UNIVERSITI PUTRA MALAYSIA

CYCLICITY OF BOUNDED LINEAR OPERATORS ON SEPARABLE BANACH SPACES AND CLOSED SUBSPACES

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CYCLICITY OF BOUNDED LINEAR OPERATORS ON SEPARABLE BANACH SPACES AND CLOSED SUBSPACES

By

NAREEN SABIH MUHEMED SAID

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirements for the Degree of Doctor of Philosophy

March 2016

DEDICATIONS

To
My father and mother
For their encouragement
and
To my husband and my kids
For their great patience

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Doctor of Philosophy

CYCLICITY OF BOUNDED LINEAR OPERATORS ON SEPARABLE BANACH SPACES AND CLOSED SUBSPACES

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March 2016

Chairman: Adem Kılıçman, PhD.
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The main focus of this thesis is to study some properties of diskcyclic operators. The similarities and differences between diskcyclic operators and the concepts of cyclicity are investigated. New classes of operators on the direct sum of Banach spaces, namely k-bitransitive, k-diskcyclic and k-compound operators, are defined to study the direct sum of diskcyclic operators. A weaker property than chaotic operators, namely semi chaotic operators, is defined and studied to show that a chaos for linear operators exists on finite dimensions. Diskcyclicity concept is extended to closed subspaces of Banach spaces, and such a concept is called a subspace-diskcyclic operator. The similarities and differences between subspace-diskcyclicity and the subspace-cyclicity are investigated. Finally, some properties of hypercyclic operators are extended to subspace-hypercyclic operators to solve some open problems in the literature. This study shows that diskcyclic, semi chaotic and subspace-diskcyclic operators have some properties that are not shared with the existing concepts of cyclicity of operators.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

KEKITARAN BAGI SEMPADAN PENGOPERASI LINEAR KE ATAS RUANG-RUANG BANACH YANG TERASING DAN SURUANG-SUBRUANG BANACH YANG TERTUTUP

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Fokus utama tesis ini adalah untuk mengkaji beberapa ciri pengoperasi kitaran-cakera. Persamaan dan perbezaan antara pengoperasi kitaran-cakera dan konsep kekitaran disiasat. Kelas baru pengoperasi terhadap hasil tambah langsung ruang Banach iaitu pengoperasi k-bitransitif, k-kitaran-cakera dan k-majmuk, ditakrifkan untuk mengkaji hasil tambah langsung pengoperasi kitaran-cakera. Suatu ciri yang lebih lemah daripada pengoperasi kalutan, yang dinamakan pengoperasi semi kalutan, ditakrifkan dan dipelajari untuk menunjukkan bahawa kalutan bagi pengoperasi linear wujud pada dimensi terhingga. Konsep kekitaran-cakera diperluaskan kepada subruang tertutup ruang Banach, dan apa-apa konsep yang sedemikian dipanggil pengoperasi subruang-kitaran-cakera. Persamaan dan perbezaan antara subruang-kekitaran-cakera dan subruang-kekitaran disiasat. Akhir sekali, beberapa ciri pengoperasi hiperkitaran diperluaskan kepada pengoperasi subruang-hiperkitaran untuk menyelesaikan beberapa masalah terbuka dalam kajian sorotan. Kajian ini menunjukkan bahawa pengoperasi kitaran-cakera, semi huru-hara dan subruang-kitaran-cakera mempunyai beberapa ciri yang tidak dikongsi dengan konsep sedia ada bagi pengoperasi kekitaran.

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I certify that a Thesis Examination Committee has met on 14 March 2016 to conduct the final examination of Nareen Sabih Muhemed Said on her thesis entitled "Cyclicity of Bounded Linear Operators on Separable Banach Spaces and Closed Subspaces" in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Doctor of Philosophy.

