UNIVERSITI PUTRA MALAYSIA

DIFFERENTIAL GAMES DESCRIBED BY INFINITE SYSTEM OF DIFFERENTIAL EQUATIONS

RISMAN MAT HASIM

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By

RISMAN MAT HASIM

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirements for the Degree of Doctor of Philosophy

April 2014

DEDICATIONS

To

My Dear Father Hj. Mat Hasim bin Maakip

My Lovely Mother Hjh. Dara binti Tamby

and

My Wife: Siti Azlina Borhan

for her great patience

and

My kids: Aiman Faris, Aiman Furqan, Aina Farehah and Aina Fasehah

and

My respected Teachers

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Doctor of Philosophy

DIFFERENTIAL GAMES DESCRIBED BY INFINITE SYSTEM OF DIFFERENTIAL EQUATIONS

By

RISMAN MAT HASIM

April 2014

Chair: Gafurjan Ibragimov, PhD
Faculty: Science

Different approaches have been used by many researchers to solve control problems for parabolic and hyperbolic partial differential equations. Some of these problems can be reduced to the ones described by infinite systems of ordinary differential equations by using the decomposition method. Therefore there is a significant relationship between control problems described by partial differential equations and those described by infinite system of differential equations.

We study three types of infinite systems. The first is infinite systems of first order differential equations. The second system is infinite system of second order differential equations and the third system is infinite system of 2-systems of first order differential equations.

In this thesis, we study the uniqueness and existence theorems for all systems then we study control and differential game problems. For the first system, we study a pursuit game of one pursuer and one evader and evasion differential game of one evader from infinitely many pursuers in the case of integral constraints. For the second system, we study an evasion differential game of one evader from finite number of pursuers in the case of geometric constraints and for the third system, we study a control problem.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

PERMAINAN PEMBEZAAN YANG DIHURAIKAN OLEH SISTEM PERSAMAAN PEMBEZAAN YANG TIDAK TERHINGGA

Oleh

RISMAN MAT HASIM

April 2014

Pengerusi: Gafurjan Ibragimov, PhD
Fakulti: Sains

Pelbagai pendekatan telah digunakan oleh penyelidik untuk menyelesaikan kawalan masalah untuk persamaan pembezaan separa parabolik dan hiperbolik. Sebahagian daripada masalah ini boleh diturunkan kepada sistem persamaan pembezaan biasa tak terhingga dengan menggunakan kaedah penguraian. Oleh itu, terdapat hubungan yang signifikan antara kawalan masalah yang digambarkan oleh persamaan pembezaan separa dan sistem persamaan pembezaan tak terhingga.

Kami mengkaji tiga jenis sistem tak terhingga. Sistem yang pertama adalah sistem tidak terhingga peringkat pertama. Sistem yang kedua adalah sistem tak terhingga peringkat kedua dan sistem yang ketiga adalah sistem tak terhingga 2-sistem peringkat pertama.

Dalam tesis ini, kami mengkaji keunikan dan kewujudan teorem untuk semua sistem tersebut dan kemudian kami mengkaji kawalan dan masalah permainan pembezaan. Untuk sistem yang pertama, kami mengkaji permainan pembezaan pengelakan dalam kes kekangan kamiran. Bagi sistem yang kedua, kami mengkaji permainan pembezaan pengelakan untuk satu mangsa dari sejumlah terhingga pemangsa dalam kes kekangan geometrik dan untuk sistem yang ketiga, kami mengkaji kawalan masalah.

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I certify that a Thesis Examination Committee has met on 10 April 2014 to conduct the final examination of Risman bin Mat Hasim on his thesis entitled "Differential Games Described by Infinite System of Differential Equations" in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Doctor of Philosophy.

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Universiti Putra Malaysia
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Faculty of Science
Universiti Putra Malaysia
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Universiti Putra Malaysia
(Internal Examiner)

Kuchkarrov Atamurat, PhD
Professor
National University of Uzbekistan
Uzbekistan
(External Examiner)

NORITAH OMAR, PhD
Professor and Deputy Dean
School of Graduate Studies
Universiti Putra Malaysia

Date: 19 September 2014

This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfilment of the requirement for the degree of Doctor of Philosophy.

