UNIVERSITI PUTRA MALAYSIA

DATA ENVELOPMENT ANALYSIS FOR RESOURCE ALLOCATION PROBLEMS IN THE PRESENCE OF RADIAL AND NON-RADIAL DATA

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By

ALI MIRSALEHY

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirement for the Degree of Doctor of Philosophy

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DEDICATION

To spirit of my caring Dad, Seyd Abbas Mirsalehy- the eternal optimist - who believed in perseverance, hardworking, a good work ethic, and the pursuit of academic excellence.

To my dear Mom, Seyd Naier Salehy, who continues to support and encourage me throughout my education, as well as everything I pursue. She has always provided peaceful guidance during difficult situations.

To my brothers Hassan, Hossein and Mohammad, who has always made me feel special and never failed to tell me how they proud of me.

Abstract of thesis presented to the senate of Universiti Putra Malaysia in fulfilment of requirement of the degree of Doctor of Philosophy

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Chairman: Mohd Rizam Abu Bakar, PhD
Faculty: Science

Resource allocation is considered to be of great significance for the development of an organization due to its limited resources. This thesis addresses two different aspects of resource allocation problems in data envelopment analysis (DEA) model including centralized resource allocation (CRA) DEA model and inverse DEA model. For each unit, targets are set separately using conventional DEA models whereas based on CRA-DEA models, a centralized decision maker is in charge of all the operating units. In such situation, the decision maker is interested in maximizing the efficiency of particular units at the same time that the aggregate output production is increased or the aggregate input consumption is decreased. The existing data CRA-DEA models are defined only for radial or non-radial data. While it was previously posited that the non-radial CRA-model is helpful in evaluating the targets of units, validation of such a model was completed once the radial inputs and outputs were proposed. It is proposed that there are some variances in the characterizations of the inputs or outputs objects (radial and non-radial inputs and outputs). While the radial inputs have to be decreased proportionately when it is intended to obtain all-outputs, the counterpart inputs need to be reduced non-proportionally. It can be then concluded that combining the two radial and non-radial approaches will yield accurate targets. In an effort to overcome the above mentioned shortcomings of the radial and non-radial CRA-models, firstly, this thesis shows that the two CRA-approaches can be connected to a number of desirable features in the previous ones. The proposed approaches are developed for the cases which consider minimizing of the aggregate input consumption and maximizing of the aggregate output production as radial and non-radial at the same time with outcomes in solving just one instead of n mathematical programing problems. Secondly, this thesis also presents a novel inverse DEA model of efficiency measurement. The proposed inverse model apparently is an aspect of resource allocation problems which is likely to concurrently consider some inputs' rises (reductions). In order to improve the efficiency scores of some units when there are changes in the output (input) amounts of an efficient unit, the current study will

elucidate the inverse DEA model with some advantage over the previous ones. The proposed model is meant for determining the utmost possible input (output) quantities of the efficient unit once its outputs (input) are changed. In this way, the modifications will bring about an increase in some units' efficiency scores.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

