda_6^{'} &= 0, & \frac{\Gamma'}{\Lambda_4^{'2}} &= \left(\frac{\Lambda_4}{\Lambda_5}\right)^3 \frac{\Gamma}{\Lambda_4^2}.
\end{aligned} \tag{B.20}$$

New parameters are represented as $\lambda_1, \lambda_2, \lambda_3$ and λ_4 to imply that $t_7(\mathbf{\Lambda}') = \frac{t_7(\mathbf{\Lambda})}{\Lambda_4\Lambda_5^3} = \lambda_1, t_8(\mathbf{\Lambda}') = \frac{t_8(\mathbf{\Lambda})}{\Lambda_4\Lambda_5^4} = \lambda_2$, $t_9(\mathbf{\Lambda}') = \frac{t_9(\mathbf{\Lambda})}{\Lambda_4\Lambda_5^5} = \lambda_3$ and $\Gamma' = \frac{\Lambda_4\Gamma}{\Lambda_5^3} = \lambda_4$. Thus all algebras from U_{15} are isomorphic to $L(0,1,1,0,\lambda_1,\lambda_2,\lambda_3,\lambda_4)$ for $\lambda_1,\lambda_2,\lambda_3,\lambda_4 \in \mathbb{C}$.

- $T_3 = \left\{ L(\mathbf{\Lambda}) \in SLb_{10}^b : \Lambda_3 = 0, \Lambda_4 \neq 0, \Lambda_5 = 0 \right\}$
The system of equations in subset T_3 becomes:

$$\begin{aligned}
\Lambda_3^{'} &= 0, & \frac{t_7(\mathbf{\Lambda}')}{\Lambda_4^{'4}} &= \frac{1}{A^3}\frac{t_7(\mathbf{\Lambda})}{\Lambda_4^4}, \\
\Lambda_4^{'} &= 1, & \frac{t_8(\mathbf{\Lambda}')}{\Lambda_4^{'5}} &= \frac{1}{A^4}\frac{t_8(\mathbf{\Lambda})}{\Lambda_4^5}, \\
\frac{\Lambda_5^{'}}{\Lambda_4^{'}} &= 0, & \frac{t_9(\mathbf{\Lambda}')}{\Lambda_4^{'6}} &= \frac{1}{A^5}\frac{t_9(\mathbf{\Lambda})}{\Lambda_4^6}, \\
\Lambda_6^{'} &= 0, & \frac{\Gamma'}{\Lambda_4^{'2}} &= \frac{1}{A^3}\frac{\Gamma}{\Lambda_4^2}.
\end{aligned} \tag{B.21}$$

(b) $T_3 = T_4 \bigcup T_5$:

- $T_4 = \left\{ L(\mathbf{\Lambda}) \in SLb_{10}^b : \Lambda_3 = 0, \Lambda_4 \neq 0, \Lambda_5 = 0, t_7(\mathbf{\Lambda}) \neq 0 \right\}$
  The system of equations in subset T_4 becomes:

$$\begin{aligned} \Lambda_3^{'} &= 0, & \frac{t_7(\mathbf{\Lambda}')}{\Lambda_4^{'4}} &= \frac{1}{A^3}\frac{t_7(\mathbf{\Lambda})}{\Lambda_4^4}, \\ \Lambda_4^{'} &= 1, & \frac{t_8(\mathbf{\Lambda}')}{\Lambda_4^{'5}} &= \frac{1}{A^4}\frac{t_8(\mathbf{\Lambda})}{\Lambda_4^5}, \\ \frac{\Lambda_5^{'}}{\Lambda_4^{'}} &= 0, & \frac{t_9(\mathbf{\Lambda}')}{\Lambda_4^{'6}} &= \frac{1}{A^5}\frac{t_9(\mathbf{\Lambda})}{\Lambda_4^6}, \\ \Lambda_6^{'} &= 0, & \frac{\Gamma'}{\Lambda_4^{'2}} &= \frac{1}{A^3}\frac{\Gamma}{\Lambda_4^2}. \end{aligned} \tag{B.22}$$

- $T_5 = \left\{ L(\mathbf{\Lambda}) \in SLb_{10}^b : \Lambda_3 = 0, \Lambda_4 \neq 0, \Lambda_5 = 0, t_7(\mathbf{\Lambda}) = 0 \right\}$
  The system of equations in subset T_5 becomes:

$$\begin{aligned} \Lambda_3^{'} &= 0, & \frac{t_7(\mathbf{\Lambda}')}{\Lambda_4^{'4}} &= 0, \\ \Lambda_4^{'} &= 1, & \frac{t_8(\mathbf{\Lambda}')}{\Lambda_4^{'5}} &= \frac{1}{A^4}\frac{t_8(\mathbf{\Lambda})}{\Lambda_4^5}, \\ \frac{\Lambda_5^{'}}{\Lambda_4^{'}} &= 0, & \frac{t_9(\mathbf{\Lambda}')}{\Lambda_4^{'6}} &= \frac{1}{A^5}\frac{t_9(\mathbf{\Lambda})}{\Lambda_4^6}, \\ \Lambda_6^{'} &= 0, & \frac{\Gamma'}{\Lambda_4^{'2}} &= \frac{1}{A^3}\frac{\Gamma}{\Lambda_4^2}. \end{aligned} \tag{B.23}$$

(c) $T_4 = U_{16} \bigcup T_6$:

- $U_{16} = \left\{ L(\mathbf{\Lambda}) \in SLb_{10}^b : \Lambda_3 = 0, \Lambda_4 \neq 0, \Lambda_5 = 0, t_7(\mathbf{\Lambda}) \neq 0, t_8(\mathbf{\Lambda}) \neq 0 \right\}$
  By reducing $\frac{t_7(\mathbf{\Lambda}')}{\Lambda_4'^4} = \frac{1}{A^3}\frac{t_7(\mathbf{\Lambda})}{\Lambda_4^4}$ and $\frac{t_8(\mathbf{\Lambda}')}{\Lambda_4'^5} = \frac{1}{A^4}\frac{t_8(\mathbf{\Lambda})}{\Lambda_4^5}$ in (B.22), then we obtain $A = \frac{t_8(\mathbf{\Lambda})}{\Lambda_4 t_7(\mathbf{\Lambda})}$. The values of B and D appear as $B = \frac{\Lambda_6 A}{3\Lambda_4^2}$ and $D = \frac{A^3}{\Lambda_4}$. Then by using value $\frac{1}{A} = \frac{\Lambda_4 t_7(\mathbf{\Lambda})}{t_8(\mathbf{\Lambda})}$, the system of equalities for U_{16} is as follows:

$$\Lambda_3^{'} = 0, \qquad \frac{t_7(\mathbf{\Lambda}')}{\Lambda_4'^4} = \left(\frac{\Lambda_4 t_7(\mathbf{\Lambda})}{t_8(\mathbf{\Lambda})}\right)^3 \frac{t_7(\mathbf{\Lambda})}{\Lambda_4^4}, \tag{B.24}$$
$$\Lambda_4^{'} = 1, \qquad \frac{t_8(\mathbf{\Lambda}')}{\Lambda_4'^5} = \left(\frac{\Lambda_4 t_7(\mathbf{\Lambda})}{t_8(\mathbf{\Lambda})}\right)^4 \frac{t_8(\mathbf{\Lambda})}{\Lambda_4^5},$$
$$\frac{\Lambda_5^{'}}{\Lambda_4^{'}} = 0, \qquad \frac{t_9(\mathbf{\Lambda}')}{\Lambda_4'^6} = \left(\frac{\Lambda_4 t_7(\mathbf{\Lambda})}{t_8(\mathbf{\Lambda})}\right)^5 \frac{t_9(\mathbf{\Lambda})}{\Lambda_4^6},$$
$$\Lambda_6^{'} = 0, \qquad \frac{\Gamma'}{\Lambda_4'^2} = \left(\frac{\Lambda_4 t_7(\mathbf{\Lambda})}{t_8(\mathbf{\Lambda})}\right)^3 \frac{\Gamma}{\Lambda_4^2}.$$

New parameters are represented as λ_1, λ_2 and λ_3 to imply that $t_7(\mathbf{\Lambda}') = \frac{t_7^4(\mathbf{\Lambda})}{\Lambda_4 t_8^3(\mathbf{\Lambda})} = \lambda_1, t_8(\mathbf{\Lambda}') = \frac{t_7^4(\mathbf{\Lambda})}{\Lambda_4 t_8^3(\mathbf{\Lambda})} = \lambda_1$, $t_9(\mathbf{\Lambda}') = \frac{t_7^5(\mathbf{\Lambda}) t_9(\mathbf{\Lambda})}{\Lambda_4 t_8^5(\mathbf{\Lambda})} = \lambda_2$ and $\Gamma' = \frac{\Lambda_4 t_7^3(\mathbf{\Lambda})\Gamma}{t_8^3(\mathbf{\Lambda})} = \lambda_3$. Thus all algebras from U_{16} are isomorphic to $L(0,1,0,0,\lambda_1,\lambda_1,\lambda_2,\lambda_3)$ for $\lambda_1 \in \mathbb{C}^*$ and $\lambda_2, \lambda_3 \in \mathbb{C}$.