The members of the Supervisory Committee were as follows:

Gafurjan Ibragimov, PhD
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Faculty of Science
Universiti Putra Malaysia
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Zanariah Abdul Majid, PhD
Associate Professor
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LIST OF ABBREVIATIONS

$\mathbb{R}$	The set of all real numbers
$\mathbb{C}$	The set of complex numbers
$\mathbb{R}^n$	The set of n vector over $\mathbb{R}$
T	Terminal time of a game problem
$det\ A$	Determinant of the matrix A
$\|x\|$	Norm or Length of a vector x
$\|\|A\|\|$	Norm of the matrix A
A^t	The Transpose of the matrix A
$\langle ., . \rangle$	An inner product
$C([a, b])$	Space of continuous real valued functions on the interval $[a, b]$
l^2_{r+1}	$\{\alpha = (\alpha_1, \alpha_2, ...) : \sum\limits_{i=1}^{\infty} \lambda_i^{r+1} \|\alpha_i\|^2 < \infty\},\ \alpha_i \in \mathbb{R}^2$
l^2_r	$\{\alpha = (\alpha_1, \alpha_2, ...) : \sum\limits_{i=1}^{\infty} \lambda_i^{r} \|\alpha_i\|^2 < \infty\},\ \alpha_i \in \mathbb{R}^2$
$\\|z\\|_{l^2_r}$	l^2_r -norm of z
$C(0, T; l^2_r)$	Space of continuous functions on $[0, T]$ with the value in l^2_r
$L_2(0,\ T;\ l^2_r)$	$\left\{ f(\cdot) = (f_1(\cdot), f_2(\cdot), \cdots) : \sum\limits_{k=1}^{\infty} \lambda_k^r \int\limits_0^T \|f_k(t)\|^2\, dt < \infty \right\}$
$L_2(0, T)$	$\left\{ f(t) : \int_0^T f^2(t) dt < \infty \right\}$
$\|\|f\|\|_{L_2(0,T;l^2_r)}$	$L_2(0, T; l^2_r)$-norm of the function f

CHAPTER 1

INTRODUCTION

1.1 Background

Differential game constitutes a group of important mathematical problems related to game theory and optimal control theory. It is a game that consists of two players, a pursuer and evader with different goals. The goal of the pursuer is to capture the evader in some sense while that of the evader is to avoid this capture. For example, capture could be minimizing the distance as much as possible between the two players. The game consists of a model describing the behavior of the players which is determined by the players's input through their respective control functions contained in the model. The model is usually a system of differential equations and each player attempts to control the state of the system so as to achieve his goal.

Differential game relates to optimal control theory in the sense that optimal control problems consists of a single control function in the model and a single criterion to be optimized. Differential game theory generalizes this to two controls and two criteria, on for each player. Therefore, optimal control problem are regarded as differential game involving only one player. Technically, control problem can be extended to a differential game problem by introducing control function of the second player to the game model. In both optimal control and differential game problems, the control functions are normally subjected to constraints to reflect a natural phenomenon.

Usually the constraints could be either geometric or integral. If players's control parameter belongs to a subset of $\mathbb{R}^n$, then it is said to be subjected to a geometric constraint. A constraint is referred to as integral if the resources of the player are bounded.

Numerous applications of differential games signify it's importance. It has been applied to solve practical problems related to military operations, economics and engineering among others. For example, it has been employed for missile guidance system and military strategy. It has been used to solve problems related market, financial and economy strategies. Other application includes searching building for Intruders, traffic control and surgical operations.

Differential game problem that requires finding conditions for which the pursuer can catch the evader is called pursuit problem. On the other hand, evasion problem requires finding conditions for which the evader can avoid catch from the pursuer. Pursuit and evasion differential games are played in an environment (space) where the solution of the system of the differential equations or game model is exists.