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LIST OF ABBREVIATIONS

X	A Banach space
H	A Hilbert space
$\dim(X)$	Dimension of the space X
$\mathbb{N}$	Set of all positive integer numbers
$\mathbb{N}_0$	Set of all non-negative integer numbers
$\mathbb{R}$	Set of all real numbers
$\mathbb{R}^+$	Set of all positive real numbers
$\mathbb{Q}$	Set of all rational numbers
$\mathbb{C}$	Set of all complex numbers
$\mathbb{Z}$	Set of all integer numbers
$\mathbb{K}$	A field
$\mathbb{D}$	The closed unit disk
$\mathbb{U}$	The open unit disk
$\mathbb{T}$	The unit circle
ℓ^p	Sequence space
$\ell^p(\mathbb{N})$	Space of all one-sided sequences
$\ell^p(\mathbb{Z})$	Space of all two-sided sequences
c_0	Space of all sequences converging to zero
e_n	An element in the standard basis of ℓ^p
$\{w_n\}$	A weight sequence
$\lvert x\rvert$	Absolute value of a real number x
$\lVert x\rVert$	Norm of a vector x
$\lVert x\rVert_p$	Norm a vector x in the space ℓ^p
$\langle x,y\rangle$	Inner product of x and y
$\sum\limits_C$	Summation over a countable set C
$\prod\limits_C$	Product over a countable set C
$\lim\{x_n\}$	Limit of a sequence $\{x_n\}$
$\sup\{x_n\}$	Supremum of a sequence $\{x_n\}$
$H(\mathbb{C})$	Space of all entire functions
G_δ	Countable intersection of open sets
$x_n \to z$	A sequence x_n converges to z
$x_n \not\to z$	A sequence x_n does not converge to z
T,S	Operators
$\mathscr{B}(X)$	Set of all bounded linear operators on a space X
T^n	Composition of T with itself n times
T^*	Adjoint of an operator T
T^{-1}	Inverse of an operator T
$\lVert T\rVert$	Norm of an operator T
$\omega(T)$	Numerical range of an operator T
$M\oplus N$	Direct sum of subspaces M and N
$X\oplus Y$	Direct sum of spaces X and Y
$T\oplus S$	Direct sum of operators T and S

$A^{\perp}$	Orthogonal complement of a set A
$\sigma_p(T)$	Point spectrum of an operator T
$\sigma(T)$	Spectrum of an operator T
A^c	Complement of a set A
$\ker(T)$	Kernel of an operator T
$R(A)$	Range of a set A
$\overline{A}$	Closure of a set A
$int(A)$	Interior of a set A
ϕ	Empty set
$B(x,r)$	An open ball with center x and radius r
$\text{span}\{A\}$	Linear span of a set A
X^*	Continuous dual space of a space X
$dist(x,A)$	Distance from a vector x to a set A
$f^{'}$	Derivative of a map f
$Orb(T,x)$	Orbit of a vector x under an operator T
$\mathbb{D}Orb(T,x)$	Disk orbit of a vector x under an operator T
$\mathbb{C}Orb(T,x)$	Scaled orbit of a vector x under an operator T
$\mathbb{K}Orb(T,x)$	Projective orbit of a vector x under an operator T
$HC(T)$	Set of all hypercyclic vectors for an operator T
$HC(X)$	Set of all hypercyclic operators on a space X
$\mathbb{D}C(T)$	Set of all diskcyclic vectors for an operator T
$\mathbb{D}C(X)$	Set of all diskcyclic operators on a space X
$SC(T)$	Set of all supercyclic vectors for an operator T
$SC(X)$	Set of all supercyclic operators on a space X
$HC(T,M)$	Set of all M-hypercyclic vectors for an operator T
$HC(M,X)$	Set of all M-hypercyclic operators on a space X
$\mathbb{D}C(T,M)$	Set of all M-diskcyclic vectors for an operator T
$\mathbb{D}C(M,X)$	Set of all M-diskcyclic operators on a space X
$SC(T,M)$	Set of all M-supercyclic vectors for an operator T
$SC(M,X)$	Set of all M-supercyclic operators on a space X
$C(A,B)$	Cross set from a set A to a set B
$J(A,B)$	Junction set from a set A to a set B

CHAPTER 1

INTRODUCTION

1.1 Basic Concepts

In this section, we give some well known definitions and concepts in functional analysis that are used in this thesis. These notions are taken from Istrăţescu (1981) and Conway (2013).

Definition 1.1.1 *Let X be a set and $d : X \times X \to \mathbb{R}$ be a function. Then d is called a metric on X if for any $x, y, z \in X$,*

(i) $d(x,y) \geq 0$ *with equality if and only if* $x = y$,

(ii) $d(x,y) = d(y,x)$,

(iii) $d(x,y) \leq d(x,z) + d(z,y)$.

In this case, X endowed with this metric is called a metric space. The number $d(x,y)$ is called the distance between x and y.

Definition 1.1.2 (normed space) *A vector space X is a normed space, if for any $x \in X$ there exists a non negative real number $\|x\|$, which is called the norm of x and satisfies the following conditions.*

(i) $\|x+y\| \leq \|x\| + \|y\|$ *for all x and y in X,*

(ii) $\|cx\| = |c|\,\|x\|$ *if $x \in X$ and c is a scalar,*

(iii) $\|x\| > 0$ *if* $x \neq 0$.

Every normed space is a metric space, where the distance $d(x,y)$ between x and y is $\|x-y\|$.