- $T_6 = \left\{L(\mathbf{\Lambda}) \in SLb_{10}^b : \Lambda_3 = 0, \Lambda_4 \neq 0, \Lambda_5 = 0, t_7(\mathbf{\Lambda}) \neq 0, t_8(\mathbf{\Lambda}) = 0\right\}$
The system of equations in subset T_6 becomes:

$$\Lambda_3^{'} = 0, \qquad \frac{t_7(\mathbf{\Lambda}')}{\Lambda_4'^4} = \frac{1}{A^3}\frac{t_7(\mathbf{\Lambda})}{\Lambda_4^4}, \tag{B.25}$$
$$\Lambda_4^{'} = 1, \qquad \frac{t_8(\mathbf{\Lambda}')}{\Lambda_4'^5} = 0,$$
$$\frac{\Lambda_5^{'}}{\Lambda_4^{'}} = 0, \qquad \frac{t_9(\mathbf{\Lambda}')}{\Lambda_4'^6} = \frac{1}{A^5}\frac{t_9(\mathbf{\Lambda})}{\Lambda_4^6},$$
$$\Lambda_6^{'} = 0, \qquad \frac{\Gamma'}{\Lambda_4'^2} = \frac{1}{A^3}\frac{\Gamma}{\Lambda_4^2}.$$

(d) $T_6 = U_{17} \bigcup U_{18}$:

- $U_{17} = \{L(\mathbf{\Lambda}) \in SLb_{10}^b :$
$\Lambda_3 = 0, \Lambda_4 \neq 0, \Lambda_5 = 0, t_7(\mathbf{\Lambda}) \neq 0, t_8(\mathbf{\Lambda}) = 0, t_9(\mathbf{\Lambda}) \neq 0\}$
By reducing $\frac{t_7(\mathbf{\Lambda}')}{\Lambda_4'^4} = \frac{1}{A^3}\frac{t_7(\mathbf{\Lambda})}{\Lambda_4^4}$ and $\frac{t_9(\mathbf{\Lambda}')}{\Lambda_4'^6} = \frac{1}{A^5}\frac{t_9(\mathbf{\Lambda})}{\Lambda_4^6}$ in (B.25), then we obtain $A = \frac{t_7^2(\mathbf{\Lambda})}{\Lambda_4^2 t_9(\mathbf{\Lambda})}$. The values of B and D appear as $B = \frac{\Lambda_6 A}{3\Lambda_4^2}$ and $D = \frac{A^3}{\Lambda_4}$. Then by using value $\frac{1}{A} = \frac{\Lambda_4^2 t_9(\mathbf{\Lambda})}{t_7^2(\mathbf{\Lambda})}$, the system of equalities for

U_{17} is as follows:

$$\begin{aligned}
\Lambda_3^{'} &= 0, & \frac{t_7\left(\mathbf{\Lambda}'\right)}{\Lambda_4'^4} &= \left(\frac{\Lambda_4^2 t_9\left(\mathbf{\Lambda}\right)}{t_7^2\left(\mathbf{\Lambda}\right)}\right)^3 \frac{t_7\left(\mathbf{\Lambda}\right)}{\Lambda_4^4}, \\
\Lambda_4^{'} &= 1, & \frac{t_8\left(\mathbf{\Lambda}'\right)}{\Lambda_4'^5} &= 0, \\
\frac{\Lambda_5^{'}}{\Lambda_4^{'}} &= 0, & \frac{t_9\left(\mathbf{\Lambda}'\right)}{\Lambda_4'^6} &= \left(\frac{\Lambda_4^2 t_9\left(\mathbf{\Lambda}\right)}{t_7^2\left(\mathbf{\Lambda}\right)}\right)^5 \frac{t_9\left(\mathbf{\Lambda}\right)}{\Lambda_4^6}, \\
\Lambda_6^{'} &= 0, & \frac{\Gamma'}{\Lambda_4'^2} &= \left(\frac{\Lambda_4^2 t_9\left(\mathbf{\Lambda}\right)}{t_7^2\left(\mathbf{\Lambda}\right)}\right)^3 \frac{\Gamma}{\Lambda_4^2}.
\end{aligned} \tag{B.26}$$

New parameters are represented as λ_1 and λ_2 to imply that $t_7\left(\mathbf{\Lambda}'\right) = \frac{\Lambda_4^2 t_9^3(\mathbf{\Lambda})}{t_7^5(\mathbf{\Lambda})} = \lambda_1$, $t_9\left(\mathbf{\Lambda}'\right) = \frac{\Lambda_4^4 t_9^6(\mathbf{\Lambda})}{t_7^{10}(\mathbf{\Lambda})} = \lambda_1^2$ and $\Gamma' = \frac{\Lambda_4^4 t_9^3(\mathbf{\Lambda})\Gamma}{t_7^6(\mathbf{\Lambda})} = \lambda_2$. Thus all algebras from U_{17} are isomorphic to $L\left(0,1,0,0,\lambda_1,0,\lambda_1^2,\lambda_2\right)$ for $\lambda_1 \in \mathbb{C}^*$ and $\lambda_2 \in \mathbb{C}$.

- $U_{18} = \left\{L\left(\mathbf{\Lambda}\right) \in SLb_{10}^b : \Lambda_3 = 0, \Lambda_4 \neq 0, \Lambda_5 = 0, t_7\left(\mathbf{\Lambda}\right) \neq 0, t_8\left(\mathbf{\Lambda}\right) = 0, t_9\left(\mathbf{\Lambda}\right) = 0\right\}$
  By reducing $\frac{t_7(\mathbf{\Lambda}')}{\Lambda_4'^4} = \frac{1}{A^3}\frac{t_7(\mathbf{\Lambda})}{\Lambda_4^4}$ in (B.25), then we obtain $A^3 = \frac{t_7(\mathbf{\Lambda})}{\Lambda_4^4}$. The values of B and D appear as $B = \frac{\Lambda_6 A}{3\Lambda_4^2}$ and $D = \frac{A^3}{\Lambda_4}$. Then by using value $\frac{1}{A^3} = \frac{\Lambda_4^4}{t_7(\mathbf{\Lambda})}$, the system of equalities for U_{18} is as follows:

$$\begin{aligned}
\Lambda_3^{'} &= 0, & \frac{t_7\left(\mathbf{\Lambda}'\right)}{\Lambda_4'^4} &= \left(\frac{\Lambda_4^4}{t_7\left(\mathbf{\Lambda}\right)}\right) \frac{t_7\left(\mathbf{\Lambda}\right)}{\Lambda_4^4}, \\
\Lambda_4^{'} &= 1, & \frac{t_8\left(\mathbf{\Lambda}'\right)}{\Lambda_4'^5} &= 0, \\
\frac{\Lambda_5^{'}}{\Lambda_4^{'}} &= 0, & \frac{t_9\left(\mathbf{\Lambda}'\right)}{\Lambda_4'^6} &= 0, \\
\Lambda_6^{'} &= 0, & \frac{\Gamma'}{\Lambda_4'^2} &= \left(\frac{\Lambda_4^4}{t_7\left(\mathbf{\Lambda}\right)}\right) \frac{\Gamma}{\Lambda_4^2}.
\end{aligned} \tag{B.27}$$

New parameters are represented as λ to imply that $\Gamma' = \frac{\Lambda_4^2\Gamma}{t_7(\mathbf{\Lambda})} = \lambda$. Thus all algebras from U_{18} are isomorphic to $L\left(0,1,0,0,1,0,0,\lambda\right)$ for $\lambda \in \mathbb{C}$.