1.2 Preliminaries

1.2.1 Hilbert Space

Definition 1.1 *Let X be a complex linear space. An inner-product on X is a function $\langle\cdot,\cdot\rangle : X \times X \to \mathbb{C}$ which satisfies the following axioms:*

1. $\langle y, x\rangle = \overline{\langle x, y\rangle}$*, the complete conjugate of* $\langle x, y\rangle$
2. $\langle x + y, z\rangle = \langle x, z\rangle + \langle y, z\rangle$
3. $\langle \lambda x, y\rangle = \lambda\langle x, y\rangle$
4. $\langle x, x\rangle \geq 0$ *and* $\langle x, x\rangle = 0$ *where* $x, y \in X$.

An inner-product space is a linear space with an inner-product on it and is denoted as $(X, \langle\cdot,\cdot\rangle)$.

(Chandrasekhara, 2002)

Example 1.1 *Euclidean space* $\mathbb{R}^n$ *with the dot product*

$$\langle (x_1, ..., x_n), (y_1, ..., y_n)\rangle = \sum_{k=1}^{n} x_k y_k,$$

is an inner product space.

(Pedersen, 2000)

Definition 1.2 *A Sequence* $\{x_n\}$ *in Hilbert space* $(X, \langle\cdot,\cdot\rangle)$ *is called Cauchy sequence if for every positive real number* $\varepsilon > 0$*, there is a positive integer* $N(\varepsilon) > 0$ *such that* $\|x_m - x_n\| < \varepsilon$ *for all natural numbers* $m, n > N(\varepsilon)$.

(Thomson et al., 2001)

Definition 1.3 *A complete inner-product space is called a Hilbert space. In other words, a Hilbert space is an inner-product space in which every Cauchy sequence in the space converges to a point in the space.*

(Ponnusamy, 2002)

Example 1.2 $\mathbb{C}$ *is a Hilbert space with inner-product*

$$\langle x, y\rangle = \sum_{k=1}^{n} x_k \bar{y}_k,$$

where $x = (x_1, x_2, ..., x_n)$ *and* $y = (y_1, y_2, ..., y_n)$ *are in* $\mathbb{C}^n$

Definition 1.4 *Let X be a linear space. A norm on X is a real-valued function $\|\cdot\|$ on X satisfying the following axioms:*

1. $\|x\| > 0 \ \forall x \in X$
2. $\|x\| = 0 \leftrightarrow x = 0$, *zero element in* X
3. $\|x + y\| \leq \|x\| + \|y\| \ \forall x, y \in X.$
4. $\|\alpha x\| = |\alpha|\|x\| \ \forall x \in X$ *and for all scalars* α

A linear space X with a norm $\|\cdot\|$ on it is called a normed space (or normed linear space). It is denoted by $(X, \|\cdot\|)$. The norm is also referred to as the length of the vector x.

(Chandrasekhara, 2002)

Definition 1.5 *The sequence space l^p $(1 \leq p \leq \infty)$ for which norm for the sequence $\{z_n\} \in l^p$ defined by*

$$\|z\|_p = \begin{cases} (\sum_{n=1}^{\infty} |z_n|^p)^{1/p} < \infty, & if \ \ 1 \leq p < \infty, \\ sup_{1 \leq n < \infty} |z_n| < \infty, & if \ \ p = \infty. \end{cases}$$

is normed space.

The norm defined for $1 \leq p < \infty$ is called l^p norm and for $p = \infty$ is called l^∞ norm.
(Ponnusamy, 2002).

Theorem 1.1 *Every inner-product space is a normed linear space with norm defined by $\|x\| = \sqrt{\langle x, x \rangle}$.*

Proof of this theorem, see (Chandrasekhara, 2002)

1.2.2 Measurable Function

Definition 1.6 *The set $\{x = (x_1, x_2, ..., x_n)| \ a < x_i < b, \ i = 1, 2, ..., n\}$ is called n-cubes.*

Definition 1.7 *A subset N of R^n is called a null set (or set of measure zero) in case N can be covered by a countable union of n-cubes whose total n-volume is less than an arbitrarily prescribed number $\epsilon > 0$.*

Example 1.3 *Any finite or countable infinite set of points in R^n has measure zero.*

Definition 1.8 *Two functions $f_1(x)$ and $f_2(x)$ defined on $A \subset R^n$ that differ in value only on a null set are said to be equal almost everywhere on A.*