Definition 1.1.3 (inner product space) *Let X be a vector space over a field $\mathbb{C}$ of complex numbers. A map $\langle \cdot, \cdot \rangle : X \times X \to \mathbb{C}$ is called an inner product in X, if for every $x, y, z \in X$ and $\alpha, \beta \in \mathbb{C}$, we have the following conditions.*

(i) $\langle x,y \rangle = \overline{\langle y,x \rangle}$ *(where $\bar{x}$ is the complex conjugate of x),*

(ii) $\langle \alpha x, y \rangle = \alpha \langle x,y \rangle$,

(iii) $\langle x+y, z \rangle = \langle x,z \rangle + \langle y,z \rangle$,

(iv) $\langle x,x\rangle \geq 0$,

(v) *if* $\langle x,x\rangle = 0$ *if and only if* $x = 0$.

An inner product space is a vector space with an inner product map.

It follows that every inner product space is a normed space with the norm defined by $\|x\|^2 = \langle x,x\rangle$.

Definition 1.1.4 (complete normed space) *A normed space X is called complete if every Cauchy sequence in X is convergent in X.*

A Banach space is a complete normed. A Hilbert space is a complete inner product space. Since every inner product space is a normed space, then every Hilbert space is a Banach space.

Both the spaces $\mathbb{C}^n$ of all complex n-tuples and $\mathbb{R}^n$ of all real n-tuples are examples of n-dimensional Hilbert spaces.

Definition 1.1.5 *Let M be a subset of a normed space X, then M is called dense if it intersects every nonempty open subset of X.*

Definition 1.1.6 *Let M be a subset of a normed space X, then M is called G_δ set if it a countable intersection of open subsets of X.*

Definition 1.1.7 (Separable Banach space) *A Banach space X is called separable if it contains a countable dense subset.*

Definition 1.1.8 *Let X be a normed space over a field $\mathbb{K}$ then the operator (map) $T : X \to X$ is called*

(i) *linear, if for any $x_1, x_2 \in X$ and for any two scalars c_1, c_2, $T(c_1x_1 + c_2x_2) = c_1T(x_1) + c_2T(x_2)$,*

(ii) *continuous at x_0 if for every $\varepsilon > 0$ there is a $\delta > 0$ such that $\|Tx - Tx_0\| < \varepsilon$ for all $x \in X$ satisfying $\|x - x_0\| < \delta$. If T continuous at every point of X, then T is continuous.*

(iii) *bounded, if there exists a positive integer number α such that $\|Tx\| \leq \alpha\|x\|$ for all $x \in X$. The smallest such α is called the operator norm $\|T\|$. If T is linear, then T is continuous if and only if T is bounded.*

The set of all bounded linear operators on X is denoted by $\mathscr{B}(X)$

Definition 1.1.9 (Linear functional) *Let X be a normed space over a field $\mathbb{K}$ then the linear map $f : X \to \mathbb{K}$ is called linear functional.*

The space of all linear functionals from a normed space X to the underlying field $\mathbb{K}$ is called continuous dual space and denoted by X^*.

Definition 1.1.10 *Let T be a bounded linear operator on a complex Banach space X, then the point spectrum of T $(\sigma_p(T))$ is the set of all eigenvalues of T.*

Definition 1.1.11 *Let T be a bounded linear operator on a Banach space X, then the spectrum of T $(\sigma(T))$ is the set of all complex numbers λ such that $T - \lambda I$ is not invertible, where I is the identity operator on X.*

The sequence space ℓ^p and shift operators: The sequence space ℓ^p is a vector space whose elements are infinite sequences of complex numbers satisfying the condition

$$\sum_n |x_n|^p < \infty,$$

for $0 < p < \infty$. If $p \geq 1$, then the norm $\|\cdot\|_p$ is defined by

$$\|x\|_p = \left(\sum_n |x_n|^p \right)^{1/p}.$$

The space ℓ^p is a complete metric space with respect to the above norm, and hence is a Banach space. However, only when $p = 2$, that is, ℓ^2 is a Hilbert space.

Shift operators are widely used in operator theory because of their favorite testing ground there. To study shift operators on ℓ^p spaces, we let $\ell^p(\mathbb{N})$ to be the space of all one-sided sequences; that is,

$$\ell^p(\mathbb{N}) = \left\{ (x_0, x_1, x_2, \cdots) : \sum_{i=0}^{\infty} |x_i|^p < \infty \right\},$$

and the space $\ell^p(\mathbb{Z})$ to be the space of all two-sided sequences; that is,

$$\ell^p(\mathbb{Z}) = \left\{ (\cdots, x_{-2}, x_{-1}, x_0, x_1, x_2, \cdots) : \sum_{i=\infty}^{\infty} |x_i|^p < \infty \right\}.$$

Remark 1.1.12 *Let A be a subset of $\ell^p(\mathbb{N})$ (or $\ell^p(\mathbb{Z})$) consisting of all sequences with finite support, that is, sequences that only have a finite number of nonzero entries, then A is dense in $\ell^p(\mathbb{N})$ (or $\ell^p(\mathbb{Z})$, respectively).*