(e) $T_5 = T_8 \bigcup T_9$:

- $T_8 = \left\{ L(\mathbf{\Lambda}) \in SLb_{10}^{b} \,:\, \Lambda_3 = 0, \Lambda_4 \neq 0, \Lambda_5 = 0, t_7(\mathbf{\Lambda}) = 0, t_8(\mathbf{\Lambda}) \neq 0 \right\}$
The system of equations in subset T_8 becomes:

$$\begin{aligned}
\Lambda_3^{'} &= 0, & \frac{t_7(\mathbf{\Lambda}')}{\Lambda_4'^4} &= 0, \\
\Lambda_4^{'} &= 1, & \frac{t_8(\mathbf{\Lambda}')}{\Lambda_4'^5} &= \frac{1}{A^4}\frac{t_8(\mathbf{\Lambda})}{\Lambda_4^5}, \\
\frac{\Lambda_5^{'}}{\Lambda_4^{'}} &= 0, & \frac{t_9(\mathbf{\Lambda}')}{\Lambda_4'^6} &= \frac{1}{A^5}\frac{t_9(\mathbf{\Lambda})}{\Lambda_4^6}, \\
\Lambda_6^{'} &= 0, & \frac{\Gamma'}{\Lambda_4'^2} &= \frac{1}{A^3}\frac{\Gamma}{\Lambda_4^2}.
\end{aligned} \tag{B.28}$$

- $T_9 = \left\{ L(\mathbf{\Lambda}) \in SLb_{10}^{b} \,:\, \Lambda_3 = 0, \Lambda_4 \neq 0, \Lambda_5 = 0, t_7(\mathbf{\Lambda}) = 0, t_8(\mathbf{\Lambda}) = 0 \right\}$
The system of equations in subset T_9 becomes:

$$\begin{aligned}
\Lambda_3^{'} &= 0, & \frac{t_7(\mathbf{\Lambda}')}{\Lambda_4'^4} &= 0, \\
\Lambda_4^{'} &= 1, & \frac{t_8(\mathbf{\Lambda}')}{\Lambda_4'^5} &= 0, \\
\frac{\Lambda_5^{'}}{\Lambda_4^{'}} &= 0, & \frac{t_9(\mathbf{\Lambda}')}{\Lambda_4'^6} &= \frac{1}{A^5}\frac{t_9(\mathbf{\Lambda})}{\Lambda_4^6}, \\
\Lambda_6^{'} &= 0, & \frac{\Gamma'}{\Lambda_4'^2} &= \frac{1}{A^3}\frac{\Gamma}{\Lambda_4^2}.
\end{aligned} \tag{B.29}$$

(f) $T_8 = U_{19} \bigcup T_{10}$:

- $U_{19} =$
$\left\{ L(\mathbf{\Lambda}) \in SLb_{10}^{b} \,:\, \Lambda_3 = 0, \Lambda_4 \neq 0, \Lambda_5 = 0, t_7(\mathbf{\Lambda}) = 0, t_8(\mathbf{\Lambda}) \neq 0, t_9(\mathbf{\Lambda}) \neq 0 \right\}$
By reducing $\frac{t_8(\mathbf{\Lambda}')}{\Lambda_4'^5} = \frac{1}{A^4}\frac{t_8(\mathbf{\Lambda})}{\Lambda_4^5}$ and $\frac{t_9(\mathbf{\Lambda}')}{\Lambda_4'^6} = \frac{1}{A^5}\frac{t_9(\mathbf{\Lambda})}{\Lambda_4^6}$ in (B.28), then we obtain $A = \frac{t_9(\mathbf{\Lambda})}{\Lambda_4 t_8(\mathbf{\Lambda})}$. The values of B and D appear as $B = \frac{\Lambda_6 A}{3\Lambda_4^2}$ and $D = \frac{A^3}{\Lambda_4}$. Then by using value $\frac{1}{A} = \frac{\Lambda_4 t_8(\mathbf{\Lambda})}{t_9(\mathbf{\Lambda})}$, the system of equalities for U_{19} is as follows:

$$
\begin{aligned}
\Lambda_3^{'} &= 0, & \frac{t_7(\mathbf{\Lambda}')}{\Lambda_4'^4} &= 0, \\
\Lambda_4^{'} &= 1, & \frac{t_8(\mathbf{\Lambda}')}{\Lambda_4'^5} &= \left(\frac{\Lambda_4 t_8(\mathbf{\Lambda})}{t_9(\mathbf{\Lambda})}\right)^4 \frac{t_8(\mathbf{\Lambda})}{\Lambda_4^5}, \\
\frac{\Lambda_5^{'}}{\Lambda_4^{'}} &= 0, & \frac{t_9(\mathbf{\Lambda}')}{\Lambda_4'^6} &= \left(\frac{\Lambda_4 t_8(\mathbf{\Lambda})}{t_9(\mathbf{\Lambda})}\right)^5 \frac{t_9(\mathbf{\Lambda})}{\Lambda_4^6}, \\
\Lambda_6^{'} &= 0, & \frac{\Gamma'}{\Lambda_4'^2} &= \left(\frac{\Lambda_4 t_8(\mathbf{\Lambda})}{t_9(\mathbf{\Lambda})}\right)^3 \frac{\Gamma}{\Lambda_4^2}.
\end{aligned} \tag{B.30}
$$

New parameters are represented as λ_1 and λ_2 to imply that $t_8(\mathbf{\Lambda}') = \frac{t_8^5(\mathbf{\Lambda})}{\Lambda_4 t_9^4(\mathbf{\Lambda})} = \lambda_1$, $t_9(\mathbf{\Lambda}') = \frac{t_8^5(\mathbf{\Lambda})}{\Lambda_4 t_9^4(\mathbf{\Lambda})} = \lambda_1$ and $\Gamma' = \frac{\Lambda_4 t_8^3(\mathbf{\Lambda})\Gamma}{t_9^3(\mathbf{\Lambda})} = \lambda_2$. Thus all algebras from U_{19} are isomorphic to $L(0,1,0,0,0,\lambda_1,\lambda_1,\lambda_2)$ for $\lambda_1 \in \mathbb{C}^*$ and $\lambda_2 \in \mathbb{C}$.

- $T_{10} = \left\{L(\mathbf{\Lambda}) \in SLb_{10}^b : \Lambda_3 = 0, \Lambda_4 \neq 0, \Lambda_5 = 0, t_7(\mathbf{\Lambda}) = 0, t_8(\mathbf{\Lambda}) \neq 0, t_9(\mathbf{\Lambda}) = 0\right\}$
The system of equations in subset T_{10} becomes:

$$
\begin{aligned}
\Lambda_3^{'} &= 0, & \frac{t_7(\mathbf{\Lambda}')}{\Lambda_4'^4} &= 0, \\
\Lambda_4^{'} &= 1, & \frac{t_8(\mathbf{\Lambda}')}{\Lambda_4'^5} &= \frac{1}{A^4}\frac{t_8(\mathbf{\Lambda})}{\Lambda_4^5}, \\
\frac{\Lambda_5^{'}}{\Lambda_4^{'}} &= 0, & \frac{t_9(\mathbf{\Lambda}')}{\Lambda_4'^6} &= 0, \\
\Lambda_6^{'} &= 0, & \frac{\Gamma'}{\Lambda_4'^2} &= \frac{1}{A^3}\frac{\Gamma}{\Lambda_4^2}.
\end{aligned} \tag{B.31}
$$

(g) $T_{10} = U_{20} \bigcup U_{37}$:

- $U_{20} = \left\{L(\mathbf{\Lambda}) \in SLb_{10}^b : \Lambda_3 = 0, \Lambda_4 \neq 0, \Lambda_5 = 0, t_7(\mathbf{\Lambda}) = 0, t_8(\mathbf{\Lambda}) \neq 0, t_9(\mathbf{\Lambda}) = 0, \Gamma \neq 0\right\}$
By reducing $\frac{t_8(\mathbf{\Lambda}')}{\Lambda_4'^5} = \frac{1}{A^4}\frac{t_8(\mathbf{\Lambda})}{\Lambda_4^5}$ and $\frac{\Gamma'}{\Lambda_4'^2} = \frac{1}{A^3}\frac{\Gamma}{\Lambda_4^2}$ in (B.31), then we obtain $A = \frac{t_8(\mathbf{\Lambda})}{\Lambda_4^3\Gamma}$. The values of B and D appear as $B = \frac{\Lambda_6 A}{3\Lambda_4^2}$ and $D = \frac{A^3}{\Lambda_4}$. Then by using value $\frac{1}{A} = \frac{\Lambda_4^3\Gamma}{t_8(\mathbf{\Lambda})}$, the system of equalities for U_{20} is as follows:

$$\Lambda_3^{'} = 0, \qquad \frac{t_7\left(\mathbf{\Lambda}'\right)}{\Lambda_4^{'4}} = 0, \tag{B.32}$$

$$\Lambda_4^{'} = 1, \qquad \frac{t_8\left(\mathbf{\Lambda}'\right)}{\Lambda_4^{'5}} = \left(\frac{\Lambda_4^3\Gamma}{t_8\left(\mathbf{\Lambda}\right)}\right)^4 \frac{t_8\left(\mathbf{\Lambda}\right)}{\Lambda_4^5},$$

$$\frac{\Lambda_5^{'}}{\Lambda_4^{'}} = 0, \qquad \frac{t_9\left(\mathbf{\Lambda}'\right)}{\Lambda_4^{'6}} = 0,$$

$$\Lambda_6^{'} = 0, \qquad \frac{\Gamma'}{\Lambda_4^{'2}} = \left(\frac{\Lambda_4^3\Gamma}{t_8\left(\mathbf{\Lambda}\right)}\right)^3 \frac{\Gamma}{\Lambda_4^2}.$$

New parameter is represented as λ to imply that $t_8\left(\mathbf{\Lambda}'\right) = \frac{\Lambda_4^7\Gamma^4}{t_8^3(\mathbf{\Lambda})} = \lambda$ and $\Gamma' = \frac{\Lambda_4^7\Gamma^4}{t_8^3(\mathbf{\Lambda})} = \lambda$. Thus all algebras from U_{20} are isomorphic to $L\left(0,1,0,0,0,\lambda,0,\lambda\right)$ for $\lambda \in \mathbb{C}^*$.