ANALISIS PENGUMPULAN DATA UNTUK MASALAH PERUNTUKAN SUMBER DALAM KEHADIRAN DATA RADIAL DAN TAK RADIAL

Oleh

ALI MIRSALEHY

Januari 2016

Pengerusi: Mohd Rizam Abu Bakar, PhD
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Peruntukan sumber dianggap sebagai membawa signifikan yang besar untuk pembangunan organisasi disebabkan oleh sumber yang terhad. Tesis ini menangani dua aspek masalah peruntukan sumber dalam data pengumpulan analisis (DPA) model termasuk peruntukan sumber berpusat (PSB) DPA model dan model songsang DPA. Bagi setiap unit, sasaran ditetapkan secara berasingan menggunakan model DPA konvensional yang berdasarkan kepada model PSB-DPA, pembuat keputusan yang berpusat bertanggungjawab dalam semua unit operasi. Dalam keadaan itu, pembuat keputusan yang berminat dalam memaksimumkan kecekapan unit tertentu pada masa yang sama di mana pengeluaran output agregat ditambah atau penggunaan input agregat menurun. Data PSB-DPA model yang sedia ada ditakrifkan hanya untuk data radial dan tak radial. Walaupun sebelum ini dikemukakan bahawa model PSB tak radial membantu dalam menilai sasaran unit, pengesahan model seperti ini telah disiapkan sekali input dan output radial telah dicadangkan. Adalah dicadangkan bahawa terdapat beberapa perbezaan dalam pencirian input atau output objek (input radial dan tak radial dan output). Walaupun input radial perlu dikurangkan secara berkadar apabila ia bertujuan untuk mendapatkan segala-output, input rakan sejawatannya perlu dikurangkan bukan secara berkadaran. Ia boleh disimpulkan bahawa menggabungkan kedua-dua pendekatan radial dan tak radial akan menghasilkan keputusan yang tepat. Dalam usaha untuk mengatasi kelemahan yang dinyatakan di atas daripada model PSB radial dan tak radial, pertamanya, tesis ini menunjukkan bahawa kedua-dua pendekatan PSB boleh bersambung dengan beberapa ciri-ciri yang diingini dalam model yang sebelumnya. Pendekatan yang dicadangkan adalah dibangunkan untuk kes-kes yang dipertimbangkan untuk mengurangkan penggunaan input agregat dan memaksimumkan pengeluaran output agregat sebagai radial dan tak radial masa yang sama dengan hasil dalam menyelesaikan hanya satu bukannya n masalah pengaturcaraan matematik. Keduanya, tesis ini juga cuba untuk membentangkan model songsang novel DPA bagi pengukuran kecekapan. Model songsang yang dicadangkan nampaknya adalah satu aspek bagi

masalah peruntukan sumber yang mungkin pada masa yang sama mengambil kira kenaikan beberapa input (pengurangan). Dalam usaha untuk meningkatkan skor kecekapan beberapa unit apabila terdapat perubahan dalam jumlah output (input) unit yang cekap, kajian semasa akan menjelaskan model DPA songsang dengan beberapa kelebihan berbanding yang sebelumnya. Model dicadangkan adalah untuk menentukan sepenuh mungkin input (output) kuantiti unit yang cekap sekali output yang (input) ditukar. Dengan cara ini, pengubahsuaian yang akan membawa kepada peningkatan dalam skor kecekapan beberapa unit.

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I certify that a Thesis Examination Committee has met on (date of viva voce) to conduct the final examination of (student's name) on his (or her) thesis entitled ("Title of Thesis") in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the (insert the name of relevant degree).

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Min and min	Minimum
Max and max	Maximum
MPSS	Most productive scale size
LP	Linear programming
DMUs	Decision making units
DM	Decision maker
MOLP	Multiple objective linear programming
DEA	Data envelopment analysis
CRA	Centralized resource allocation
PPS	Production possibility set
CRS	Constant returns to scale
VRS	Variable Returns to scale
IRS	Increasing returns to scale
DRS	Decreasing returns to scale
GRS	General returns to scale
SE	Scale efficiency
I-Oriented	Input oriented
O-Oriented	output oriented
N-Oriented	Non-oriented

CHAPTER 1

INTRODUCTION

The aim of this chapter is to establish an overview of the research. Background information of relevant notions, scope and contributions of the research are discussed. A brief overview on each chapter of the thesis is also presented.

1.1 Data Envelopment Analysis

In today's highly competitive business environment, organizations must strive for efficiency to ensure its survival and growth. Thus, it is necessary for organizations to carefully plan their pursuit for efficiency in order to remain competitive and successful. However, to do so requires the use of accurate measurement and evaluation tools. Although considering different perspectives is sufficient in many instances, a more precise method must be applied when accurate measurements are needed to meet the complexity of today's systems. The past thirty years have seen increasingly rapid advances in the field of data envelopment analysis (DEA) for evaluating the performance of decision making units (DMUs).

DMU is a generic and flexible concept that is widely practiced by a set of peer entities ranging from non-profit making agencies and government sectors to financial and educational institutions. The broad definition of DMU has led to its applications for various purposes from the evaluation of banks to the assessment of a university's performance.

The initial DEA model, as originally presented in Charnes, Cooper, & Rhodes (1978) (CCR), was built on the earlier work of Farrell (1957). Farrell proposed an activity analysis approach to correct what he believed were deficiencies in the commonly used index number approaches to productivity (and the like) measurements. His main concern was to generate an overall measure of efficiency which reflects the measurements of multiple inputs and outputs.