Now, suppose that

$$e_n = (\delta_{n,k})_{k\in\mathbb{N}_0} = \Big(0,\cdots,0,\underset{n}{1},0,\cdots\Big) \text{ where } n \in \mathbb{N}_0. \tag{1.1}$$

Then $\{e_n : n \in \mathbb{N}_0\}$ is a standard basis for $\ell^p(\mathbb{N})$. The unilateral forward weighted shift operator with a weight sequence $\{w_n\}_{n\in\mathbb{N}_0}$ is defined as

$$F(e_n) = w_n e_{n+1} \text{ for all } n \in \mathbb{N}_0.$$

If $w_n = 1$ for all $n \in \mathbb{N}_0$, the operator is simply called unilateral forward shift.

The unilateral backward weighted shift operator with a weight sequence $\{w_n\}_{n\in\mathbb{N}_0}$ is defined as

$$Be_n = w_n e_{n-1} \text{ for all } n \geq 1 \text{ and } Be_0 = 0.$$

If $w_n = 1$ for all $n \in \mathbb{N}_0$, the operator is simply called unilateral backward shift.

The kth-power of the unilateral forward weighted shift F with weight sequence $\{w_n\}_{n\in\mathbb{N}_0}$ is defined as follows

$$F^k e_n = \left(\prod_{j=0}^{k-1} w_{n+j}\right) e_{n+k} \text{ for all } n \in \mathbb{N}_0. \tag{1.2}$$

The kth-power of the unilateral backward weighted shift B with weight sequence $\{w_n\}_{n\in\mathbb{N}_0}$ is defined as follows

$$B^k e_n = \begin{cases} \left(\prod_{j=0}^{k-1} w_{n-j}\right) e_{n-k} \text{ if } n \geq k, \\ 0 \text{ if } n < k. \end{cases} \tag{1.3}$$

Now, if

$$e_n = (\delta_{n,k})_{k\in\mathbb{Z}} = \Big(0,\cdots,0,\underset{n}{1},0,\cdots\Big) \text{ where } n \in \mathbb{Z}.$$

Then $\{e_n : n \in \mathbb{Z}\}$ is a standard basis for $\ell^p(\mathbb{Z})$. The bilateral forward weighted shift operator with a weight sequence $\{w_n\}_{n\in\mathbb{Z}}$ is defined as

$$T(e_n) = w_n e_{n+1} \text{ for all } n \in \mathbb{Z}.$$

If $w_n = 1$ for all $n \in \mathbb{Z}$, the operator is simply called bilateral forward shift.

The bilateral backward weighted shift operator with a weight sequence $\{w_n\}_{n\in\mathbb{Z}}$ is defined as

$$S(e_n) = w_n e_{n-1} \text{ for all } n \in \mathbb{Z}$$

If $w_n = 1$ for all $n \in \mathbb{Z}$, the operator is simply called bilateral backward shift.

The kth-power of the bilateral forward weighted shift T with weight sequence $\{w_n\}$ is defined as follows

$$T^k e_n = \left(\prod_{j=0}^{k-1} w_{n+j} \right) e_{n+k} \text{ for all } n \in \mathbb{Z}. \tag{1.4}$$

and the kth-power of the bilateral backward weighted shift S with weight sequence $\{w_n\}$ is defined as follows

$$S^k e_n = \left(\prod_{j=0}^{k-1} w_{n-j} \right) e_{n-k} \text{ for all } n \in \mathbb{Z}. \tag{1.5}$$

we will always consider the weight sequence to be bounded (otherwise the operator is not bounded) and positive (the shifts with weight sequence $\{w_n\}$ are unitarily equivalent to those of weight sequence $\{|w_n|\}$) in the sequel.

Definition 1.1.13 (adjoint) *Let H_1 and H_2 be Hilbert spaces, and let $T : H_1 \to H_2$ be a bounded linear operator. The Hilbert-adjoint (adjoint, for short) T^* of T is the operator $T^* : H_2 \to H_1$ defined as follows:*

$$\langle Tx, y \rangle = \langle x, T^* y \rangle ,$$

for all $x \in H_1$ and $y \in H_2$.