Definition 1.9 *The measurable sets of R^n are defined as the members of the smallest family of sets of R^n that contains all open sets, all closed sets, all null sets of R^n and also every difference, and countable union, and countable intersection of its members.*

Definition 1.10 *A real-valued function $h(t)$ on a real interval J is called measurable in case for all real α and β, the set $\{t|\ t \in J,\ \alpha < h(t) < \beta\}$ is measurable in R^1.*

If $h(t)$ is measurable on J, there exist a closed subset C on which $h(t)$ is continuous, and we can require that the measure of $J - C$ be arbitrarily small.
If $h(t)$ is measurable on J then we can define the Lebesgue integral $\int_J h(t)dt$ by considering appropriate limits of sums. The function $h(t)$ is called integrable on J if the integration of $|h(t)|$ is finite real number.
A change of the values of $h(t)$ on a null set does not effect the value of the integral.
If $h(t)$ is piecewise continuous and J is compact the value of the Lebesgue integral of $h(t)$ is the same as that of the usual Riemann integral.

Definition 1.11 *Let $h(t)$ be integrable on the interval $J = (t_0, t_1)$ and consider*

$$H(t) = \int_{t_0}^{t} h(s)ds \ for\ t_0 \leq t \leq t_1.$$

Such an indefinite integral $H(t)$ defines an absolutely continuous function. An absolutely continuous function can be proved to be continuous and to have a derivative almost everywhere (everywhere on J except a null set) and there

$$\frac{d}{dt}H(t) = h(t).$$

Every Lipschitz continuous function $H(t)$ is absolutely continuous

Definition 1.12 *The set of all real functions $u(t)$ on an interval J for which $\int_J |u(t)|^p dt < \infty,\ 1 \leq p < \infty$, defines the space L_p. The norm $\|u\|_p = \left(\int_J |u(t)|^p dt\right)^{1/p}$ of the vector space L_p acquires a complete metric and L_p is a Banach Space (upon identifying functions differing only on null sets, L_p becomes a complete normed vector space).*

1.3 Objectives of the thesis

The following are the objectives of the thesis:

- to prove existence and uniqueness theorem for infinite system of 2-systems of first order Differential Equations.

$$\begin{cases} \dot{x_k} = -\alpha_k x_k - \beta_k y_k + w_{1k}, & x_k(0) = x_{k0}, \\ \dot{y_k} = -\beta_k x_k - \alpha_k y_k + w_{2k}, & y_k(0) = x_{k0}, \end{cases} \quad k = 1, 2, ...$$

in the Hilbert Space l_2, α_k, β_k are real numbers, $\alpha_k \geq 0, x_0 = (x_{10}, x_{20}, ...) \in l_2, y_0 = (y_{10}, y_{20}, ...) \in l_2$.

- to obtain sufficient conditions of completion of pursuit in differential game with coordinate-wise integral constraints and describes by infinite system of differential equations of first order.

$$\dot{z}_i + \lambda_i z_i = u_i - v_i, \ \ i = 1, 2, \dots .$$

  where z_i, u_i, $v_i \in R^{n_i}$, n_i is a positive integer, λ_i are given nonnegative numbers, $u = (u_1, u_2, ...)$ and $v = (v_1, v_2, ...)$ are control parameters of the pursuer and the evader respectively. Integral constraints are imposed on the control functions of players.

- to obtain sufficient conditions of evasion from many pursuers in the game described by infinite system of differential equations of second order in the case of geometric constraints.

$$\ddot{z}_{ik} = -\lambda_k z_{ik} - u_{ik} + v_k, \ \ z_{ik}(0) = z_{ik}^0, \ \ \dot{z}_{ik}(0) = z_{ik}^1, k = 1, 2, ...,$$

  where

$$z_{ik}, u_{ik}, v_k, z_{ik}^0, z_{ik}^1 \in R^1,$$

$$z_i^0 = (z_{i1}^0, z_{i2}^0, ..) \in l_{r+1}^2, \ \ z_i^1 = (z_{i1}^1, z_{i2}^1, ..) \in l_r^2, ||z_i^0||_{r+1} + ||z_i^1||_r \neq 0,$$

  $u_i = (u_{i1}, u_{i2}, ...)$ is control parameter of ith pursuer, $i = 1, 2, ..., m$, and $v = (v_1, v_2, ...)$ is control parameter of the evader. Geometric constraints are imposed on the control functions of players.