Charnes et al. (1978) described DEA as a "mathematical programming model applied to observational data that provides a new way of obtaining empirical estimates of relationships – such as the production functions and/or efficient production possibility surfaces that are the cornerstones of modern economics" (Cooper et al., 2011).

In fact, DEA is a 'data oriented' approach for evaluating the performance of a set of peer entities which provides a single efficiency score while simultaneously considering multiple inputs and multiple outputs. Because it requires few assumptions, DEA has opened up possibilities for use in cases which have been resistant to other approaches because of the complex (often unknown) nature of the relationships between the multiple inputs and multiple outputs involved in the operation of the DMUs (Cooper et al., 2007).

Researchers in a number of fields have recognized that DEA is an excellent methodology for modelling operational processes, and its empirical orientation and minimization of a priori assumptions have resulted in its use in a number of studies involving efficient frontier estimations. DEA encompasses a variety of applications in evaluating the performances of different kinds of entities such as hospitals, universities, cities, courts, business firms and banks among others. Emrouznejad et al. (2008) examined a compilation that covers the first 30 years of published research since Charnes et al. (1978) initially proposed the DEA method. Over 4,000 research documents including journal articles, book chapters, and documents published through conference proceedings are included in this bibliography. Such rapid growth and widespread (and almost immediate) acceptance of the methodology of DEA is a testimony to its strengths and applicability.

1.2 The Concept of Efficiency

A clear understanding of what is meant by efficiency is necessary before any measurements and comparisons are performed. Askin and Standridge (1993) define efficiency as 'doing a task right'. To go a step further, a system is efficient when it transforms input resources into output with minimal waste, i.e., it seeks to achieve 'more for less' (Sink et al., 1989). Historically, efficiency can be measured by taking the ratio of output produced to input utilised as shown by the following expression:

$$\frac{Output}{Input}. \qquad (1.1)$$

The higher this ratio is, the more efficient the processes are as more output is being produced with less input. To better understand the concept of efficiency, the simplest case of the above formula is shown below in Table 1.1.

Table 1.1 Single Input and Single Output Case

DMU	***A***	***B***	***C***	***D***	***E***	***F***	***G***	***H***
Inputs (x)	2	4	4	5	6	7	7	8
Outputs (y)	1	3	4	2	5	4	5	7
$\frac{x}{y}$	0.5	0.75	1	0.4	0.83	0.57	0.71	0.87

Primarily, assume there are 8 DMUs which are labeled as A to H at the first row in Table 1.1. Each makes use of one input (x) for producing one output (y) as indicated in each column. Also, the last row of Table 1.1 indicates the efficiency for each DMU by expression (1.1). Based on the results obtained from this measure, D can be considered as the least efficient DMU while C as the most efficient.

These data are also illustrated in Figure 1.1 by plotting ‘value of outputs’ on the vertical axis and ‘value of inputs’ on the horizontal axis. The slope of the line connecting each point to the origin corresponds to expression (1.1). The line with the highest slope is called the ‘efficient frontier’. The efficient frontier is achieved when a point makes a connection or touches the straight line that cuts through the chart from the origin as illustrated by point C in Figure 1.1. As can be seen from Figure 1.1, all other points lie below this line. This frontier is said to literally ‘envelops’ all DMUs. In fact, the term DEA originates from this property because in mathematical parlance, such a frontier is said to ‘envelop’ these points.

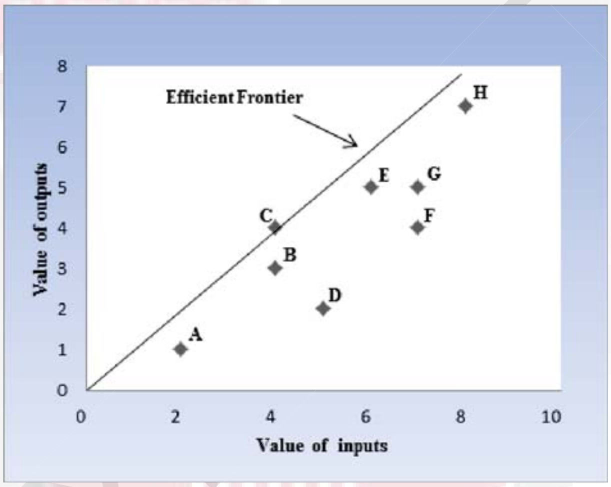

Figure 1.1. Comparison of DMUs

In this example, the challenge is to move the inefficient DMUs up into the efficient frontier, or in other words, on how to make the inefficient DMUs, efficient. For instance, DMU D, seen as an inefficient DMU in Figure 1.2 can be improved through several ways. One possible way is by decreasing the input to D1 (2, 2) on the efficient frontier. The other way to attain this is by increasing the output to D2 (5, 5). Note that D1 and D2 are considered as an input target and output target for DMU D, respectively. Any point between the two points, D1 and D2, suggests an opportunity to influence the result in a way whereby the output is not reduced and the input is not raised while creating the efficient DMU. A detailed information is further explained in Cooper et al. (2000).