Definition 1.1.14 (inverse) *Let X_1, X_2 be normed spaces, and let $T : X_1 \to X_2$ be a bijective bounded linear operator. Then the inverse of T is the operator $T^{-1} : X_2 \to X_1$ such that*

$$T^{-1} T x = x \text{ and } T T^{-1} y = y \text{ for all } x \in X_1, y \in X_2$$

Proposition 1.1.15 *Let F and B be forward and backward unilateral weighted shift respectively. Let T and S be forward and backward bilateral weighted shift respectively.*

(i) If $Fe_n = w_n e_{n+1}$ for all $n \in \mathbb{N}_0$, then $F^* = B$ such that $Be_n = w_{n-1} e_{n-1}$ for all $n \geq 1$ and $Be_0 = 0$,

(ii) If $Be_n = w_n e_{n-1}$ for all $n \geq 1$ and $Be_0 = 0$, then $B^* = F$ such that $Fe_n = w_{n+1} e_{n+1}$ for all $n \in \mathbb{N}_0$,

(iii) If $Te_n = w_n e_{n+1}$, then $T^* = S$ such that $Se_n = w_{n-1} e_{n-1}$ for all $n \in \mathbb{Z}$,

(iv) If $Se_n = w_n e_{n-1}$, then $S^* = T$ such that $Fe_n = w_{n+1} e_{n+1}$ for all $n \in \mathbb{Z}$.

The unilateral weighted shift operators are not invertible, because they are not bijection. However, we have the following proposition.

Proposition 1.1.16 *Let F, B be unilateral weighted shifts on $\ell^p(\mathbb{N})$, then*

(i) If $Fe_n = w_n e_{n+1}$ for all $n \in \mathbb{N}_0$, then the left inverse to F is B where $Be_n = \frac{1}{w_{n-1}} e_{n-1}$ for all $n \geq 1$ and $Be_0 = 0$; that is, $BF = I$,

(ii) If $Be_n = w_n e_{n-1}$ for all $n \geq 1$ and $Be_0 = 0$, then the right inverse to B is F where $Fe_n = \frac{1}{w_{n+1}} e_{n+1}$ for all $n \in \mathbb{N}_0$; that is, $BF = I$.

For bilateral weighted shift operators, a bilateral weighted shift is invertible if and only if there exists an $c > 0$ such that $|w_n| > c$ for all $n \in \mathbb{Z}$ (Feldman, 2003). Therefore, we have the following proposition.

Proposition 1.1.17 *Let T and S be forward and backward invertible bilateral weighted shift respectively, then*

(i) if $Te_n = w_n e_{n+1}$, then $T^{-1} = S$ where $Se_n = \frac{1}{w_{n-1}} e_{n-1}$,

(ii) if $Se_n = w_n e_{n-1}$, then $S^{-1} = T$ where $Te_n = \frac{1}{w_{n+1}} e_{n+1}$.

Direct sums:
There are two types of direct sums, internal and external, defined on vector spaces.

Definition 1.1.18 (Internal direct sum) *A vector space X is the internal direct sum of two its subspaces M and W, written*

$$X = M \oplus W,$$

if every $x \in X$, x has a unique representation

$$x = y + z \qquad y \in M, z \in W.$$

Then W is called an algebraic complement of M and vice versa, and (M, W) is called a complementary pair of subspaces in X.

Note that, if X is a Hilbert space and M is a closed subspace (a subspace that is a closed set in the original topology), then

$$X = M \oplus M^{\perp}$$

where $M^{\perp} = \{x \in X : \langle x, y \rangle = 0 \text{ for all } y \in M\}$ is the orthogonal complement of M. This allow us to define the orthogonal projection operator.

Definition 1.1.19 (Orthogonal projection operator) *Let H be a Hilbert space and M be a closed subspace of H. Then the orthogonal projection operator (or projection, for short) onto M is denoted by P and defined as follows:*

$$Px = y \text{ where } x = y + z,\ y \in M \text{ and } z \in M^{\perp}$$

The vector y is called the projection of x onto M. P_M refers to the projection onto a subspace M.

Definition 1.1.20 (External direct sum) *The external direct sum of two vector spaces X and Y (over the same field $\mathbb{K}$) is the set*

$$X \oplus Y = \{(x,y) : x \in X, y \in Y\},$$

with the operations $(x_1,y_1)+(x_2,y_2) = (x_1+x_2, y_1+y_2)$ and $\alpha(x,y) = (\alpha x, \alpha y)$; $\alpha \in \mathbb{K}$. By the same way, the direct sum of more than two vector spaces is defined.

Note that, we can represent internal direct sum of two subspaces as the external direct sum of two vector spaces; therefore, both of them are isomorphism. In the sequel, we always deal with external direct sum even when we have subspaces of a vector spaces, and we simply called it direct sum. If X and Y are Banach spaces, then the norm that we use on $X \oplus Y$ is defined by

$$\|(x,y)\|_{X\oplus Y} = \|x\|_X + \|y\|_Y,$$

or simply,

$$\|(x,y)\| = \|x\| + \|y\|.$$

If X and Y are Hilbert spaces, then the inner product on $X \oplus Y$ is defined by

$$\langle (x_1,x_2),(y_1,y_2)\rangle_{X\oplus Y} = \langle x_1,y_1\rangle_X + \langle x_2,y_2\rangle_Y,$$

or simply,

$$\langle (x_1,x_2),(y_1,y_2)\rangle = \langle x_1,y_1\rangle + \langle x_2,y_2\rangle.$$

It is easy to show that $X \oplus Y$ is a Banach space (or Hilbert space) if and only if both factors are Banach spaces (or Hilbert spaces, respectively).