- $U_{37} = \Big\{L\left(\mathbf{\Lambda}\right) \in SLb_{10}^b : \Lambda_3 = 0, \Lambda_4 \neq 0, \Lambda_5 = 0, t_7\left(\mathbf{\Lambda}\right) = 0, t_8\left(\mathbf{\Lambda}\right) \neq 0, t_9\left(\mathbf{\Lambda}\right) = 0, \Gamma = 0\Big\}$

  By reducing $\frac{t_8(\mathbf{\Lambda}')}{\Lambda_4'^5} = \frac{1}{A^4}\frac{t_8(\mathbf{\Lambda})}{\Lambda_4^5}$ in (B.31), then we obtain $A^4 = \frac{t_8(\mathbf{\Lambda})}{\Lambda_4^5}$. The values of B and D appear as $B = \frac{\Lambda_6 A}{3\Lambda_4^2}$ and $D = \frac{A^3}{\Lambda_4}$. Then by using value $\frac{1}{A^4} = \frac{\Lambda_4^5}{t_8(\mathbf{\Lambda})}$, the system of equalities for U_{37} is as follows:

$$\Lambda_3^{'} = 0, \qquad \frac{t_7\left(\mathbf{\Lambda}'\right)}{\Lambda_4^{'4}} = 0, \tag{B.33}$$

$$\Lambda_4^{'} = 1, \qquad \frac{t_8\left(\mathbf{\Lambda}'\right)}{\Lambda_4^{'5}} = \left(\frac{\Lambda_4^5}{t_8\left(\mathbf{\Lambda}\right)}\right) \frac{t_8\left(\mathbf{\Lambda}\right)}{\Lambda_4^5},$$

$$\frac{\Lambda_5^{'}}{\Lambda_4^{'}} = 0, \qquad \frac{t_9\left(\mathbf{\Lambda}'\right)}{\Lambda_4^{'6}} = 0,$$

$$\Lambda_6^{'} = 0, \qquad \frac{\Gamma'}{\Lambda_4^{'2}} = 0.$$

Thus all algebras from U_{37} are isomorphic to $L\left(0,1,0,0,0,1,0,0\right)$.

(h) $T_9 = T_{11} \bigcup T_{12}$:

- $T_{11} = \Big\{ L(\mathbf{\Lambda}) \in SLb^b_{10} \,:\, \Lambda_3 = 0, \Lambda_4 \neq 0, \Lambda_5 = 0, t_7(\mathbf{\Lambda}) = 0, t_8(\mathbf{\Lambda}) = 0,$ $t_9(\mathbf{\Lambda}) \neq 0 \}$

  The system of equations in subset T_{11} becomes:

$$\begin{aligned} \Lambda_3^{'} &= 0, & \frac{t_7(\mathbf{\Lambda}')}{\Lambda_4'^4} &= 0, \\ \Lambda_4^{'} &= 1, & \frac{t_8(\mathbf{\Lambda}')}{\Lambda_4'^5} &= 0, \\ \frac{\Lambda_5^{'}}{\Lambda_4^{'}} &= 0, & \frac{t_9(\mathbf{\Lambda}')}{\Lambda_4'^6} &= \frac{1}{A^5}\frac{t_9(\mathbf{\Lambda})}{\Lambda_4^6}, \\ \Lambda_6^{'} &= 0, & \frac{\Gamma'}{\Lambda_4'^2} &= \frac{1}{A^3}\frac{\Gamma}{\Lambda_4^2}. \end{aligned} \tag{B.34}$$

- $T_{12} = \Big\{ L(\mathbf{\Lambda}) \in SLb^b_{10} \,:\, \Lambda_3 = 0, \Lambda_4 \neq 0, \Lambda_5 = 0, t_7(\mathbf{\Lambda}) = 0, t_8(\mathbf{\Lambda}) = 0,$ $t_9(\mathbf{\Lambda}) = 0 \}$

  The system of equations in subset T_{12} becomes:

$$\begin{aligned} \Lambda_3^{'} &= 0, & \frac{t_7(\mathbf{\Lambda}')}{\Lambda_4'^4} &= 0, \\ \Lambda_4^{'} &= 1, & \frac{t_8(\mathbf{\Lambda}')}{\Lambda_4'^5} &= 0, \\ \frac{\Lambda_5^{'}}{\Lambda_4^{'}} &= 0, & \frac{t_9(\mathbf{\Lambda}')}{\Lambda_4'^6} &= 0, \\ \Lambda_6^{'} &= 0, & \frac{\Gamma'}{\Lambda_4'^2} &= \frac{1}{A^3}\frac{\Gamma}{\Lambda_4^2}. \end{aligned} \tag{B.35}$$

(i) $T_{11} = U_{21} \bigcup U_{38}$:

- $U_{21} = \Big\{ L(\mathbf{\Lambda}) \in SLb^b_{10} \,:\, \Lambda_3 = 0, \Lambda_4 \neq 0, \Lambda_5 = 0, t_7(\mathbf{\Lambda}) = 0, t_8(\mathbf{\Lambda}) = 0,$ $t_9(\mathbf{\Lambda}) \neq 0, \Gamma \neq 0 \}$

  By reducing $\frac{t_9(\mathbf{\Lambda}')}{\Lambda_4'^6} = \frac{1}{A^5}\frac{t_9(\mathbf{\Lambda})}{\Lambda_4^6}$ and $\frac{\Gamma'}{\Lambda_4'^2} = \frac{1}{A^3}\frac{\Gamma}{\Lambda_4^2}$ in (B.34), then we obtain $A = \frac{\Lambda_4^2\Gamma^2}{t_9(\mathbf{\Lambda})}$. The values of B and D appear as $B = \frac{\Lambda_6 A}{3\Lambda_4^2}$ and $D = \frac{A^3}{\Lambda_4}$. Then by using value $\frac{1}{A} = \frac{t_9(\mathbf{\Lambda})}{\Lambda_4^2\Gamma^2}$, the system of equalities for U_{21} is as follows:

$$\Lambda_3^{'} = 0, \qquad \frac{t_7(\mathbf{\Lambda'})}{\Lambda_4'^4} = 0, \tag{B.36}$$
$$\Lambda_4^{'} = 1, \qquad \frac{t_8(\mathbf{\Lambda'})}{\Lambda_4'^5} = 0,$$
$$\frac{\Lambda_5^{'}}{\Lambda_4^{'}} = 0, \qquad \frac{t_9(\mathbf{\Lambda'})}{\Lambda_4'^6} = \left(\frac{t_9(\mathbf{\Lambda})}{\Lambda_4^2\Gamma^2}\right)^5 \frac{t_9(\mathbf{\Lambda})}{\Lambda_4^6},$$
$$\Lambda_6^{'} = 0, \qquad \frac{\Gamma'}{\Lambda_4'^2} = \left(\frac{t_9(\mathbf{\Lambda})}{\Lambda_4^2\Gamma^2}\right)^3 \frac{\Gamma}{\Lambda_4^2}.$$

New parameter is represented as λ to imply that $t_9(\mathbf{\Lambda'}) = \frac{t_9^6(\mathbf{\Lambda})}{\Lambda_4^{16}\Gamma^{10}} = \lambda^2$ and $\Gamma' = \frac{t_9^3(\mathbf{\Lambda})}{\Lambda_4^8\Gamma^5} = \lambda$. Thus all algebras from U_{21} are isomorphic to $L\left(0,1,0,0,0,0,\lambda^2,\lambda\right)$ for $\lambda \in \mathbb{C}^*$.