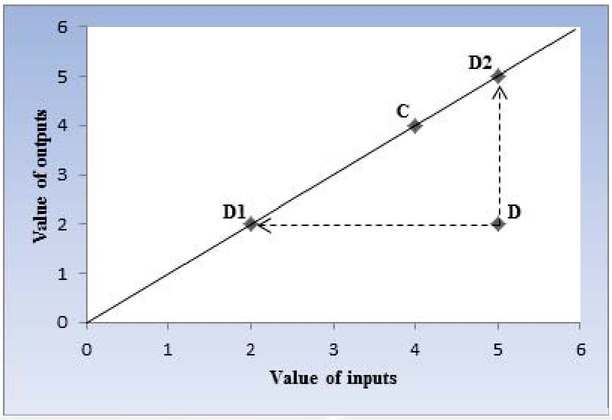

Figure 1.2. Improvement of DMU D

In general, there are two types of efficiency, relative and absolute efficiency, which have different properties. Let n be the numbers of similar DMUs; $Y_o \left(o \in \{1,\ldots,n\}\right)$ be the amount of output produced by an input unit and Y^* be the amount of output produced by an input unit based on the ideal standard. Then, the absolute efficiency for the o-th DMU is evaluated as follows: $\frac{Y_o}{Y^*}$. To clarify this efficiency, suppose that the marks obtained by three students in a special course are 15, 10 and 5 out of 20 (note that 20 is accounted for an ideal standard). Then, the absolute efficiency for them will be $\frac{15}{20}$, $\frac{10}{20}$, and $\frac{5}{20}$, respectively. This makes evaluating DMUs hard because it is not easy for DMs to specify the total and ideal standard for each DMU. The relative efficiency is comparable to the absolute efficiency; however, the former outperforms the latter in evaluating the efficiency of DMUs.

Now, if assume that the j-th $\left(j \in \{1,\ldots,n\}\right)$ DMUs consumes inputs X_j to generate outputs Y_j, then the relative efficiency for the o-th DMU can be defined as following formulations, which presents the evaluated DMU in the best light in comparison to the other DMUs:

$$RE_o = \frac{Y_o / X_o}{max\,\{Y_j / X_j : j = 1,\ldots,n\}}.$$

For instance, the relative efficiency of B in Table 1.1 is as:

$$RE_B = \frac{3/4}{max\,\{1/2,\ 3/4,\ 4/4,\ 2/5,\ 5/6,\ 4/7,\ 5/7,\ 7/8\}} = \frac{3/4}{1} = 0.75\,.$$

1.3 Basic Features of DEA Models

Literature shows the availability of numerous DEA models to estimate efficiency scores. However, in general, these models differ broadly in four aspects with regards to their: (i) approach in measuring technical efficiency, (ii) orientations in efficiency analysis, (iii) assumptions on production frontiers, and (iv) ability to handle different data types.

1.3. 1 Radial and Non-Radial Approaches

In terms of measuring efficiency, DEA models take either a radial approach or a non-radial approach. In the radial approach, inputs and outputs are assumed to change proportionally (e.g., the CCR model: Charnes et al. (1978)). This approach is therefore prone to neglect non-radial input and output slacks. Because it does not detect input excesses and output shortfalls, radial models can only classify each DMU as weakly-efficient or inefficient. In contrast, non-radial DEA models directly deal with input excesses and output shortfalls, and thus, are capable of distinguishing efficient DMUs from inefficient ones (e.g., the SBM model: Tone (2001)).