The direct sum of bounded linear operators on Banach spaces is defined as follows.

Definition 1.1.21 *Let T_1 and T_2 be bounded linear operators on Banach spaces X_1 and X_2 respectively. Then the direct sum of T_1 and T_2 is $T_1 \oplus T_2 : X_1 \oplus X_2 \to X_1 \oplus X_2$ which is defined as follows*

$$(T_1 \oplus T_2)(x,y) = (T_1x, T_2y).$$

By the same way, the direct sum of any finite number of operators is defined.

1.2 Linear Dynamics

The dynamical systems theory is a concept in mathematics which deals with the long-term behavior of evolving systems. For example, take a scientific calculator and input any number then striking one of the function key over and over again. That iterative procedure is an example of discrete dynamical system. The main question is that if we have a function $T : X \to X$ and an initial value x_0 what happens to the sequence of iterates

$$x_0, T(x_0), T(T(x_0)), \cdots .$$

Definition 1.2.1 *If T is a map on X, then the orbit of a vector x under T is defined as follows*

$$Orb(T,x) = \{T^n x : n \geq 0\} .$$

The basic goal of the theory of dynamical system is to understand the eventual behavior of the orbit of a vector as n becomes large. That is, where do points go and what do they do when they get there?

An example of dynamical system, consider the size of population given by $c_{i+1} = T(c_i)$ at discrete time $i = 0, 1, 2, \cdots$. The size of population at $i+1$ depends only on its size at time i, where T is a suitable map. With i application of T, we get

$$c_i = (T \circ \cdots \circ T)(c_0) \text{ for all } i = 1, 2, \cdots .$$

Therefore the behaviour of the size of population is found by its initial point c_0 and the map T. In general, if the elements x_i from X are the states of the system at time $i \geq 0$ and $T : X \to X$ is the evolution of the system. Then

$$x_{i+1} = T(x_i) \text{ for all } i \geq 0.$$

The underlying space X should be a metric space since we have to determine the changes in the values of x_i. Also, the map T should be continuous since we only want the small changes in x_i implies to small changes in x_{i+1}.

Definition 1.2.2 *A dynamical system is a pair (X,T) consisting of a metric space X and a continuous map $T : X \to X$.*

As we have seen that the map T does not need to be linear. However, we will deal only with linear dynamical system , or linear dynamics, for short. The area of linear dynamics is a connection between the areas of operator theory and dynamical systems. It is an interesting area since it connects with various areas in mathematics, for example complex analysis, harmonic analysis and geometry of Banach spaces. The aim of this area is to to study the dynamics of continuous linear operators on Banach or Hilbert spaces.

In the sequel, we will always suppose that X is a separable Banach space over the field $\mathbb{C}$ of complex numbers, unless otherwise stated.

Hypercyclicity is one of the main concepts in linear dynamics. A bounded linear operator T on a Banach space X is called hypercyclic if there exists a vector $x \in X$ such that its orbit $Orb(T,x)$ is dense in X, such an x is called hypercyclic vector. The concepts of diskcyclicity , supercyclicity and cyclicity are related to the density of sets obtained by expanding the orbits in a natural way. If there is a vector $x \in X$ whose scaled orbit $\mathbb{C}Orb(T,x) = \{\alpha T^n x : n \geq 0, \alpha \in \mathbb{C}\}$ is dense in X then T is called supercyclic operator and x is called supercyclic vector. If there is a vector $x \in X$ whose disk orbit $\mathbb{D}Orb(T,x) = \{\alpha T^n x : n \geq 0, \alpha \in \mathbb{D} \setminus \{0\}\}$ is dense in X (where $\mathbb{D}$ is the closed unit disk), then T is called diskcyclic operator and the associated vector is called diskcyclic vector for T. A bounded linear operator T on a Banach space X is called cyclic if there exists a vector $x \in X$ such that the closed linear span generated by the orbit of x is dense in X, such an x is called cyclic vector for T.

Recently, the notions of subspace-hypercyclicity and subspace-supercyclicity were defined. A bounded linear operator T on a Banach space X is called subspace-hypercyclic for a subspace M of X (or M-hypercyclic, for short) if there is a vector $x \in X$ whose orbit $Orb(T,x)$ is dense in M, such an x is called subspace-hypercyclic (or M-hypercyclic, for short) vector for T. Similarly, a bounded linear operator T on a Banach space X is called subspace-supercyclic for a subspace M (or M-supercyclic, for short) if there is a vector $x \in X$ whose scaled orbit $\mathbb{C}Orb(T,x)$ is dense in M, such an x is called subspace-supercyclic (or M-supercyclic, for short) vector for T.