- to obtain sufficient conditions of evasion (problem) and completion of pursuit (problem) in the game of many pursuers and one evader described by infinite system of differential equations of first order in the case of integral constraints.

$$\dot{z}_{ik} = -\lambda_k z_{ik} - u_{ik} + v_k, \ z_{ik}(t_0) = z_{ik}^0, \ k = 1, 2, ...,$$

  where z_{ik}, u_{ik}, v_k, t_0, $z_{ik}^0 \in R^1$, $z_i^0 = (z_{i1}^0, z_{i2}^0, ...) \in l_{r+1}^2$, $z_i^0 \neq 0$, $u_i = (u_{i1}, u_{i2}, ...)$ is control parameter of the ith Pursuer, and $i = 1, 2, ...,$ $v = (v_1, v_2, ...)$ is that of the Evader. Suppose that $u_i(\cdot)$, $v(\cdot) \in L_2(t_0, T; \ l_r^2)$, where T is fixed positive number. In both cases, integral constraints are imposed on the control functions of players.

1.4 Outlines of the thesis

In this thesis, we study the differential game problems described by an infinite system of differential equations in Hilbert Space. The players influence on the system is by the use of control parameters which are subject to various constraints.

In chapter 2 is review about a history of the differential game where the players influence on the system is by the use of control parameters which are subject to various constraints. Some important works by the researches which related to our works are also presented.

Chapter 3, we proof the existence and uniqueness theorem for infinite system of 2-systems of first order Differential Equations. Here we showed that the system has a unique solution and we showed the continuity of the function.

In chapter 4 focuses on the pursuit game with coordinate wise integral constraints on control functions of players. This game is the solution for the system infinite systems of first order differential equations in early chapter 3.

In chapter 5 we focuses on evasion game from many pursuers. We investigate a game problem of m pursuers and one evader described by infinite systems of differential equations of second order. Geometric constraints are imposed on the controls of players.

Chapter 6 we study in both cases, pursuit and evasion differential game in Hilbert Space described by the same system. First part, in evasion Differential game with integrals constraints on the control function of players. A group consisting of countable number of pursuers tries to force the states of the systems toward the origin against any action of the evader. Second part, pursuit differential game with countable many pursuers. Chapter 7 proposes some future studies as an extension to this research.

BIBLIOGRAPHY

Azamov, A.A. 1974. An Elementary Introduction to the Theory of Differential Games. National Uzbek of Uzbekistan.

Advonin, S.A. and Ivanov, S.A. 1989. The Controllability of Systems with Distributed Parameters and Families of Exponentials. UMKVO, Kiev.

Basar, T. and Olsder, G. 1998. Dynamic Noncooperative Game Theory. Philadelphia. SIAM, Second edition.

Berkovitz, L.D. 1964. A Variational Approach to Differential Games. Advances in Game Theory. *Annals of Mathematics Studies* 52: 127–175.

Berkovitz, L.D. 1967. A Survey of Differential Games, Mathematical Theory of Control, Edited by A. V. Balakrishnan and L. W. Neustadt. *Academic Press, New York* 373–385.

Borowko, P., Rzymowski, W. and Stachura, A. 1988. Evasion from Many Pursuers in the Simple Motion Case. *Journal of Mathematical Analysis and Applications* 135: 170–180.

Butkovskiy, A.G. 1975. Control Methods in Systems with Distributed Parameters. Nauka, Moscow..

Chandrasekhara, R.K. 2002. Functional Analysis. Alpha Science International Ltd.

Chernous'ko. 1976. A Problem of Evasion From Many Pursuers. *Journal of Applied Mathematics Mechanics* 40(1): 14–24.

Chernous'ko, F.L. and Zak, V. 1985. On Differential Games of Evasion from Many Pursuers. *Journal of Optimization Theory and Applications* 46: 461–470.

Chernous'ko, F.L. 1992. Bounded Controls in Systems with Distributed Parameters. *Journal of Applied Mathematics Mechanics* 56(5): 810–826.