1.3.2 Orientation

DEA models can be classified as output-oriented, input-oriented or base-oriented. While output-oriented DEA models (e.g., the BCC-O model: Banker et al. (1984)) focus on output augmentation to achieve efficiency (outputs are controllable), input-oriented DEA models (e.g., the BCC-I model: Banker et al., (1984)) aim to minimise the amount of inputs required for producing a certain amount of outputs (inputs are controllable). Non-oriented DEA models (e.g., the ADD model: Charnes et al. (1985)), on the other hand, are concerned with determining the optimal mix of inputs and outputs (both inputs and outputs are controllable).

1.3.3 Returns to Scale

The other basis for variation among DEA models is returns to scale, which (in economics) describes what happens when the scale of production increases over the long run when all input levels are variables (chosen by the organization). There are two basic types of returns to scale: constant returns to scale (CRS) and variable returns to scale (VRS). Models that assume CRS production technology (e.g., the CCR model: Charnes et al. (1978)) presume that the size of a DMU does not affect its efficiency. More precisely, a DMU operates under CRS technology if an increase in its inputs results in a proportionate increase in its outputs. If it is suspected that an increase in inputs does not result in a proportional change in outputs, models that assume VRS production technology (e.g., the BCC model: Banker et al. (1984)) should be considered. In terms of linear programming, the production possibility set for a VRS model is spanned by the convex hull of input and output variables. The VRS specification, in general, is a safer option if the DEA model does not include all the variables deemed to be relevant in the analysis (Galagedera and Silvapulle, 2003).

1.3.4 Units and Translation Invariant Properties

Two important properties in DEA models are the units invariant property and the translation invariant property. A DEA model is considered units invariant (e.g., the Hybrid model: Tone (2004)) if it yields an efficiency score that is independent of the measurement units of the inputs and outputs. The translation invariant property (e.g., the ADD model: Charnes et al. (1985)) allows a DEA model to handle negative data (Pastor and Ruiz, 2007). Formally, a DEA model is said to be translation invariant if translating the original input and/or output data yields a new problem with the same optimal solution as the old one. Being a VRS model is a key condition for having this property. Therefore, when dealing with negative data in DEA, an implicit assumption is that the production technology satisfies VRS. Not all VRS models, however, have the translation invariant property, and a good example of this is the basic additive model introduced in the next chapter.

In Table 1.2, these four aspects are briefly illustrated for basic DEA models. This information reveals that, generally, different models achieve different projected points for inefficient DMUs and hence a different level of efficiency measurement (Ali, 1994).

Table 1.2. Characteristics of Basic DEA Models

Model	CCR	BCC	ADD	SBM	Hybrid
Radial or Non-Radial	Radial	Radial	Non-Radial	Non-Radial	Radial and Non-Radial
Orientation*	I-Oriented O-Oriented	I-Oriented O-Oriented	N-Oriented	N-Oriented I-Oriented O-Oriented	N-Oriented I-Oriented O-Oriented
Returns to Scale	CRS	VRS	VRS	CRS VRS	CRS VRS
Units Invariant	Yes	Yes	No	Yes	Yes
Translation Invariant (X)	No	No (I-Oriented) Yes (O-Oriented)	Yes	No	No
Translation Invariant (Y)	No	Yes (I-Oriented) No (O-Oriented)	Yes	No	No

*: The symbols I-Oriented, O-Oriented, and N-Oriented indicate the input-oriented, output-oriented and non-oriented, respectively.

1.4 Problem Statements

As mentioned above (1.1.2.1), DEA models can be categorized into two basic approaches: radial approach and non-radial approach. Preliminary work on the radial approaches was undertaken by Debreu (1951) and Farrell (1957). Radial approaches (e.g., the CCR and BCC models) are the most common and efficient approaches to deal with proportional improvement in inputs and outputs data. To date, numerous desirable features have been identified in radial approaches. For instance, they can generally obtain the relative development in inputs and outputs. Moreover, they have the potential to estimate the efficiency based on the attainable data, or they can provide an obvious economic explanation without considering the cost. However, these models suffer from the following drawbacks:

- They assess the efficiency based on the existing data without considering the precedence knowledge of the decision-maker's (DM's).
- Due to the proportional improvement in these models, they cannot be used in cases where inputs involve labours, materials and capital.
- DM does not have the flexibility to select a reference unit for an inefficient unit.
- They are unable to achieve an efficient target in DEA.