- $U_{38} = \Big\{L(\mathbf{\Lambda}) \in SLb_{10}^b : \Lambda_3 = 0, \Lambda_4 \neq 0, \Lambda_5 = 0, t_7(\mathbf{\Lambda}) = 0, t_8(\mathbf{\Lambda}) = 0, t_9(\mathbf{\Lambda}) \neq 0, \Gamma = 0\}$

  By reducing $\frac{t_9(\mathbf{\Lambda'})}{\Lambda_4'^6} = \frac{1}{A^5}\frac{t_9(\mathbf{\Lambda})}{\Lambda_4^6}$ in (B.34), then we obtain $A^5 = \frac{t_9(\mathbf{\Lambda})}{\Lambda_4^6}$. The values of B and D appear as $B = \frac{\Lambda_6 A}{3\Lambda_4^2}$ and $D = \frac{A^3}{\Lambda_4}$. Then by using value $\frac{1}{A^5} = \frac{\Lambda_4^6}{t_9(\mathbf{\Lambda})}$, the system of equalities for U_{38} is as follows:

$$\Lambda_3^{'} = 0, \qquad \frac{t_7(\mathbf{\Lambda'})}{\Lambda_4'^4} = 0, \tag{B.37}$$
$$\Lambda_4^{'} = 1, \qquad \frac{t_8(\mathbf{\Lambda'})}{\Lambda_4'^5} = 0,$$
$$\frac{\Lambda_5^{'}}{\Lambda_4^{'}} = 0, \qquad \frac{t_9(\mathbf{\Lambda'})}{\Lambda_4'^6} = \left(\frac{\Lambda_4^6}{t_9(\mathbf{\Lambda})}\right) \frac{t_9(\mathbf{\Lambda})}{\Lambda_4^6},$$
$$\Lambda_6^{'} = 0, \qquad \frac{\Gamma'}{\Lambda_4'^2} = 0.$$

  Thus all algebras from U_{38} are isomorphic to $L(0,1,0,0,0,0,1,0)$.

(j) $T_{12} = U_{39} \bigcup U_{40}$:

- $U_{39} = \Big\{L(\mathbf{\Lambda}) \in SLb_{10}^b : \Lambda_3 = 0, \Lambda_4 \neq 0, \Lambda_5 = 0, t_7(\mathbf{\Lambda}) = 0, t_8(\mathbf{\Lambda}) = 0, t_9(\mathbf{\Lambda}) = 0, \Gamma \neq 0\}$

  By reducing $\frac{\Gamma'}{\Lambda_4'^2} = \frac{1}{A^3}\frac{\Gamma}{\Lambda_4^2}$ in (B.35), then we obtain $A^3 = \frac{\Gamma}{\Lambda_4^2}$. The values of B and D appear as $B = \frac{\Lambda_6 A}{3\Lambda_4^2}$ and $D = \frac{A^3}{\Lambda_4}$.

Then by using value $\frac{1}{A^3} = \frac{\Lambda_4^2}{\Gamma}$, the system of equalities for U_{39} is as follows:

$$\begin{aligned}
\Lambda_3^{'} &= 0, & \frac{t_7(\boldsymbol{\Lambda}')}{\Lambda_4'^4} &= 0, \\
\Lambda_4^{'} &= 1, & \frac{t_8(\boldsymbol{\Lambda}')}{\Lambda_4'^5} &= 0, \\
\frac{\Lambda_5^{'}}{\Lambda_4^{'}} &= 0, & \frac{t_9(\boldsymbol{\Lambda}')}{\Lambda_4'^6} &= 0, \\
\Lambda_6^{'} &= 0, & \frac{\Gamma'}{\Lambda_4'^2} &= \left(\frac{\Lambda_4^2}{\Gamma}\right)\frac{\Gamma}{\Lambda_4^2}.
\end{aligned} \tag{B.38}$$

Thus all algebras from U_{39} are isomorphic to $L(0,1,0,0,0,0,0,1)$.

- $U_{40} = \Big\{L(\boldsymbol{\Lambda}) \in SLb_{10}^b \,:\, \Lambda_3 = 0, \Lambda_4 \neq 0, \Lambda_5 = 0, t_7(\boldsymbol{\Lambda}) = 0, t_8(\boldsymbol{\Lambda}) = 0, t_9(\boldsymbol{\Lambda}) = 0, \Gamma = 0\}$
  All algebras from U_{40} are isomorphic to $L(0,1,0,0,0,0,0,0)$.

(ii) $T_2 = \Big\{L(\boldsymbol{\beta}) \in SLb_{10}^b \,:\, \beta_3 = 0, \beta_4 = 0\Big\}$
The subset of T_2 brings the following system of equations:

$$\begin{aligned}
\beta_3^{'} &= 0, & \beta_7^{'} &= \frac{1}{A^5}\left(\frac{D}{A}\right)\beta_7, \\
\beta_4^{'} &= 0, & \beta_8^{'} &= \frac{1}{A^6}\left(\frac{D}{A}\right)\left[\beta_8 - 4\left(\frac{B}{A}\right)\beta_5^2\right], \\
\beta_5^{'} &= \frac{1}{A^3}\left(\frac{D}{A}\right)\beta_5, & \beta_9^{'} &= \frac{1}{A^7}\left(\frac{D}{A}\right)\left[\beta_9 + \left(\frac{B}{A}\right)\gamma - 9\left(\frac{B}{A}\right)\beta_5\beta_6\right], \\
\beta_6^{'} &= \frac{1}{A^4}\left(\frac{D}{A}\right)\beta_6, & \gamma^{'} &= \frac{1}{A^7}\left(\frac{D}{A}\right)^2\gamma.
\end{aligned} \tag{B.39}$$

(a) $T_2 = T_{13} \bigcup T_{14}$:

- $T_{13} = \Big\{L(\boldsymbol{\beta}) \in SLb_{10}^b \,:\, \beta_3 = 0, \beta_4 = 0, \beta_5 \neq 0\Big\}$
  The system of equations of subset T_{13} is as following:

$$\beta_3^{'} = 0, \qquad \frac{\beta_7^{'}}{\beta_5^{'}} = \frac{1}{A^2}\frac{\beta_7}{\beta_5}, \tag{B.40}$$
$$\beta_4^{'} = 0, \qquad \frac{\beta_8^{'}}{\beta_5^{'}} = \frac{1}{A^3}\left[\frac{\beta_8}{\beta_5} - 4\left(\frac{B}{A}\right)\beta_5\right],$$
$$\beta_5^{'} = 1, \qquad \frac{\beta_9^{'}}{\beta_5^{'}} = \frac{1}{A^4}\left[\frac{\beta_9}{\beta_5} + \left(\frac{B}{A}\right)\frac{\gamma}{\beta_5} - 9\left(\frac{B}{A}\right)\beta_6\right],$$
$$\frac{\beta_6^{'}}{\beta_5^{'}} = \frac{1}{A}\frac{\beta_6}{\beta_5}, \qquad \frac{\gamma^{'}}{\beta_5^{'2}} = \frac{1}{A}\frac{\gamma}{\beta_5^2}.$$

- $T_{14} = \left\{L(\boldsymbol{\beta}) \in SLb_{10}^b \,:\, \beta_3 = 0, \beta_4 = 0, \beta_5 = 0\right\}$
The subset of T_{14} brings the following system of equations:

$$\beta_3^{'} = 0, \qquad \beta_7^{'} = \frac{1}{A^5}\left(\frac{D}{A}\right)\beta_7, \tag{B.41}$$
$$\beta_4^{'} = 0, \qquad \beta_8^{'} = \frac{1}{A^6}\left(\frac{D}{A}\right)\beta_8,$$
$$\beta_5^{'} = 0, \qquad \beta_9^{'} = \frac{1}{A^7}\left(\frac{D}{A}\right)\left[\beta_9 + \left(\frac{B}{A}\right)\gamma\right],$$
$$\beta_6^{'} = \frac{1}{A^4}\left(\frac{D}{A}\right)\beta_6, \qquad \gamma^{'} = \frac{1}{A^7}\left(\frac{D}{A}\right)^2\gamma.$$

(b) $T_{13} = U_{22} \bigcup T_{15}$:

- $U_{22} = \left\{L(\boldsymbol{\beta}) \in SLb_{10}^b \,:\, \beta_3 = 0, \beta_4 = 0, \beta_5 \neq 0, \beta_6 \neq 0\right\}$
By reducing $\frac{\beta_6^{'}}{\beta_5^{'}} = \frac{1}{A}\frac{\beta_6}{\beta_5}$ in (B.40), then we obtain $A = \frac{\beta_6}{\beta_5}$. The values of B and D appear as $B = \frac{\beta_8 A}{4\beta_5^2}$ and $D = \frac{A^4}{\beta_5}$. Then by using value $\frac{1}{A} = \frac{\beta_5}{\beta_6}$ and $\frac{B}{A} = \frac{\beta_8\beta_6}{4\beta_5^3}$, the system of equalities for U_{22} is as follows:

$$\beta_3^{'} = 0, \tag{B.42}$$
$$\beta_4^{'} = 0,$$
$$\beta_5^{'} = 1,$$
$$\frac{\beta_6^{'}}{\beta_5^{'}} = \left(\frac{\beta_5}{\beta_6}\right)\frac{\beta_6}{\beta_5},$$
$$\frac{\beta_7^{'}}{\beta_5^{'}} = \left(\frac{\beta_5}{\beta_6}\right)^2\frac{\beta_7}{\beta_5},$$
$$\frac{\beta_8^{'}}{\beta_5^{'}} = 0,$$
$$\frac{\beta_9^{'}}{\beta_5^{'}} = \left(\frac{\beta_5}{\beta_6}\right)^4\left[\frac{\beta_9}{\beta_5} + \left(\frac{\beta_8\beta_6}{4\beta_5^3}\right)\frac{\gamma}{\beta_5} - 9\left(\frac{\beta_8\beta_6}{4\beta_5^3}\right)\beta_6\right],$$
$$\frac{\gamma^{'}}{\beta_5^{'2}} = \left(\frac{\beta_5}{\beta_6}\right)\frac{\gamma}{\beta_5^2}.$$

New parameters are represented as λ_1, λ_2 and λ_3 to imply that $\beta_7^{'} = \frac{\beta_5\beta_7}{\beta_6^2} = \lambda_1$, $\beta_9^{'} = \frac{1}{4\beta_6^4}\left(4\beta_5^3\beta_9 + \beta_6\beta_8\gamma - 9\beta_5\beta_6^2\beta_8\right) = \lambda_2$ and $\gamma' = \frac{\gamma}{\beta_5\beta_6} = \lambda_3$. Thus all algebras from U_{22} are isomorphic to $L\left(0,0,1,1,\lambda_1,0,\lambda_2,\lambda_3\right)$ for $\lambda_1, \lambda_2, \lambda_3 \in \mathbb{C}$.