Tukhtasinov, M. and Mamatov, M. S. 2009. On Transfer Problems in Control Systems. *Differential Equations* 45: 439–444.

Chikrii, A.A. 1979. Quasilinear Differential Games with Many Participants. *Doklad of the. USSR. Academy of. Sciences* 246(6): 1306–1310.

Dongxu, L. 2006. Multi-player Pursuit-Evasion Differential Games. *PhD thesis* The Ohio State University.

Elliott, R.J. and Kalton, N.J. 1972a. The Existence of Value in Differential Games. Memoirs of the American Mathematical Society.

Elliott, R.J. and Kalton, N.J. 1972b. Values in Differential Games. *Bulletin of the American Mathematical Society* 78: 427-431.

Friedman, A. 1971. Differential Games, John. Wiley and Sons, New York.

Fleaming, W.H. 1961. The Convergence Problem for Differential Games. *Journal of Mathematical Analysis and Applications* 3: 102–106.

Fleaming, W.H. 1964. The Convergence Problem for Differential Games, Part 2. *Advances in Game Theory, Annals of Mathematics Studies* 52: 195–210.

Gutman, S., Esh, M. and Gefen, M. 1987. Simple Linear Pursuit-Evasion Games. *Computers and Mathematics with Applications* 13: 83–95.

Hagedorn, P. and Breakwell, J. 1976. A Differential Game with two Pursuers and one Evader. *Journal of Optimization Theory and Applications* 18: 15–29.

Ibragimov G.I. 1998. A Game of Optimal Pursuit of One Object by Several. *Journal. Applied Mathematics Mechanics* 62(2): 187–192.

Ibragimov G.I. 2003. A Problem Of Optimal Pursuit in Systems with Distributed Parameters. *Journal. Applied Mathematics Mechanics* 66(5): 719–724.

Ibragimov G.I. 2004a. On Possibility of Evasion on a Differential Game Described by Infinite System of Differential Equations. *Uzbek Mathematics Journal* 1: 50–55.

Ibragimov G.I. 2004b. Collective Pursuit with Integral Constraints on the Controls of Players. *Siberian Advances in Mathematics* 14(2): 14–26.

Ibragimov G.I. 2005a. Problem of Relaxation of Oscillation Systems in Presence of Disturbance. *Uzbek Mathematics Journal* 2: 34–50.

Ibragimov G.I. 2005b. A Group Pursuit Game. *Automation and Remote Control* 66(8): 1214–1223.

Ibragimov G.I. 2005c. Optimal Pursuit with Countable Many Pursuers and One Evader. *Differential Equations* 41(5): 627–635.

Ibragimov G.I. and Rikhsiev, B.B. 2006. On some Sufficient Conditions for Optimality of the Pursuit Time in the Differential Game with Multiple Pursuers. *Automation and Remote Control* 67(4): 529–537.

Ibragimov G.I. and Leong, W.J. 2008. A Differential Game of Multiperson Pursuit in the Hilbert Space. *Communications in Mathematical Analysis* 21(6): 61–66.

Ibragimov G.I. and Husin, N.A. 2010. A Pursuit-Evasion Differential Game with Many Pursuers and One Evader. *Malaysian Journal of Mathematical Sciences* 4: 183–194.

Ibragimov G.I. and Ja'afaru, A.B. 2011. On Control Problem Described by Infinite System of First-Order Differential Equations. *Australian Journal of Basic and Applied Sciences* 5(10): 736–742.

Ivanov, S.A. 1980. Extremal Strategies in Differential Games with Integral Constraints. *J. Prikl. Mat. Mekh.*

Ichikawa, A. 1976. Linear-Quadratic Differential Games in a Hilbert Space. *Journal on Control* 14(1): 120–136.

Isaacs, R. 1965. Differential Game, New York.

Ja'afaru, A.B and Ibragimov G.I. 2011. Differential Games Described by Infinite System of Differential Equations of Second Order. The Case of Negative Coefficients. *International Journal of Pure and Applied Mathematics* 70(7): 927–938.

Krasovskii, N.N. and Subbotin, A.I. 1988. Game-theoretical Control Problems. NewYork. Springer.