On the other hand, the non-radial approaches were the central focus of studies by Koopmans (1951) and Russell (1985) in which inputs and outputs data improve non-proportionally. Yet, numerous studies have attempted to explain the non-radial approaches. For example, Tone (2002) employed a new and suitable synthetic procedure to obtain a non-radial approaches which was termed the 'slacks-based measure' (SBM) in which both the input and output slacks could be maximized. These models have a number of attractive features, e.g., they put aside the supposition of proportional reduction in the inputs and target at earning maximum amounts of contraction in inputs which might abandon the changing rates of the original input resources. Nevertheless, in spite of the safety and efficacy of the non-radial approaches, they suffer from several major drawbacks listed below:

- When evaluating changes in the efficiency during the time, the non-zero pattern of the slacks at time period t can meaningfully differ from that of the time period $t+1$. Therefore, it cannot be ascertained whether the pattern is rational or not.
- When the primary proportionality is missing, it would be unsuitable for investigation.
- In models such as the SBM model, the optimum slacks would exhibit an acute conflict in catching the positive and zero amounts.

Meanwhile, recent developments in the field of DEA have led to resource allocation problems. The process of allotting the restricted resources to different units of an organization for the purpose of fulfilling the overall goals is referred to as resource allocation. Hence, resource allocation is considered to be of great significance for the development of an organization. Due to this, resource allocation has turned into a new subject despite being a classical practice in management science. Resource allocation problems in DEA model can be categorized into centralized resource allocation (CRA) DEA and inverse DEA models.

In a large longitudinal study by Lozano and Villa (2004), CRA-DEA models were proposed. For each unit, targets are set separately using the conventional DEA models; whereas, based on CRA-DEA models, a centralized DM is in charge of all the operating DMUs. In such situation, the DMs are interested in maximizing the efficiency of the particular units while at the same time, the aggregate output production is increased, or the aggregate input consumption is decreased. The existing CRA-DEA models are used only for radial or non-radial approaches; therefore, they share the same weaknesses listed for radial or non-radial approaches described earlier.

Although it was previously posited that the non-radial approaches are beneficial in evaluating the targets of units, the validation of these approaches can only be done once the radial inputs and outputs have been proposed after considering the radial or non-radial problems. Since variance exists in the characterizations of the inputs or outputs objects (radial and non-radial inputs and outputs), the radial inputs have to be decreased proportionately when it is intended to obtain all-outputs, and the counterpart inputs need to be reduced non-proportionally.

Moreover, the inverse DEA was initially proposed by Wei et al. (2000) with the aim of addressing this question: if given inputs (outputs) of a certain unit are elevated amid a set of DMUs, how much rise should be given to the outputs (inputs) of that same unit while the DMU is maintained? One of the problems with the instrument used to examine inverse data in the studies was that the efficiency scores of DMUs remain constant when there are changes in the output (input) amounts of an efficient unit. So far, this problem has been discarded by the preceding studies although there have been various methods on the inverse DEA model.

Eliminating the weaknesses of these approaches can lead to a more efficient approach for yielding accurate targets. This thesis intends to develop new DEA models to address some of the shortcomings of the existing models and methods. These new DEA models are expected to outperform the existing DEA models in solving problems such as in determining relative efficiency and resource allocation problems.

1.5 Motivation

Applying the radial and non-radial approaches for CRA methods in DEA clearly can be important when these two approaches are mixed in the problem. Based on the literature review in chapter 2, no research has considered radial and non-radial approaches in a unified framework for CRA methods. Thus, the first research motivation being considered by this research study is how the connecting radial and non-radial approaches can be measured to suit the requirements of the user, and how they can be addressed to open a way for a more comprehensive and accurate benchmarks of estimation to be used.

Many published works address the problem of Wei et al. (2000) for invers DEA in resource allocation problems. Several ideas have been proposed and have had a significant impact on this problem. However, previous studies merely considered that the efficiency scores of DMUs remain constant when there are changes in the output (input) amounts of an efficient DMU. Thus, the second research motivation being considered by this research study is how the efficiency scores of DMUs can be improved when there are changes in the output (input) amounts of an efficient DMU.

1.6 Aim and Objectives

This thesis studies the target models and methods in data envelopment analysis. It highlights several drawbacks of the existing models and proposes recommendations to rectify the methods for determining efficiency and resource allocation problems via new approaches. The primary objective of this thesis is to investigate centralized resource allocation problems by linking radial and non-radial approaches in the conventional DEA models and exploring the relationship these two approaches in two proposed models.