- $T_{15} = \left\{L\left(\boldsymbol{\beta}\right) \in SLb_{10}^b : \beta_3 = 0, \beta_4 = 0, \beta_5 \neq 0, \beta_6 = 0\right\}$
  The system of equations of subset T_{15} is as following:

$$\beta_3^{'} = 0, \qquad \frac{\beta_7^{'}}{\beta_5^{'}} = \frac{1}{A^2}\frac{\beta_7}{\beta_5}, \tag{B.43}$$
$$\beta_4^{'} = 0, \qquad \frac{\beta_8^{'}}{\beta_5^{'}} = 0,$$
$$\beta_5^{'} = 1, \qquad \frac{\beta_9^{'}}{\beta_5^{'}} = \frac{1}{A^4}\left[\frac{\beta_9}{\beta_5} + \left(\frac{B}{A}\right)\frac{\gamma}{\beta_5}\right],$$
$$\frac{\beta_6^{'}}{\beta_5^{'}} = 0, \qquad \frac{\gamma^{'}}{\beta_5^{'2}} = \frac{1}{A}\frac{\gamma}{\beta_5^2}.$$

(c) $T_{15} = T_{16} \bigcup T_{17}$:

- $T_{16} = \left\{L\left(\boldsymbol{\beta}\right) \in SLb_{10}^b : \beta_3 = 0, \beta_4 = 0, \beta_5 \neq 0, \beta_6 = 0, \beta_7 \neq 0\right\}$

The system of equations of subset T_{16} is as following:

$$\begin{aligned} \beta_3^{'} &= 0, & \frac{\beta_7^{'}}{\beta_5^{'}} &= \frac{1}{A^2}\frac{\beta_7}{\beta_5}, \\ \beta_4^{'} &= 0, & \frac{\beta_8^{'}}{\beta_5^{'}} &= 0, \\ \beta_5^{'} &= 1, & \frac{\beta_9^{'}}{\beta_5^{'}} &= \frac{1}{A^4}\left[\frac{\beta_9}{\beta_5} + \left(\frac{B}{A}\right)\frac{\gamma}{\beta_5}\right], \\ \frac{\beta_6^{'}}{\beta_5^{'}} &= 0, & \frac{\gamma^{'}}{\beta_5^{'2}} &= \frac{1}{A}\frac{\gamma}{\beta_5^2}. \end{aligned} \tag{B.44}$$

- $T_{17} = \left\{L(\boldsymbol{\beta}) \in SLb_{10}^b : \beta_3 = 0, \beta_4 = 0, \beta_5 \neq 0, \beta_6 = 0, \beta_7 = 0\right\}$
  The system of equations of subset T_{17} is as following:

$$\begin{aligned} \beta_3^{'} &= 0, & \frac{\beta_7^{'}}{\beta_5^{'}} &= 0, \\ \beta_4^{'} &= 0, & \frac{\beta_8^{'}}{\beta_5^{'}} &= 0, \\ \beta_5^{'} &= 1, & \frac{\beta_9^{'}}{\beta_5^{'}} &= \frac{1}{A^4}\left[\frac{\beta_9}{\beta_5} + \left(\frac{B}{A}\right)\frac{\gamma}{\beta_5}\right], \\ \frac{\beta_6^{'}}{\beta_5^{'}} &= 0, & \frac{\gamma^{'}}{\beta_5^{'2}} &= \frac{1}{A}\frac{\gamma}{\beta_5^2}. \end{aligned} \tag{B.45}$$

(d) $T_{16} = U_{23} \bigcup T_{18}$:

- $U_{23} = \left\{L(\boldsymbol{\beta}) \in SLb_{10}^b : \beta_3 = 0, \beta_4 = 0, \beta_5 \neq 0, \beta_6 = 0, \beta_7 \neq 0, \gamma \neq 0\right\}$
  By reducing $\frac{\beta_6^{'}}{\beta_5^{'}} = \frac{1}{A}\frac{\beta_6}{\beta_5}$ in (B.44), then we obtain $A = \frac{\beta_5\beta_7}{\gamma}$. The values of B and D appear as $B = \frac{\beta_8 A}{4\beta_5^2}$ and $D = \frac{A^4}{\beta_5}$. Then by using value $\frac{1}{A} = \frac{\gamma}{\beta_5\beta_7}$ and $\frac{B}{A} = \frac{\beta_7\beta_8}{4\beta_5\gamma}$, the system of equalities for U_{23} is as follows:

$$
\begin{aligned}
\beta_3' &= 0, \\
\beta_4' &= 0, \\
\beta_5' &= 1, \\
\frac{\beta_6'}{\beta_5'} &= 0, \\
\frac{\beta_7'}{\beta_5'} &= \left(\frac{\gamma}{\beta_5\beta_7}\right)^2 \frac{\beta_7}{\beta_5}, \\
\frac{\beta_8'}{\beta_5'} &= 0, \\
\frac{\beta_9'}{\beta_5'} &= \left(\frac{\gamma}{\beta_5\beta_7}\right)^4 \left[\frac{\beta_9}{\beta_5} + \left(\frac{\beta_7\beta_8}{4\beta_5\gamma}\right)\frac{\gamma}{\beta_5}\right], \\
\frac{\gamma'}{\beta_5'^2} &= \left(\frac{\gamma}{\beta_5\beta_7}\right)\frac{\gamma}{\beta_5^2}.
\end{aligned}
\tag{B.46}
$$

New parameters are represented as λ_1 and λ_2 to imply that $\beta_7' = \frac{\gamma^2}{\beta_5^3\beta_7} = \lambda_1$, $\beta_9' = \frac{\gamma^4}{4\beta_5^6\beta_7^4}(4\beta_5\beta_9 + \beta_7\beta_8) = \lambda_2$ and $\gamma' = \frac{\gamma^2}{\beta_5^3\beta_7} = \lambda_1$. Thus all algebras from U_{23} are isomorphic to $L(0,0,1,0,\lambda_1,0,\lambda_2,\lambda_1)$ for $\lambda_1 \in \mathbb{C}^*$ and $\lambda_2 \in \mathbb{C}$.

- $T_{18} = \left\{L(\boldsymbol{\beta}) \in SLb_{10}^b : \beta_3 = 0, \beta_4 = 0, \beta_5 \neq 0, \beta_6 = 0, \beta_7 \neq 0, \gamma = 0\right\}$
  The system of equations of subset T_{18} is as following:

$$
\begin{aligned}
\beta_3' &= 0, & \frac{\beta_7'}{\beta_5'} &= \frac{1}{A^2}\frac{\beta_7}{\beta_5}, \\
\beta_4' &= 0, & \frac{\beta_8'}{\beta_5'} &= 0, \\
\beta_5' &= 1, & \frac{\beta_9'}{\beta_5'} &= \frac{1}{A^4}\frac{\beta_9}{\beta_5}, \\
\frac{\beta_6'}{\beta_5'} &= 0, & \frac{\gamma'}{\beta_5'^2} &= 0.
\end{aligned}
\tag{B.47}
$$

(e) $T_{18} = U_{24} \bigcup U_{41}$:

- $U_{24} = \left\{L(\boldsymbol{\beta}) \in SLb_{10}^b : \beta_3 = 0, \beta_4 = 0, \beta_5 \neq 0, \beta_6 = 0, \beta_7 \neq 0, \gamma = 0, \beta_9 \neq 0\right\}$
  By reducing $\frac{\beta_7'}{\beta_5'} = \frac{1}{A^2}\frac{\beta_7}{\beta_5}$ and $\frac{\beta_9'}{\beta_5'} = \frac{1}{A^4}\frac{\beta_9}{\beta_5}$ in (B.47), then we obtain $A^2 = \frac{\beta_9}{\beta_7}$.