Leitmann, G. 1968. Simple Differential Game. *Journal of Optimization Theory and Applications* 2: 220–225.

Leitmann, G. 1969. A Differential Game of Pursuit and Evasion. *International Journal of Non-Linear Mechanics* 4: 1–9.

Lewin, J. 1994. Differential Games, Springer, Berlin, Germany.

Mishchenko, E.F., Nikol'skii, M.S. and Satimov, N.Y. 1977. A Problem of Escaping Contact in Many-Person Differential Games. *USSR Academy of Sciences, Proceedings of the Mathematical Institute* 143: 105–128.

Neumann, J. and Morgenstern, O. 1944. Theory of Games and Economic Behavior. Princeton University Press.

Osipov Yu. 1975. Differential Game Theory in Systems with Distributed Parameters. *Dokl. Akad. Nauk* 223(6): 1314–1317.

Pachter, M. 1987. Simple Motion Pursuit-Evasion in the Half Plane. *Computers and Mathematics with Applications* 13: 69–82.

Pedersen, M. 2000. Functional Analysis in Applied Mathematics and Engineering. Chapman and Hall/CRC..

Petrosyan, L.A. 1993. Differential Games of Pursuit. St. Petersburg State University.

Ponnusamy, S. 2002. Foundations of Functional Analysis. Alpha Science International Ltd.

Pontrayagin, L.S. 1988. Collected works. Nauka, Moscow.

Pshenichniy, B.P. 1976. Simple Pursuit by Several Objects. *Cybernetic* 3: 145–146.

Satimov, N.Y., Rikhsiev, B.B. and Khamdamov, A.A. 1983. On a Pursuit Problem for n-person Linear Differential and Discrete Games with Integral Constraints. *Math. USSR Sbornik* 46(4): 459–471.

Satimov, N.Y. and Rikhsiev, B.B. 2000. Methods of Solving of Evasions Problems in Mathematics Control Theory. Tashkent.

Satimov, N.Y. and Tukhtasinov, M. 2005a. On Some Game Problems for First-Order Controlled Evolution Equations. *Differential Equations* 41: 1169–1177.

Satimov, N.Y. and Tukhtasinov, M. 2005b. Some game problems in distributed controlled systems. *Journal of Applied Mathematics and Mechanics* 69: 885–890.

Satimov, N.Y. and Tukhtasinov, M. 2006. Game Problems on a Fixed Interval in Controlled First-Order Evolution Equations. *Mathematical Notes* 80: 578–589.

Satimov, N.Y., Tukhtasinov, M. and Ismatkhodzhaev, S. 2007. On and Evasion Problem on a Semi-Infinite Interval for a Class of Controlled Distributed Systems. *Mathematical Notes* 81: 260–267.

Thomson, B.S., Bruckner, J.B. and Brucner, A.M. 2001. Elementary Real Analysis. Prentice-Hall, Inc. New Jersey.

Tukhtasinov, M. 1995. On Some Problems in the Theory of Differential Pursuit Games in Systems with Distributed Parameters. *Prikl. Mat. Mekh* 59: 979–984.

Tukhtasinov, M. and Mamatov, M.S. 2008. On Pursuit Problems in Controlled Distributed Systems. *Mathematical Notes* 84: 256–262.

Tukhtasinov, M. and Mamatov, M.S. 2009. On Transfer Problems in Control Systems. *Differential Equations* 45: 439–444.

Vagin, D.A. and Petrov, N.N. 2001. A Problem of Group Pursuit With Phase Contraints,. *Journal of Applied Mathematics Mechanics* 66(2): 225–232.

Zak, V.L. 1985. Problem of Evasion from Many Pursuers Controlled by Acceleration. *Izvestiya Akademii Nauk SSSR, Tekhnicheskaya Kibernetika* 2: 57–71.

Zaremba, L.S. 1979. On the Existence of Value in the Varaiya-Lin Sense in Differential Games of Pursuit and Evasion. *Journal of Optimization Theory and Applications* 29: 135–145.

Zaremba, L.S. 1980. Existence of Value in Pursuit-Evasion with Restricted Phase Coordinates. *Journal of Optimization Theory and Applications* 30: 451–470.