The second purpose of this research is to develop traditional DEA models for examining the relative efficiency of DMUs and centralized resource allocation by connecting the framework for radial and non-radial approaches. These models explain the differences in theoretical concepts in relative efficiency and targets among relevant DEA models. Finally, the current research attempts to present a novel inverse DEA model of efficiency measurement and its efficiency level in comparison with the other DMUs. Indeed, the proposed inverse model is an aspect of the resource allocation problems which improve the efficiency scores of some units when there are changes in the output (input) amount of an efficient unit.

The specific objectives of this thesis are as follows:

1. To define the centralized resource allocation in DEA for connecting the two basic radial and non-radial models in an integrated structure with the intention of controlling the proportionality of slacks.

This is achieved by converting connected-SBM model, proposed by Avkiran et al. (2008), to a novel CRA-DEA model in chapter 3.

2. To delineate the new model which solves the problem of traditional radial and non-radial DEA models by developing a traditional DEA model for measuring relative efficiency.

This is done by first specifying the directional distance function within the full input-output space and then defining a formal and unambiguous concept of relative efficiency with the existence of both radial and non-radial inputs and outputs in chapter 4.

3. To determine a new and generalised DEA model for centralized resource allocation with the aim of attaining more pragmatic results via the suggested method which has been introduced in chapter 4.

This is accomplished by converting proposed model in chapter 4, to a new CRA-DEA model in chapter 5.

4. To identify the new and efficient inverse DEA model which improves the efficiency score for some DMUs.

This is attained by introducing a new inverse DEA model in chapter 6 while lending its basis to the efficiency directional SBM which is a special case of the proposed model in chapter 4.

1.7 Scope and Limitations

This study focuses on the relation between radial and non-radial approaches in DEA, so it will exclude other aspects of DEA. Many studies in resource allocation problems focus mainly on radial or non-radial approaches with regards to the applications of DEA in projecting DMUs onto the efficient frontier; however, this thesis aims to utilize new measures in dealing with resource allocation problems. It explains the connecting framework for radial and non-radial approaches in DEA models that are completed by converting the suggested model to a linear programming problem. Also, inverse DEA models are another subcategory of resource allocation problems where the current study will clarify an inverse DEA model with regards to its advantages over the previous models. Furthermore, due to availability constraints for real data in Malaysia, the data presented in previous investigations were employed in the present study.

1.8 Organization of Chapters

The thesis is organized into seven chapters, including this introductory chapter. Chapter 2 provides an overview of the most pertinent works related to this research. This chapter presents the historical origins of DEA and examines various theoretical concepts and relevant DEA models.

Chapter 3 examines an alternative approach to the CRA-DEA models that all the units are under the control of an entity of the centralized DM. The proposed approach is a technique for connecting the two basic radial CRA-BCC and non-radial CRA-SBM models in an integrated structure called the ‘connected CRA-SBM’. In this model, exchanging the amount of the two parameters allows the analysis between the CRA-BCC and the CRA-SBM models to be made which deals with the weaknesses inherent in such models. Numerical examples emphasise the significance of the method presented.

Chapter 4 presents a hybrid measure of the DEA method for defining the efficiency score with the existence of both radial and non-radial inputs and outputs based on the directional distance function. It is called a directional hybrid measure (DHM). The empirical examples emphasize the consequence of the proposed measure.

Chapter 5 aims to provide a new CRA model by utilizing the proposed DHM model discussed in Chapter 4. It exhibits a connecting framework for radial and non-radial approaches in CRA-DEA models. The numerical examples demonstrate the applicability and efficacy of the proposed method.

Chapter 6 introduces a novel technique which lends its basis to the directional slack-based measure for the inverse DEA models. In practice, it endeavors to explain the inverse directional slack-based measure model within a new production possibility set. On one occasion, there is a modification imposed on the output (input) quantities of an efficient decision making unit. In detail, the efficient decision making unit in this method was omitted from the present production possibility set but substituted by the considered efficient DMU while its input and output quantities were subsequently modified. The significance of the represented model is accentuated through the numerical and empirical examples presented.

Chapter 7 summarises the previous chapters. It also explains the proposed method / approach which is the extension of the work explained. A general conclusion and convolution of the dissertation and some directions for future research are also presented.

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