The values of B and D appear as $B = \frac{\beta_8 A}{4\beta_5^2}$ and $D = \frac{A^4}{\beta_5}$. Then by using value $\frac{1}{A^2} = \frac{\beta_7}{\beta_9}$, the system of equalities for U_{24} is as follows:

$$\begin{aligned}
\beta_3^{'} &= 0, & \frac{\beta_7^{'}}{\beta_5^{'}} &= \left(\frac{\beta_7}{\beta_9}\right)\frac{\beta_7}{\beta_5}, \\
\beta_4^{'} &= 0, & \frac{\beta_8^{'}}{\beta_5^{'}} &= 0, \\
\beta_5^{'} &= 1, & \frac{\beta_9^{'}}{\beta_5^{'}} &= \left(\frac{\beta_7}{\beta_9}\right)^2\frac{\beta_9}{\beta_5}, \\
\frac{\beta_6^{'}}{\beta_5^{'}} &= 0, & \frac{\gamma^{'}}{\beta_5^{'2}} &= 0.
\end{aligned} \tag{B.48}$$

New parameter is represented as λ to imply that $\beta_7^{'} = \frac{\beta_7^2}{\beta_5\beta_9} = \lambda$ and $\beta_9^{'} = \frac{\beta_7^2}{\beta_5\beta_9} = \lambda$. Thus all algebras from U_{24} are isomorphic to $L(0,0,1,0,\lambda,0,\lambda,0)$ for $\lambda \in \mathbb{C}^*$.

- $U_{41} = \left\{L(\boldsymbol{\beta}) \in SLb_{10}^b : \beta_3 = 0, \beta_4 = 0, \beta_5 \neq 0, \beta_6 = 0, \beta_7 \neq 0, \gamma = 0, \beta_9 = 0\right\}$

  By reducing $\frac{\beta_7^{'}}{\beta_5^{'}} = \frac{1}{A^2}\frac{\beta_7}{\beta_5}$ in (B.47), then we obtain $A^2 = \frac{\beta_7}{\beta_5}$. The values of B and D appear as $B = \frac{\beta_8 A}{4\beta_5^2}$ and $D = \frac{A^4}{\beta_5}$. Then by using value $\frac{1}{A^2} = \frac{\beta_5}{\beta_7}$, the system of equalities for U_{41} is as follows:

$$\begin{aligned}
\beta_3^{'} &= 0, & \frac{\beta_7^{'}}{\beta_5^{'}} &= \left(\frac{\beta_5}{\beta_7}\right)\frac{\beta_7}{\beta_5}, \\
\beta_4^{'} &= 0, & \frac{\beta_8^{'}}{\beta_5^{'}} &= 0, \\
\beta_5^{'} &= 1, & \frac{\beta_9^{'}}{\beta_5^{'}} &= 0, \\
\frac{\beta_6^{'}}{\beta_5^{'}} &= 0, & \frac{\gamma^{'}}{\beta_5^{'2}} &= 0.
\end{aligned} \tag{B.49}$$

  Thus all algebras from U_{41} are isomorphic to $L(0,0,1,0,1,0,0,0)$.

The subsets in SLb_{10}^b are illustrated by the following graph trees.

Figure B.2: **Graph tree of** SLb_{10}^{b}

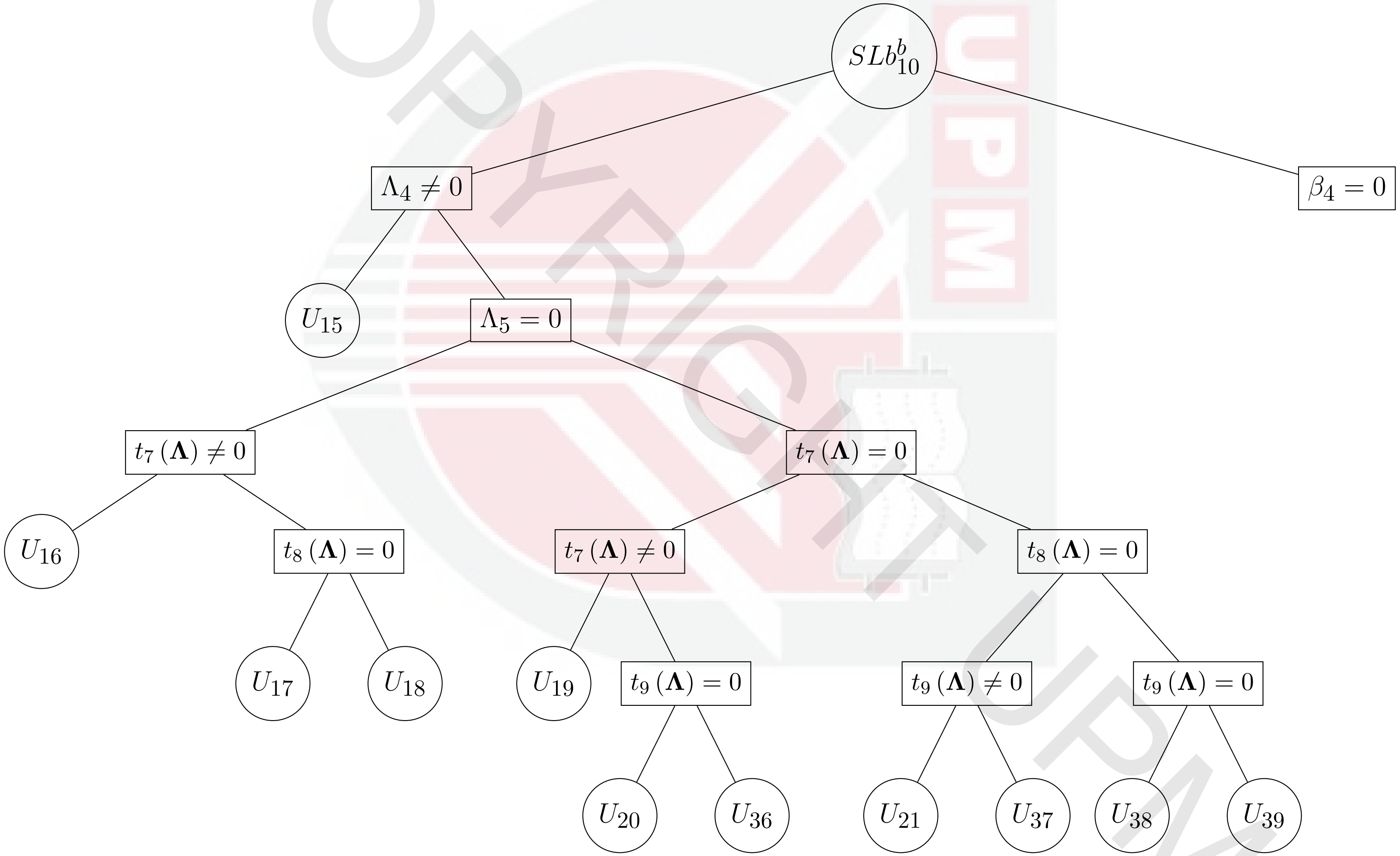
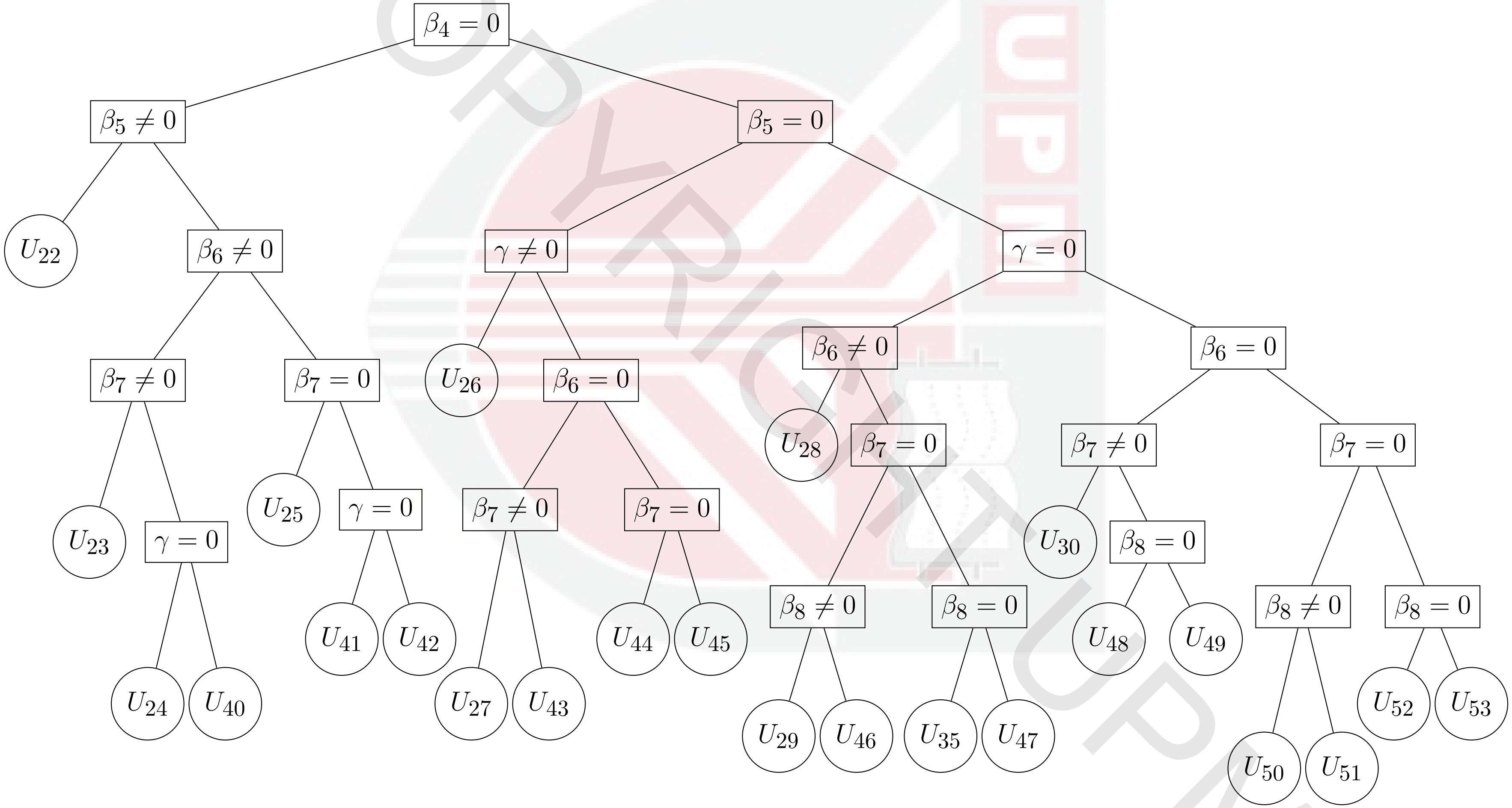