The density of orbits plays an essential role in all branches of area of dynamical systems; especially chaotic systems. For a long time, the theory of chaos was associated with non linearity. However, it has been showed that some linear system are chaotic as well. However, such a behaviour exists only on infinite dimensional spaces. A commonly used definition of chaos goes back to Devaney (1989). A continuous mapping of a topological space is called chaotic in the sense of Devaney (or chaotic, for short) if it is topological transitive, has a dense set of periodic points and has a sensitive dependence on initial conditions. On a separable Banach spaces, an immediate application of the Baire category theorem yields that an operator is hypercyclic if and only if it is topological transitive. As a consequence, Godefroy and Shapiro (1991) indicated that a hypercyclic operator on a Banach space is sensitive to initial conditions. Therefore, a hypercyclic operator is chaotic if and only if it has a dense set of periodic points.

Another reason for the importance of the cyclicity of operators is that, they are clearly related to the both famous invariant subspace and invariant subset problems.

1.3 Invariant Subspace and Subset Problems

The invariant subspace problem states whether every operator on a Banach space X has a nontrivial invariant closed subspace (a subspace M is invariant under T if $TM \subseteq M$). For any bounded linear operator T, the smallest invariant closed subspace containing the vector x is the closure of the linear span of its orbit. It follows that T lacks nontrivial invariant closed subspace if and only if every nonzero vector $x \in$

X is cyclic for T. Many interesting findings according to the invariant subspace problem were obtained. For example, if X is a finite dimensional over the field of complex numbers, then each operator has an eigenvector, and therefore it has an invariant closed subspace. Moreover, by spectral theorem, all normal operators admits invariant closed subspace. Also, it is easy to show that if X is not separable then the question has positive answer by taking any nonzero vector x, the linear span generated by $Orb(T,x)$ is invariant under T and not dense in X. Lomonosov (1973) proved that all operators which commutes with a compact operator have a common nontrivial invariant closed subspace. However, Enflo (1987) constructed an operator on a specific Banach space without nontrivial invariant closed subspace which means that the answer is negative in some Banach spaces. Recently, Argyros and Haydon (2011) constructed a Banach space such that the invariant subspace problem has a positive answer.

Similar to cyclicity, hypercyclicity is mainly concerned to the invariant subset problem which states whether every bounded linear operator on a Banach space X has a nontrivial invariant closed subset. It is clear that for any vector $x \in X$, the smallest closed invariant subset for the operator T containing the vector x is the closure of $Orb(T,x)$. It immediately follows that an operator T lacks nontrivial invariant closed subset if and only if all nonzero vectors $x \in X$ are hypercyclic for T. Read (1988) solved invariant subset problem on Banach space. He constructed an operator on the Banach space ℓ^1 without nontrivial invariant closed subset ; i.e, every nonzero vector is hypercyclic. However, for Hilbert spaces, both problems have been open for over eighty years.

1.4 Problem statements

This study addressed the following problems regarding diskcyclic operators, chaotic operators and subspace-hypercyclic operators, which are summarized as follows:

On diskcyclic operators:
Since the diskcyclic bilateral weighted shifts on $\ell^2(\mathbb{Z})$ were totally described by Zeana (2002), it is natural to ask:

1. What kinds of unilateral weighted shifts on $\ell^2(\mathbb{N})$ are diskcyclic?

Kitai (1984) proved that hypercyclic operators do not exist on finite dimensional spaces. However, Herzog (1992) showed that supercyclic operators can be found on one-dimensional complex Banach spaces. For diskcyclicity, we have the following question.

2. Do diskcyclic operators exist on finite dimensional spaces?

It is known that an operator is hypercyclic (or supercyclic) if and only if its inverse is hypercyclic (or supercyclic respectively). The following question can be asked:

3. Do the inverse of invertible diskcyclic operators have to be diskcyclic?

Since the set of all diskcyclic vectors is dense G_δ, then the following question is raised :

4. Is it possible to find some linear structure in this set?

k-bitransitive and k-compound operators:
Zeana (2002) proved that if the direct sum of finite collection of operators is diskcyclic then every operator is diskcyclic. However, the converse is unknown, that is,

5. Is the direct sum of n diskcyclic operators again diskcyclic?

Semi chaotic operators:
Since all notions of chaos for linear operators are strictly infinite dimensional phenomenon, we have this question:

6. Is it possible to define a chaos for linear operators in such a way to be exist on finite dimensional Banach spaces?

On subspace-diskcyclic operators:
Both hypercyclicity and supercyclicity concepts were extended to subspaces; however, diskcyclicity was not. It is natural to ask:

7. Is it possible for a disk orbit of an operator to be dense in a subspace?

On subspace-hypercyclic operators:
All weighted shifts operators that are hypercyclic were totally described by Salas (1995). Therefore, one may ask:

8. What kinds of weighted shift operators are subspace-hypercyclic?

Kitai (1984) proved that if the direct sum of two operators is hypercyclic, then both operators are hypercyclic. Now, we have the following question.

