UNIVERSITI PUTRA MALAYSIA

SEQUENTIAL AND PARALLEL METHODS FOR NUMERICAL SOLUTIONS OF DELAY DIFFERENTIAL EQUATIONS

FUZIYAH ISHAK

IPM 2009 9

SEQUENTIAL AND PARALLEL METHODS FOR NUMERICAL SOLUTIONS OF DELAY DIFFERENTIAL EQUATIONS

By

FUZIYAH ISHAK

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirements for the Degree of Doctor of Philosophy

October 2009

To my family.

To the memories of my mother and my father.

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Doctor of Philosophy

SEQUENTIAL AND PARALLEL METHODS FOR NUMERICAL SOLUTIONS OF DELAY DIFFERENTIAL EQUATIONS

By

FUZIYAH ISHAK

October 2009

Chair : Dato' Mohamed Suleiman, PhD

Institute : Institute for Mathematical Research

This thesis describes the development of sequential and parallel methods for solving delay differential equations. A new sequential code for the numerical solution of delay differential equations is considered. The variable order variable stepsize formulae based on the Adams-Bashforth-Moulton methods are represented in divided difference form. Derivative discontinuities are detected by local error estimate at the grid points. Large magnitude of the local error estimate indicates the presence of derivative discontinuity. Stepsize is then reduced and eventually the discontinuity point is included in the grid. The formulae representation proves to be efficient when compared with the existing method in modified divided difference form.

We also consider the development of two-point block methods on sequential and parallel computers. Formulae for three two-point block methods for solving delay differential equations are derived. The implicit block methods are implemented using variable stepsize variable order technique. The formulae for two-point diagonally and triangularly implicit block methods using predictor-corrector application are represented in divided difference form. Meanwhile, the predictor-corrector formulae for two-point fully implicit block method are calculated beforehand and stored at the beginning of the code. All of the block methods rely on the local error estimates to detect derivative discontinuities. In all of the developed methods, regions of absolute stability are presented and compared. Comparison among the developed methods indicates that all of the methods achieve the desired accuracy. Block methods are efficient when compared with the sequential non-block method as the total steps taken can be reduced. The new block methods are then used for the parallel implementation in solving large system of delay differential equations. The parallel programs using Message Passing Interface are run on Sun Fire V1280 using two processors. Numerical results indicate that parallel implementation increases the performance of the block methods.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia
sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

KAEDAH-KAEDAH JUJUKAN DAN SELARI UNTUK PENYELESAIAN BERANGKA PERSAMAAN PEMBEZAAN LENGAH

Oleh

FUZIYAH ISHAK

Oktober 2009

Pengerusi : Dato' Mohamed Suleiman, PhD

Institut : Institut Penyelidikan Matematik

Tesis ini menghuraikan proses pembangunan kaedah-kaedah jujukan dan selari bagi penyelesaian persamaan pembezaan lengah. Satu kod berjujukan yang baru untuk penyelesaian berangka persamaan pembezaan lengah adalah dipertimbangkan. Formula saiz langkah dan peringkat berbeza-beza berdasarkan kaedah-kaedah Adams-Bashforth-Moulton dipersembahkan dalam bentuk beza terbahagi. Ketakselanjaran terbitan dikesan oleh nilai anggaran ralat setempat pada titik-titik grid. Magnitud yang besar bagi anggaran ralat setempat menunjukkan kehadiran ketakselanjaran terbitan. Saiz langkah kemudiannya dikurangkan supaya titik ketakselanjaran adalah tergolong dalam titik grid. Apabila dibandingkan dengan kaedah sedia ada dalam bentuk beza terbahagi berubah, formula yang dipersembahkan dalam bentuk beza terbahagi menghasilkan satu kaedah yang cekap.

Kami juga mempertimbangkan kaedah-kaedah blok dua-titik yang dilaksanakan pada komputer selari dan tak selari. Tiga formula bagi kaedah-kaedah blok dua-titik diterbitkan. Kaedah-kaedah blok tersirat ini dilaksanakan dengan menggunakan teknik saiz langkah dan peringkat berbeza-beza. Formula bagi kaedah-kaedah blok tersirat dua-titik secara pepenjuru dan bentuk segitiga menggunakan aplikasi pembetul-peramal yang diwakili dalam bentuk beza terbahagi. Sementara itu, formula pembetul-peramal bagi kaedah blok dua-titik sepenuhnya tersirat dikira terlebih dahulu dan disimpan di awal kod. Semua kaedah-kaedah blok tersebut bergantung kepada nilai anggaran ralat setempat untuk mengesan ketakselanjaran terbitan. Rantau kestabilan bagi kaedah-kaedah yang dibangunkan adalah dibentangkan. Perbandingan di antara kaedah-kaedah yang dibangunkan menunjukkan bahawa semua kaedah-kaedah tersebut mencapai tahap kejituan yang dikehendaki. Kaedah-kaedah blok adalah cekap apabila dibandingkan dengan kaedah jujukan tanpa blok kerana jumlah bilangan langkah yang diambil dapat dikurangkan. Kaedah-kaedah blok kemudiannya digunakan di dalam pelaksanaan selari bagi menyelesaikan sistem persamaan pembezaan lengah yang besar. Pelaksanaan program selari dijalankan dengan menggunakan komputer Sun Fire V1280 bersama dua buah pemproses dibantu oleh protokol mesej penghantar Message Passing Interface. Keputusan berangka menunjukkan bahawa teknik selari meningkatkan prestasi kaedah-kaedah blok tersebut.