Figure B.3: **Graph tree of SLb_{10}^{b} (continued from Figure B.2)**



APPENDIX C

This part includes two examples of computer program to represent the list of algebras of SLb_{10} into table of multiplications. Here orbits U_{30} and U_{52} are used to ilustrate the examples.

Calling Sequence:
`SC` represents the list of algebras of SLb_{10} either in structural constants $\beta_3, \beta_4, ..., \beta_9, \gamma$ or $\Lambda_3, \Lambda_4, f_5(\mathbf{\Lambda}), ..., f_9(\mathbf{\Lambda}), \Gamma$ or $\Lambda_3, \Lambda_4, \Lambda_5, t_6(\mathbf{\Lambda}), ..., t_9(\mathbf{\Lambda}), \Gamma$.

Parameters:
$\texttt{isom}_1$ – e_0e_1
$\texttt{isom}_2$ – e_1e_1
$\texttt{isom}_3$ – e_2e_1
$\texttt{isom}_4$ – e_3e_1
$\texttt{isom}_5$ – e_4e_1
$\texttt{isom}_6$ – e_5e_1
$\texttt{isom}_7$ – e_6e_1
$\texttt{isom}_8$ – e_7e_1

Example of U_{52}:
<u>Program</u>

$\texttt{SC:=[}\beta_3 = 0, \beta_4 = 0, \beta_5 = 0, \beta_6 = 0, \beta_7 = 0, \beta_8 = 0, \beta_9 = 1, \gamma = 0\texttt{:}$
$\texttt{isom}_1\texttt{:=eval(}\beta_3e_3 + \beta_4e_4 + \beta_5e_5 + \beta_6e_6 + \beta_7e_7 + \beta_8e_8 + \beta_9e_9, \texttt{SC);}$
$\texttt{isom}_2\texttt{:=eval(}\gamma e_9, \texttt{SC);}$
$\texttt{isom}_3\texttt{:=eval(}\beta_3e_4 + \beta_4e_5 + \beta_5e_6 + \beta_6e_7 + \beta_7e_8 + \beta_8e_9, \texttt{SC);}$
$\texttt{isom}_4\texttt{:=eval(}\beta_3e_5 + \beta_4e_6 + \beta_5e_7 + \beta_6e_8 + \beta_7e_9, \texttt{SC);}$
$\texttt{isom}_5\texttt{:=eval(}\beta_3e_6 + \beta_4e_7 + \beta_5e_8 + \beta_6e_9, \texttt{SC);}$
$\texttt{isom}_6\texttt{:=eval(}\beta_3e_7 + \beta_4e_8 + \beta_5e_9, \texttt{SC);}$
$\texttt{isom}_7\texttt{:=eval(}\beta_3e_8 + \beta_4e_9, \texttt{SC);}$
$\texttt{isom}_8\texttt{:=eval(}\beta_3e_9, \texttt{SC);}$

Result

$$isom_1 := e_9$$
$$isom_2 := 0$$
$$isom_3 := 0$$
$$isom_4 := 0$$
$$isom_5 := 0$$
$$isom_6 := 0$$
$$isom_7 := 0$$
$$isom_8 := 0$$

Example of U_{30}:

Program

SC:=[$\beta_3 = 0, \beta_4 = 0, \beta_5 = 0, \beta_6 = 0, \beta_7 = 1, \beta_8 = 1, \beta_9 = \lambda, \gamma = 0$:
isom$_1$:=eval($\beta_3 e_3 + \beta_4 e_4 + \beta_5 e_5 + \beta_6 e_6 + \beta_7 e_7 + \beta_8 e_8 + \beta_9 e_9$, SC);
isom$_2$:=eval(γe_9, SC);
isom$_3$:=eval($\beta_3 e_4 + \beta_4 e_5 + \beta_5 e_6 + \beta_6 e_7 + \beta_7 e_8 + \beta_8 e_9$, SC);
isom$_4$:=eval($\beta_3 e_5 + \beta_4 e_6 + \beta_5 e_7 + \beta_6 e_8 + \beta_7 e_9$, SC);
isom$_5$:=eval($\beta_3 e_6 + \beta_4 e_7 + \beta_5 e_8 + \beta_6 e_9$, SC);
isom$_6$:=eval($\beta_3 e_7 + \beta_4 e_8 + \beta_5 e_9$, SC);
isom$_7$:=eval($\beta_3 e_8 + \beta_4 e_9$, SC);
isom$_8$:=eval($\beta_3 e_9$, SC);

Result

$$isom_1 := e_7 + e_8 + \lambda e_9$$
$$isom_2 := 0$$
$$isom_3 := e_8 + e_9$$
$$isom_4 := 0$$
$$isom_5 := 0$$
$$isom_6 := 0$$
$$isom_7 := 0$$
$$isom_8 := 0$$

BIODATA OF STUDENT

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LIST OF PUBLICATIONS

Publications that arise from the study are:

1. S. Mohd Kasim, N. S. Mohamed, I. S. Rakhimov and S. K. Said Husain. *Isomorphism Criteria of Leibniz Algebras Arising from the Naturally Graded Non-Lie Filiform Leibniz Algebra*, Proceedings of 2^{nd} Regional Conference on Applied and Engineering Mathematics 2012, p225-228, 2012. (Penang, Malaysia)

2. S. Mohd Kasim, N. S. Mohamed, I. S. Rakhimov and S. K. Said Husain. *Algorithm for Constructing the Isomorphism Criteria for 10-Dimensional Complex Filiform Leibniz Algebras*, Proceedings of the Fundamental Science Congress 2012, p51-52, 2012. (UPM Serdang, Selangor, Malaysia)

3. S. Mohd Kasim, I. S. Rakhimov and S. K. Said Husain. *Appearence of Odd Parts of Catalan Numbers in the Classification Problem of Complex Filiform Leibniz Algebras*, Proceedings in the International Science Postgraduate Conference 2012, p465-477, 2012. (UTM Skudai, Johor, Malaysia)

4. S. Mohd Kasim, I. S. Rakhimov and S. K. Said Husain. *On Isomorphism Classes and Invariants of A Subclass Filiform Leibniz Algebras Arising from Naturally Graded Non-Lie Filiform Leibniz Algebras in Dimension 10*, Proceedings of the Fundamental Science Congress 2013, p512-516, 2013. (UPM Serdang, Selangor, Malaysia)

5. S. Mohd Kasim, I. S. Rakhimov and S. K. Said Husain. *Isomorphism Classes of 10-Dimensional Filiform Leibniz Algebras*, AIP Conference Proceedings, 1602, p708-715, 2014.