9. Is the direct sum of two operators subspace-hypercyclic if and only if both operators are subspace-hypercyclic?

10. Does every subspace-hypercyclic operator have a dense invariant subspace whose nonzero elements are subspace-hypercyclic vectors? Question 5 of Rezaei (2013).

11. Is every hypercyclic operator also subspace-hypercyclic for some subspaces? Question iii of Madore and Martínez-Avendaño (2011).

12. Is T subspace-hypercyclic if and only if T^{-1} is? Question i of Madore and Martínez-Avendaño (2011).

13. If T is subspace-hypercyclic and $\lambda \in \mathbb{T}$, is λT subspace-hypercyclic? Question ii of Madore and Martínez-Avendaño (2011).

14. If T is subspace-hypercyclic, is T^n subspace-hypercyclic for all $n > 1$? Question 2 of Rezaei (2013).

1.5 Research objectives

The objectives of this research are to investigate some properties of diskcyclic operators and subspace-hypercyclic operators, and to define and study new classes of linear operators in order to find solutions to the problems in Section 1.4. More specifically,

- to characterize all diskcyclic unilateral weighted shift operators on $\ell^2(\mathbb{N})$.
- to investigate some properties of diskcyclic operators.
- to study the behaviour of diskcyclic vectors for some diskcyclic operators.
- to study the direct sum of n diskcyclic operators.
- to define and study a new notion of chaos for linear operators.
- to extend diskcyclicity to closed subspaces of Banach spaces.
- to characterize all subspace-hypercyclic weighted shift operators.
- to find the relation between the direct sum of n operators and subspace-hypercyclicity.
- to solve some open problems, concerning subspace-hypercyclicity, totally and partially.

The next section shows how these objectives are organised in this study.

1.6 Organisation of the Thesis

This thesis is organised as follows: In Chapter 2, we provide a brief review on topics, which are used through this thesis.

In Chapter 3, we show that hypercyclicity and diskcyclicity coincide for unilateral weighted shifts. We give further properties of diskcyclic operators, some of them are quite surprising. We study the adjoint and inverse of diskcyclic operators. Also, we define diskcyclic subspace, that is, a subspace of all diskcyclic vectors for an operator, we show that in some cases a diskcyclic operator has a subspace diskcyclic.

In Chapter 4, we extend the notion of diskcyclic operators on a Banach space into a direct sum of Banach spaces. In particular, we define and study new classes of operators which are called k-bitransitive, k-compound operators and k-diskcyclic, to study the direct sum of diskcyclic operators. We use these operators to prove that in some cases if n operators are diskcyclic, then the direct sum of them is k-bitransitive. Also, we show that, for any $r \geq 2$, T satisfies the diskcyclic criterion if and only if T is r-diskcyclic.

In Chapter 5, we define and study new classes of operators on Banach spaces which are weaker form of chaotic operators, and we call them semi chaotic operators. In contrast with chaotic operators, we show semi chaotic exists on finite dimensions. Then we characterize all semi chaotic weighted shift operators on $\ell^p(\mathbb{N})$ and $\ell^2(\mathbb{Z})$ in term of their weight sequences. We use these characterizations to show that semi chaotic operators do not need to be chaotic.

In Chapter 6, we study the possibility of the disk orbit of an operator to be dense in a subspace, and we call such operators subspace-diskcyclic operators. We study some of their properties. For example, we find the properties that are shared between subspace-diskcyclicity and other cyclicity, and the properties that are not shared. Then, we characterize all subspace-diskcyclic weighted shift operators on $\ell^2(\mathbb{N})$ and $\ell^2(\mathbb{Z})$ in term of their weight sequences.

In Chapter 7, we characterize all subspace-hypercyclic unilateral weighted shift operators on $\ell^2(\mathbb{N})$ and subspace-hypercyclic bilateral weighted shifts on $\ell^2(\mathbb{Z})$ in term of their weight sequences. We show that under certain conditions the direct sum of two operators is subspace-hypercyclic if and only if both operators are subspace-hypercyclic. Also, we show that under certain conditions if $T \oplus T$ is subspace-hypercyclic, then T satisfies subspace-hypercyclic criterion. Then, we find more properties of subspace-hypercyclic operators, some of them answer some open problems in the literature partially. Moreover, we study relative dense sets on Banach spaces, to give a complete answer to an open problem in the literature, and partial answer to some other open problems.

In Chapter 8, we give a summary of the research of this thesis, and we give some open problems for further studies.

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