ACKNOWLEDGEMENTS

Bismillahirrahmanirrahim. First and foremost of all, I would like to express my sincere gratitude and thanks to my supervisor, Y. Bhg. Dato' Professor Dr. Mohamed bin Suleiman for his invaluable guidance and assistance during my study. I am very grateful to Prof. Mohamed for his advice which had directed me toward this research specifically and numerical analysis in general. My deepest thanks are also extended to the members in the supervisory committee, Associate Professor Dr. Fudziah bt. Ismail, Dr. Zanariah bt. Majid and Dr. Khairil Iskandar bin Othman for their generosity and patience.

This entire journey would not have been a memorable one without numerous help, support and encouragement from my dearest friends and colleagues. I am truly blessed with friends that continuously provide me with helpful suggestions and insights prior to the final preparation of this thesis.

I wish to thank the Institute for Mathematical Research (INSPEM), Universiti Putra Malaysia (UPM) for providing a place that is conducive for me to perform my experiments. Many thanks to Universiti Teknologi MARA (UiTM) and Ministry of Higher Education Malaysia for the study leave and sponsorship which enable me to pursue my degree.

Last but not least, I would like to express my gratitude to my family. I thank my daughters Elisa Farhana, Sarah, Nadira and son Zareef Hidayat for their love, patience and compromise. I am grateful for their understanding and support over this entire journey. My deepest and sincerest thank goes out to my husband Zulkiflee for believing in me. Without his encouragement, this thesis would not have been completed.

I certify that an Examination Committee has met on 30th October 2009 to conduct the final examination of Fuziyah binti Ishak on her Doctor of Philosophy thesis entitled "Sequential and Parallel Methods for Numerical Solutions of Delay Differential Equations" in accordance with Universiti Pertanian Malaysia (Higher Degree) Act 1980 and Universiti Pertanian Malaysia (Higher Degree) Regulations 1981. The Committee recommends that the student be awarded the degree of Doctor of Philosophy.

Members of the Examination Committee were as follows:

Azmi Jaafar, PhD
Associate Professor
Faculty of Computer Science and Information Technology
Universiti Putra Malaysia
(Chairman)

Malik Hj. Abu Hassan, PhD
Professor
Faculty of Science
Universiti Putra Malaysia
(Internal Examiner)

Leong Wah June, PhD
Senior Lecturer
Faculty of Science
Universiti Putra Malaysia
(Internal Examiner)

Bahrom Sanugi, PhD
Professor
Center for Industrial Relation and Service
Universiti Sains Islam Malaysia
(External Examiner)

____

BUJANG KIM HUAT, PhD
Professor and Deputy Dean
School of Graduate Studies
Universiti Putra Malaysia

Date:

This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfilment of the requirement for the degree of Doctor of Philosophy. The members of the Supervisory Committee were as follows:

DATO' MOHAMED SULEIMAN, PhD
Professor
Faculty of Science
Universiti Putra Malaysia
(Chairman)

FUDZIAH ISMAIL, PhD
Associate Professor
Faculty of Science
Universiti Putra Malaysia
(Member)

ZANARIAH ABDUL MAJID, PhD
Senior Lecturer
Faculty of Science
Universiti Putra Malaysia
(Member)

KHAIRIL ISKANDAR OTHMAN, PhD
Lecturer
Faculty of Computer and Mathematical Sciences
Universiti Teknologi MARA
(Member)

HASANAH MOHD GHAZALI, PhD
Professor and Dean
School of Graduate Studies
Universiti Putra Malaysia

Date: 14 January 2010

DECLARATION

I declare that the thesis is my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously, and is not concurrently, submitted for any other degree at Universiti Putra Malaysia or at any other institution.

FUZIYAH ISHAK

Date: 30 January 2010

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES